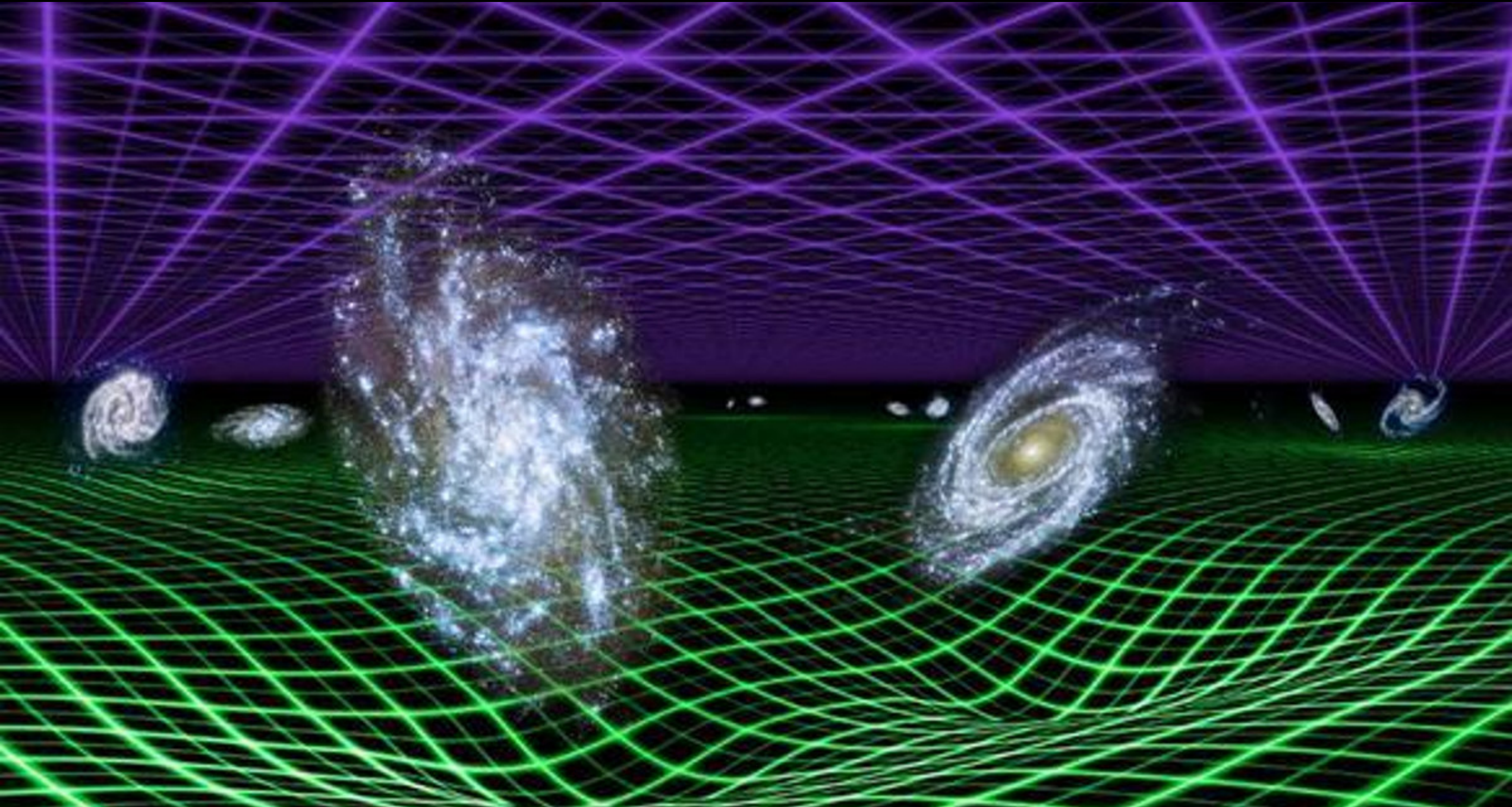
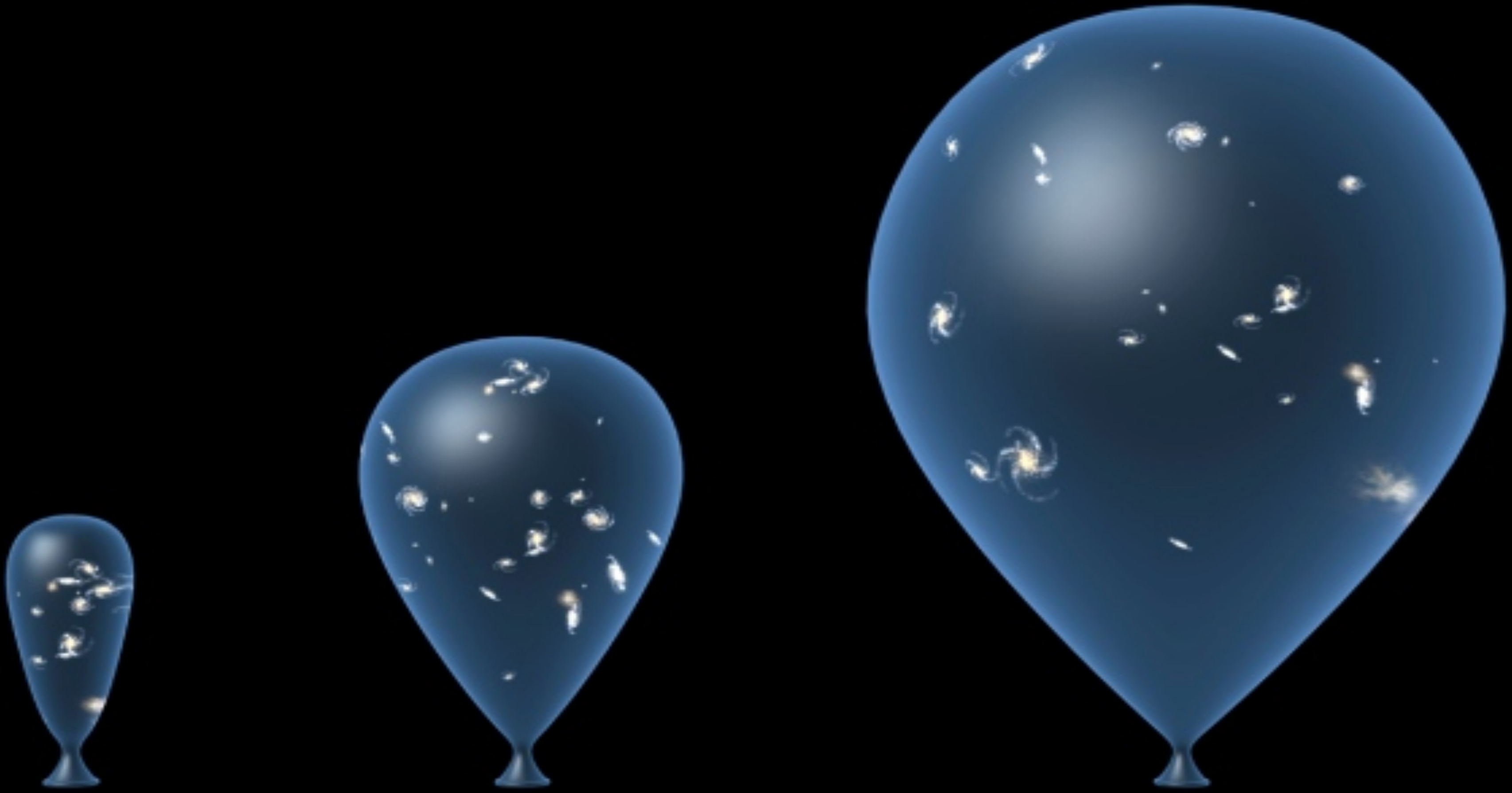
Ay1 – Lecture 17

The Expanding Universe

Introduction to Cosmology



17.1 The Expanding Universe



ANNALEN DER PHYSIK.

BEGRÜNDET UND FORTGEFÜHRT DURCH
F. A. C. GREN, L. W. GILBERT, J. C. POGGENDORFF, G. u. E. WIEDEMANN, P. DRUDE.

VIERTE FOLGE.

BAND 49.

DER GANZEN REIHE 354. BAND.

UNTER MITWIRKUNG
DER DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT

HERAUSGEGEBEN VON
W. WIEN UND M. PLANCK.

MIT EINEM PORTRÄT UND ZEHN FIGURENTAFELN.

LEIPZIG, 1916.
…N AMBROSIUS BARTH.

ANNALEN DER PHYSIK.
VIERTE FOLGE. BAND 49.

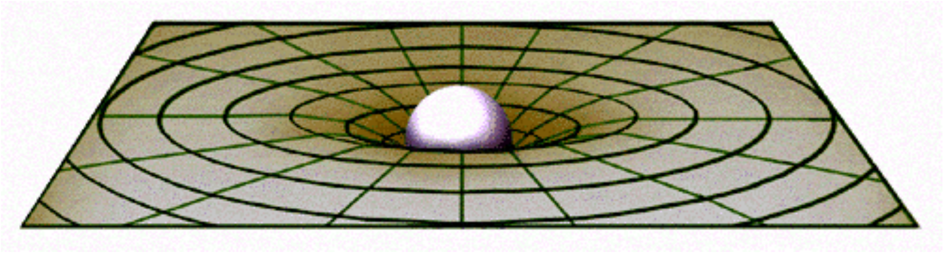
1. *Die Grundlage der allgemeinen Relativitätstheorie;*
von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem „absoluten Differentialkalkül", welcher auf den Forschungen von Gauss, Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, so daß ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichun[gen] der Gravitation unterstützte.

General Relativity (1915)

- A fundamental change in viewing the physical space and time, and matter/energy
- Postulates equivalence among **all** frames of reference (including accelerated ones)
- Introduces curvature of space, predicts a number of new effects:
  - o Light deflection by masses
  - o Gravitational redshift

  … etc.



Presence of mass/energy determines the geometry of space

Geometry of space determines the motion of mass/energy

GR is essentially the modern theory of gravity on large scales.

Since gravity is the only important force on cosmological scales, GR is the theoretical basis of modern cosmology

Discovery of the Expanding Universe

Based on an early work byVesto Melvin Slipher ->

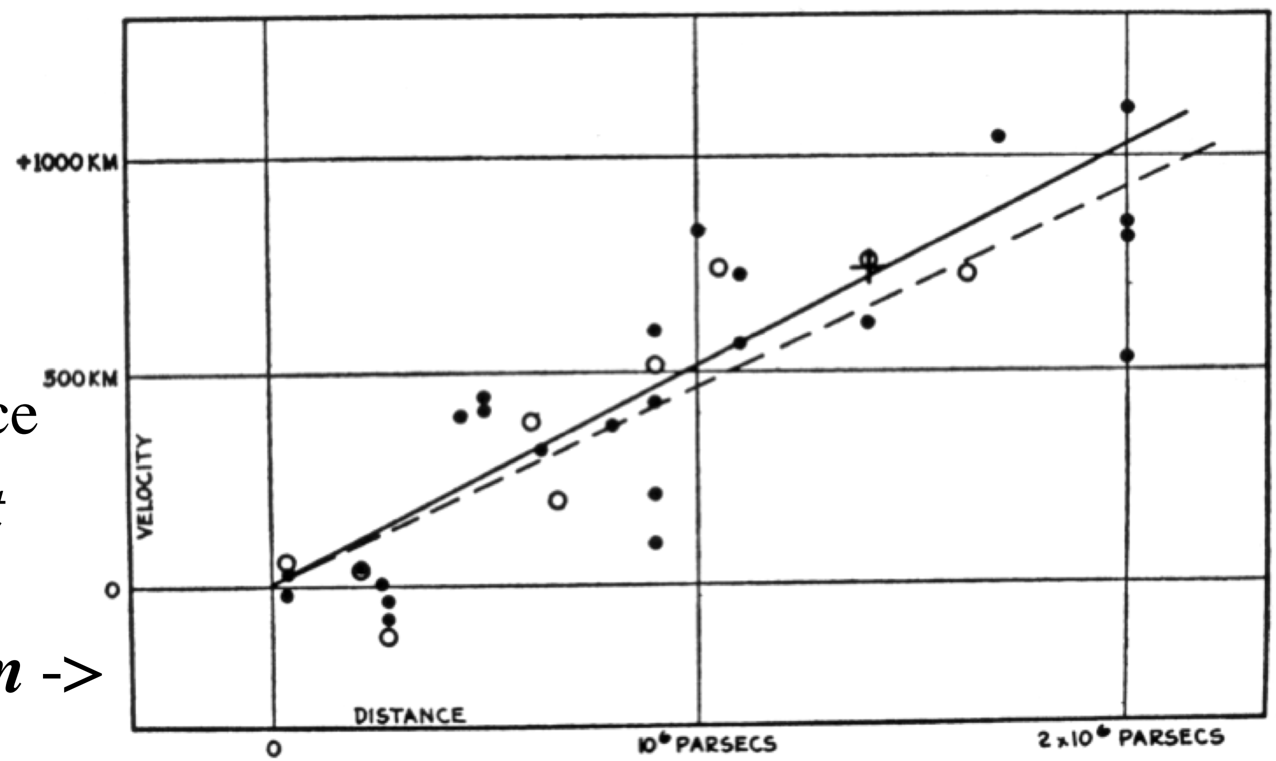
< - Edwin Hubble discovered that galaxies recede from us with a velocity that is proportional to the distance

The *Hubble's law*:

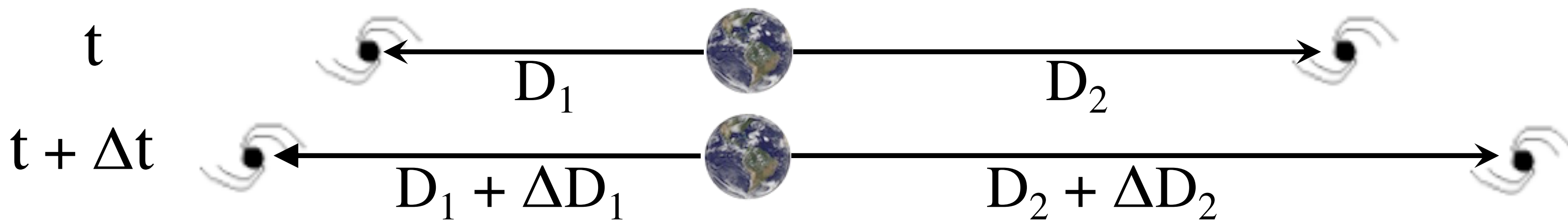
$$V = H\,D$$

Velocity — Distance — ***Hubble's constant***

The ***Hubble diagram*** ->

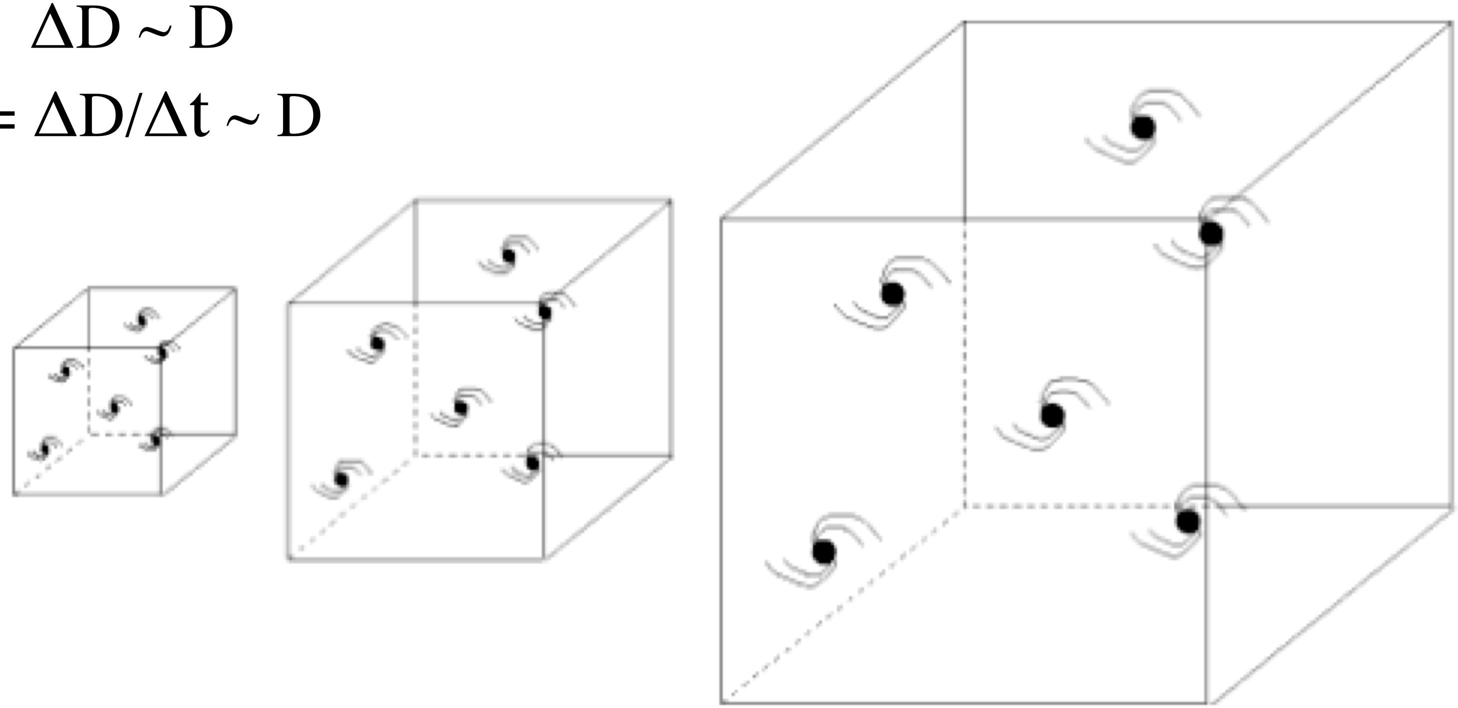


Expansion of the Universe



$\Delta D \sim D$

$V = \Delta D/\Delta t \sim D$



The ***space itself expands***, and carries galaxies apart

In a homogeneous, isotropic universe, there is no preferred center

The Cosmological Principle

Relativistic cosmology uses some symmetry assumptions or principles in order to make the problem of "solving the universe" viable. The **Cosmological Principle** states that

At each epoch, the universe is the same at all locations and in all directions, except for local irregularities

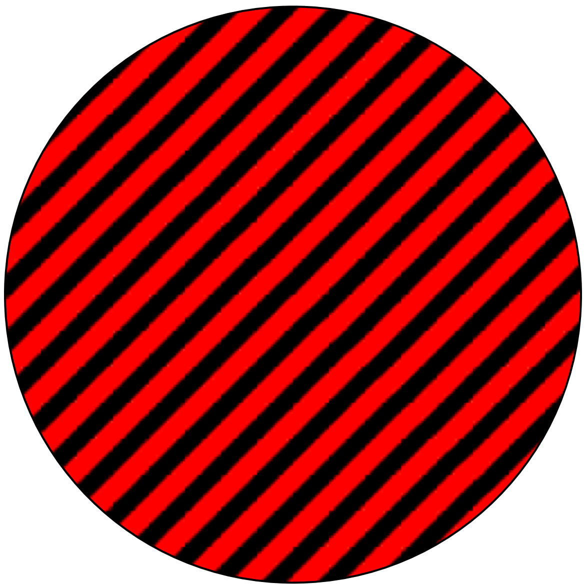
Therefore, globally the Universe is assumed to be **homogeneous** and **isotropic** at any given time; and its dynamics should be the same everywhere

Note: the **Perfect Cosmological Principle** states that the Universe appears the same at all times and it is unchanging - it is also homogeneous in time - this is the basis of the "Steady State" model

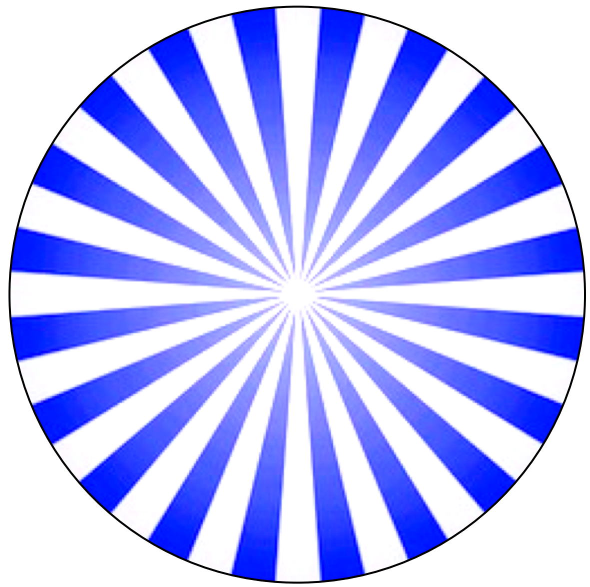
Homogeneity and Isotropy

(in *space*, but not in *time*)

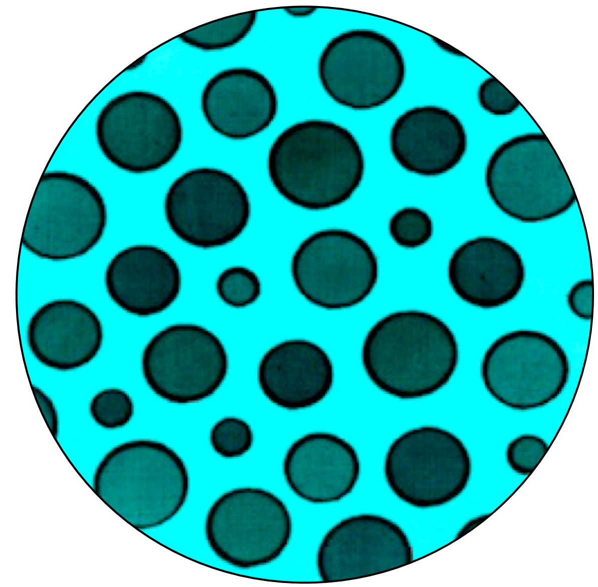
Homogeneous but not Isotropic



Isotropic but not Homogeneous



Isotropic and Homogeneous

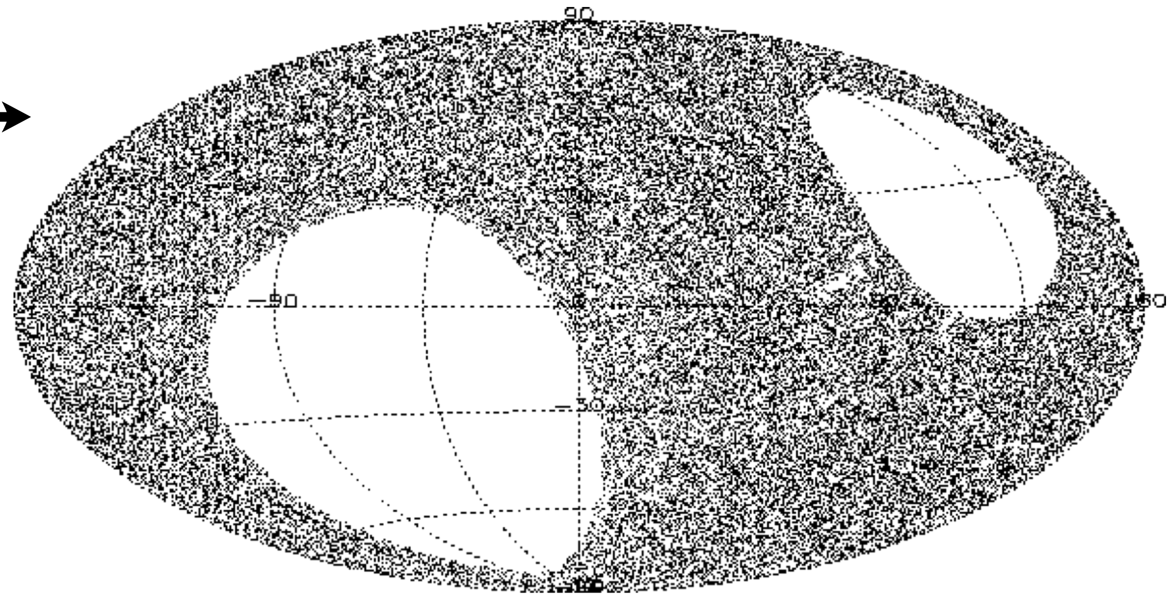


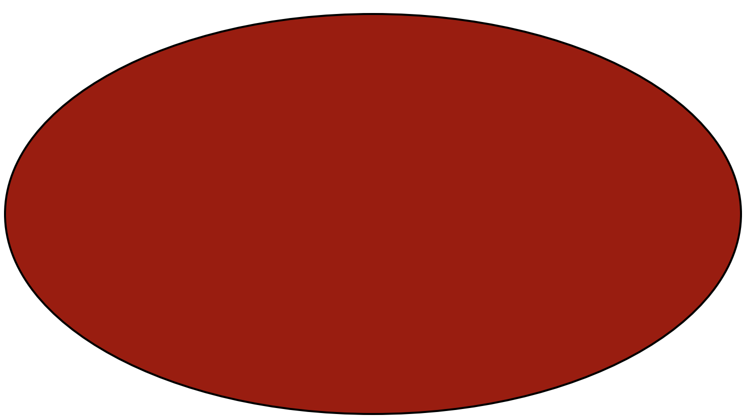
This simplifies the modeling, since only the radial coordinate matters, and the density of any mass/energy component is the same everywhere at a given time

So, is the Universe Really Homogeneous and Isotropic?

Globally, on scales larger than ~ 100 Mpc, say, it is - so the cosmological principle is valid

Distribution on the sky → of 65000 distant radio sources from the Texas survey, a cosmological population

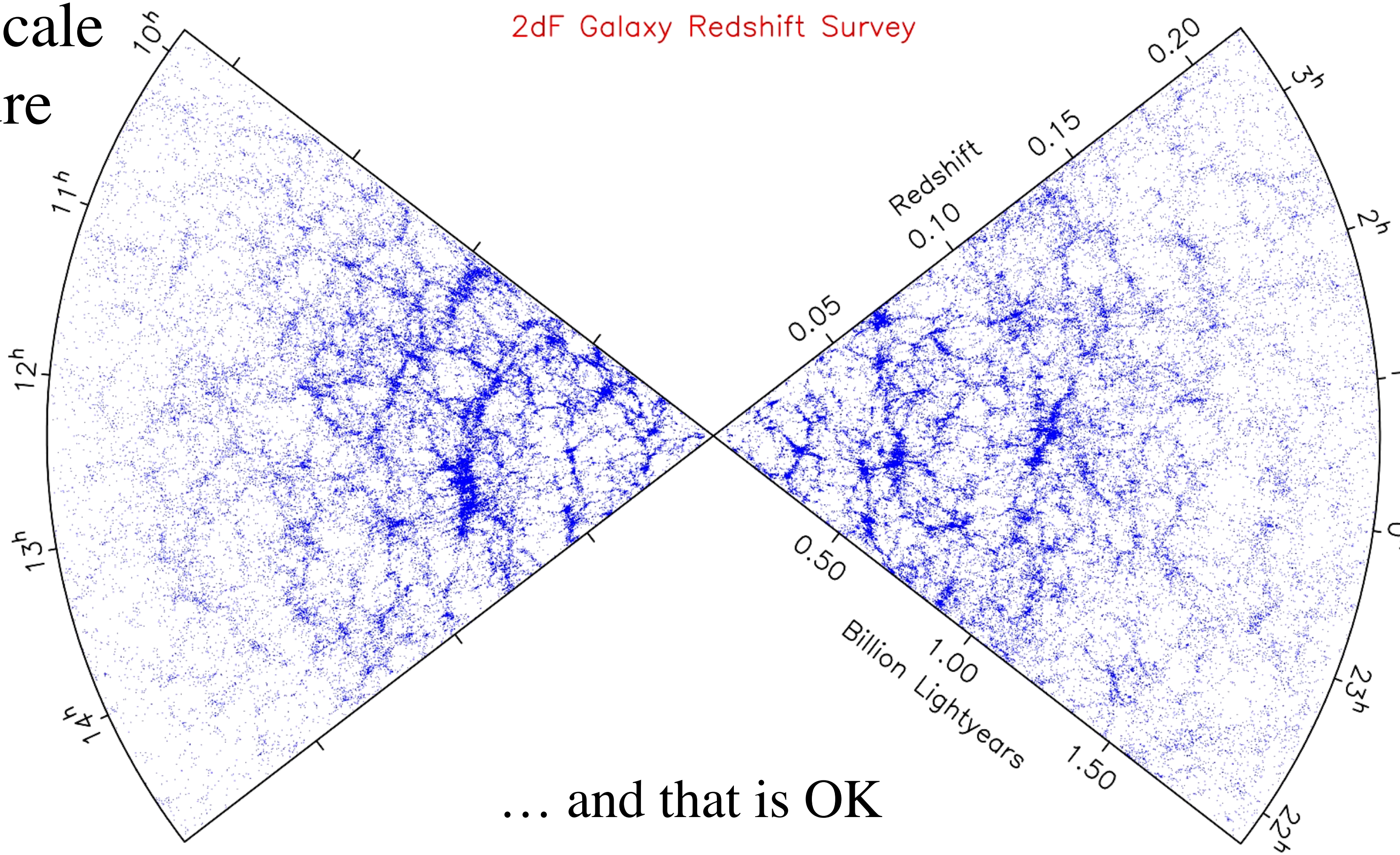




… and of course the CMBR, uniform to better than $\Delta T/T < 10^{-5}$, after taking the dipole out

So, is the Universe Really Homogeneous and Isotropic?

But not so on scales up to ~ 100 Mpc, as shown by the large-scale structure



… and that is OK

Expansion Relative to What? Comoving and Proper Coordinates

There are fundamentally two kinds of coordinates in a GR cosmology:

- ***Comoving coordinates*** = expand with the universe

  Examples:

  – Unbound systems, e.g., any two distant galaxies

  – Wavelengths of massless quanta, e.g., photons

- ***Proper coordinates*** = stay fixed, space expands relative to them. Examples:

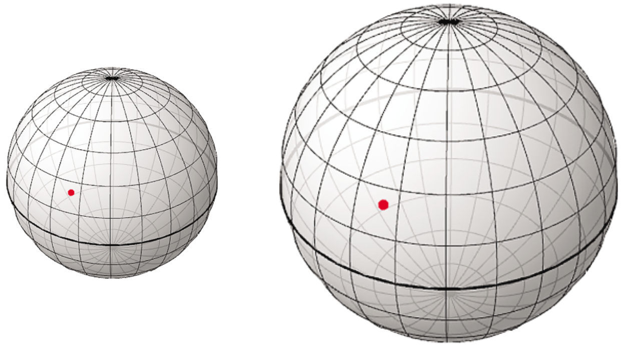
  – Sizes of atoms, molecules, solid bodies

  – Gravitationally bound systems, e.g., Solar system, stars, galaxies …

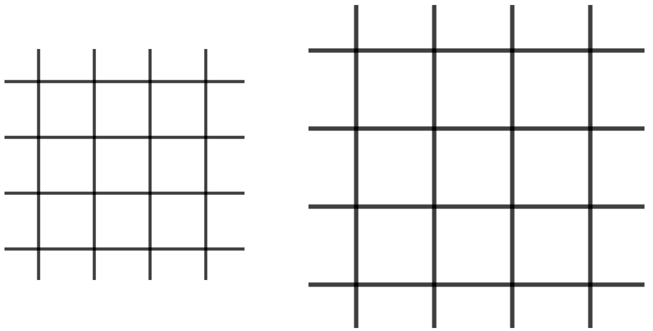
Expansion into What?

Into itself. There is nothing "outside" the universe
(Let's ignore the multiverse hypothesis for now)

A positive curvature universe is like the surface of a sphere, but in one extra dimension. Its volume is finite, but changes with the expansion of space



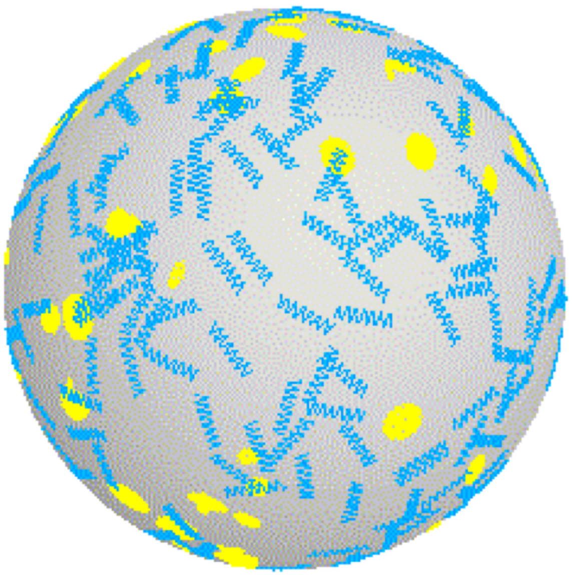
A flat or a negative curvature universe is infinite in all directions; the comoving coordinate grid stretches relative to the proper coordinates

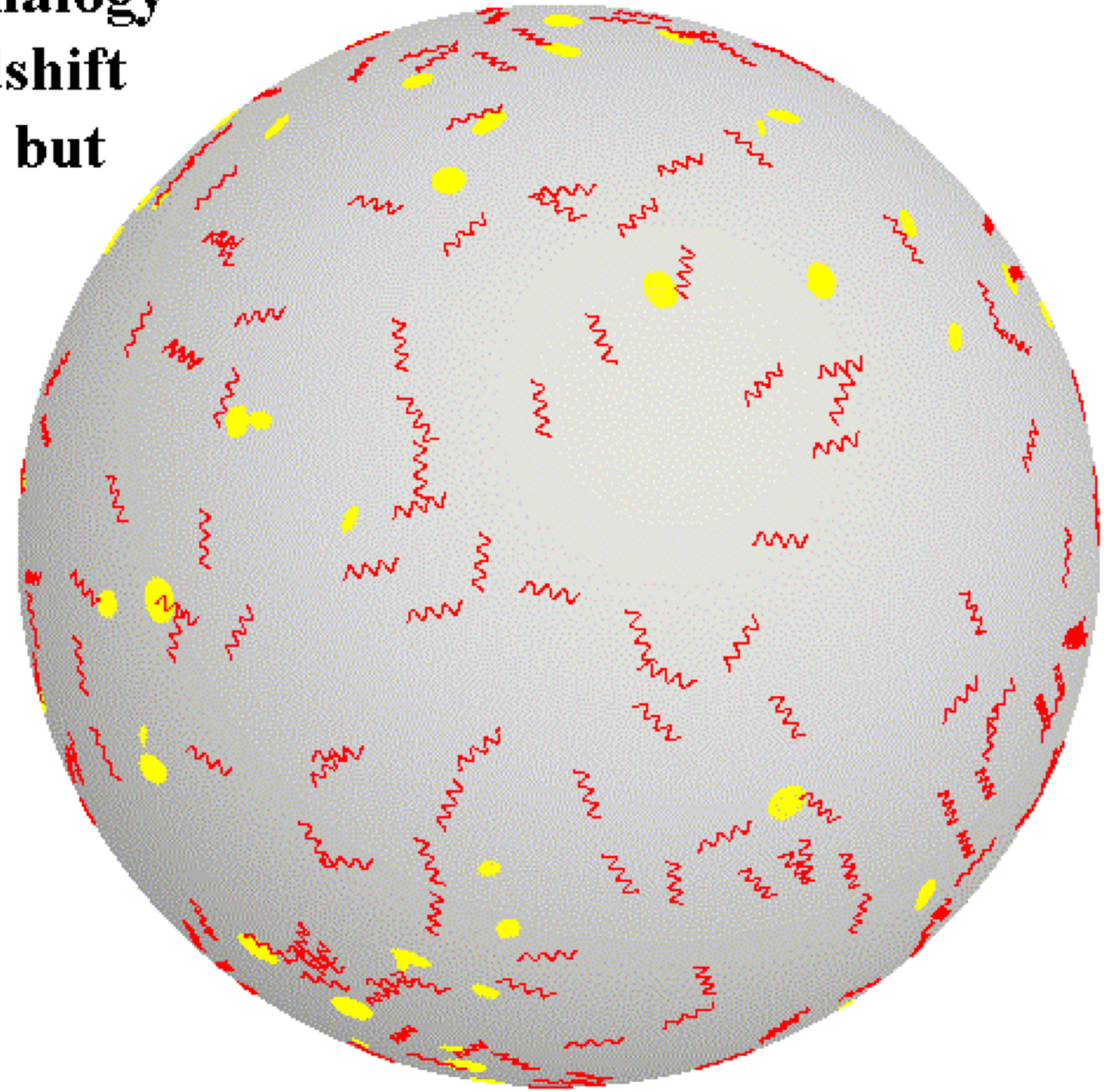


In either case, there is no "edge", and there is no center
(homogeneity and isotropy)

The Cosmological Redshift

Expanding Balloon Analogy
Photons move and redshift
Galaxies spread apart but stay the same size





Redshift as Doppler Shift

We define **doppler redshift** to be the shift in spectral lines due to motion:

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

which, in the case of **v<<c** reduces to the familiar

$$z = \frac{v}{c}$$

The **cosmological redshift** is something different, although we are often sloppy and refer to it in the same terms of the doppler redshift. The cosmological redshift is actually due to the **expansion of space** itself.

Cosmological Redshift

A more correct approach is to note that the wavelengths of photons expand with the universe:

$$\frac{R(t_0)}{R(t_e)} = \frac{\lambda_0}{\lambda_e}$$

Where *R(t)* is a separation between any two comoving points

Or, by our definition of redshift:

$$z = \frac{\Delta\lambda}{\lambda}$$

We get:

$$\boxed{\frac{R(t_0)}{R(t_e)} = (1+z)}$$

Thus, by measuring redshifts, we measure directly how much has the universe expanded since then

The two approaches are actually equivalent

Is Energy Conserved in an Expanding (or Contracting) Universe?

No!

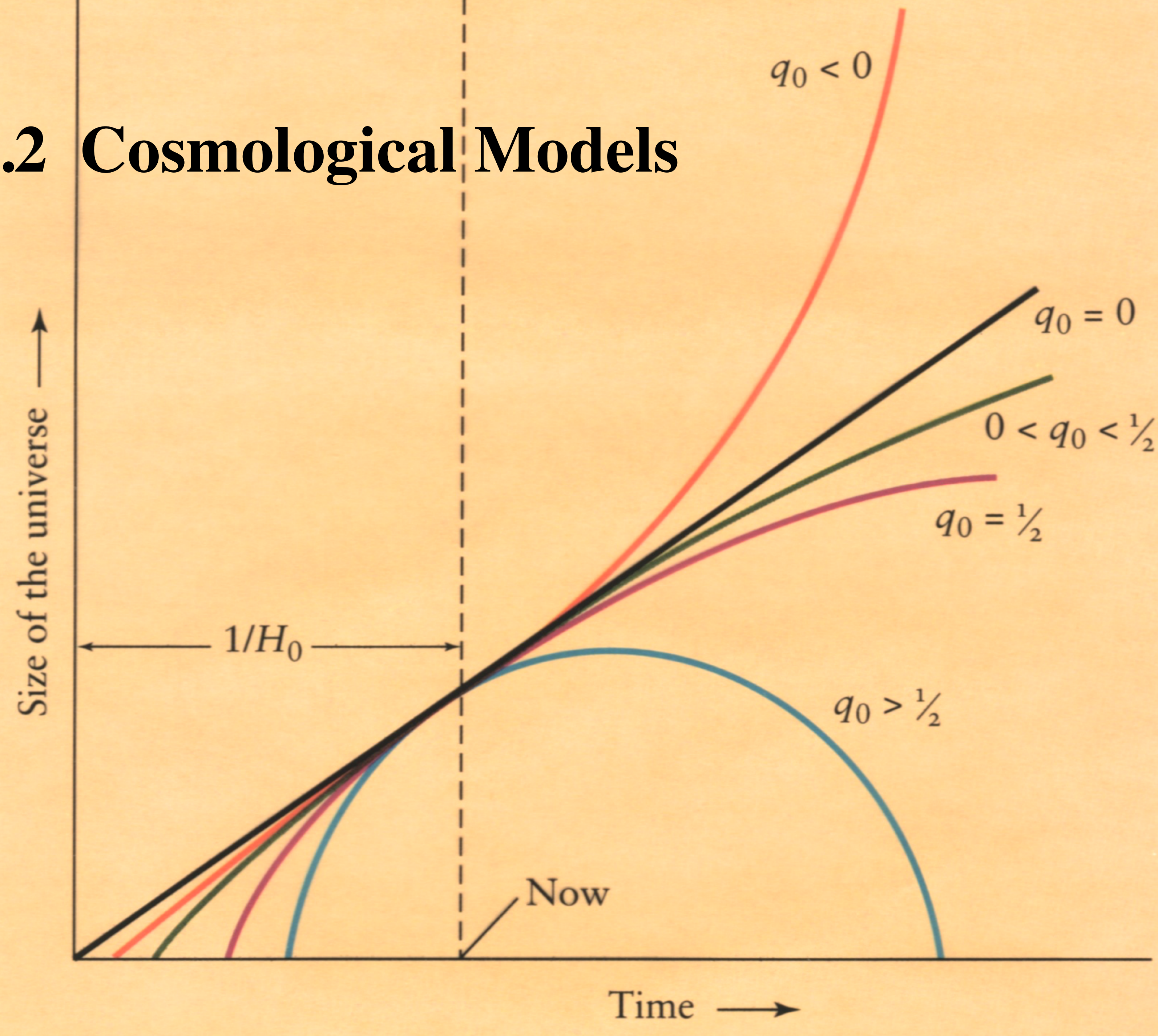
- Consider energies of photons
- Consider potential energies of unbound systems

?

Cosmological Redshift is:

A. The Doppler effect due to the universal expansion
B. A consequence of General Relativity
C. A measure of the expansion factor
D. Due to the curvature of space
E. Due to the non-conservation of energy

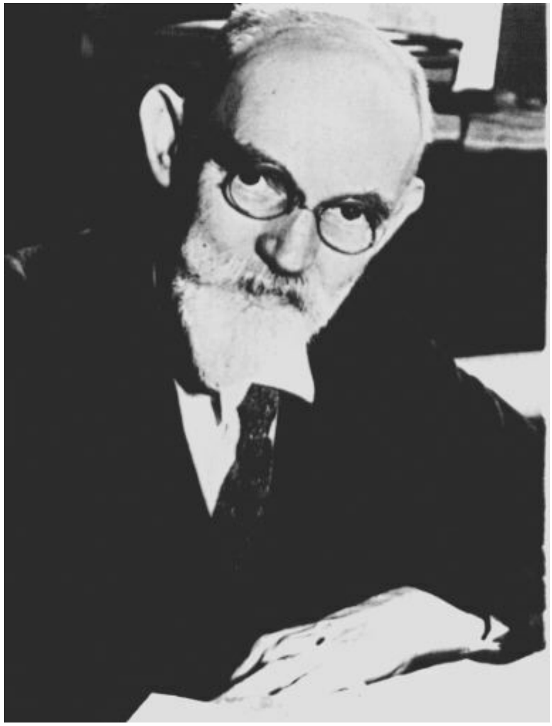
17.2 Cosmological Models



The Early Cosmological Models

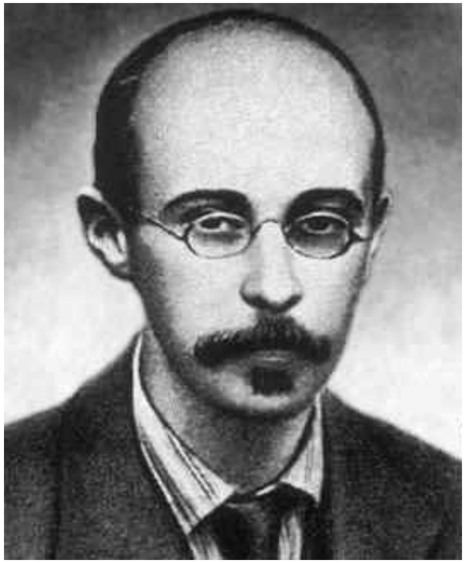
Einstein in 1917 constructed the first relativistic cosmological models. Thinking that the universe is static, he introduced the cosmological constant term to balance the force of gravity. This model was unstable.

Willem De Sitter in 1917 also developed a similar model, but also obtained solutions of Einstein equations for a nearly empty, ***expanding*** universe.



In 1932, Einstein & De Sitter jointly developed another, simple cosmological model which bears their names.

The Friedmann and Lemaitre Models



⇦ **Alexander Friedmann**

In 1922 developed the GR-based, expanding universe model. It was not taken very seriously at the time, since the expansion of the universe has not yet been established.

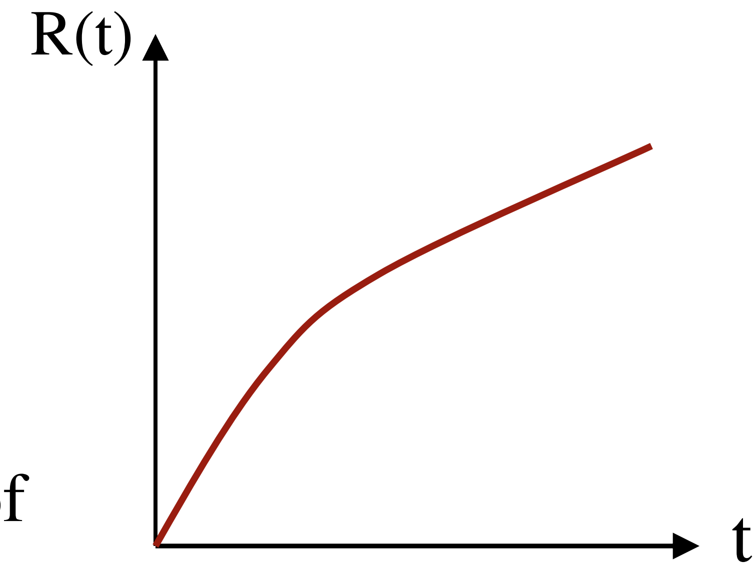
Georges Lemaitre ⇨

In 1927 independently developed cosmological models like Friedmann's. In 1933, he "ran the film backwards" to a hot, dense, early state of the universe he called "the cosmic egg". This early prediction of the Big Bang was largely ignored.

They used the homogeneity and isotropy to reduce the full set of 16 Einstein equations of GR to one: the Friedmann-Lemaitre eqn.

Kinematics of the Universe

We introduce a **scale factor**, commonly denoted as **R(t)** or **a(t):** a spatial distance between any two unaccelerated frames which move with their comoving coordinates



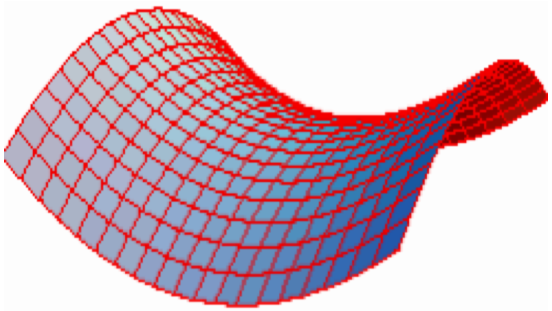
This fully describes the evolution of a homogeneous, isotropic universe

Computing **R(t)** and various derived quantities defines the **cosmological models**. This is accomplished by solving the **Friedmann (or Friedmann-Lemaitre) Equation**

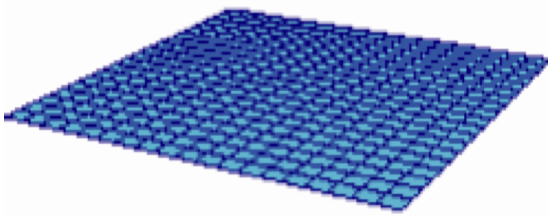
The equation is parametrized (and thus the models defined) by a set of **cosmological parameters**

Geometry and the Fate of the Universe

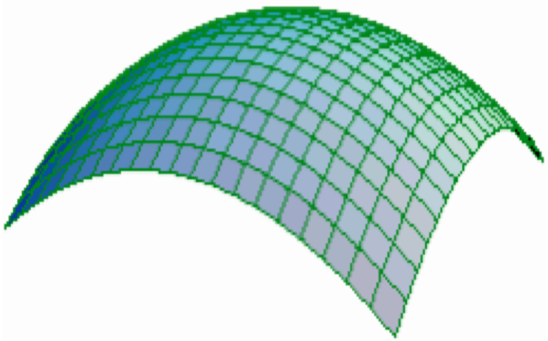
Matter and energy content of the universe determines its geometry (curvature of space), and the ultimate fate



$\rho < \rho_{crit}$, $k = -1$
negative curvature
expands for ever

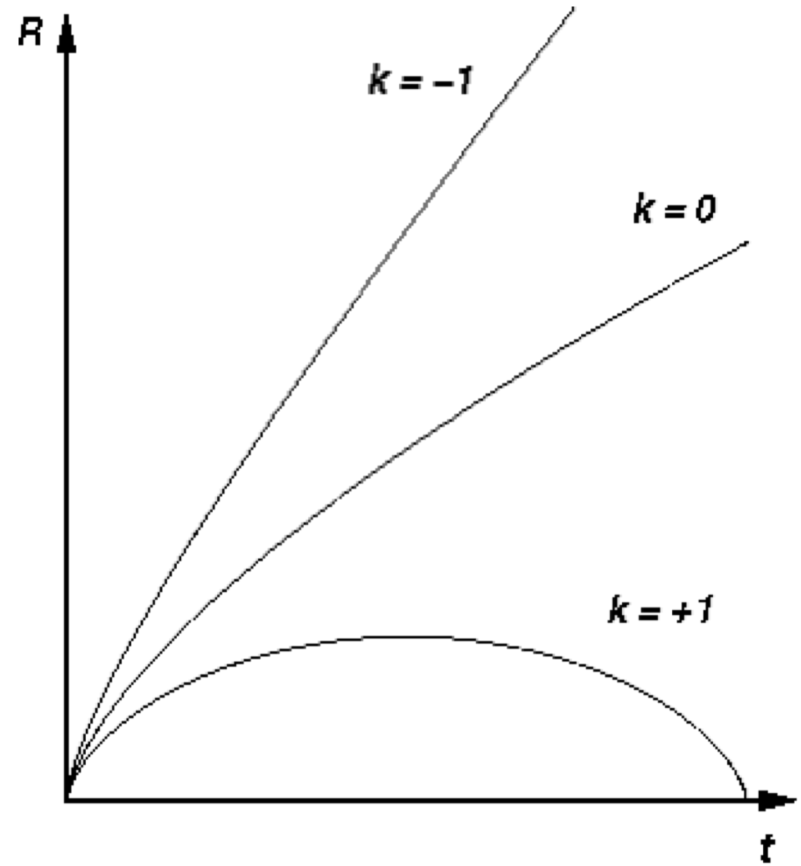


$\rho = \rho_{crit}$, $k = 0$
flat (Euclidean)
expands for ever



$\rho > \rho_{crit}$, $k = +1$
positive curvature
collapses

Possible expansion histories:



Cosmological Parameters

Cosmological models are typically defined through several handy key parameters:

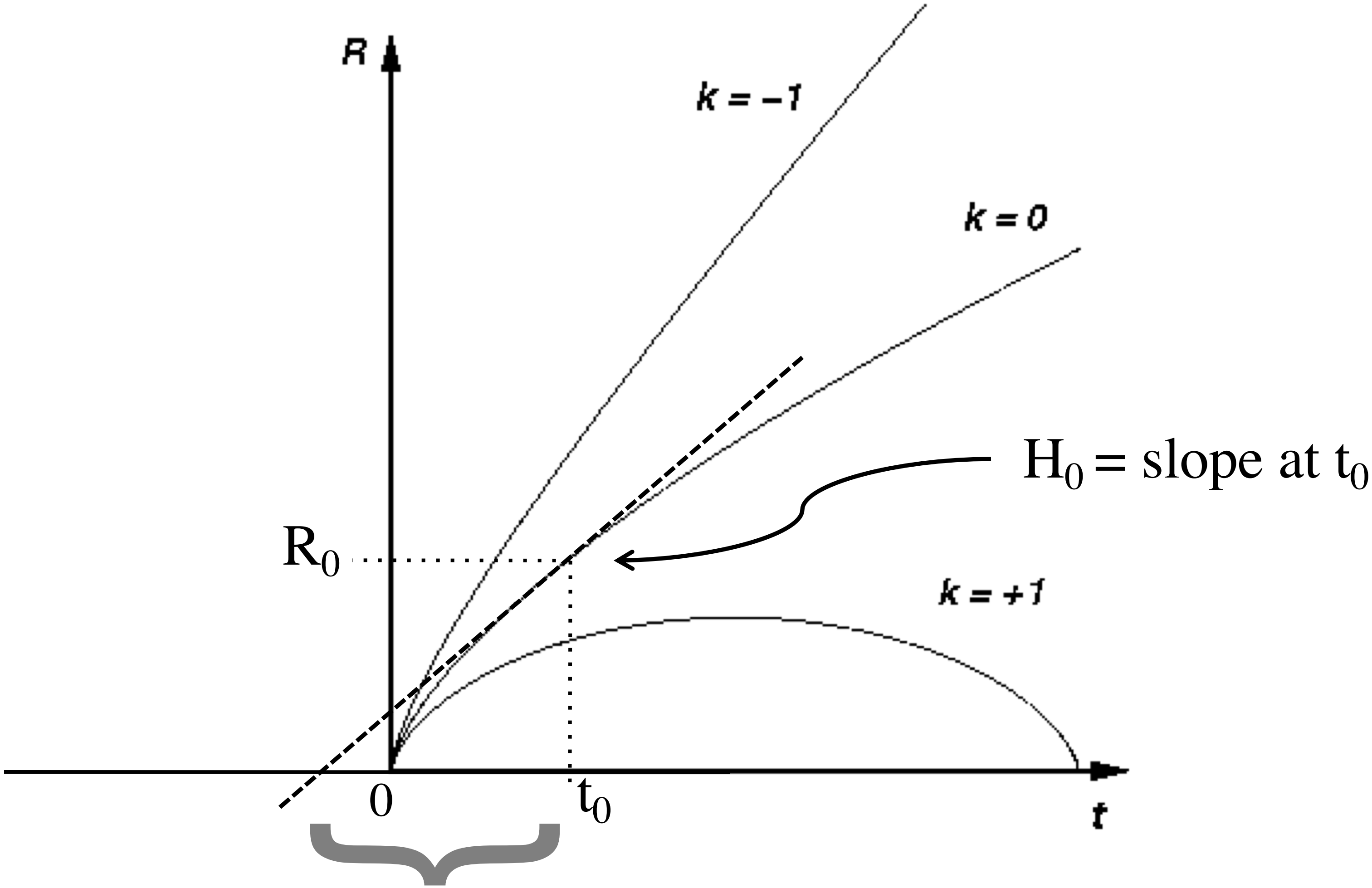
1. The Hubble Parameter

The **Hubble parameter** is the normalized rate of expansion:

$$H \equiv \frac{\dot{R}}{R}$$

Note that the Hubble parameter is not a constant! The **Hubble constant** is the Hubble parameter measured today -- we denote its value by H_0. Current estimates are in the range of H_0 = 65-75 km/s/Mpc -- we will discuss these efforts in more detail later.

Hubble Constant Defines the Scale of the Universe



1 / H_0 = Hubble time

c / H_0 = Hubble length

Cosmological Parameters

2. The Matter Density Parameter.

Rewriting the Friedmann Eqn. using the Hubble parameter, and for now set $\Lambda = 0$:

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R}$$

The Universe is flat if k=0, or if it has a critical density of

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the **matter density parameter** as

$$\Omega_M = \frac{\rho}{\rho_{crit}}$$

Cosmological Parameters

3. The "dark energy" density parameter

We can express a similar density parameter for lambda again by using the Friedmann equation and setting $\rho = 0$. We then get

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}$$

The total density parameter is then

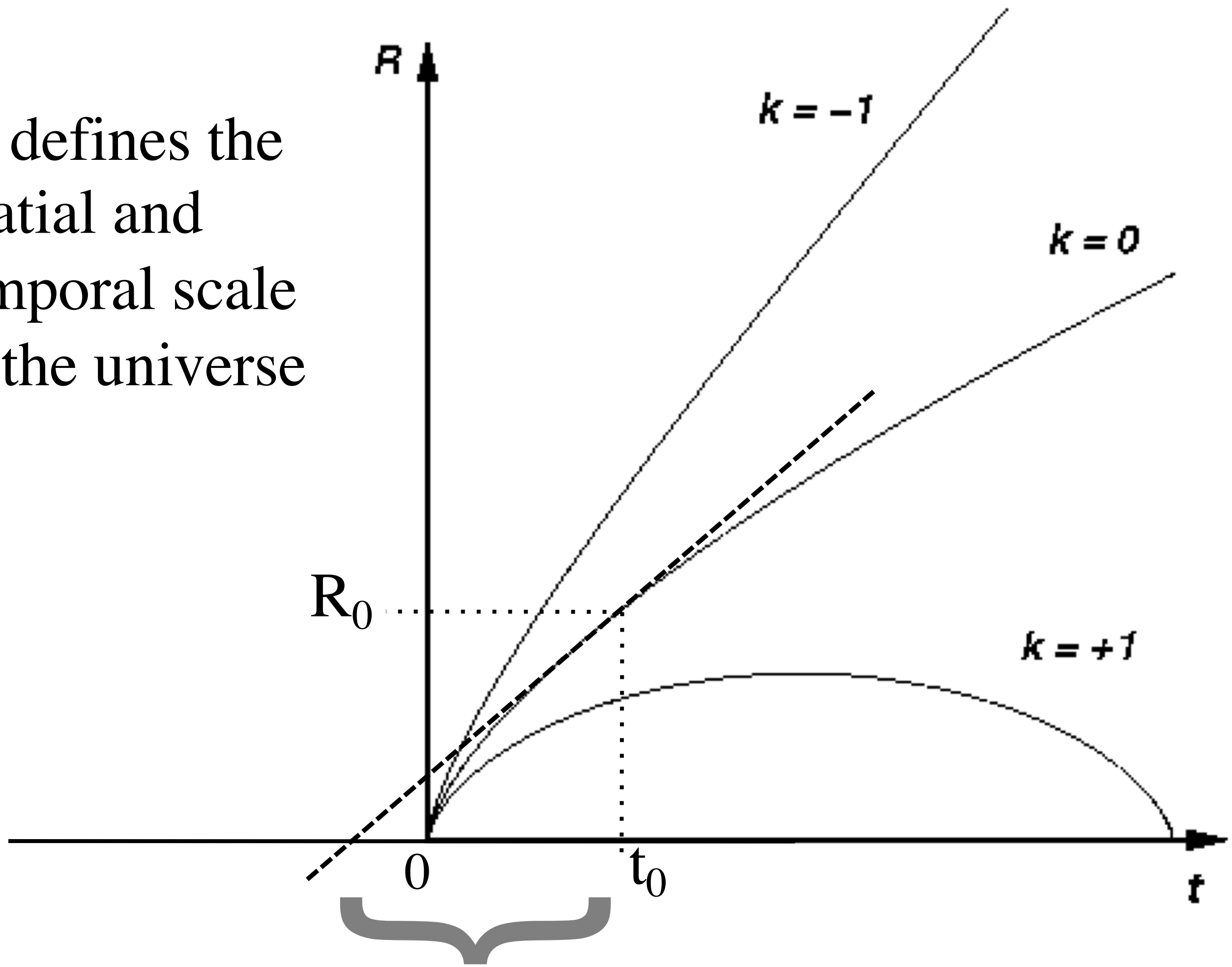
$$\Omega = \Omega_M + \Omega_L$$

4. The deceleration parameter

$$q = -R\ddot{R}/\dot{R} = \frac{\Omega_M}{2} - \Omega_L$$

Cosmological Parameters

H_0 defines the spatial and temporal scale of the universe



The other parameters (Ω_x) determine the shape of the R(t) curves

$1 / H_0$ = Hubble time

c / H_0 = Hubble length

Cosmological Parameters

A few notes:

The Hubble parameter is usually called the Hubble constant (even though it changes in time!) and it is often written as: **h** = H_0 / (100 km s^{-1} Mpc^{-1}), or $\mathbf{h_{70}}$ = H_0 / (70 km s^{-1} Mpc^{-1})

The current physical value of the critical density is

$$\rho_{0,crit} = 0.921 \times 10^{-29}\, h_{70}^2 \ \text{g cm}^{-3}$$

The density parameter(s) can be written as:

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1$$

where Ω_k is a fictitious "curvature density"

Evolution of the Density

Densities of various matter/energy components change with the stretching of the volume (~ R^3) according to their equation of state (EOS):

$$\rho \sim R^{-3(w+1)}$$

where w is the EOS parameter (need not be constant):

- **Matter** dominated ($w = 0$): $\rho \sim R^{-3}$
- **Radiation** dominated ($w = 1/3$): $\rho \sim R^{-4}$
- **Cosmological constant** ($w = -1$): $\rho = constant$
- Dark energy with $w < -1$ e.g., $w = -2$: $\rho \sim R^{+3}$
  - Energy density *increases* as is stretched out!
  - Eventually would dominate over even the energies holding atoms together! ("Big Rip")

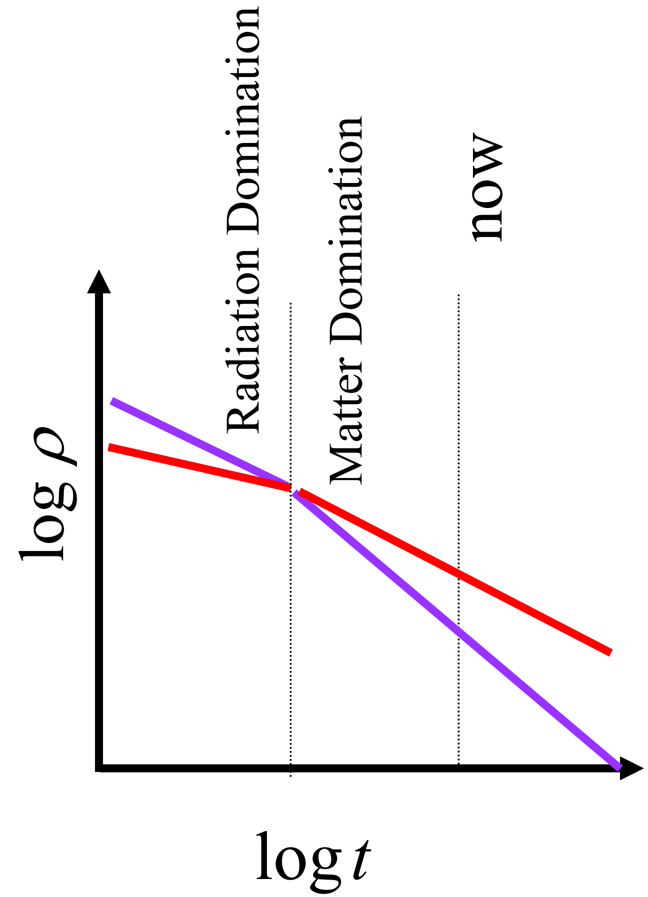
In a mixed universe, different components will dominate the global dynamics at different times

Models With Both Matter & Radiation

Harder to solve for $\rho\ (t)$

However, to a good approximation, we can assume that $k = 0$ and either radiation or matter dominate

	γ-dom	m-dom
$a(t)$	$\propto t^{1/2}$	$\propto t^{2/3}$
$\rho_{\mathrm{m}} \propto a^{-3}$	$\propto t^{-3/2}$	$\propto t^{-2}$
$\rho_{\gamma} \propto a^{-4}$	$\propto t^{-2}$	$\propto t^{-8/3}$



Generally, $$\frac{8\pi G\rho}{3} = H_0^{\ 2}\left(\Omega_{\Lambda,0} + \Omega_{\mathrm{m},0}a^{-3} + \Omega_{\gamma,0}a^{-4}\right)$$

What is Dominant When?

Matter dominated ($w = 0$): $\rho \sim R^{-3}$

Radiation dominated ($w = 1/3$): $\rho \sim R^{-4}$

Dark energy ($w \sim -1$): $\rho \sim constant$

- Radiation density decreases the fastest with time
  - Must increase fastest on going back in time
  - Radiation must dominate early in the Universe
- Dark energy with $w \sim -1$ dominates last; it is the dominant component now, and in the (infinite?) future

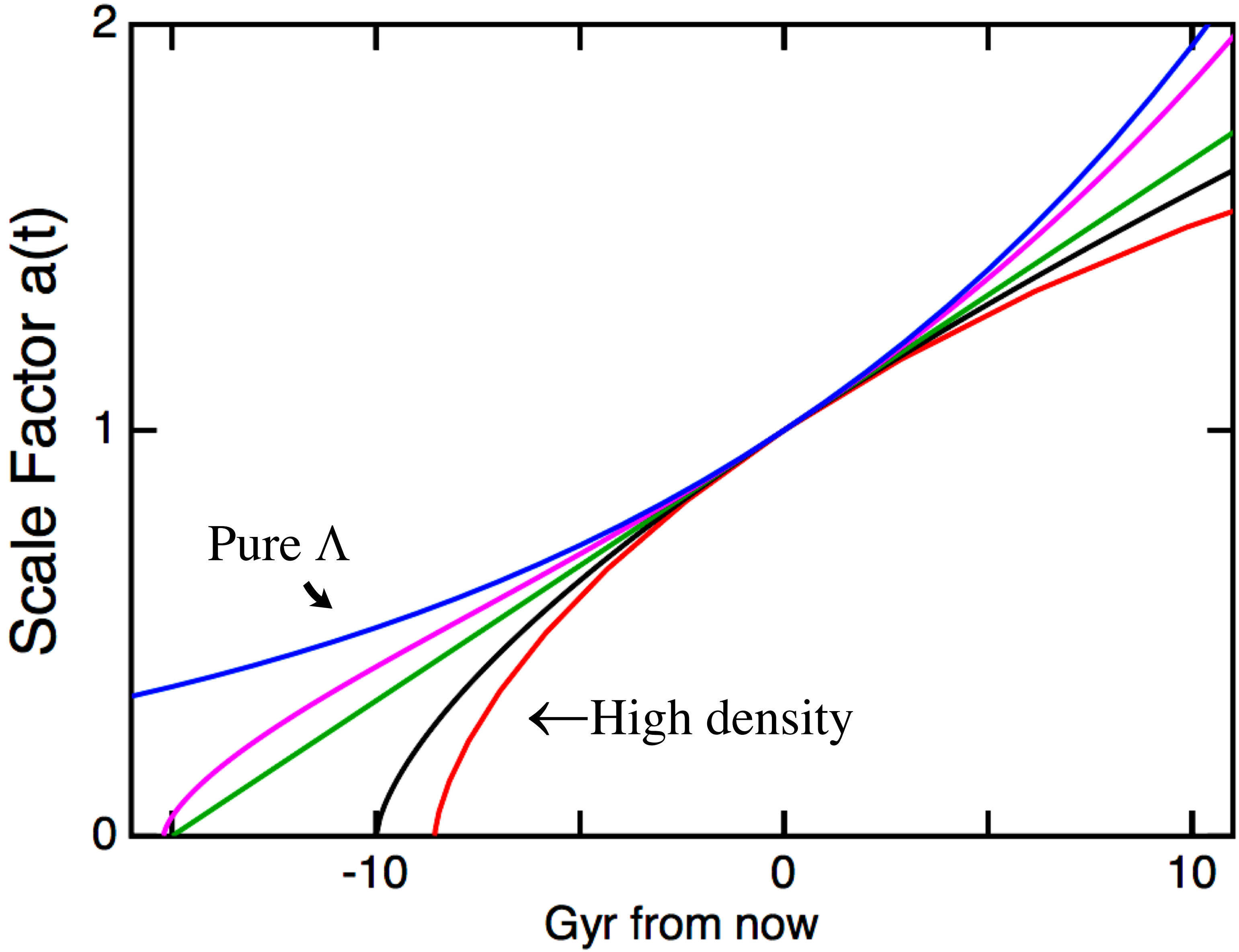


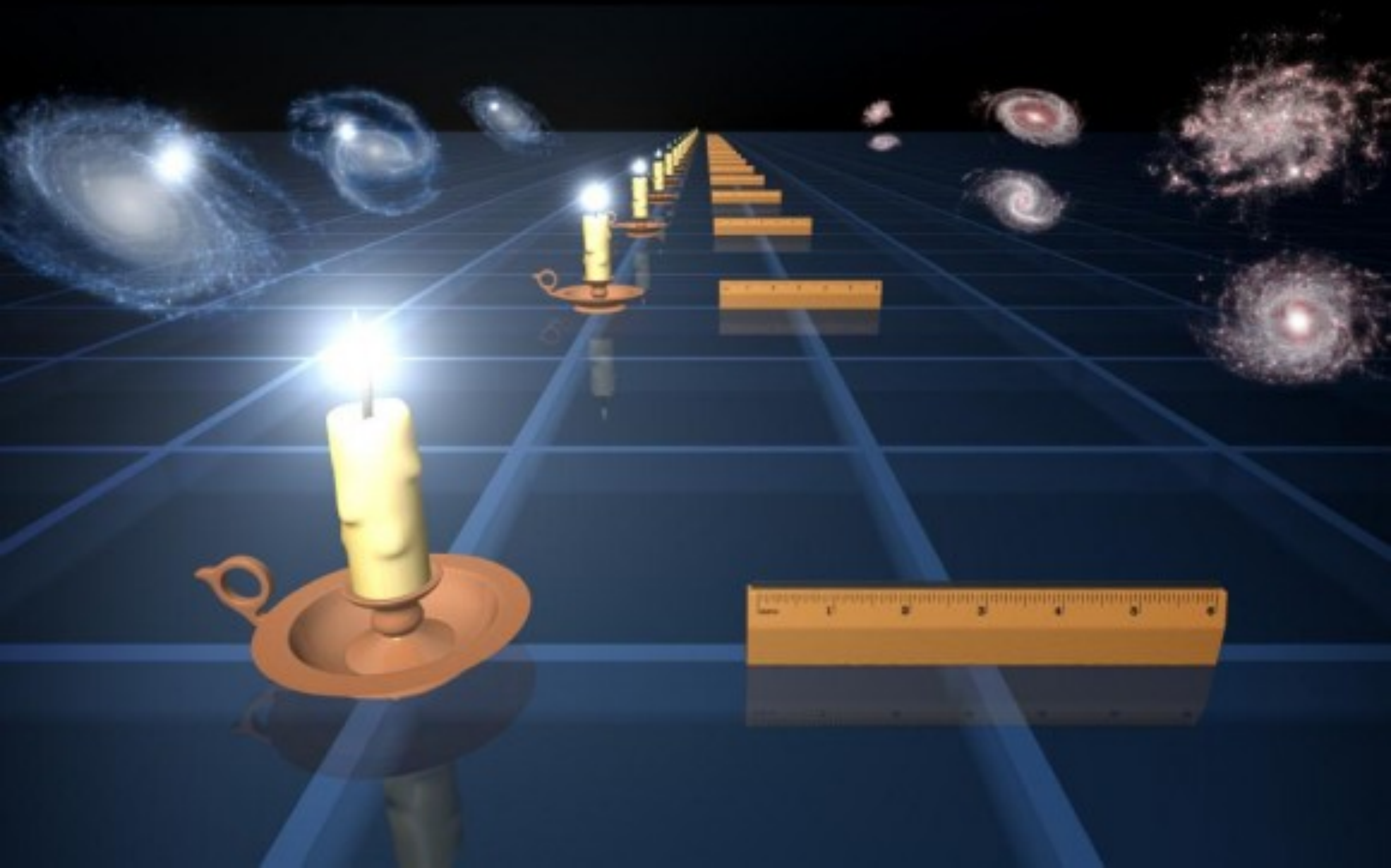
Fig. 10.— Scale factor *vs.* time for 5 different models: from top to bottom having $(\Omega_{m\circ}, \Omega_{v\circ}) = (0, 1)$ in blue, (0.25,0.75) in magenta, (0,0) in green, (1,0) in black and (2,0) in red. All have $H_\circ = 65$.

?

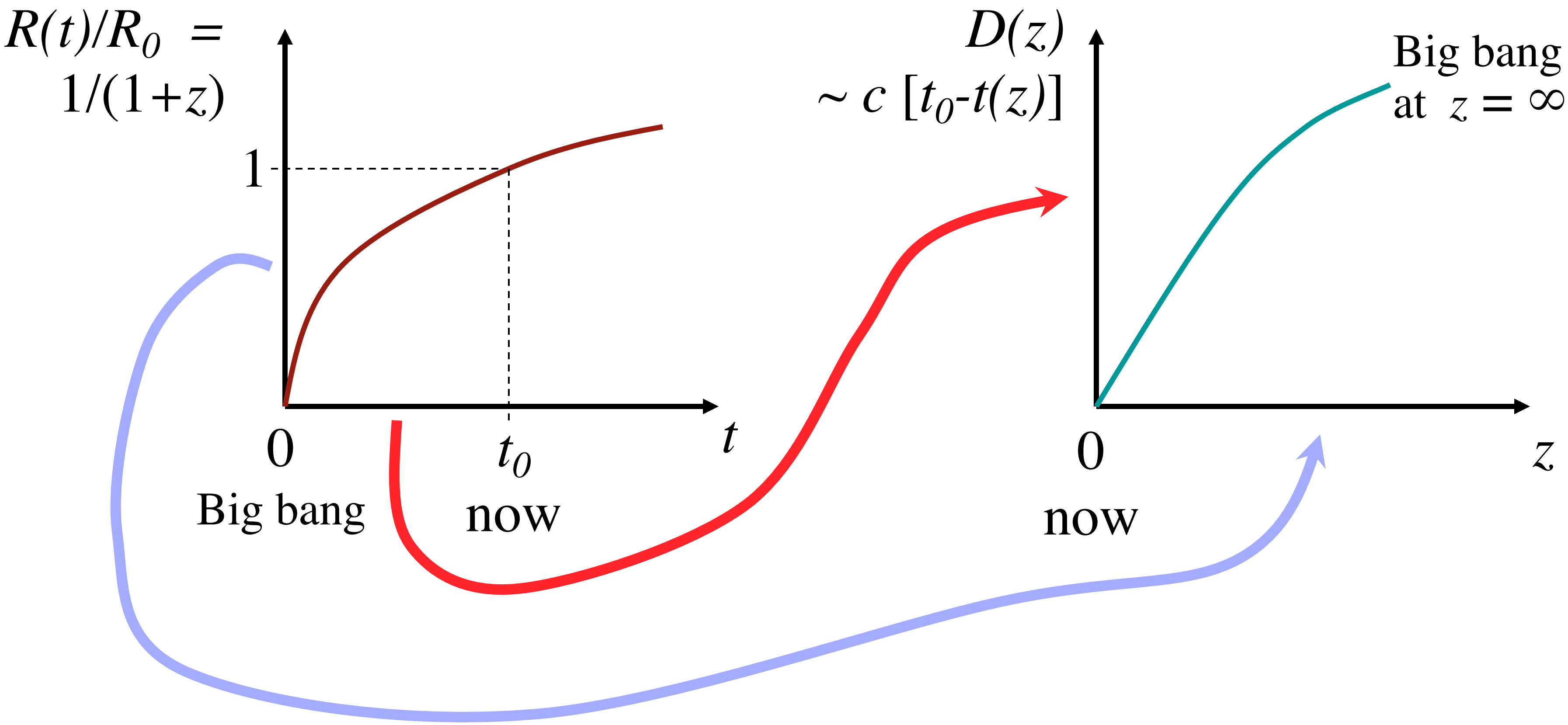
The Dark Energy dominates the expansion rate:

A. In the early universe
B. In the distant future
C. Only if the matter density is less than critical
D. Only if the total density is equal to critical
E. At some point regardless of the matter density

17.3 Distances in Cosmology



The Basis of Cosmological Tests



All cosmological tests essentially consist of comparing some measure of (relative) distance (or look-back time) to redshift. Absolute distance scaling is given by the H_0.

Distances in Cosmology

A convenient unit is the **Hubble distance**,

$$D_H = c\,/\,H_0 = 4.283\ h_{70}^{-1}\ \text{Gpc} = 1.322\times10^{28}\ h_{70}^{-1}\ \text{cm}$$

and the corresponding **Hubble time**,

$$t_H = 1\,/\,H_0 = 13.98\ h_{70}^{-1}\ \text{Gyr} = 4.409\times10^{17}\ h_{70}^{-1}\ \text{s}$$

At low z' s, distance $D \approx z\, D_H$. But more generally, the comoving distance to a redshift z is:

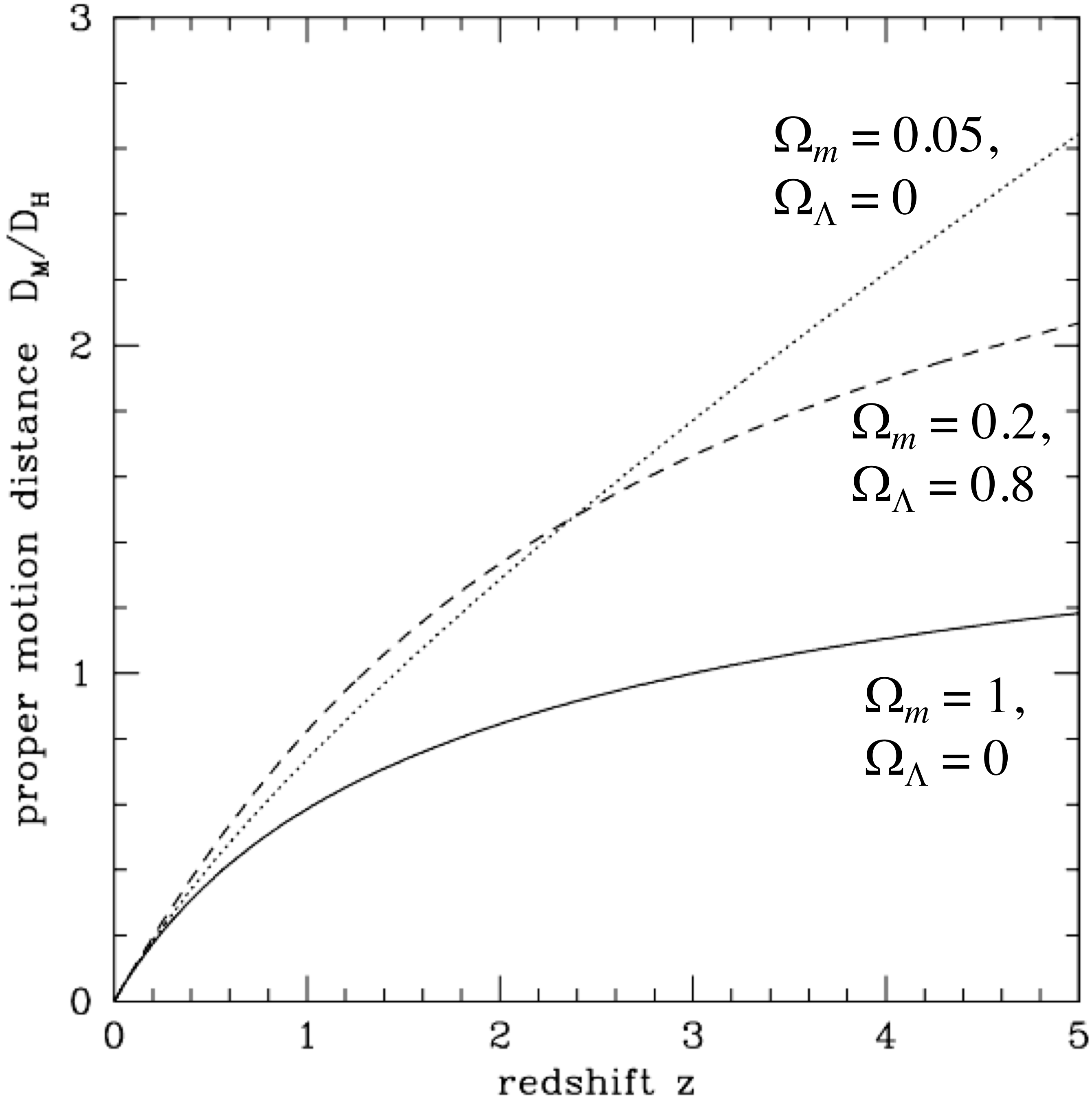
$$D_{\rm C} = D_{\rm H} \int_0^z \frac{dz'}{E(z')}$$

where

$$E(z) \equiv \sqrt{\Omega_{\rm M}\,(1+z)^3 + \Omega_k\,(1+z)^2 + \Omega_\Lambda}$$

Comoving Distance

Derived by solving the Friedmann-Lematre eqn. for a particular choice of cosmological parameters



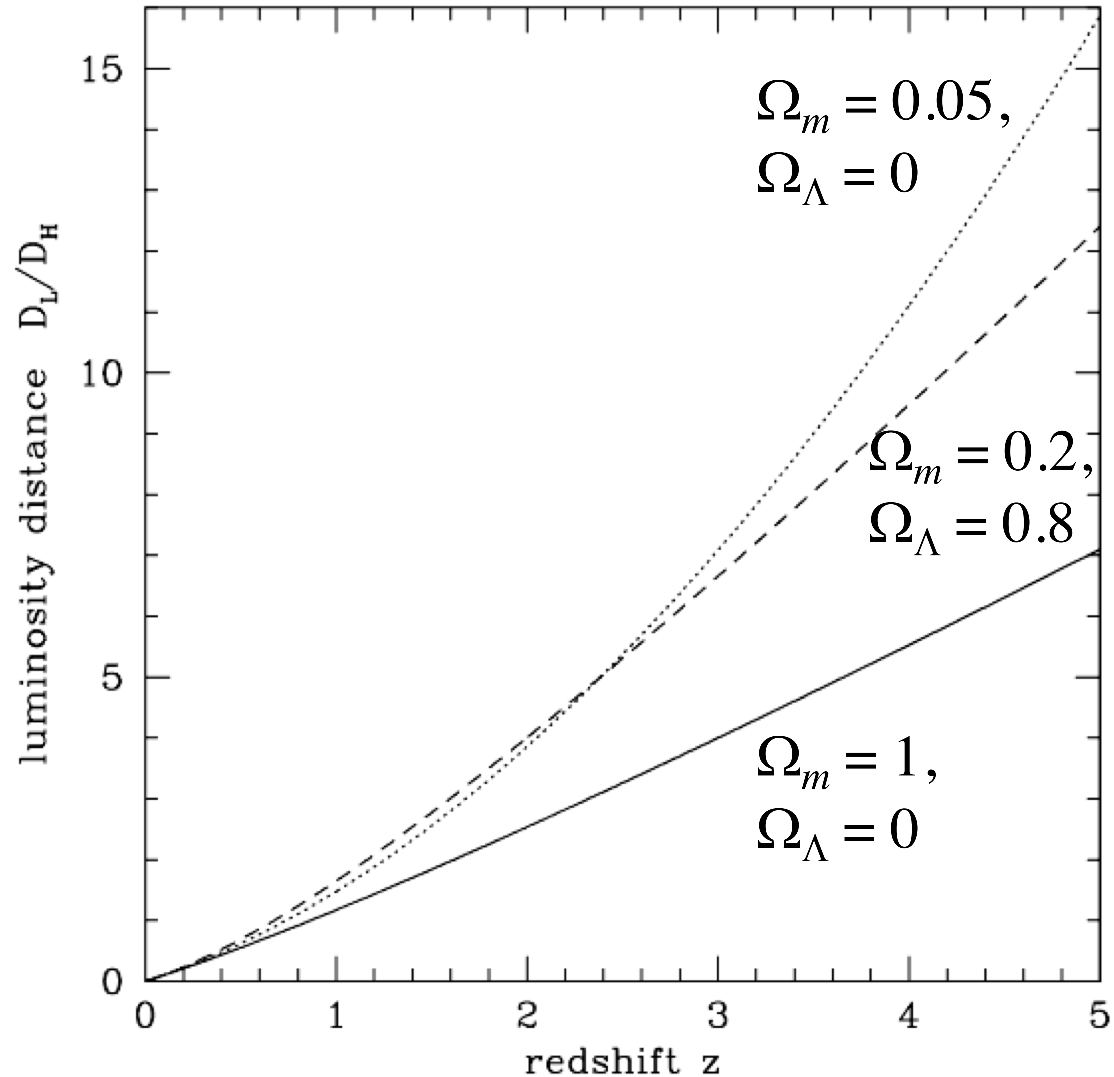
Luminosity Distance

In relativistic cosmologies, observed flux (bolometric, or in a finite bandpass) is:

$$f = L \,/\, [\,(4\pi\, D^2)\,(1+z)^2\,]$$

One factor of $(1+z)$ is due to the energy loss of photons, and one is due to the time dilation of the photon rate.

A luminosity distance is defined as $D_L = D\,(1+z)$, so that $f = L \,/\, (4\pi\, D_L^2)$

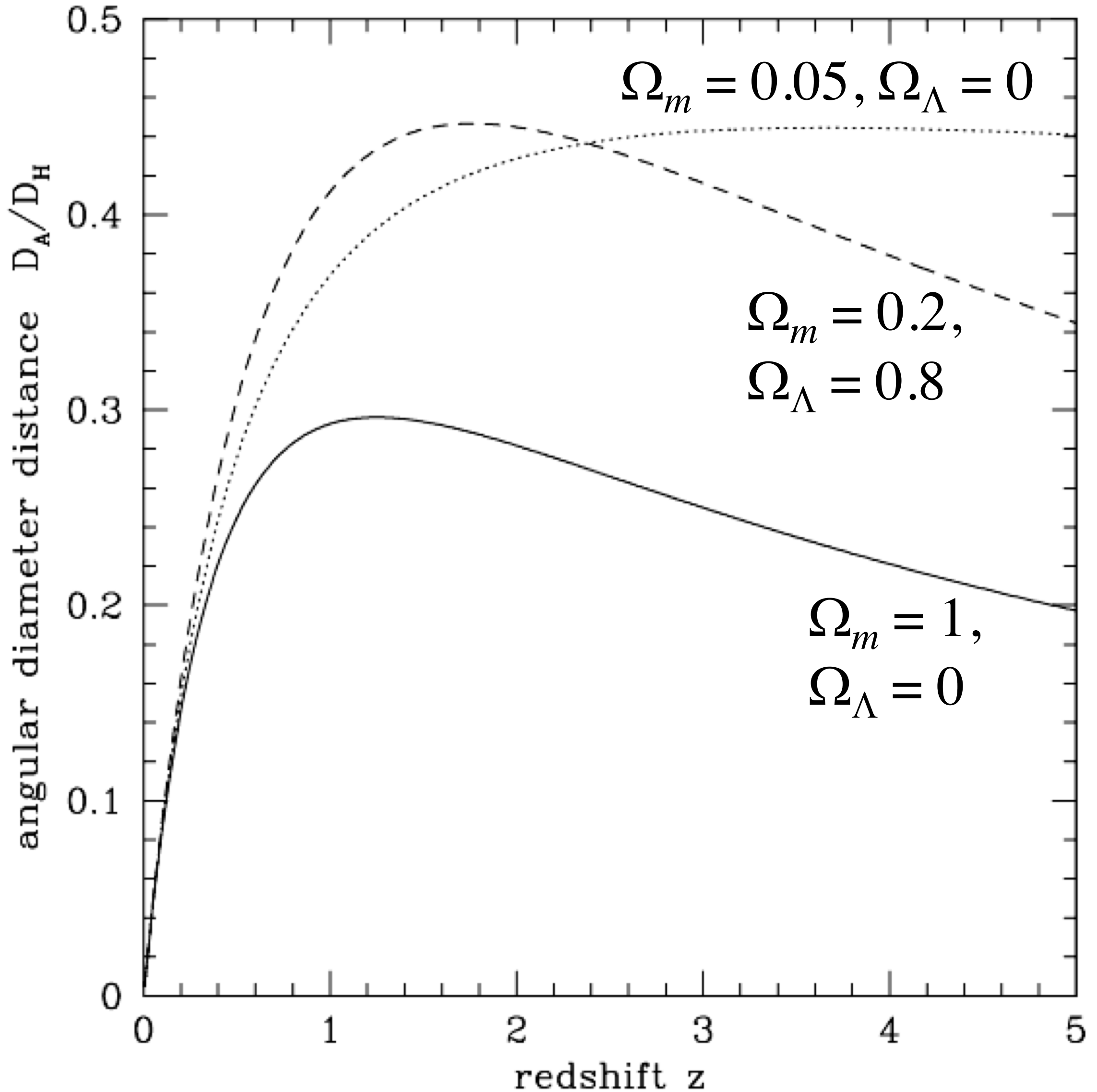


Angular Diameter Distance

Angular diameter of an object with a fixed *comoving* size X is by definition $\theta = X / D$

A fixed *proper* size X is $(1+z)$ times larger than in the comoving coord's, so its angular diameter will be $\theta = (1+z)\, X / D$

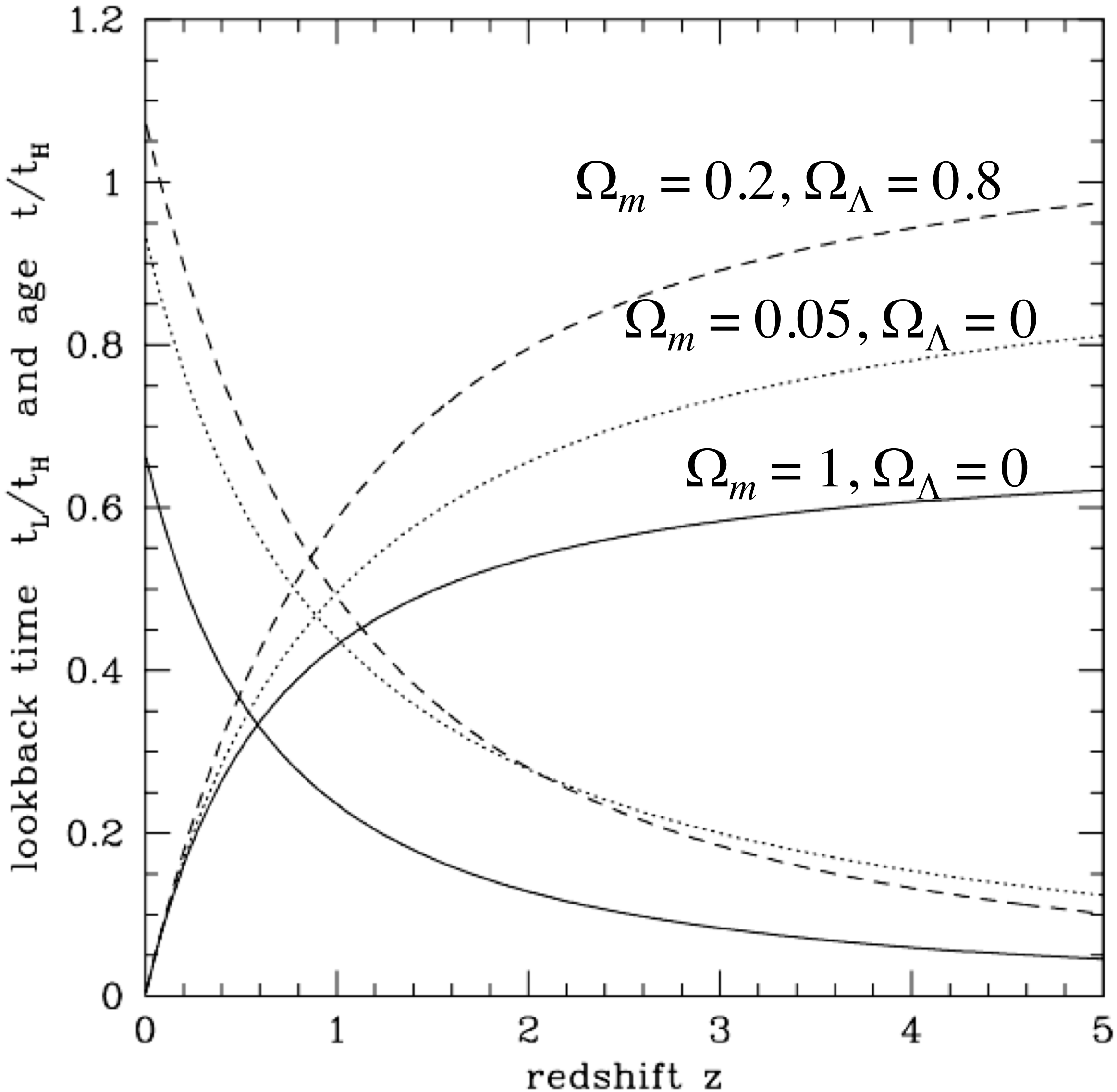
We define the angular diameter distance $D_A = D / (1+z)$, so that the angular diameter of an object whose size is fixed in proper coord's is $\theta = X / D_A$



Age and Lookback Time

The time elapsed since some redshift z is:

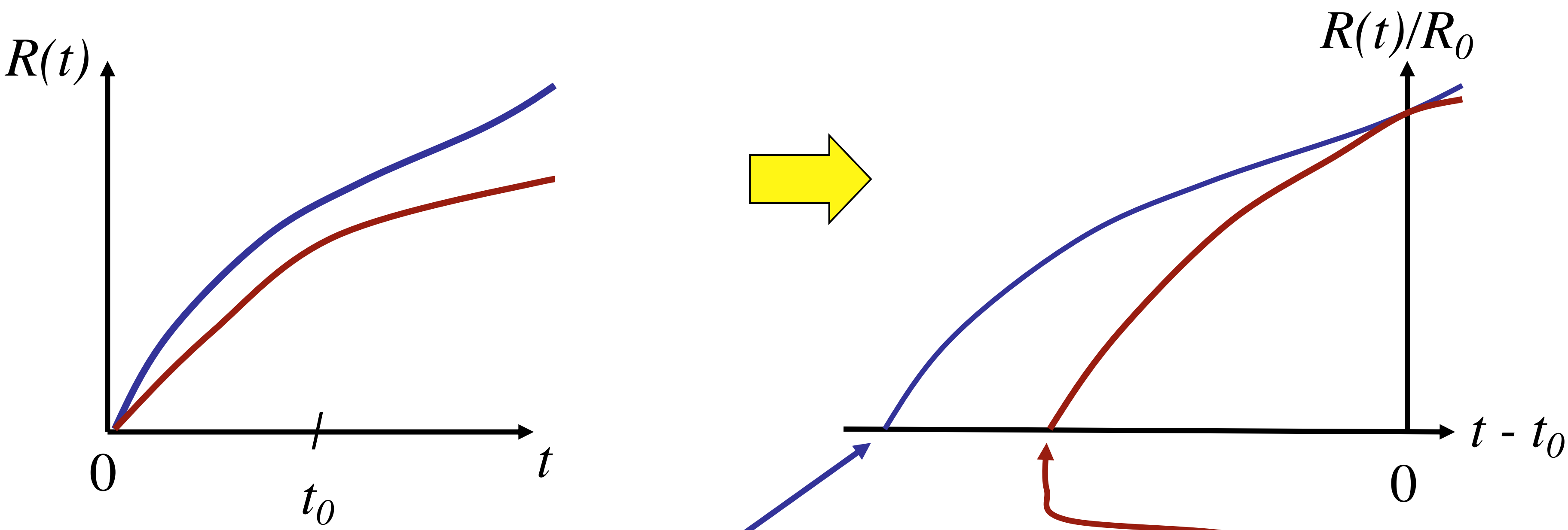
$$t_L = t_H \int_0^z \frac{dz'}{(1+z')\,E(z')}$$



Generally it has to be integrated numerically, except in special cases

Integrating to infinity gives the age of the universe, and the difference is the age at a given redshift

Cosmological Tests: Expected Generic Behavior of Various Models



Models with a lower density and/or positive Λ expand faster, are thus larger, older today, have more volume and thus higher source counts, at a given *z* sources are further away and thus appear fainter and smaller

Models with a higher density and lower Λ behave exactly the opposite

?

The Hubble distance is:

A. The distance to the Big Bang
B. The distance to the Hubble Space Telescope
C. The distance to the most distant galaxies
D. About 14 billion light years
E. About 14 Gigaparsec
F. Changes in time