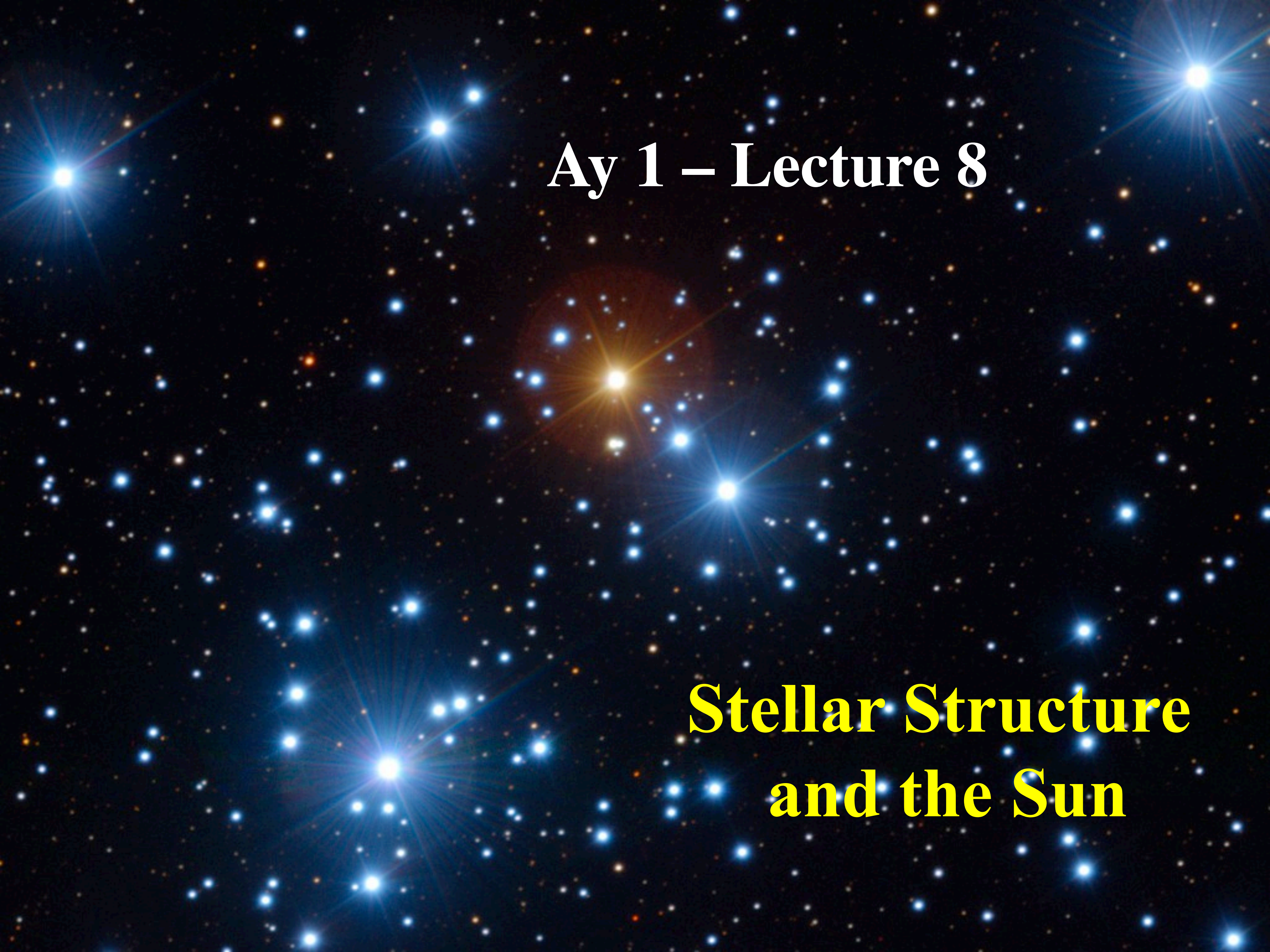
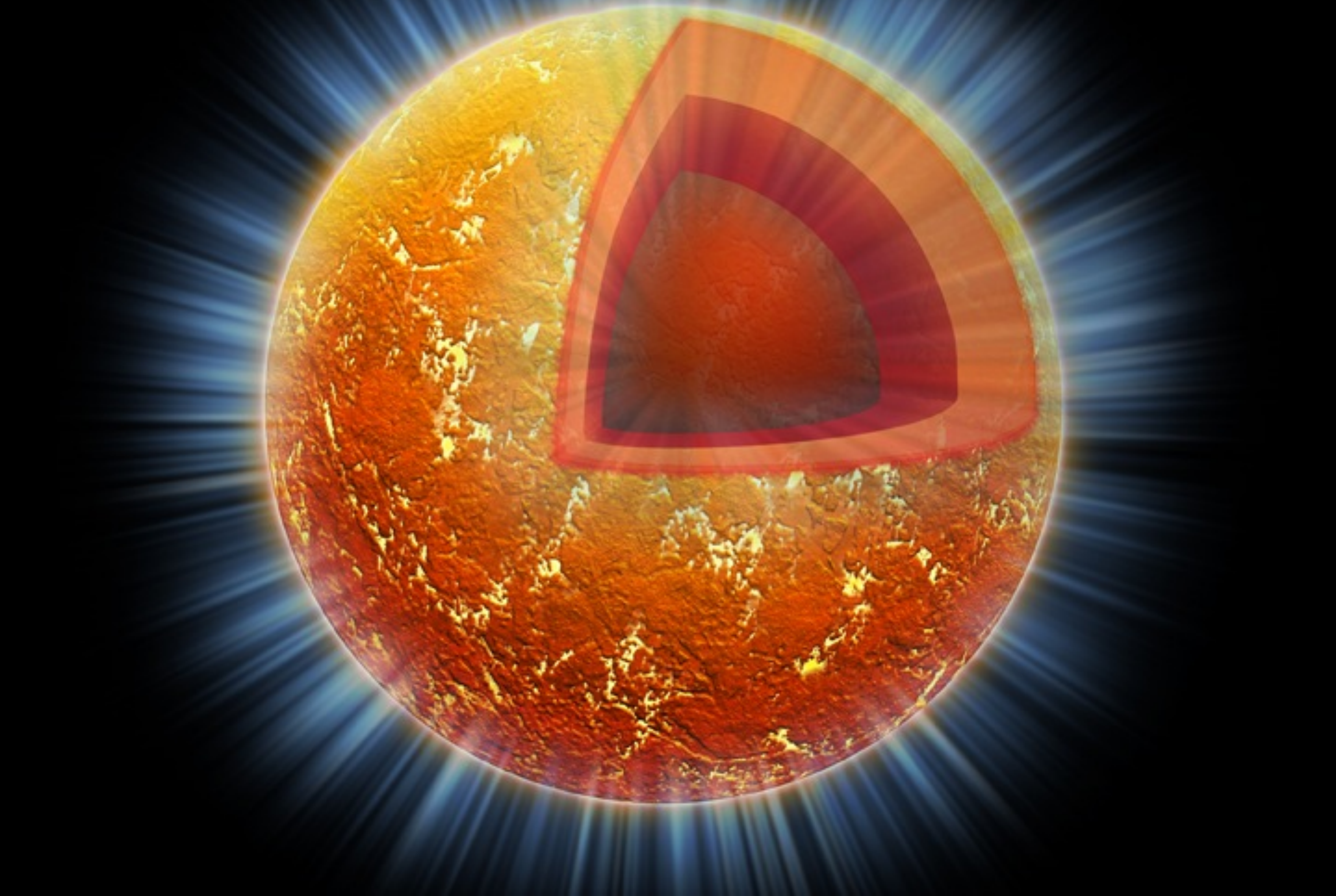


Ay 1 – Lecture 8

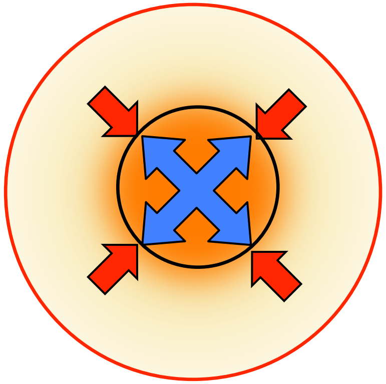
Stellar Structure and the Sun

8.1 Stellar Structure Basics

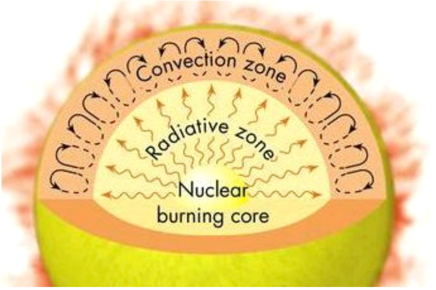


How Stars Work

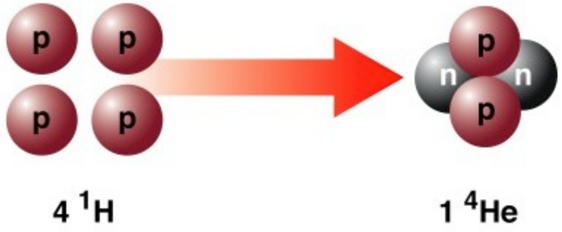
- **Hydrostatic Equilibrium**: gas and radiation pressure balance the gravity



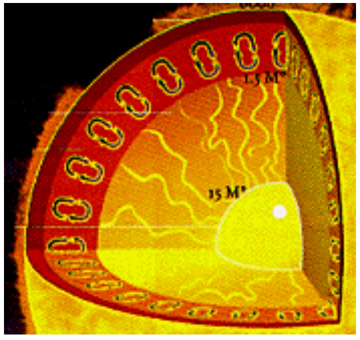
- **Thermal Equilibrium:** Energy generated = Energy radiated



- **Thermonuclear Reactions:** The source of the energy



- **Energy Transport:** How does it get from the core to the surface

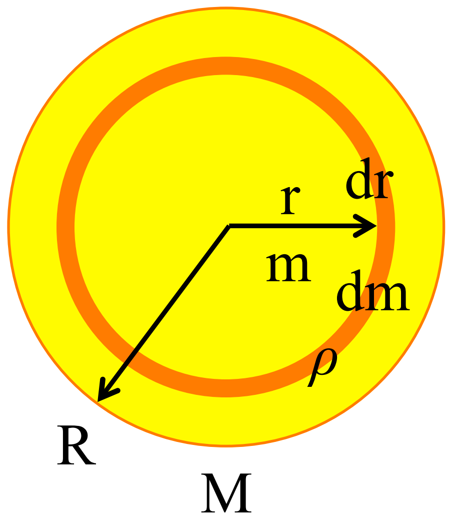


Equations of the Stellar Structure

Mass vs. radius:

$$dm = 4\pi r^2 \rho dr$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$



Luminosity vs. radius:

$$dL = 4\pi r^2 \rho dr \times q$$

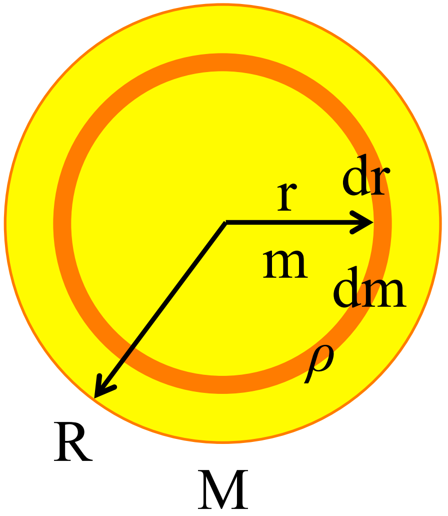
$$\frac{dL}{dr} = 4\pi r^2 \rho q$$

q = rate of energy generation per unit mass

Hydrostatic Equilibrium

At a given radius, the gravitational force on a shell is:

$$F_g = -\frac{Gm\Delta m}{r^2}$$



The weight of that mass shell over the area has to be the difference between the pressures on an inner and an outer surface of the shell.

$$dm = 4\pi r^2 \rho dr$$

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

Pressure of What?

Total pressure: $P = P_{gas} + P_{radiation}$

The equation of state for an ideal gas is: $P_{gas} = nkT$

n = the number of particles per unit volume (ions and electrons)

Or: $$P_{gas} = \frac{\rho}{\mu m_H} \times kT$$

ρ = mass density
m_H = the mass of hydrogen atom
μ = average particle mass in units of m_H

↑

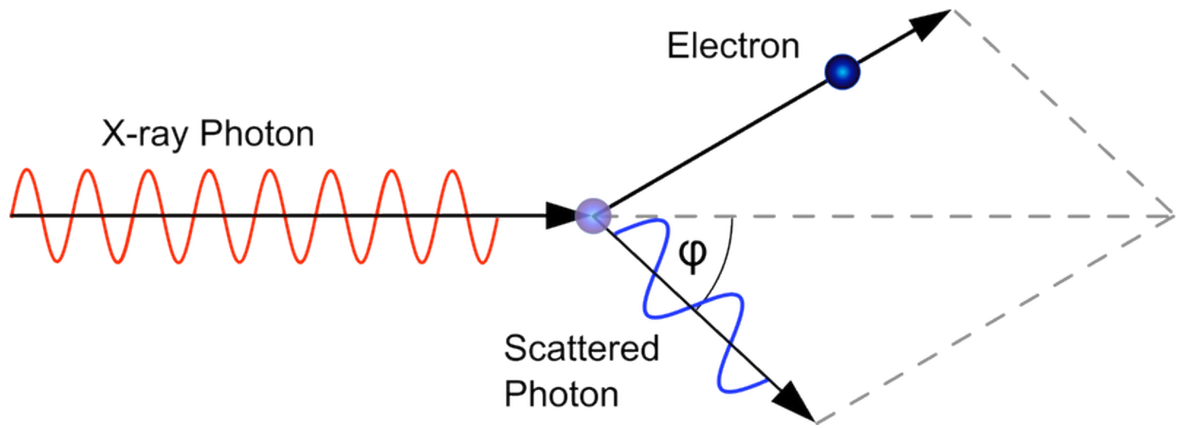
Depends on the chemical composition. Typically μ ~ 0.8

The ideal gas constant: $$R \equiv \frac{k}{m_H}$$

Thus: $$P_{gas} = \frac{R}{\mu}\rho T$$

Radiation Pressure

Momentum exchange between photons and electrons/ions results in a radiation pressure



For blackbody radiation: $$P_r = \frac{1}{3} a T^4$$

…where a is the *radiation constant*:

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3} = \frac{4\sigma}{c} = 7.565 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

Radiation pressure dominates over the gas pressure inside very massive stars, which are hotter

Equations of Stellar Structure

At radius r in a static, spherically symmetric star and the density ϱ:

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

Mass conservation

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

Hydrostatic equilibrium

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

Energy transport due to radiation (only)

$$\frac{dL}{dr} = 4\pi r^2 \rho q$$

Energy generation

4 equations with 4 unknowns - enough for a solution once we know P(ϱ ,T), opacity κ, and q

Some Order-of-Magnitude Estimates

Let's see if we can estimate roughly the conditions in the Solar core. **Pressure** $P = F / A$:

$$F \approx G\, M_{\odot}^2 / R_{\odot}^2$$

$$A \approx 4 \pi R_{\odot}^2$$

$$P \approx G\, M_{\odot}^2 / 4 \pi R_{\odot}^4$$

($M_{\odot} \approx 2 \times 10^{33}$ g, $R_{\odot} \approx 7 \times 10^{10}$ cm, $G \approx 6.7 \times 10^{-8}$ cgs)

Thus: $P_{est} \sim 10^{15}$ dyn / cm^2 -- and surely an underestimate

True value: $P_c \approx 2 \times 10^{17}$ dyn / cm^2

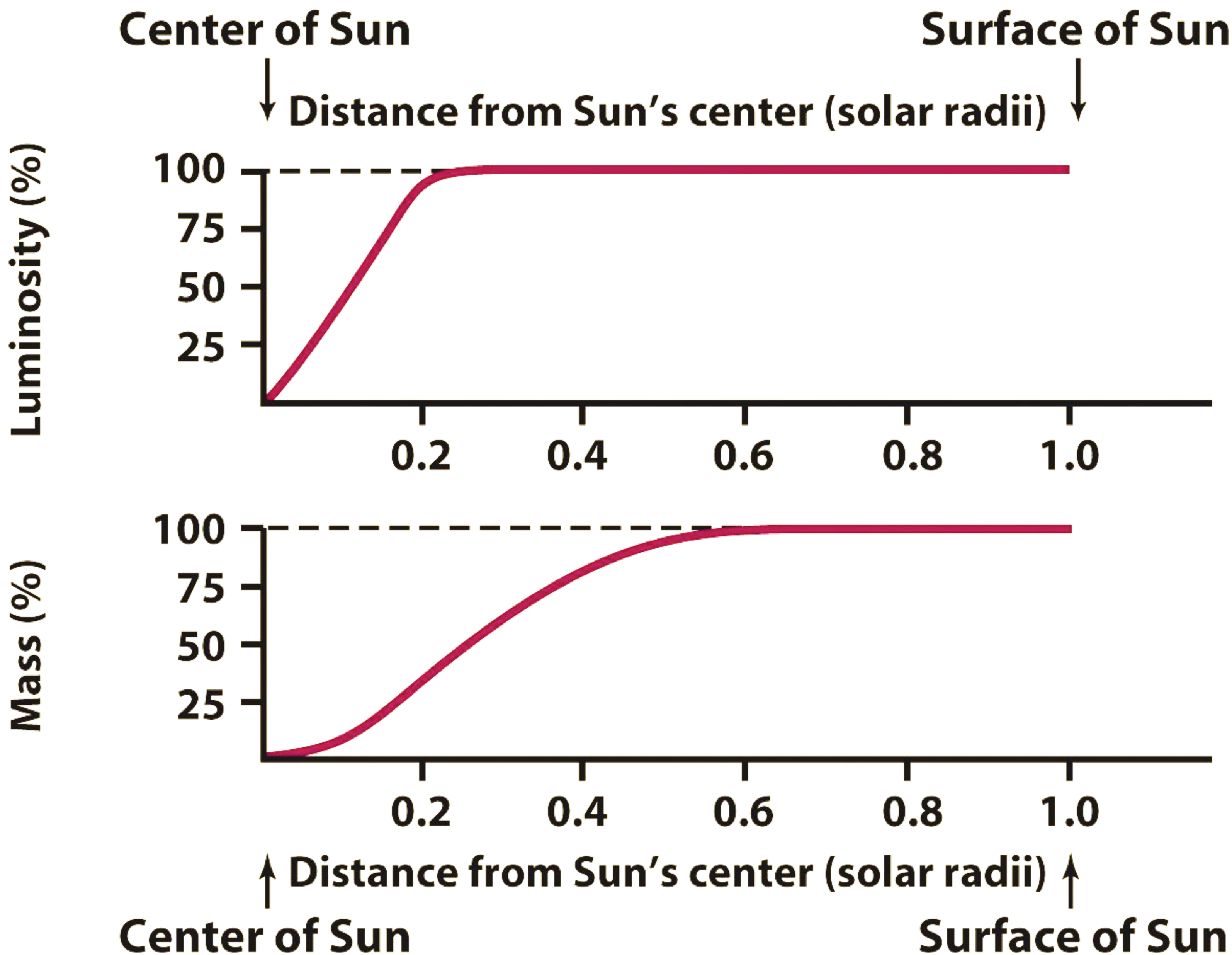
Now the **temperature:** $3/2\, k\, T \approx G\, m_p\, M_{\odot} / R$

($k \approx 1.4 \times 10^{-16}$ erg/K, $m_p \approx 1.7 \times 10^{-24}$ g)

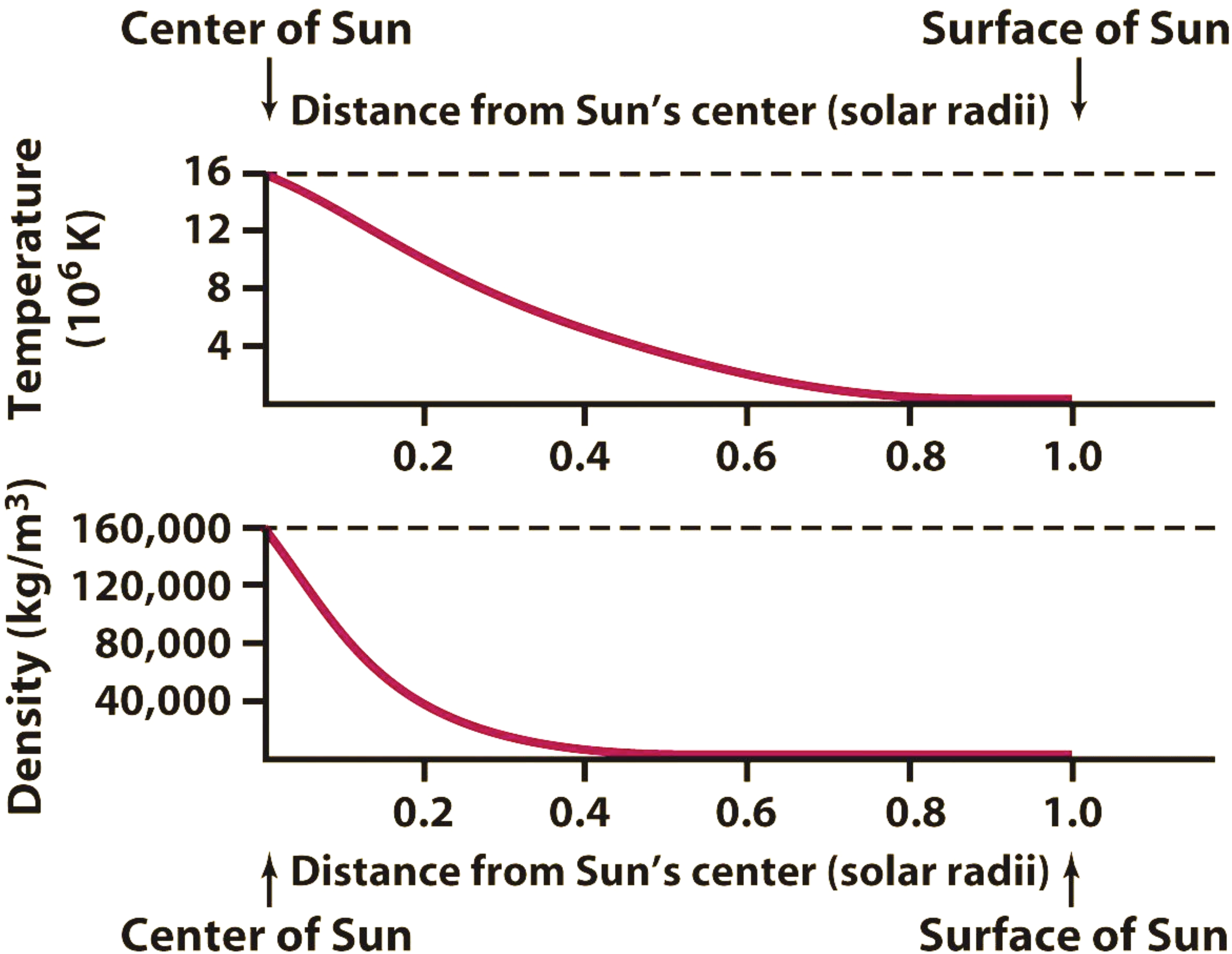
Thus: $T_{est} \approx 1.6 \times 10^{7}$ K

True value: $T_c \approx 1.57 \times 10^{7}$ K -- not bad!

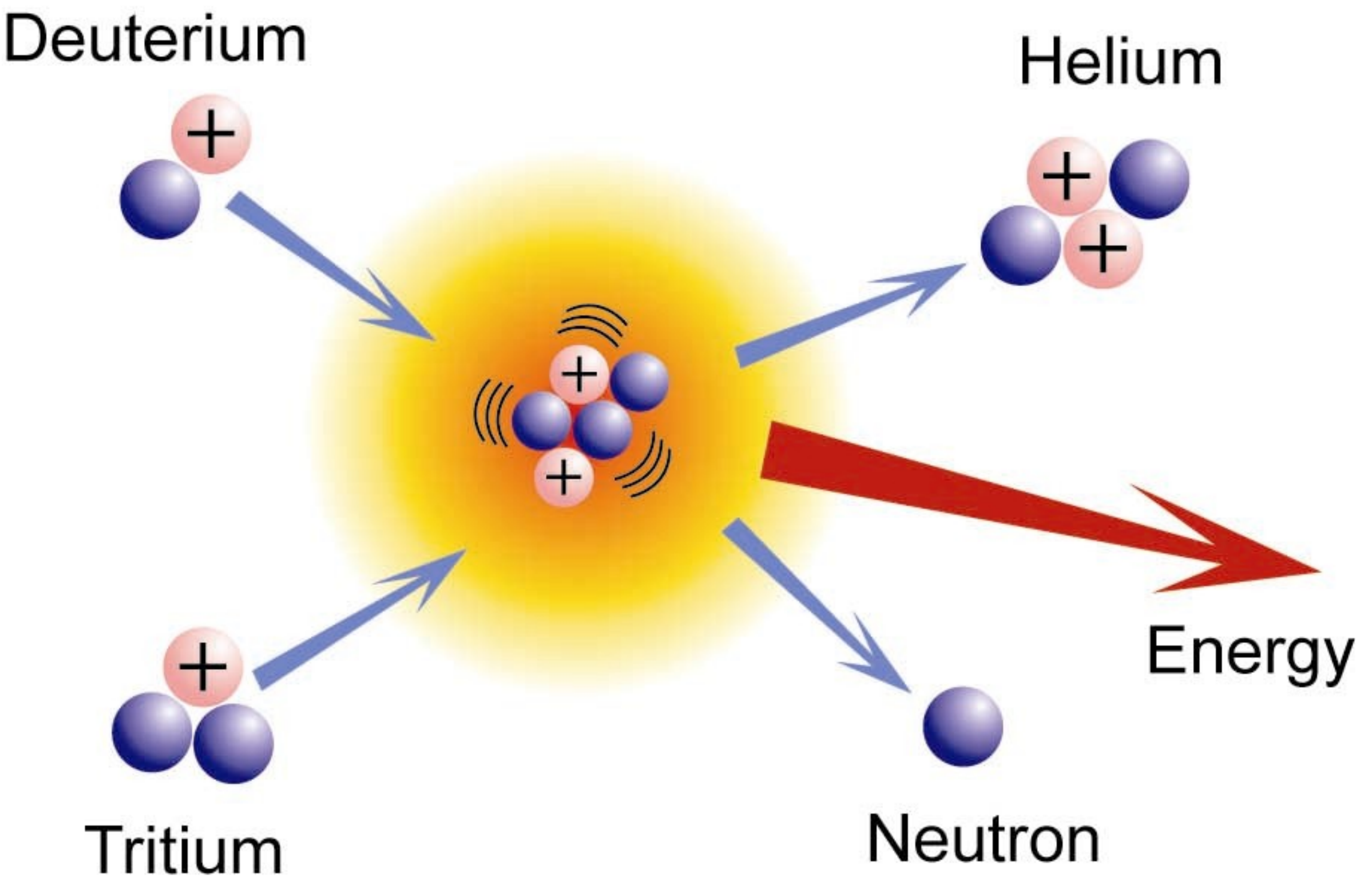
Standard Solar Model



Standard Solar Model



8.2 Energy Generation in Stars



Energy Production in Stars: Thermonuclear Reactions

Mass of nuclei with several protons and / or neutrons does not exactly equal mass of the constituents - slightly smaller because of the **binding energy** of the nucleus

The main process is hydrogen fusion into helium:

$$4 \times {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \text{photons and neutrinos}$$

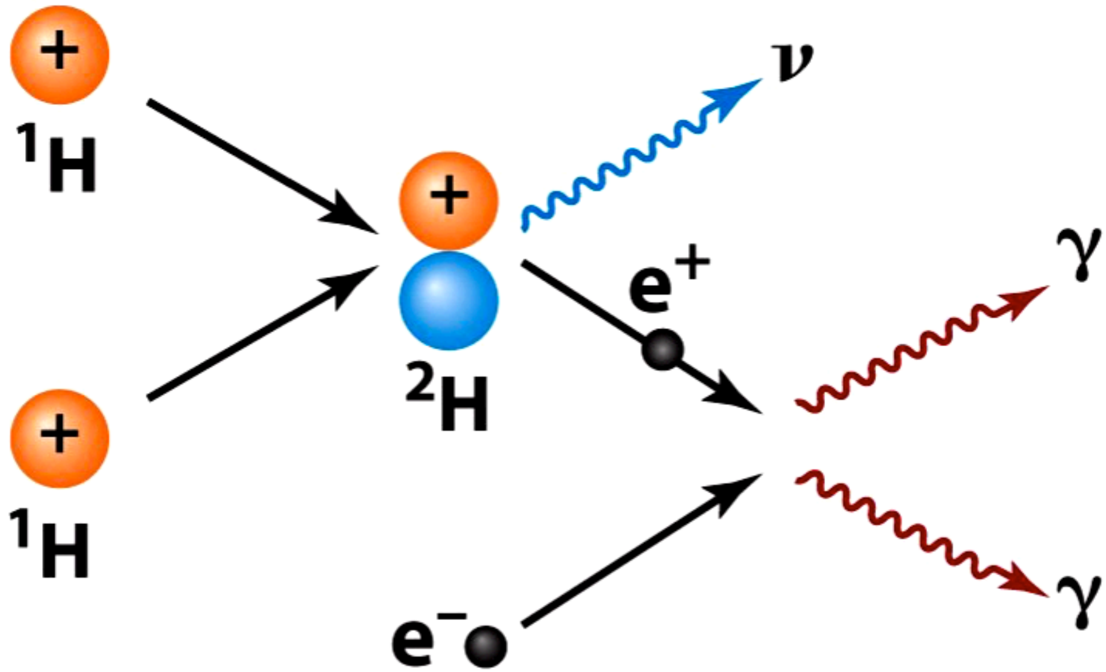
4 protons, total mass = $4 \times 1.0081 = 4.0324$ amu

helium nucleus, mass = 4.0039 amu

Mass difference: 0.0285 amu = 4.7×10^{-26} g

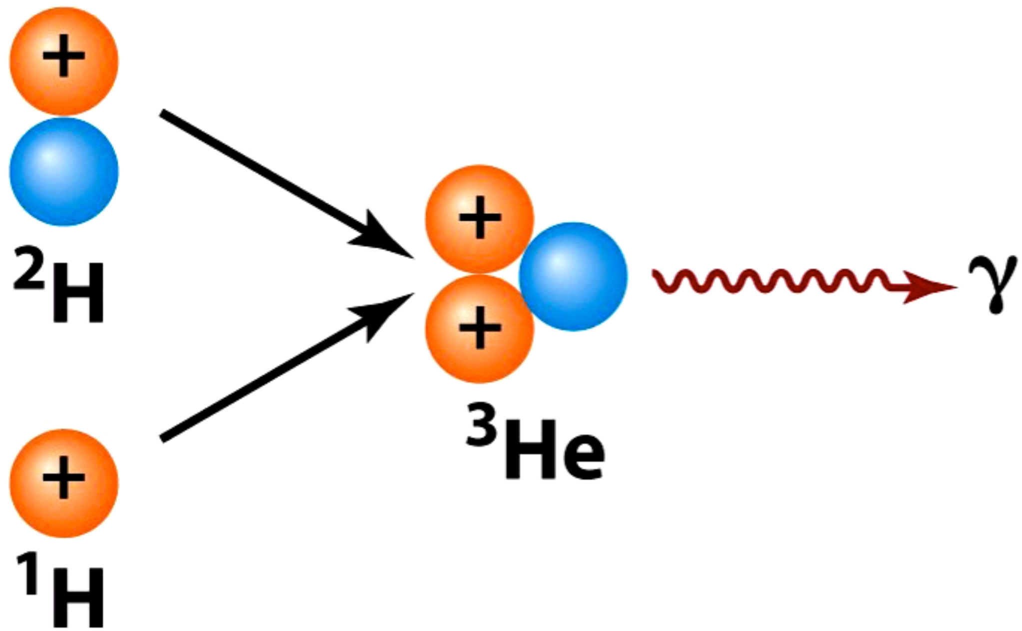
$$\Delta E = \Delta M c^2 = 4.3 \times 10^{-5} \text{ erg} = 27 \text{ MeV}$$

Or about 0.7% of the total rest mass



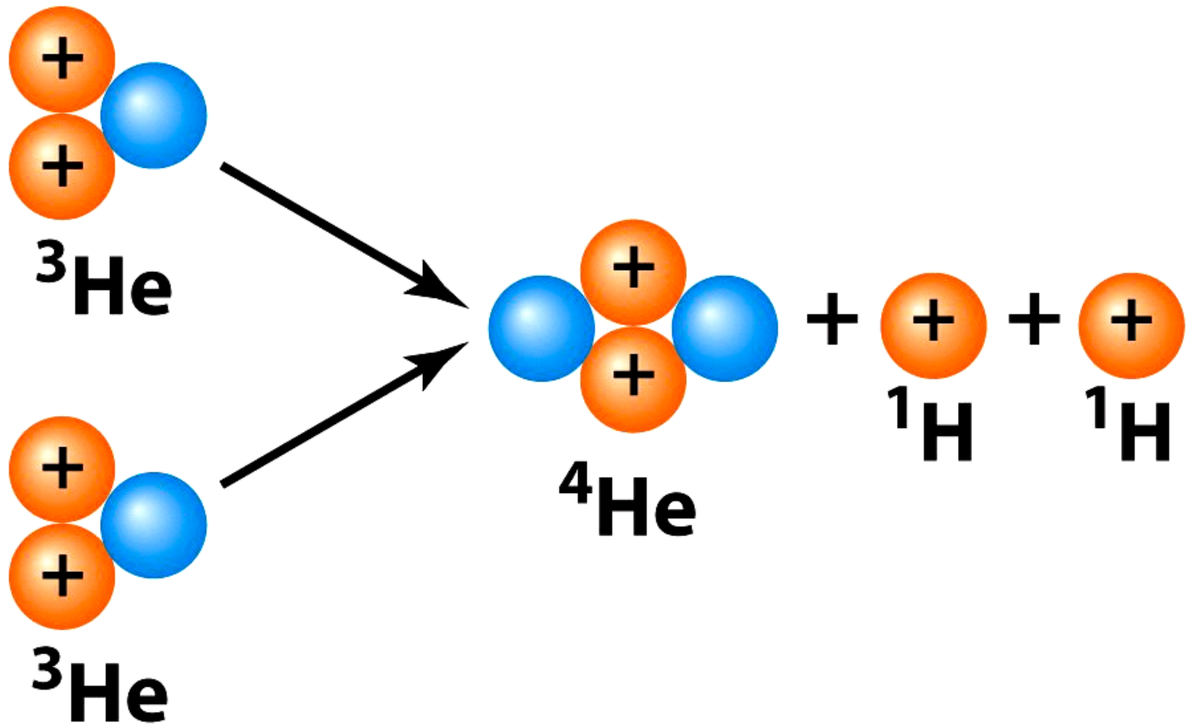
Step 1:

- **Two protons (hydrogen nuclei, ^1H) collide.**
- **One of the protons changes into a neutron (shown in blue), a neutral, nearly massless neutrino (ν), and a positively charged electron, or positron (e^+).**
- **The proton and neutron form a hydrogen isotope (^2H).**
- **The positron encounters an ordinary electron (e^-), annihilating both particles and converting them into gamma-ray photons (γ).**



Step 2:

- The ^2H nucleus from the first step collides with a third proton.
- A helium isotope (^3He) is formed and another gamma-ray photon is released.



Step 3:

- Two ^3He nuclei collide.
- A different helium isotope with two protons and two neutrons (^4He) is formed and two protons are released.

The P-P Cycle

$$^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_e$$

$$^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + \gamma$$

69%

$$^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

(PP I)

31%

$$^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$$^{7}_{4}\mathrm{Be} + e^{-} \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

$$^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP II)

0.3%

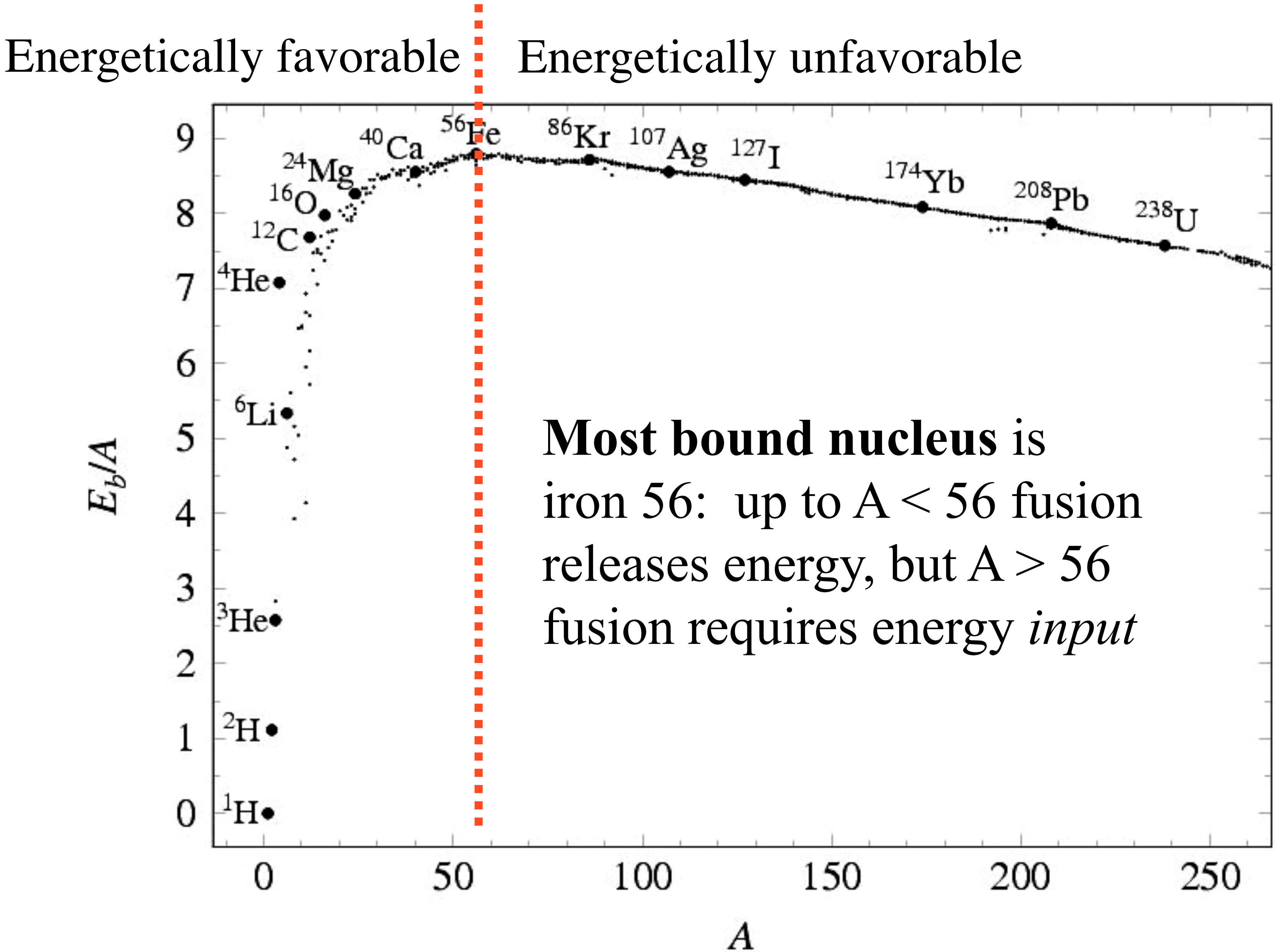
$$^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

$$^{8}_{5}\mathrm{B} \longrightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

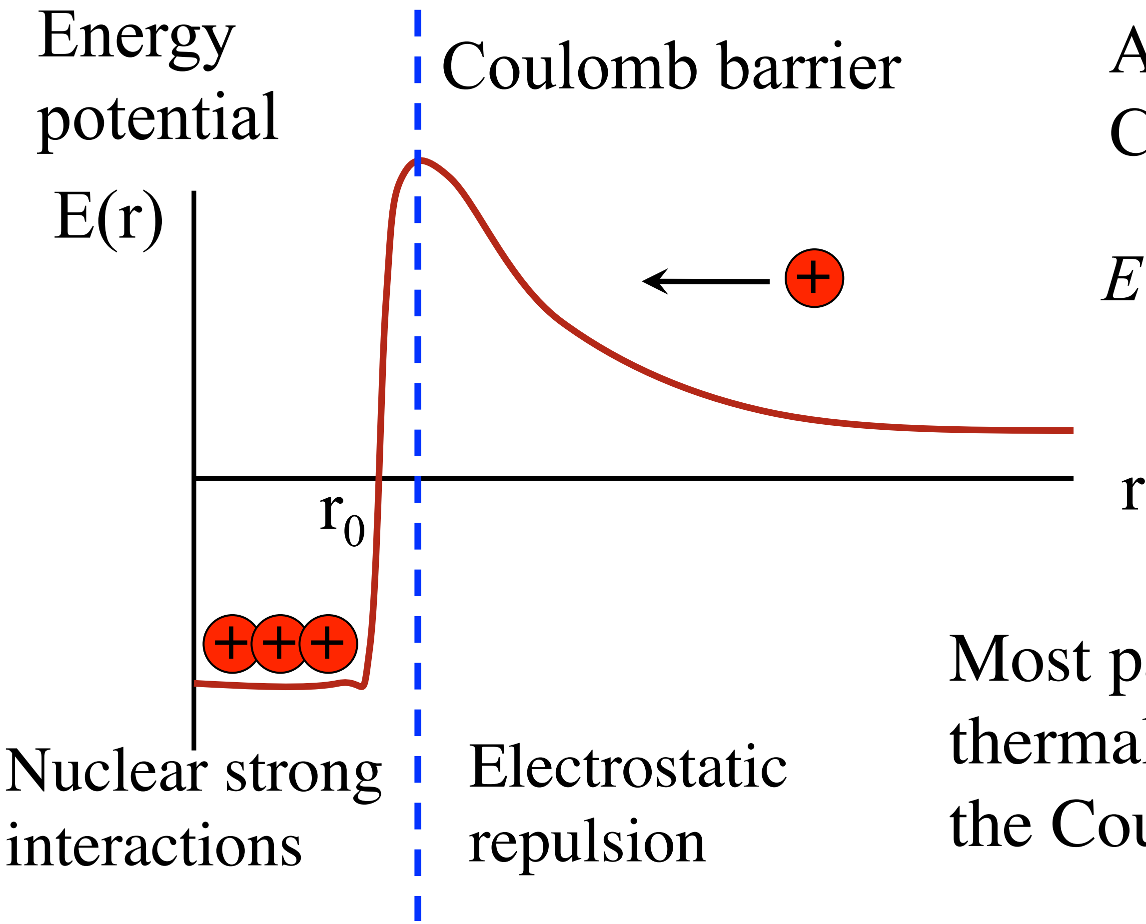
$$^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP III)

Binding Energy Per Nucleon vs. Atomic Number



Overcoming the Electrostatic Barrier



At r = r_0, height of the Coulomb barrier is:

$$E = \frac{Z_1 Z_2 e^2}{r_0} \sim Z_1 Z_2 \text{ MeV}$$

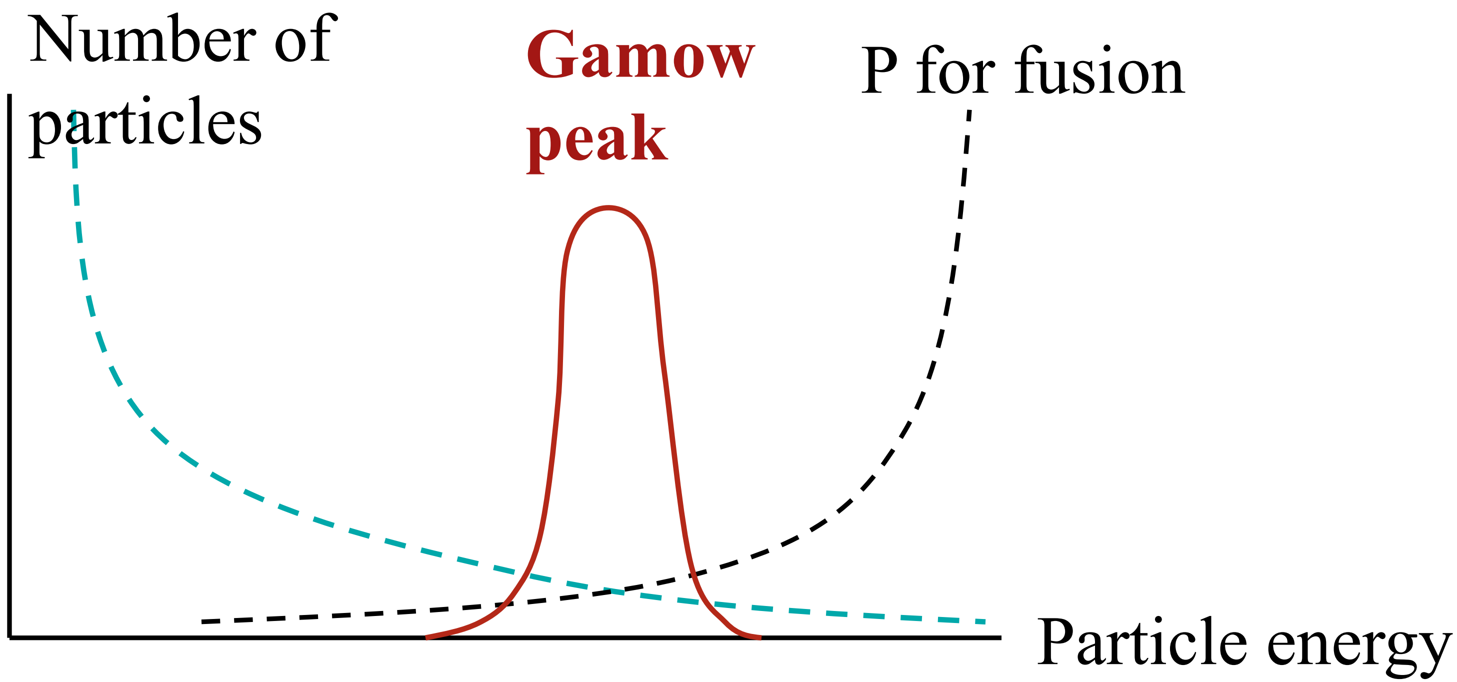
Most particles have too low thermal energies to overcome the Coulomb barrier!

What makes the fusion possible is the ***quantum tunneling*** effect

Probability of tunneling increases steeply with particle energy

The Gamow Peak

The most energetic nuclei are the most likely to fuse, but very few of them in a thermal distribution of particle speeds:



Narrow range of energies around the Gamow peak where significant numbers of particles in the plasma are able to fuse. Energy is >> typical thermal energy, so fusion is slow

Thermonuclear Reactions (TNR)

- Burning of H into He is the only energy generation process on the Main Sequence, where stars spend most of their lives; all others happen in post-MS evolutionary stages
  - Solar luminosity ~ 4.3 million tons of H into He per second
- In addition to the **p-p cycle**, there is the **CNO Cycle**, in which the C, N, O, nuclei catalyze the burning of H into He
- The rates of TNR are usually very steep functions of temperature, due to high potential barriers
- Generally, more massive stars achieve higher T_c , and can synthesize elements up to Fe; beyond Fe, it happens in SN explosions

Self-Regulation in Stars

Suppose the fusion rate increases slightly. Then,

(1) Temperature increases

(2) Pressure increases

(3) Core expands

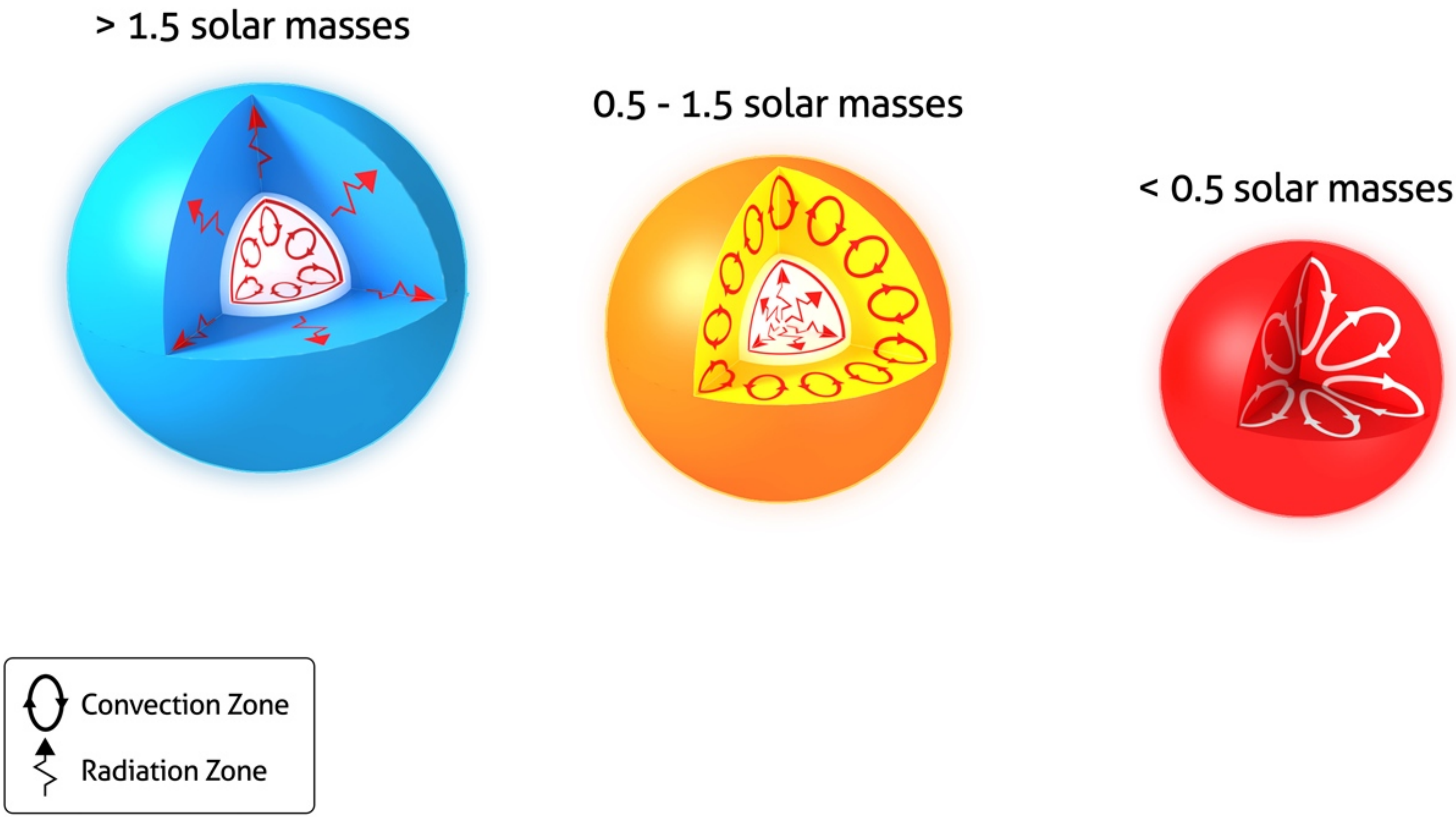
(4) Density and temperature decrease

(5) Fusion rate decreases

So there's a feedback mechanism which prevents the fusion rate from skyrocketing upward

This is the inverse of the core collapse mechanism discussed for the protostars

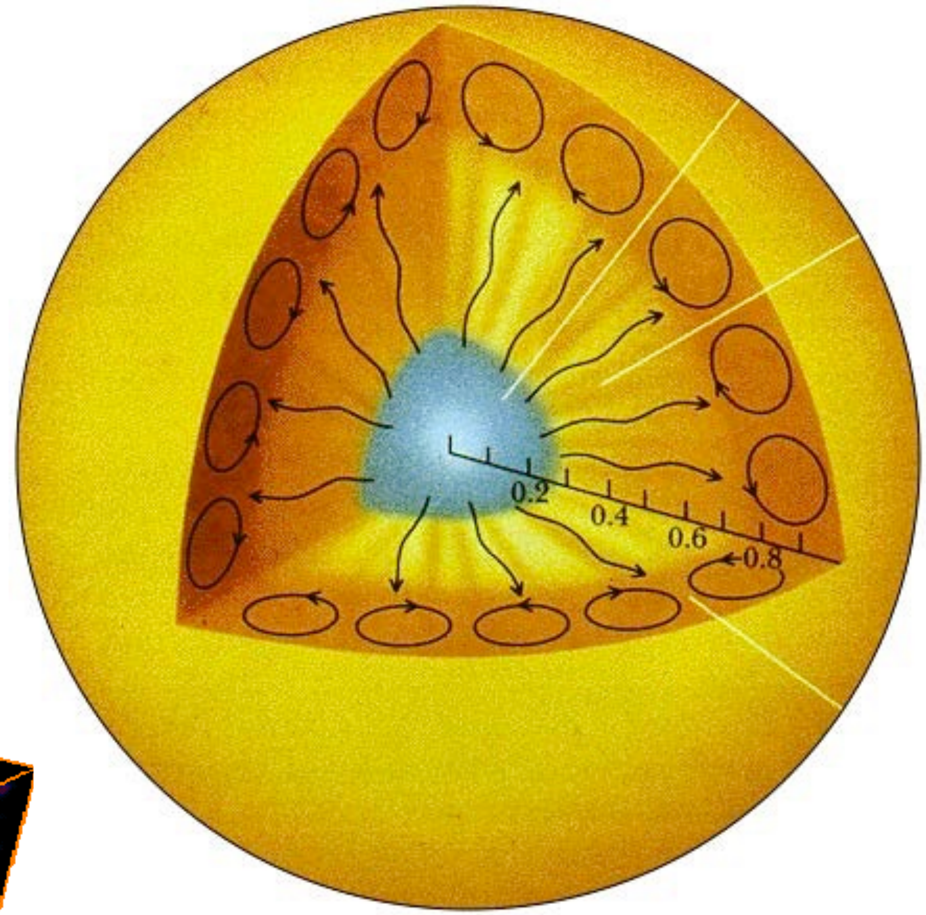
8.3 Energy Transport in Stars

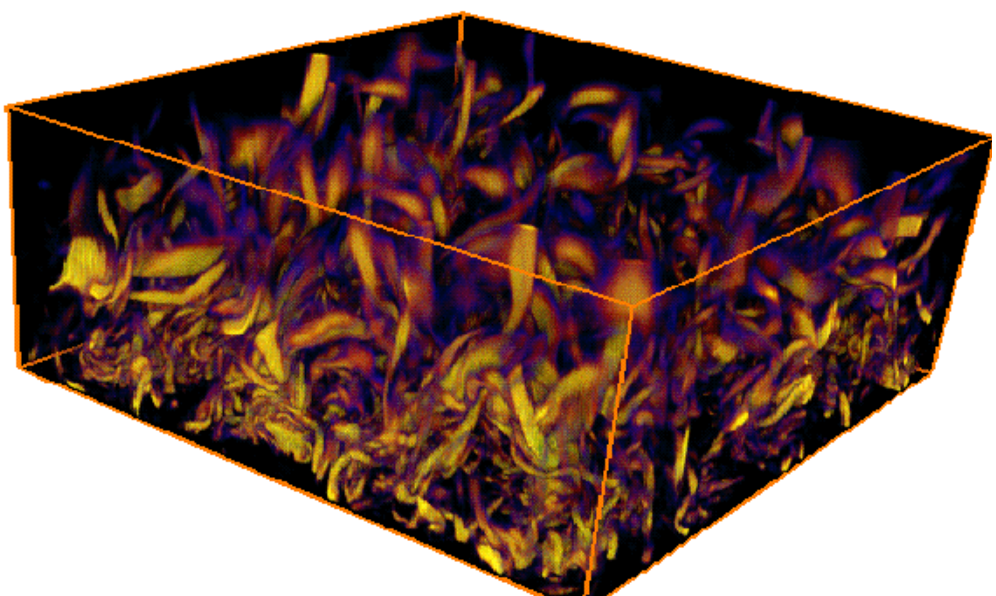


Energy Transport Mechanisms in Stars

How does the energy get out?

1. **Radiatively** (photon diffusion)
2. **Convectively**
3. Conduction (generally not important in stars)

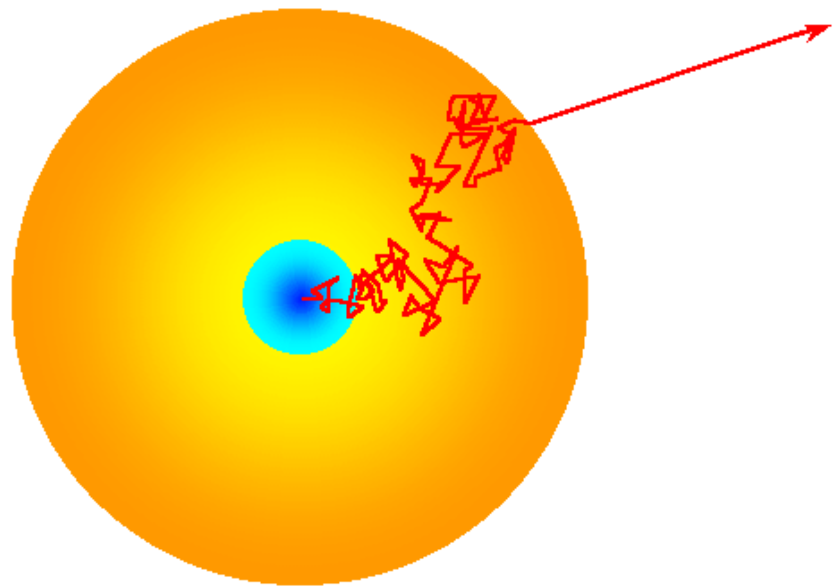




… and the reality is fairly complex

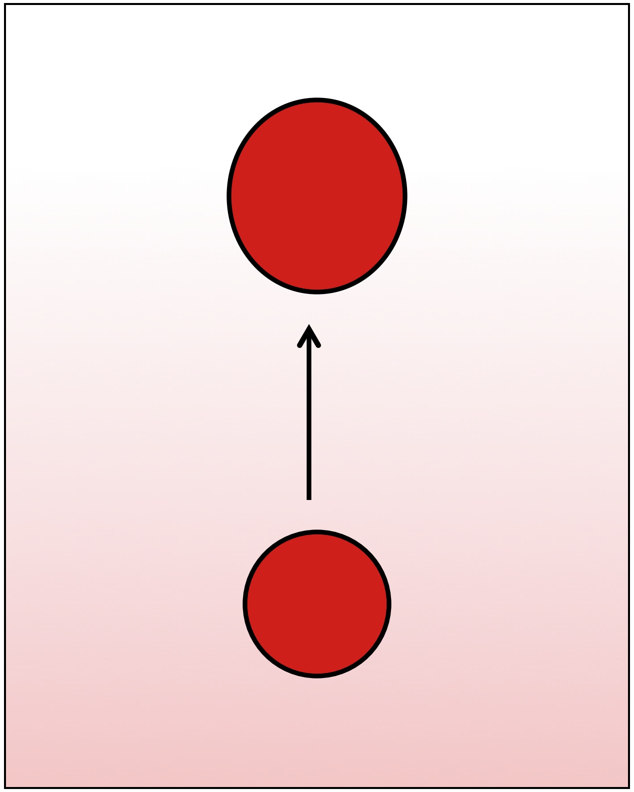
Radiative Energy Transfer

- As the heat diffuses from the core outwards, the photons are scattered by the dense plasma inside the star
- For the Sun, it takes ~ 250,000 years for the energy to reach the surface



- The opacity of the plasma depends on the temperature, density, and chemical composition
- If the plasma is too opaque, convection becomes a more efficient mechanism for the energy transfer

When Does the Convection Happen?



The Schwarzschild criterion: Imagine displacing a small mass element vertically upward by a distance dr.   Assume that **no heat** is exchanged with the surrounding, i.e. the process is **adiabatic**:

- Element expands to stay in pressure balance
- New density will *not* generally equal the new ambient density

If this mechanical energy transport is more efficient than the radiative case, the medium will be **convectively unstable**

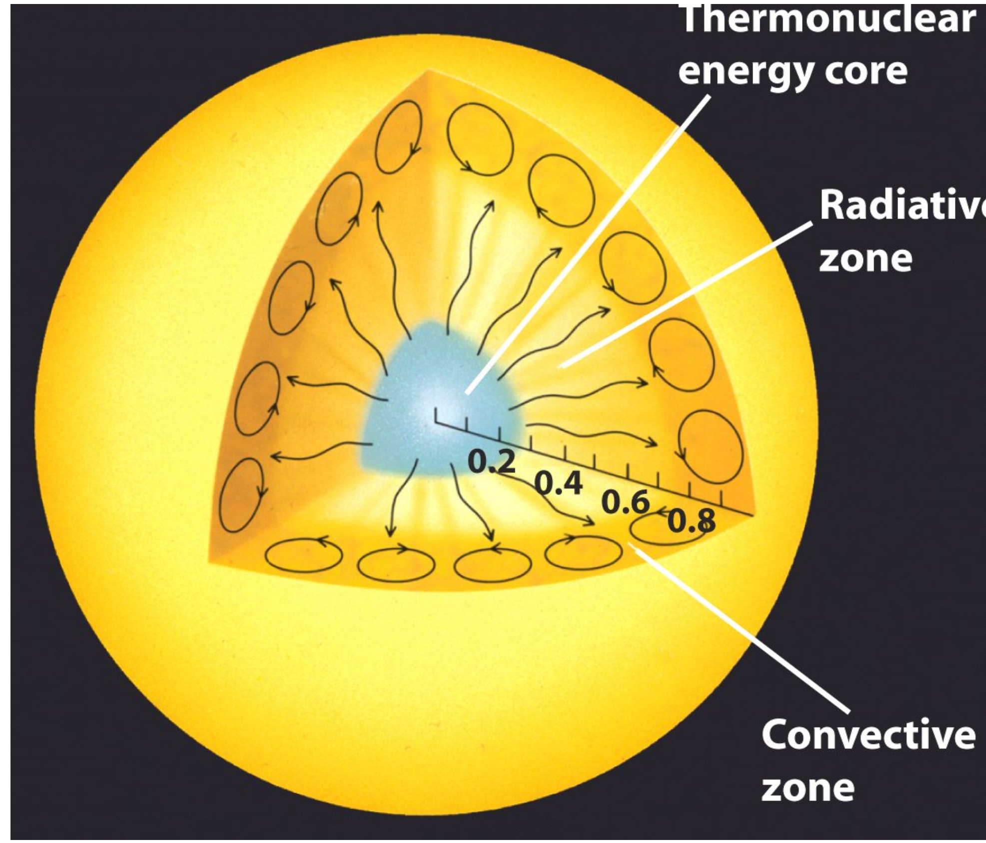
8.4 The Sun, Our Star

Why Study the Sun?

- **The nearest star -** can study it in a greater detail than any others. This can help us understand better the overall stellar physics and phenomenology
  – Radiation transfer, convection
  – Photospheric and chromospheric activity
- Kind of **important for the life on Earth** …
  – Solar activity has terrestrial consequences
- **A gateway to the neutrino astronomy** (and physics)
  – Thermometry of stellar cores and the standard model
  – Neutrino oscillations

A Theoretical Model of the Energy Transfer in the Sun

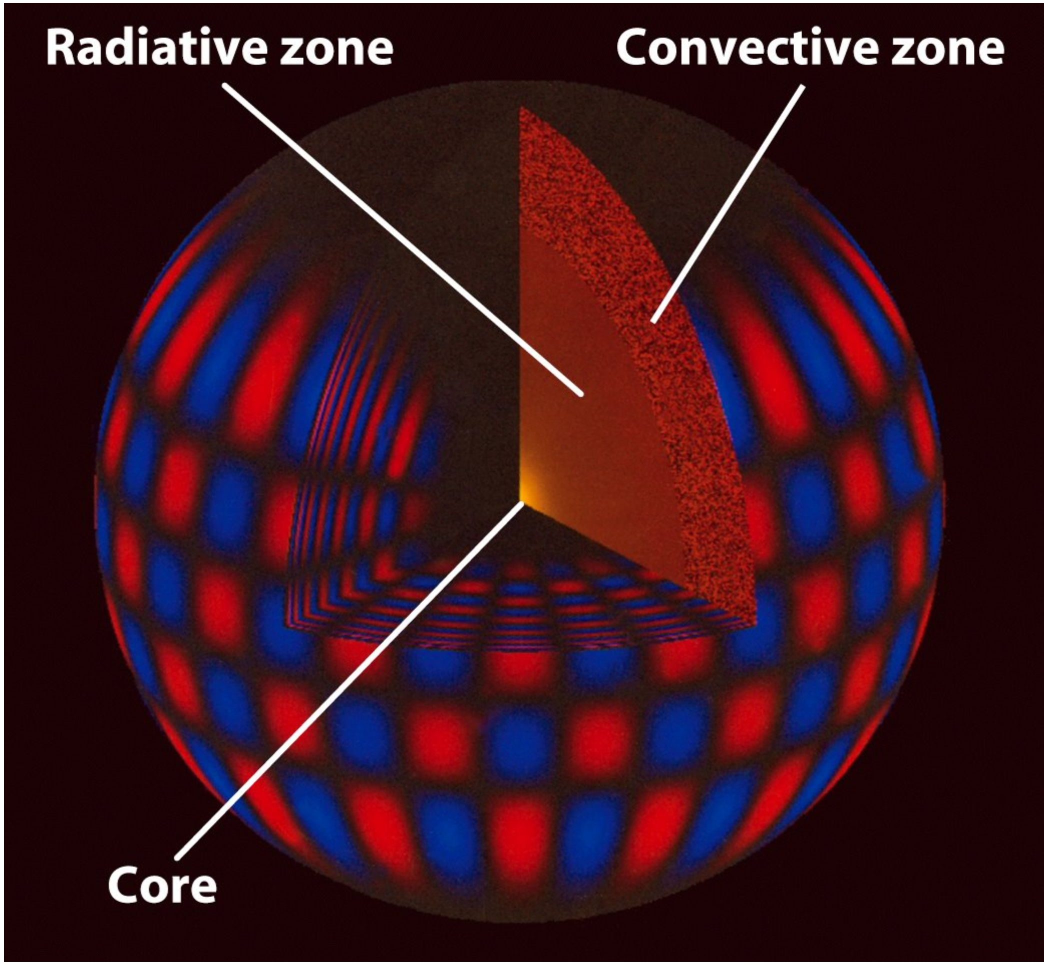
- Hydrogen fusion takes place in a core extending out to about 0.25 solar radius



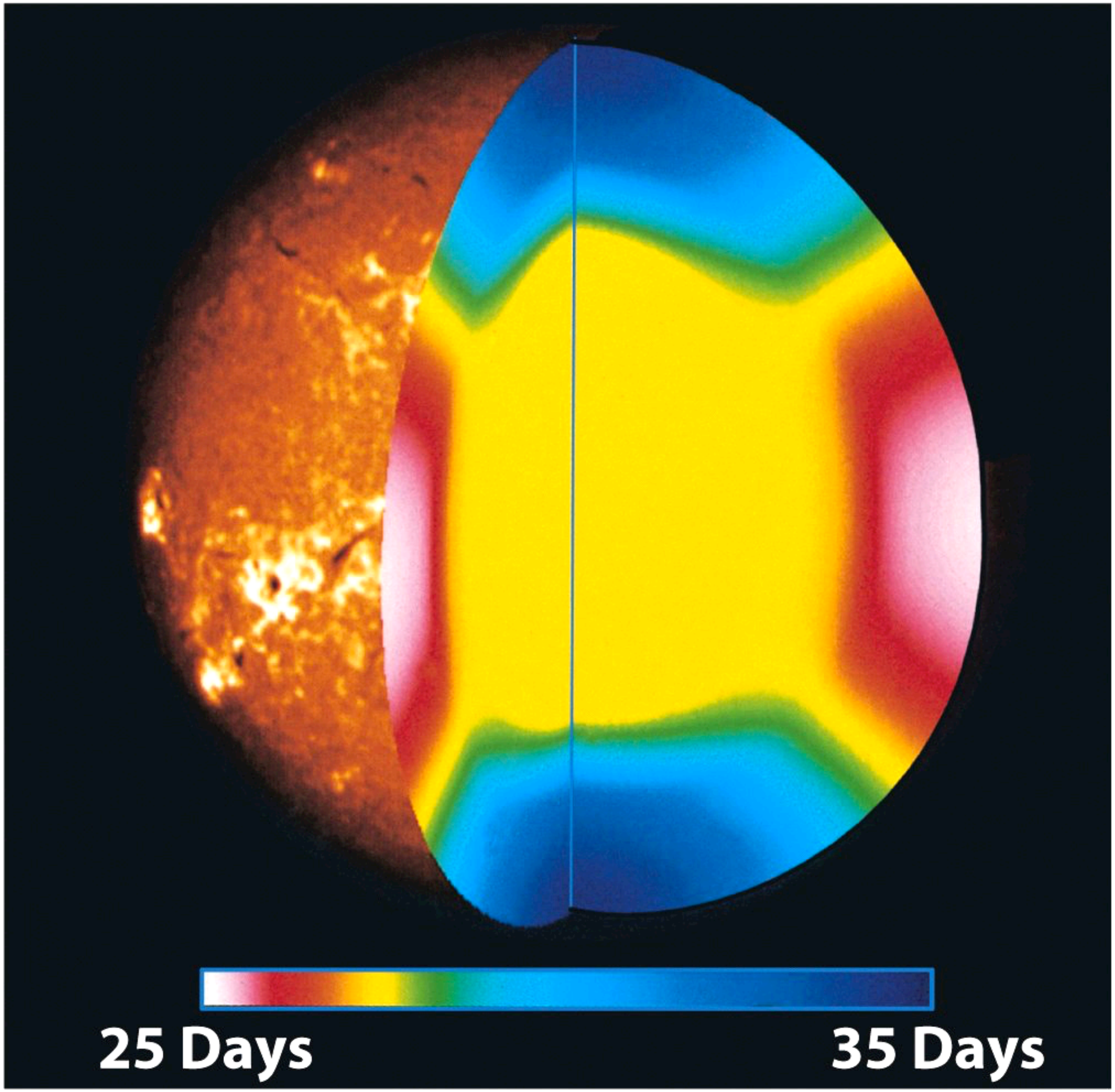
- The **core** is surrounded by a radiative zone extending to about 0.71 solar radius. Energy transfer through *radiative diffusion*
- The **radiative zone** is surrounded by a rather opaque **convective zone** of gas at relatively low temperature and pressure. Energy transfer through *convection*

We can probe the solar interior using the Sun's own vibrations

- Helioseismology is the study of how the Sun vibrates
- These vibrations have been used to infer pressures, densities, chemical composition, and rotation rates within the Sun
- Major contributions from Caltech (Libbrecht et al.)



Rotation of the Solar Interior



Solar Atmosphere / Surface Layers

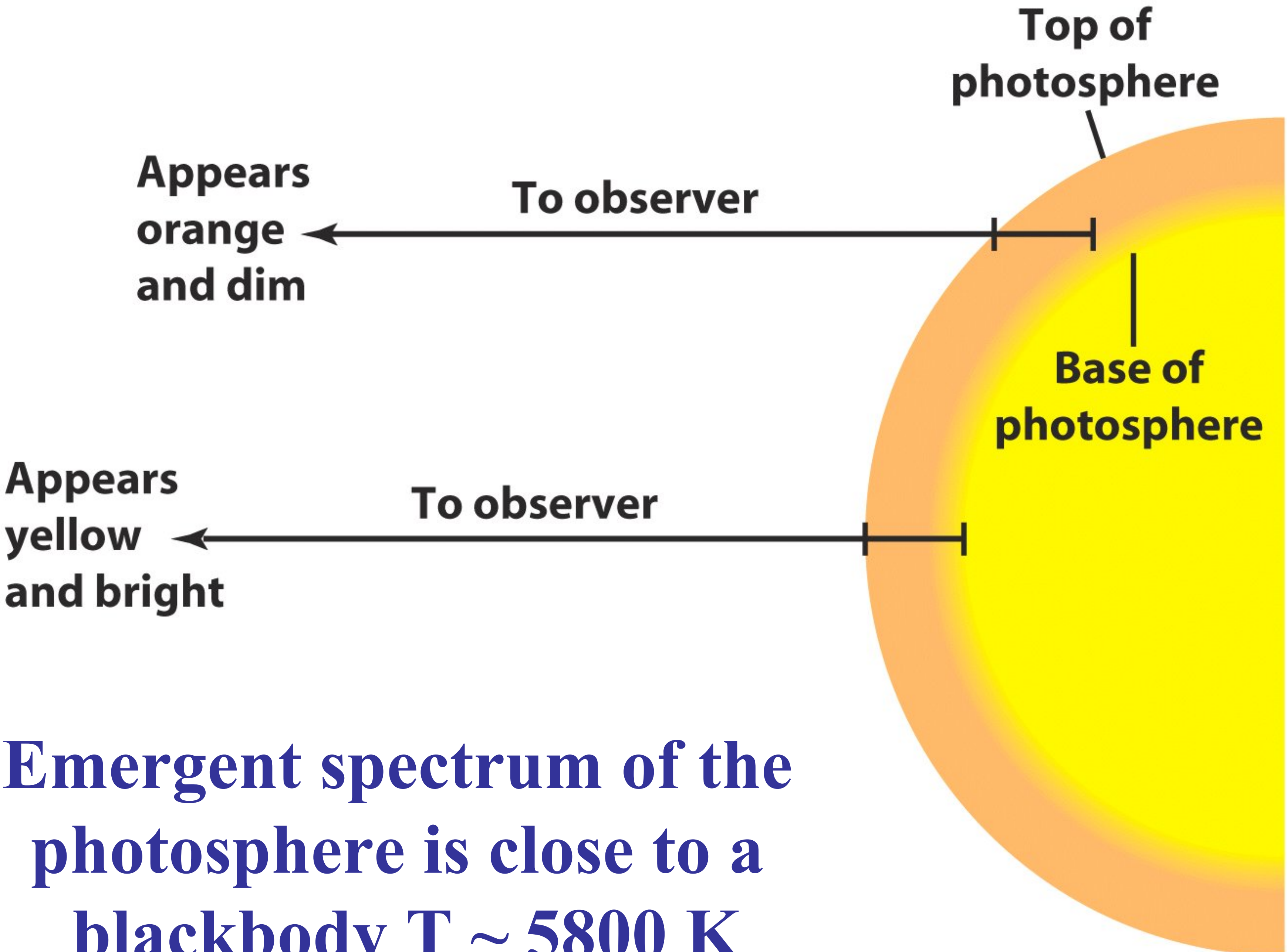
- The Sun's atmosphere has three main layers:

  1. the photosphere

  2. the chromosphere

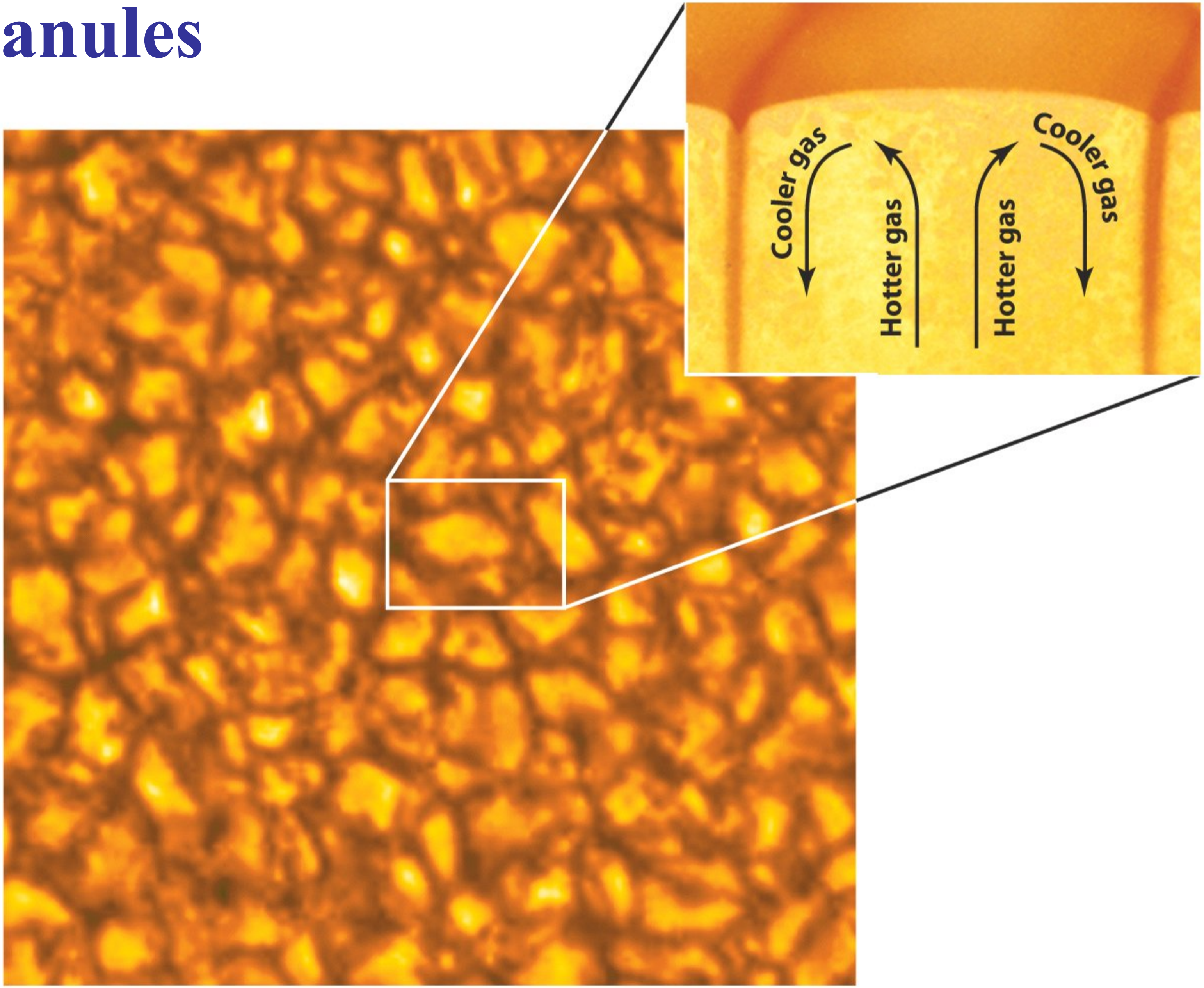
  3. the corona

- The visible surface of the Sun, the photosphere, is the lowest layer in the solar atmosphere

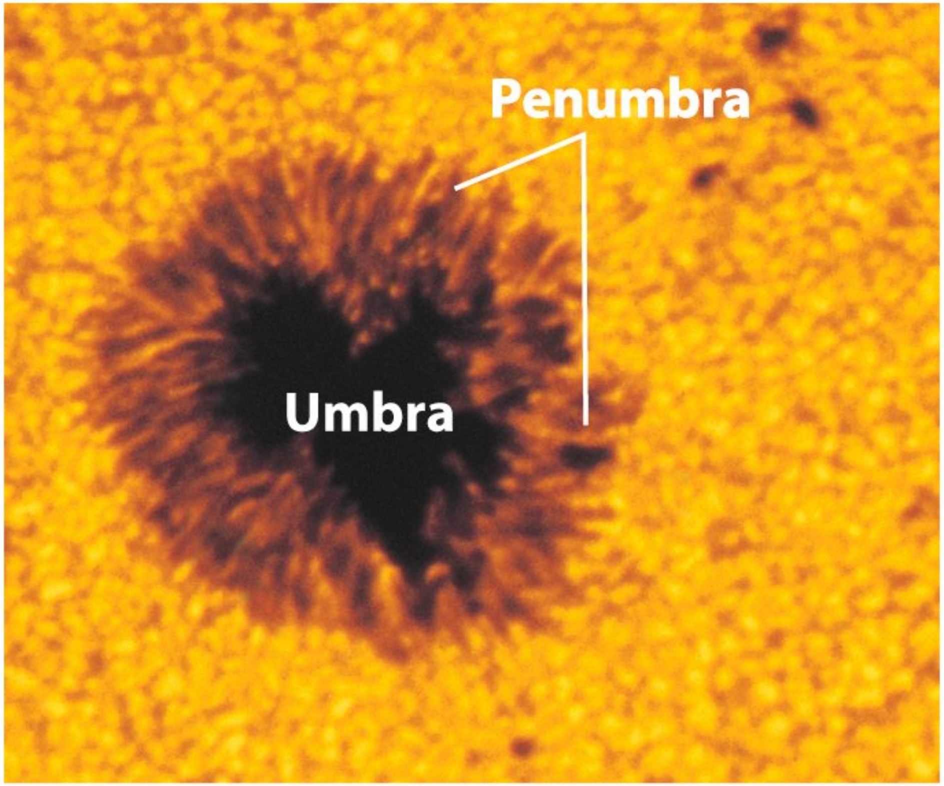


Emergent spectrum of the photosphere is close to a blackbody T ~ 5800 K

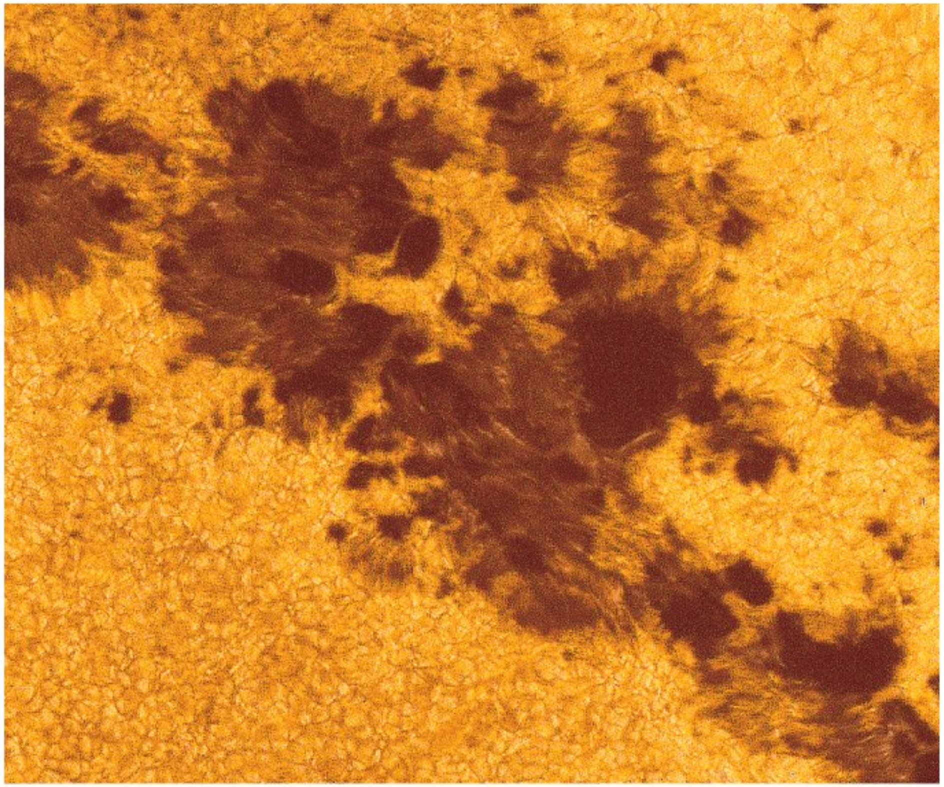
Convection in the photosphere produces granules



Sunspots are low-temperature regions in the photosphere



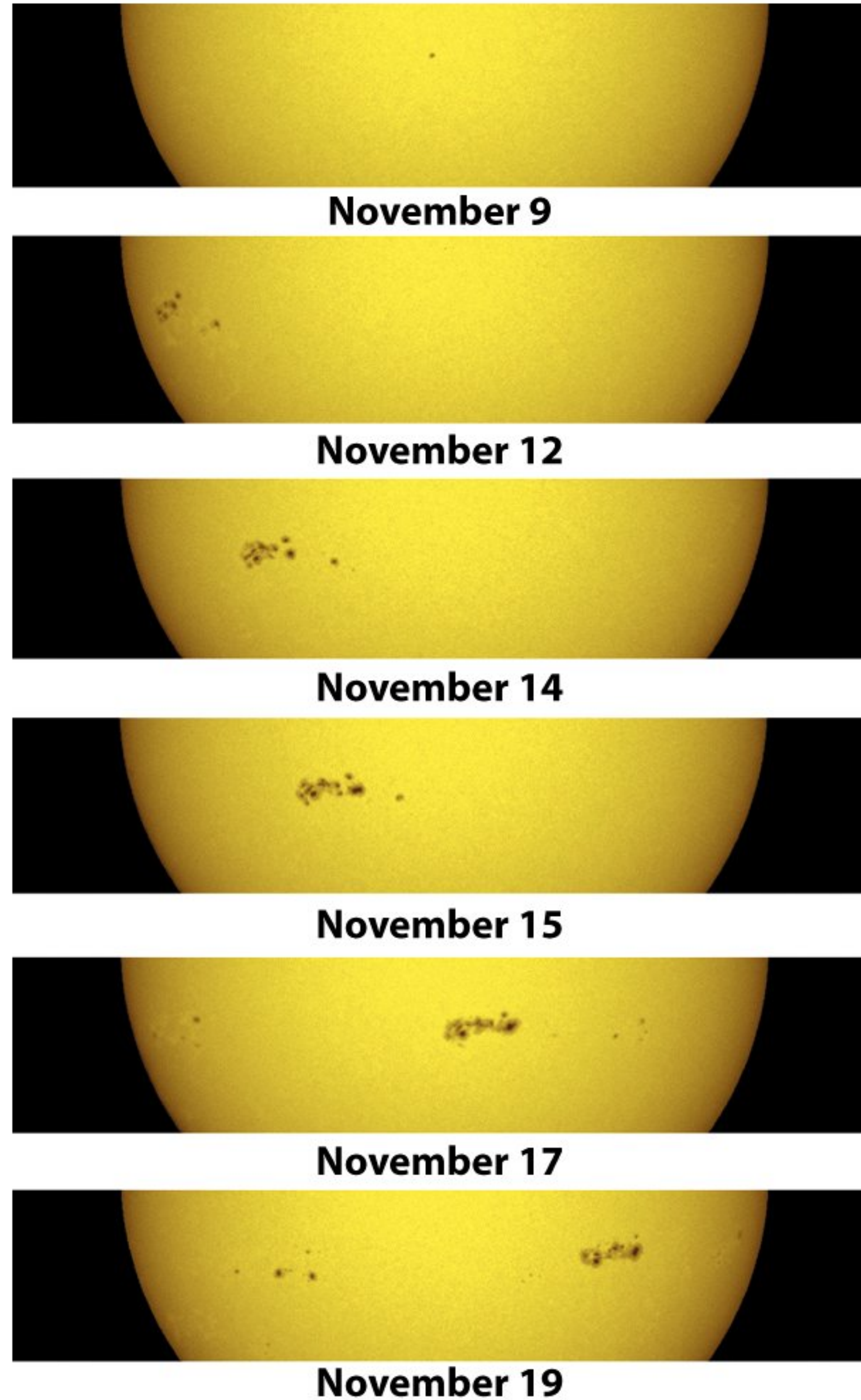
(a)



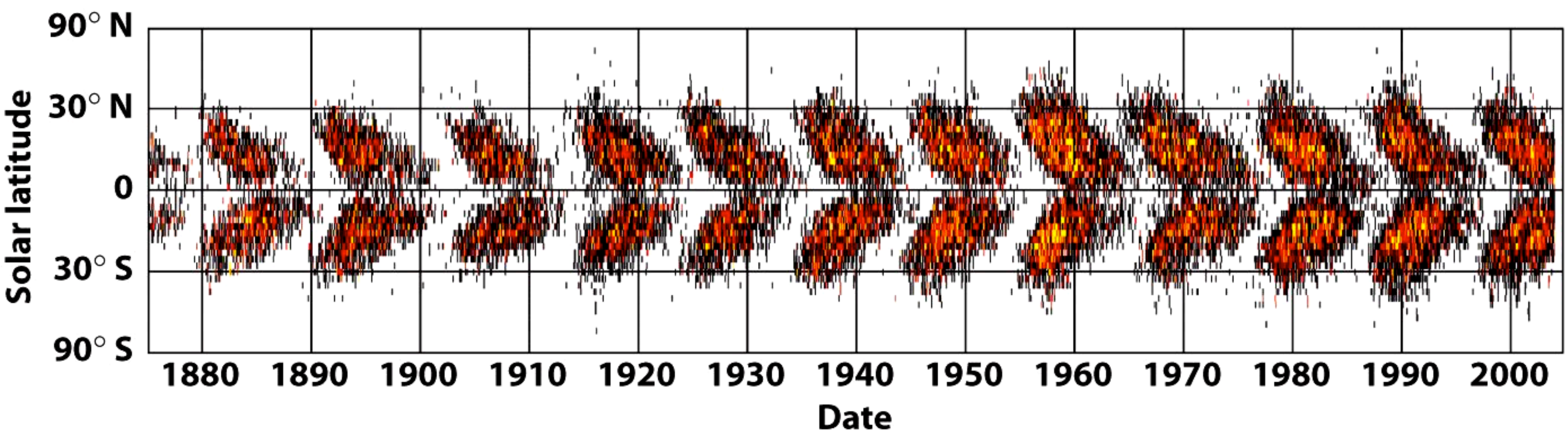
(b)

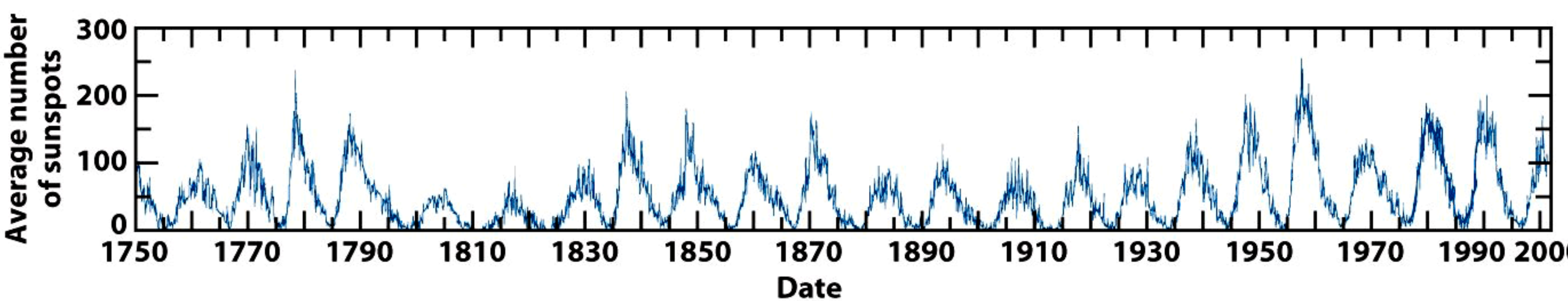
Sunspots come in groups, and follow the Sun's differential rotation. They start from the higher latitudes and migrate towards the Solar equator.

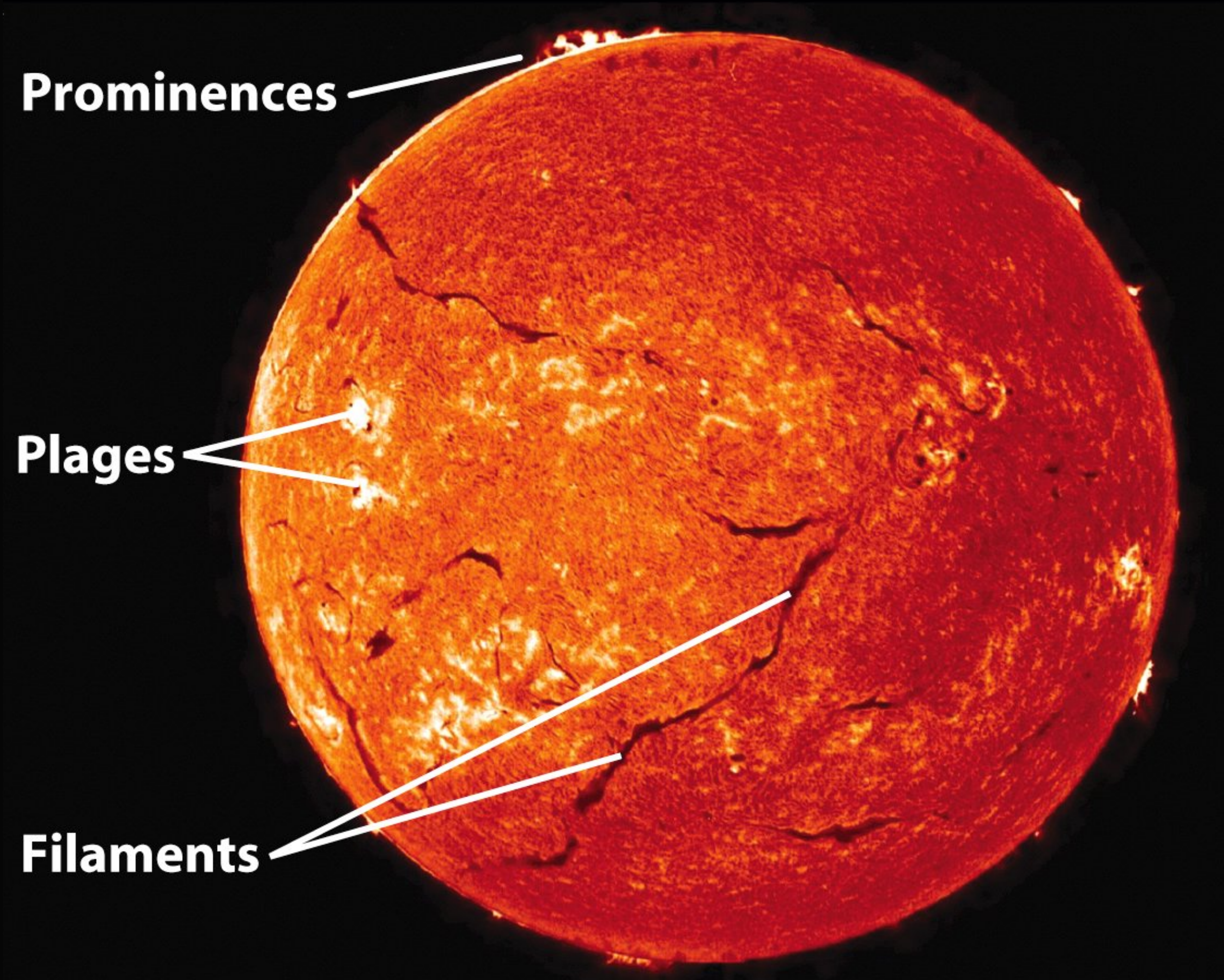
They correlate well with other manifestations of the Solar activity.

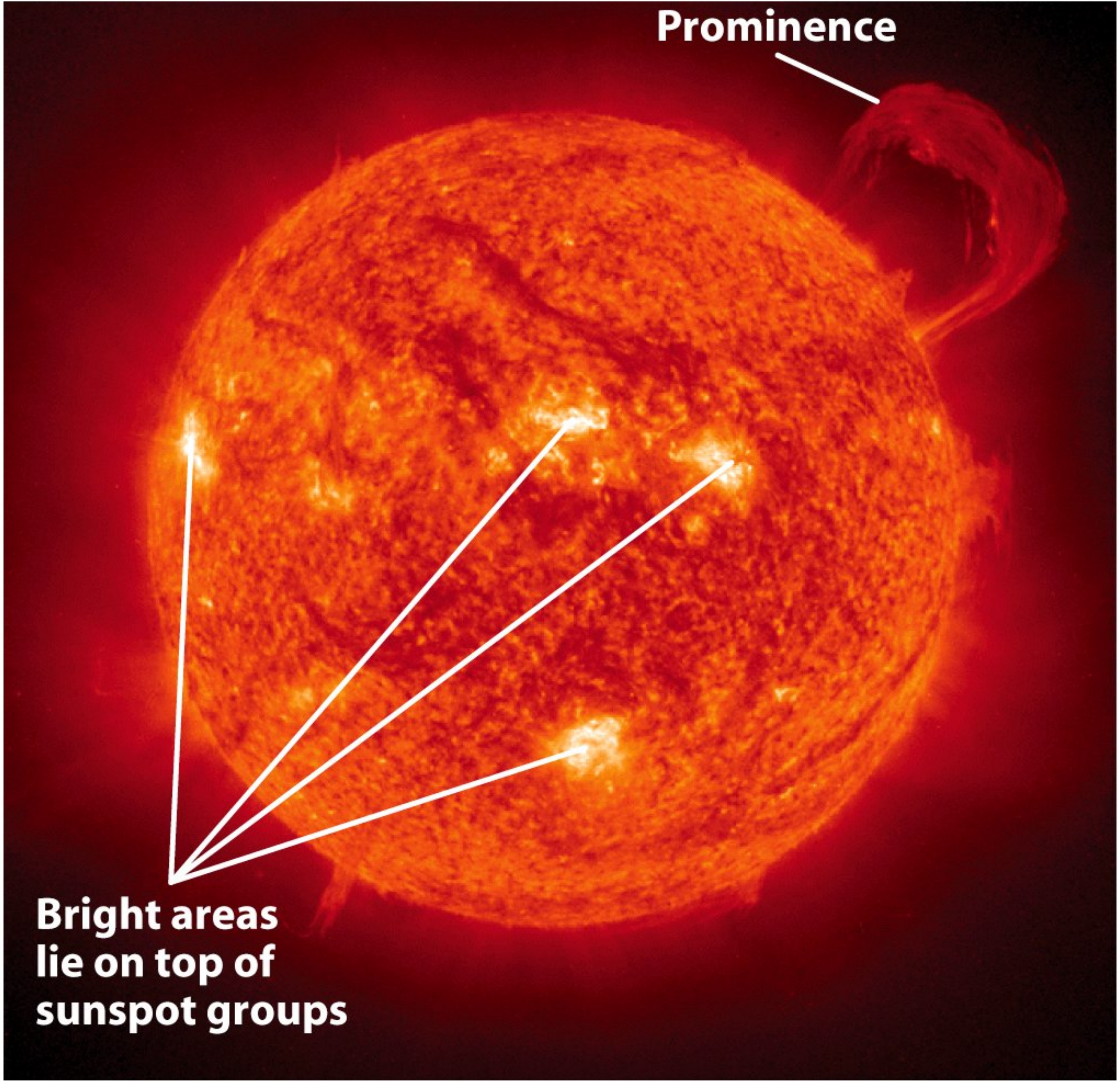


- The Sun's surface features (including sunspot numbers) vary in an ***11-year cycle***; it is really a ***22-year cycle*** in which the surface magnetic field increases, decreases, and then increases again with the opposite polarity
- There are probably also longer period cycles



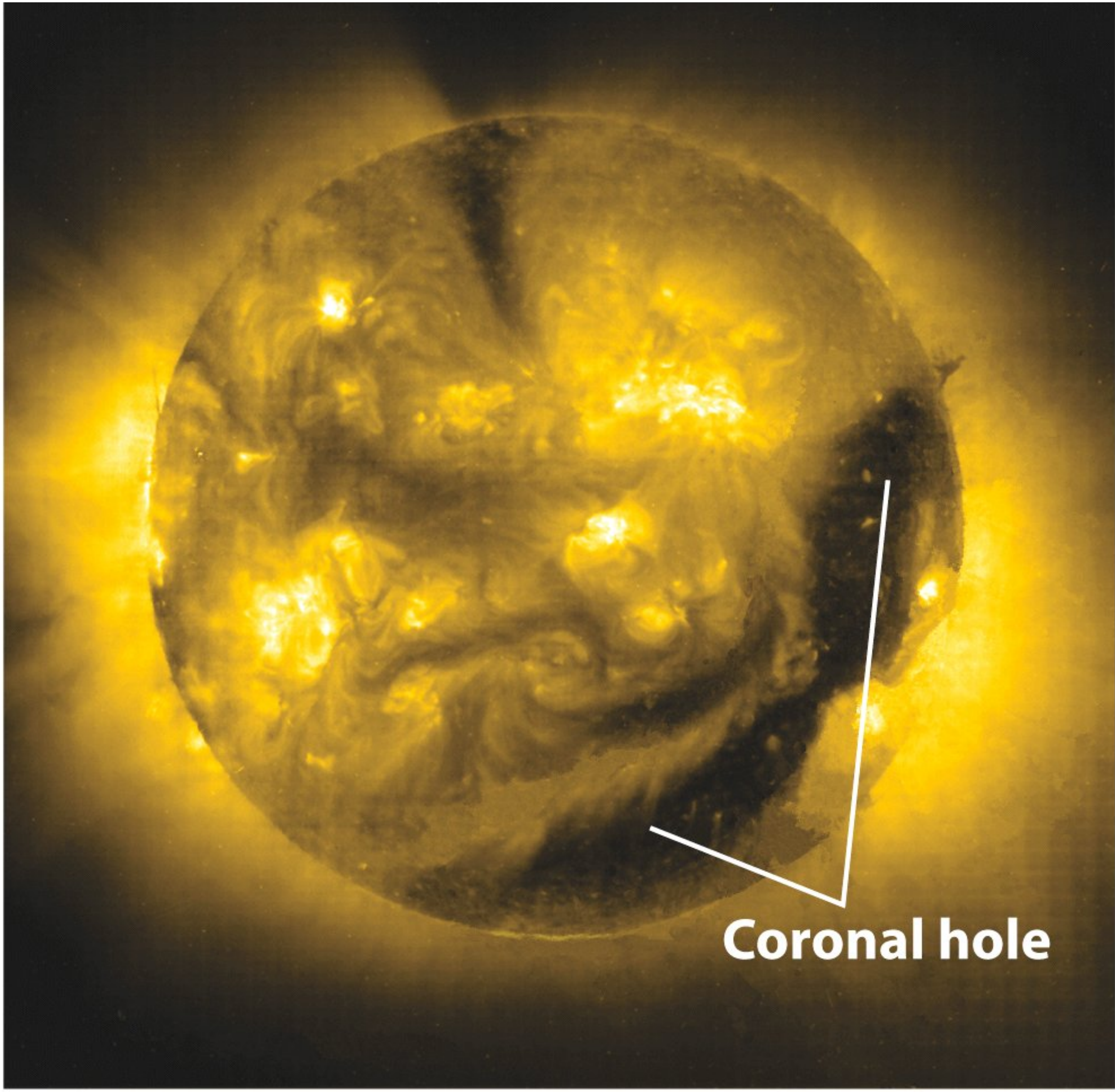



Prominences
Plages
Filaments



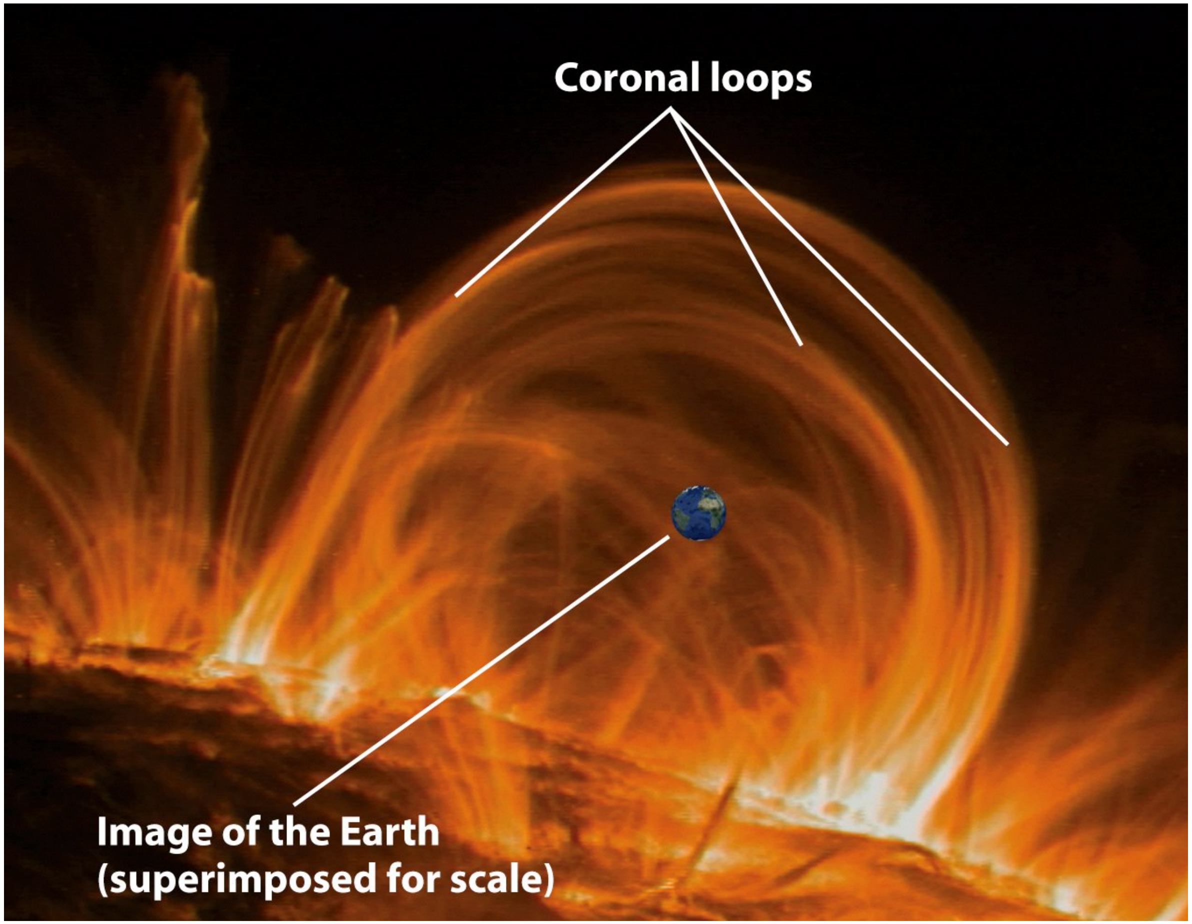
Note the scale: Earth's diameter is < 1% of the Solar diameter

Activity in the corona includes coronal mass ejections and coronal holes

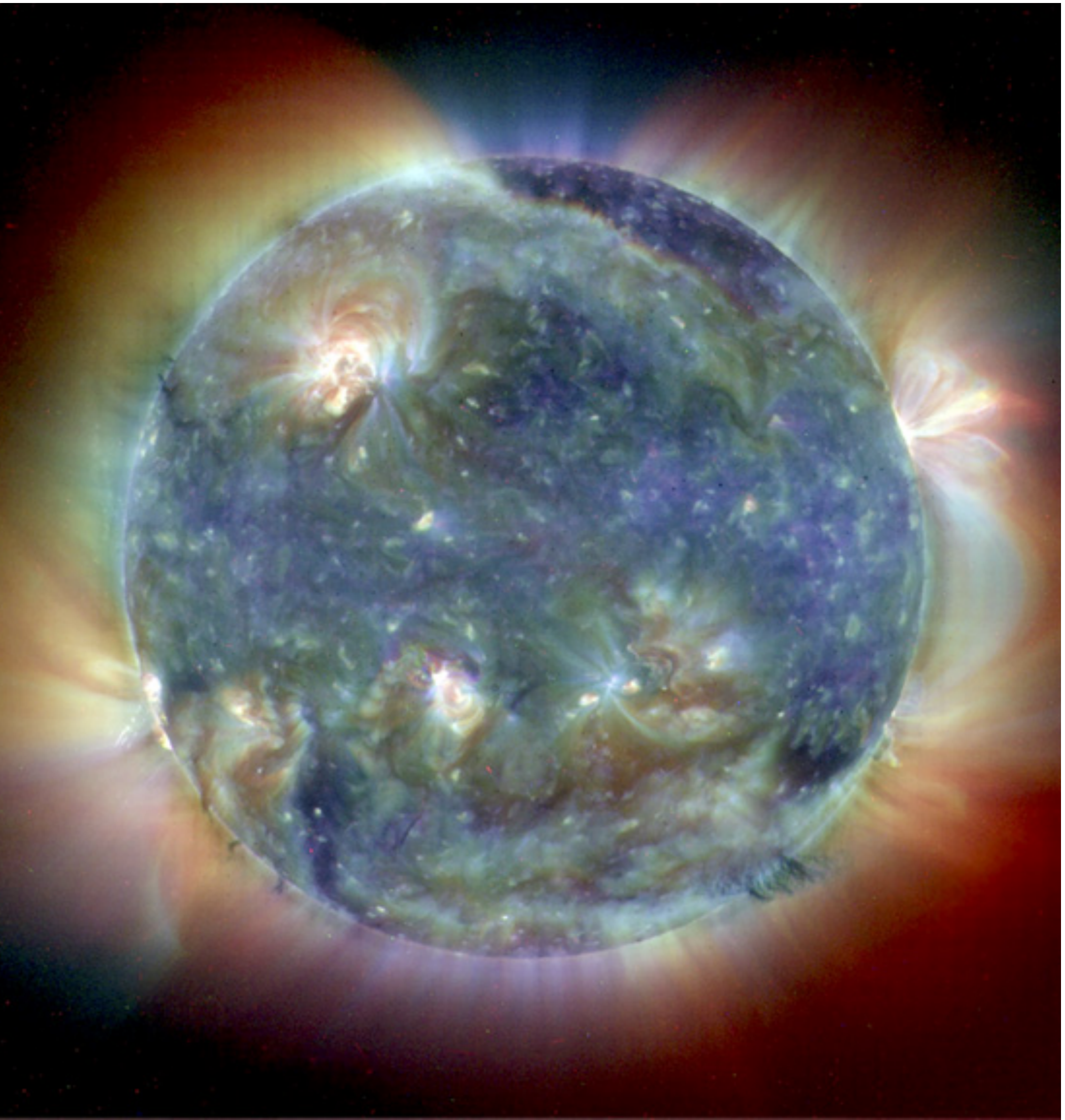


The Sun's magnetic field also produces other forms of solar activity

- A solar flare is a brief eruption of hot, ionized gases from a sunspot group
- A coronal mass ejection is a much larger eruption that involves immense amounts of gas from the corona

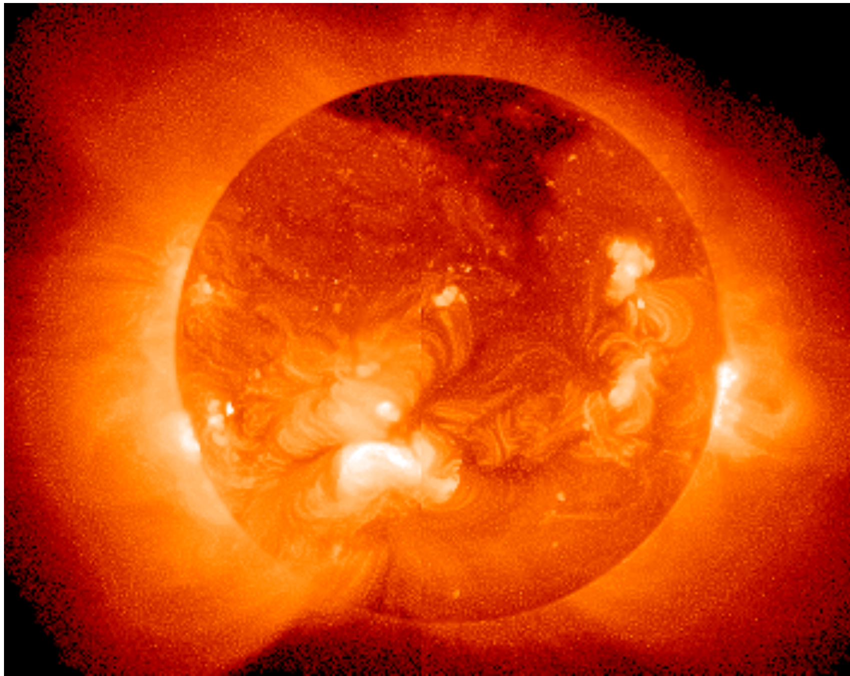


The Sun is Now Being Monitored by a Number of Satellites and Ground-Based Observatories



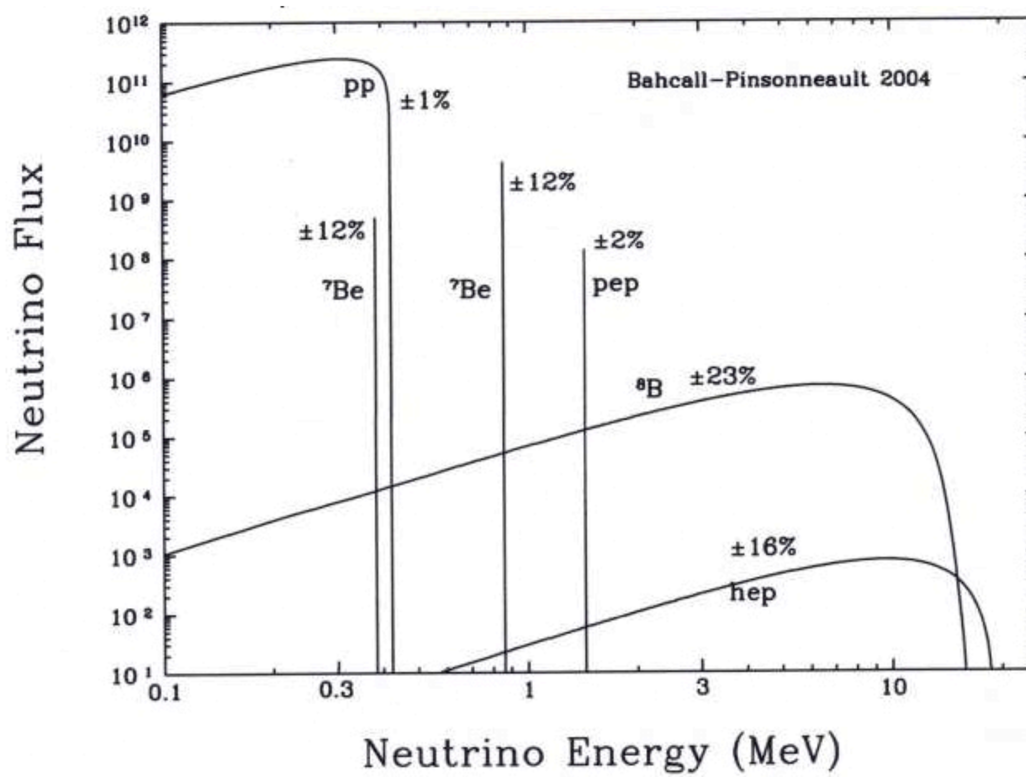
←SOHO far-UV composite

Yohkoh soft X-ray ↓



Solar Neutrinos and the Birth of Neutrino Astronomy

- Detection of Solar neutrinos offers a unique probe of deep stellar interiors - a fundamental test of our understanding of stars and their energy production



- For many years, there was a factor of 3 discrepancy between the theoretical predictions and the experiment (the "Solar neutrino problem"). The resolution of it provided a fundamental physical insight (neutrino oscillations)

Solar Neutrino Flux

Neutrinos from the main p-p chain are of very low energy. Less important reactions (energetically) yield a smaller flux of higher energy neutrinos:

$$p + e^- + p \rightarrow {}^2H + \nu_e$$ `pep' - 1.4 MeV neutrino

$${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$$ 0 - 18.8 MeV

$$e^- + {}^7Be \rightarrow {}^7Li + \nu_e$$ 0.383, 0.861 MeV

$${}^8B \rightarrow {}^8Be + e^+ + \nu_e$$ 0 - 15 MeV

Since we get 2 neutrinos for each 28 MeV of energy, we can use Solar luminosity to calculate neutrino flux at Earth:

$$\text{Neutrino flux} = \frac{2L_{sun}}{28\text{ MeV}} \times \frac{1}{4\pi d^2} \sim 6 \times 10^{10}\text{ neutrinos/s/cm}^2$$

Homestake Mine Detector

First attempt to detect Solar neutrinos began in the 1960s:

Detector is a large tank containing 600 tons of C_2Cl_4, situated at 1500m depth in a mine in South Dakota.

Neutrinos interact with the chlorine to produce a radioactive isotope of argon:

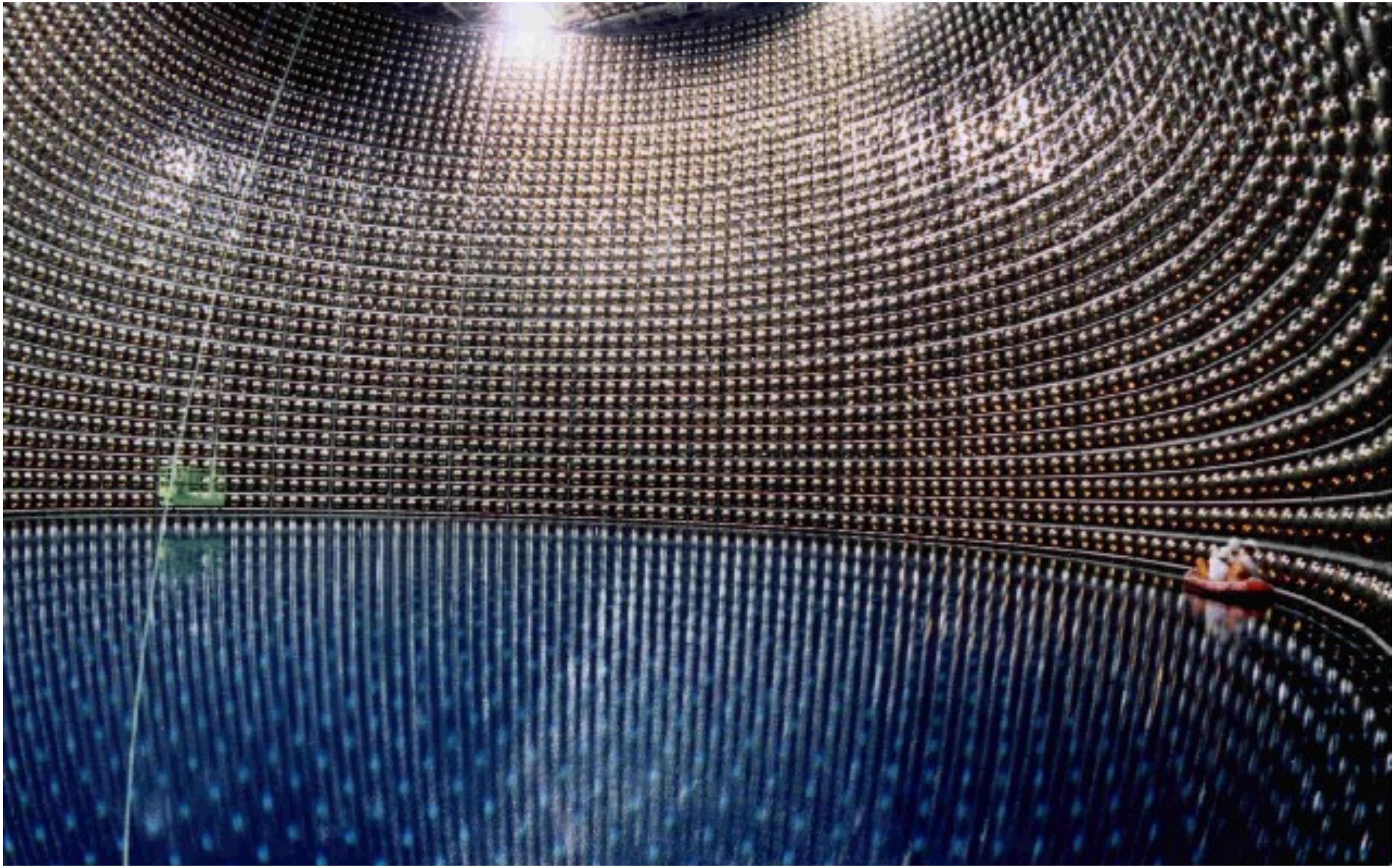
$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$$

+ an electron which is not observed.

Super Kamiokande

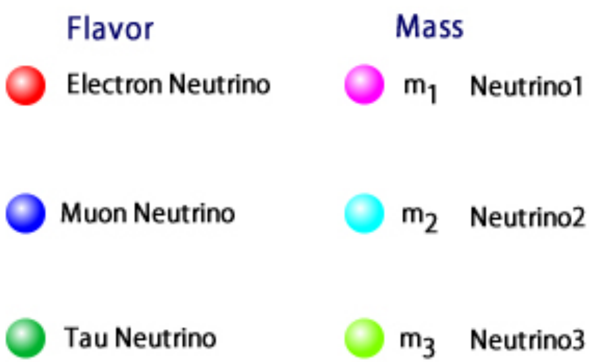
look for a Cherenkov radiation from the high energy electrons in ultra-pure water

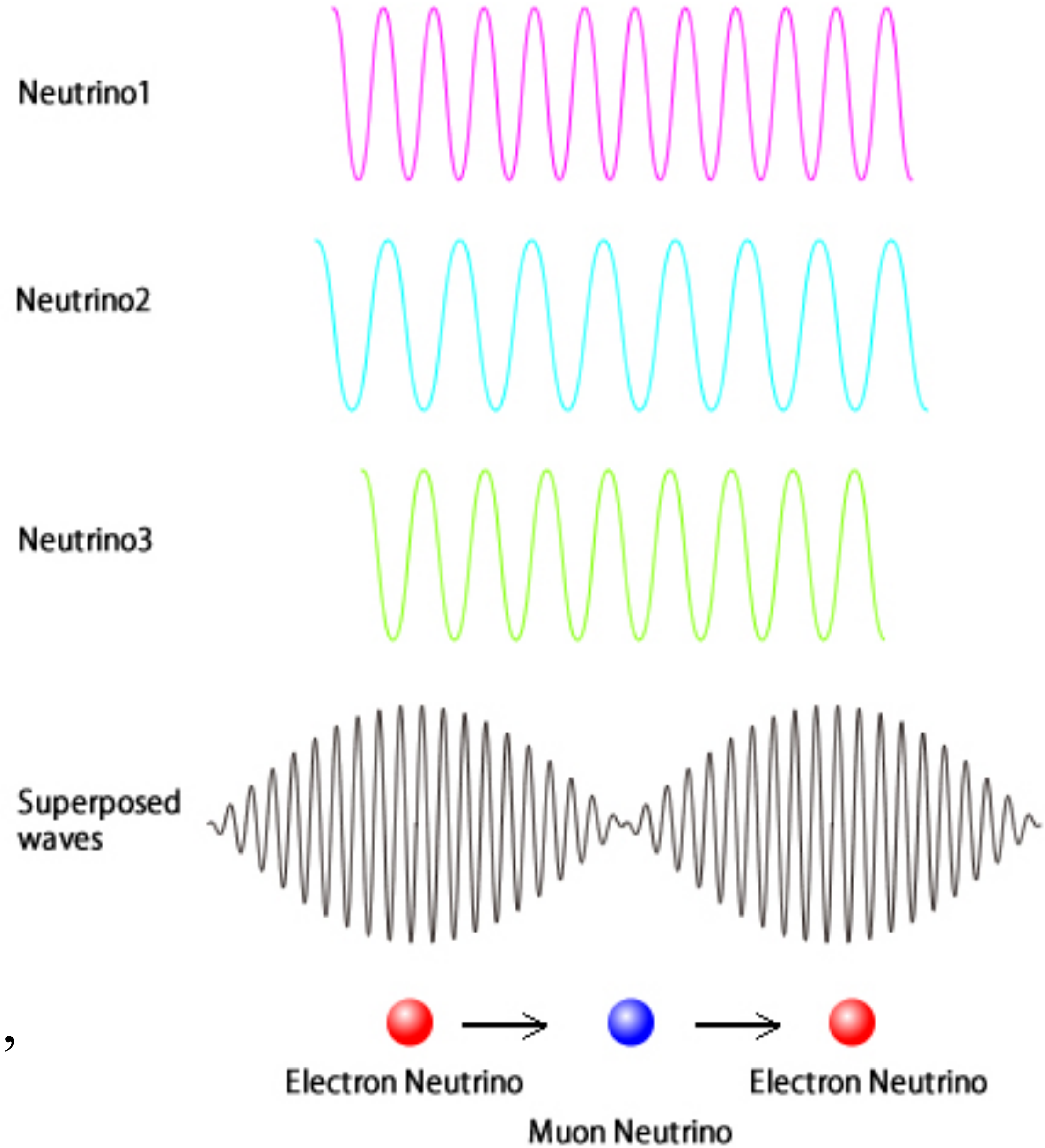
Measure: $\nu_e + e^- \rightarrow \nu_e + e^-$



Neutrino Oscillations

There are 3 flavors of leptons (electron, muon, tau), and the corresponding types of neutrinos:





Neutrinos are a quantum superposition of the 3 types, and oscillate between them