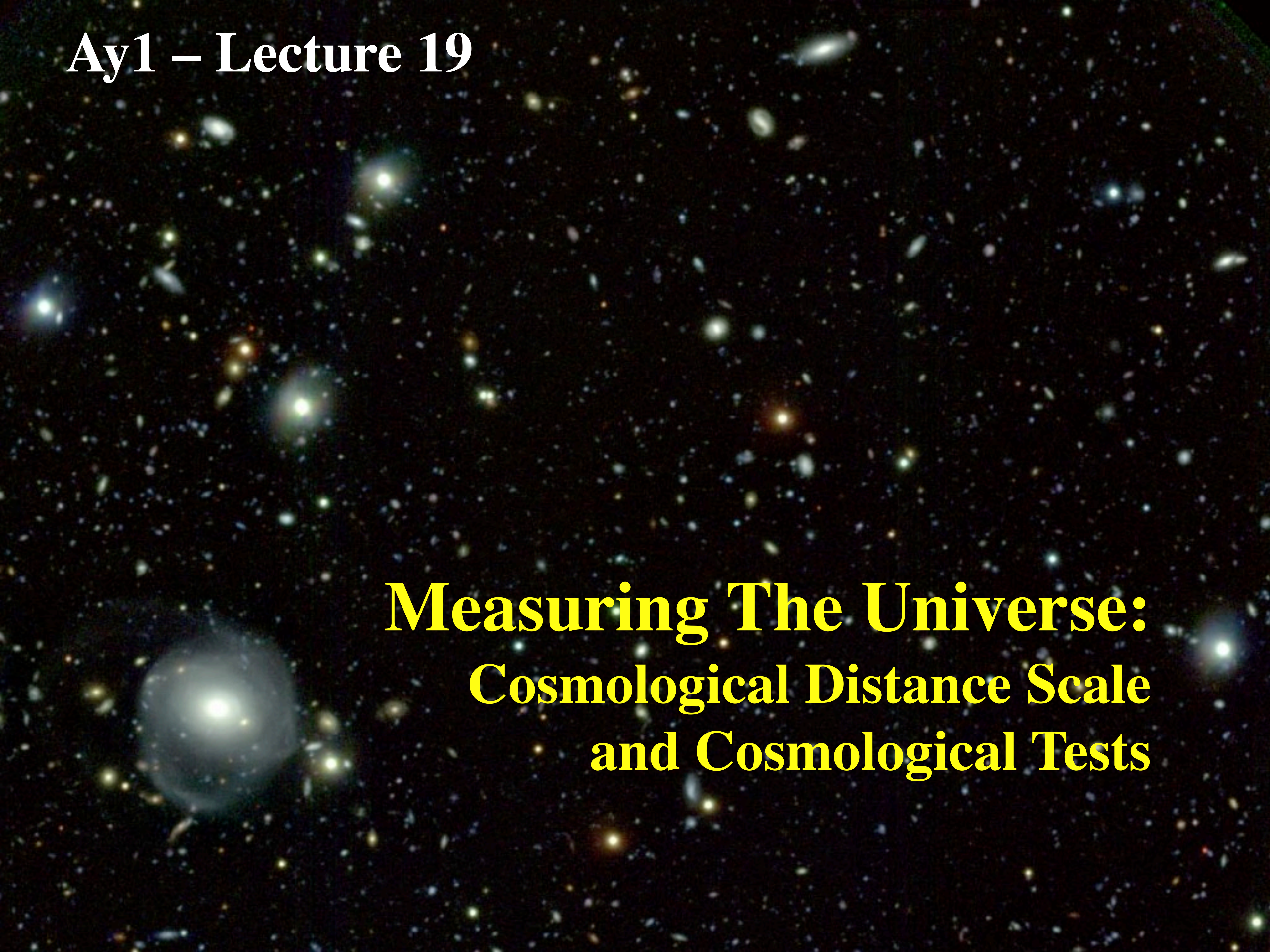
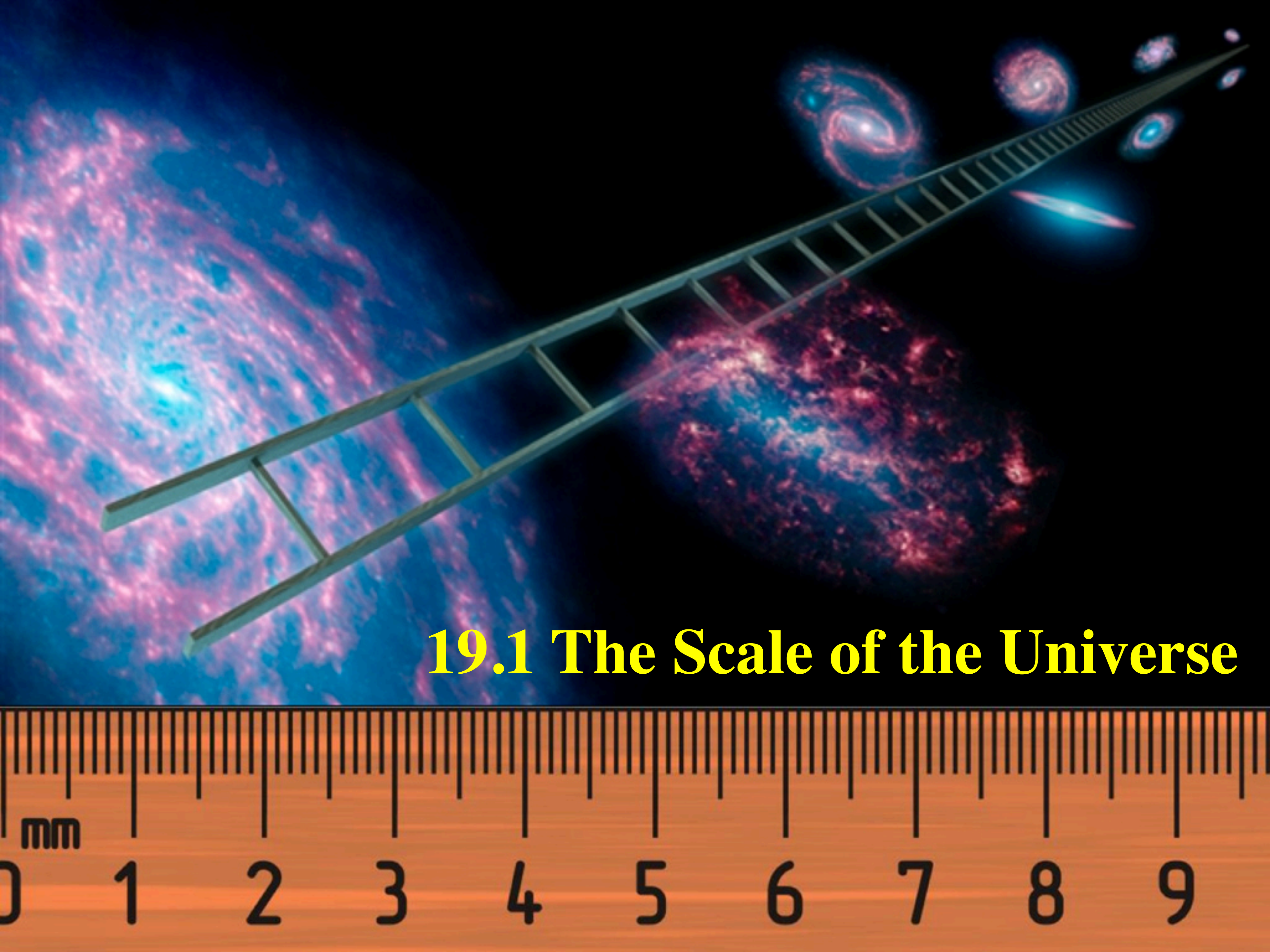


Ay1 – Lecture 19

Measuring The Universe:
Cosmological Distance Scale and Cosmological Tests

19.1 The Scale of the Universe



The Scale of the Universe

- The **Hubble length**, $D_H = c/H_0$, and the **Hubble time**, $t_H = 1/H_0$ give the approximate spatial and temporal scales of the universe
- H_0 is independent of the "shape parameters" (expressed as density parameters) Ω_m, Ω_Λ, Ω_k, w, etc., which govern the global geometry and dynamics of the universe
- Distances to galaxies, quasars, etc., scale linearly with H_0, $D \approx cz / H_0$. They are necessary in order to convert observable quantities (e.g., fluxes, angular sizes) into physical ones (luminosities, linear sizes, energies, masses, etc.)

Measuring the Scale of the Universe

- The only clean-cut distance measurements in astronomy are from trigonometric parallaxes. Everything else requires physical modeling and/or a set of calibration steps (the ***"distance ladder"***), and always some statistics:

Use parallaxes to calibrate some set of distance indicators

→ Use them to calibrate another distance indicator further away

→ And then another, reaching even further

→ etc. etc.

→ Until you reach a "pure Hubble flow"

- The age of the universe can be constrained independently from the H_0, by estimating ages of the oldest things one can find around (e.g., globular clusters, heavy elements, white dwarfs)

Distance Ladder: Methods

Methods yielding absolute distances:

Parallax (trigonometric, secular, and statistical)
The moving cluster method - has some assumptions
Baade-Wesselink method for pulsating stars
Expanding photosphere method for Type II SNe
Sunyaev-Zeldovich effect
Gravitational lens time delays

Model dependent!

kpc
Mpc
Gpc

Secondary distance indicators: ***"standard candles"*****,** requiring a calibration from an absolute method applied to local objects - ***the distance ladder*****:**

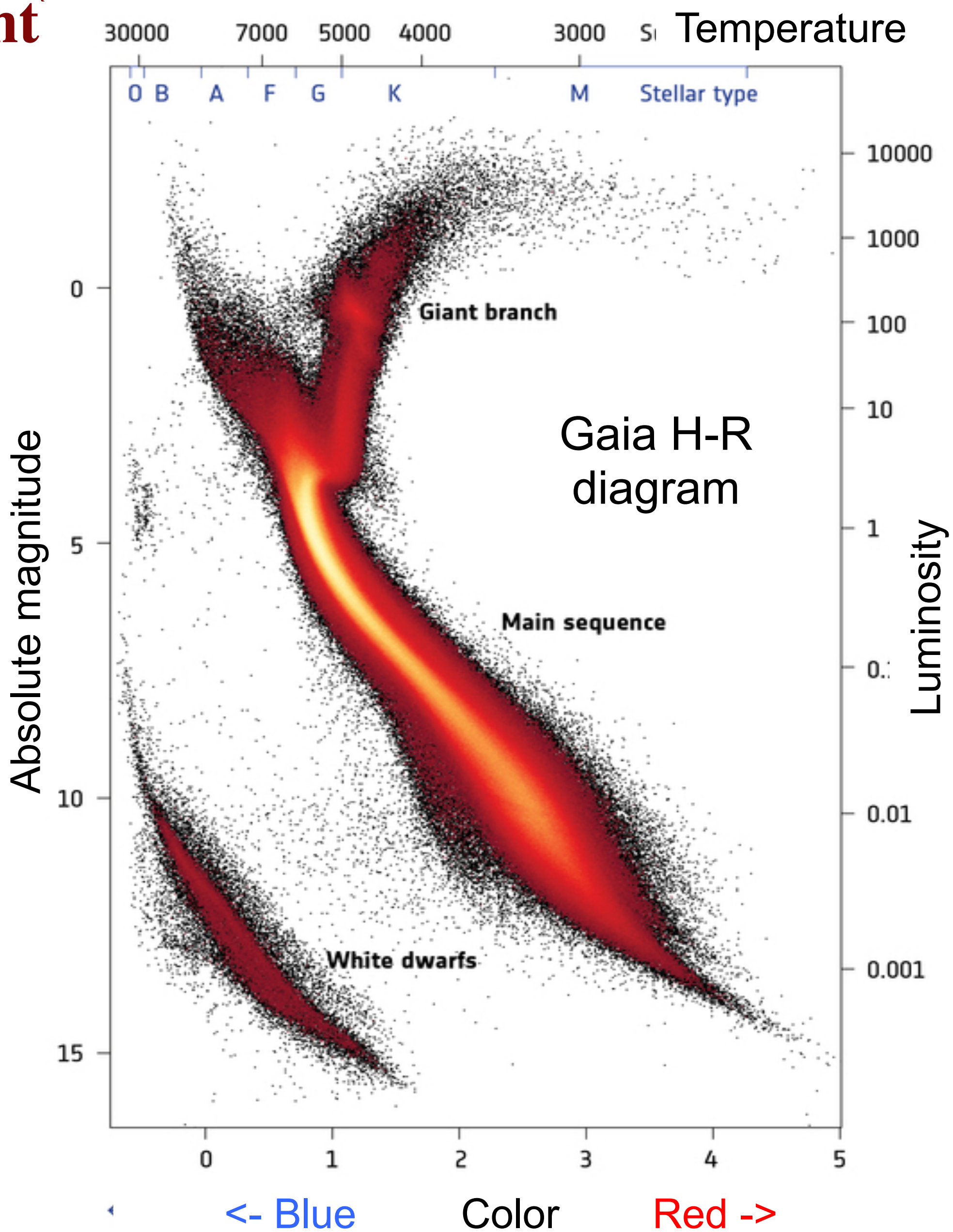
Pulsating variables: Cepheids, RR Lyrae, Miras
Main sequence fitting to star clusters
Brightest red giants
Planetary nebula luminosity function
Globular cluster luminosity function
Surface brightness fluctuations
Tully-Fisher, D_n-σ, FP scaling relations for galaxies
Type Ia Supernovae
… etc.

kpc
Mpc
Gpc

Main Sequence Fitting for Star Clusters

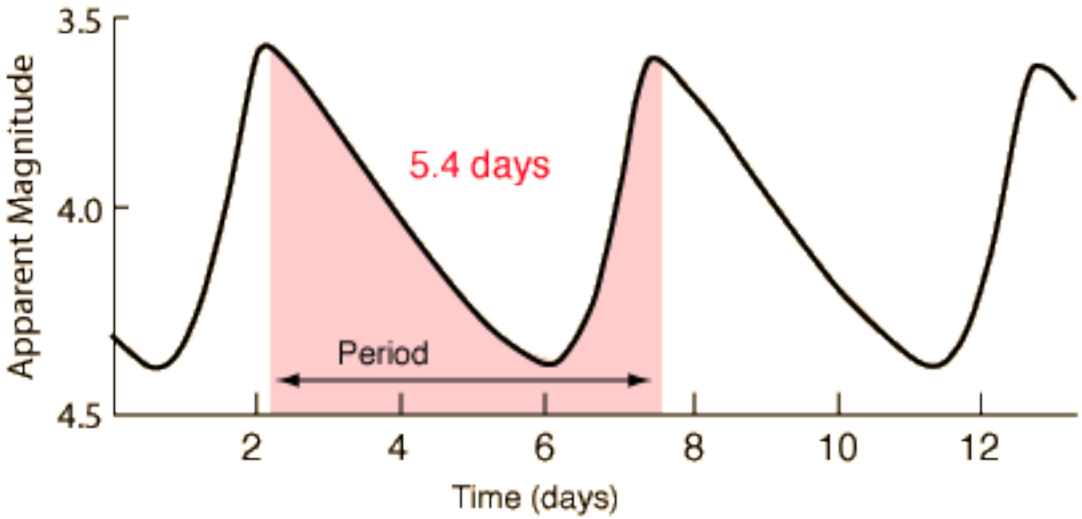
Luminosity (distance dependent) vs. temperature or color (distance independent)

- Measure distance to star clusters (open or globular) by fitting their main sequence of a cluster with a known distance (e.g., Hyades)
- The apparent magnitude difference gives the ratio of distances, as long as we know the reddening!
- There are no parallaxes to GCs (no nearby globulars) so we use parallaxes to nearby subdwarfs (metal-poor main sequence stars)

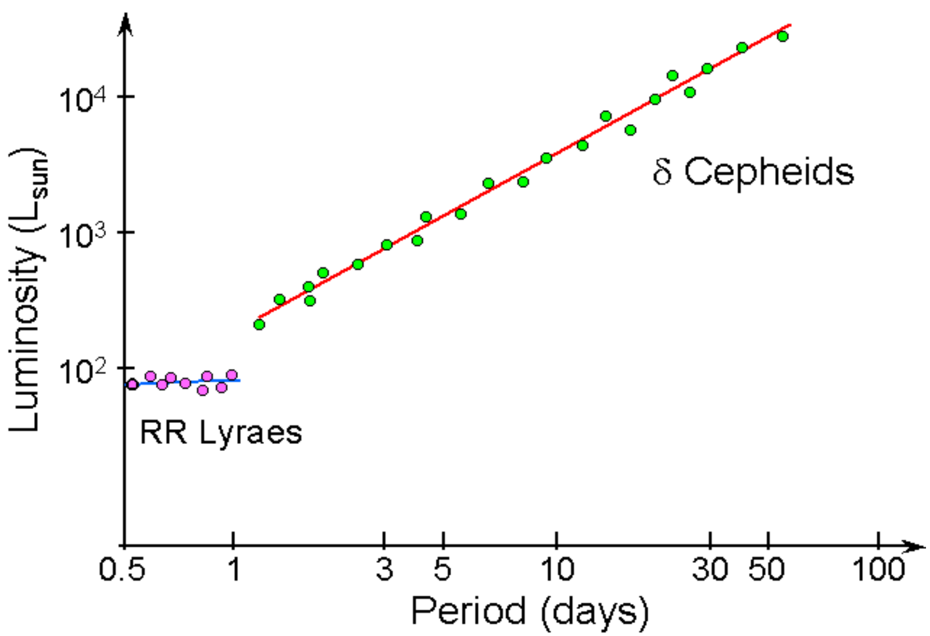


Cepheids

- Luminous ($M \sim$ -4 to -7 mag), pulsating variables, evolved high-mass stars on the instability strip in the H-R diagram
  - Can be observed out to a few tens of Mpc



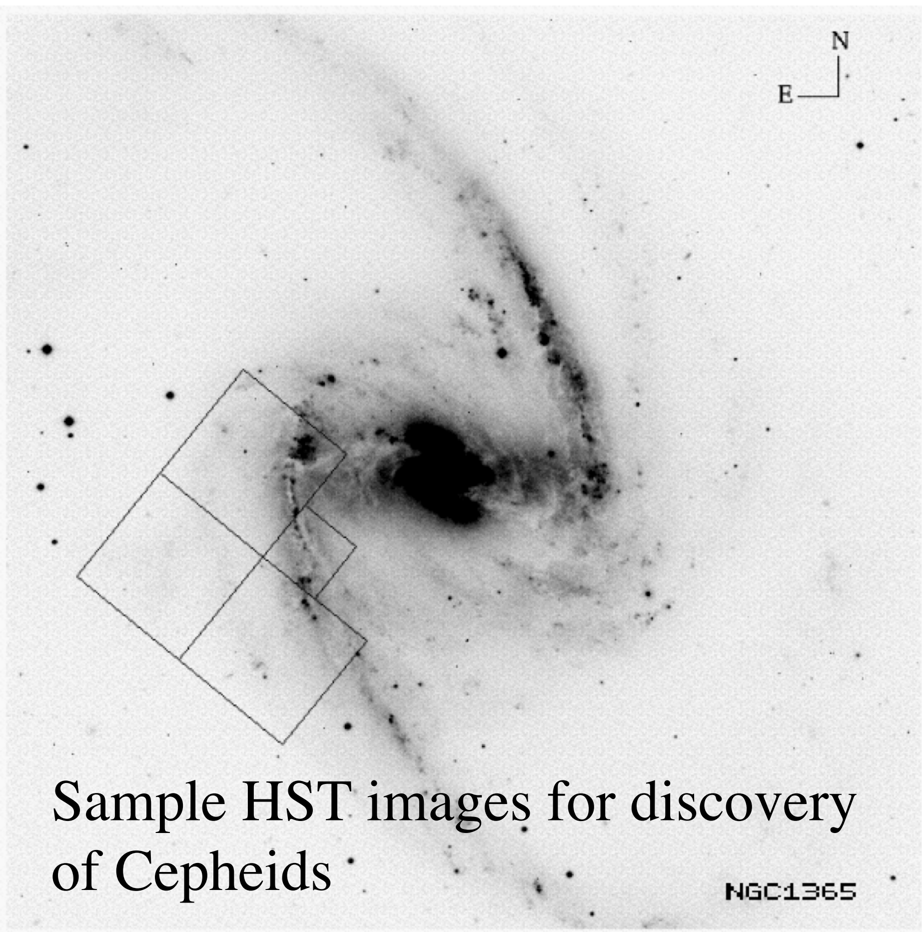
- Obey a period-luminosity relation (P-L): brighter Cepheids have longer periods than fainter ones
  - ✧ Calibrated using Hipparcos parallaxes

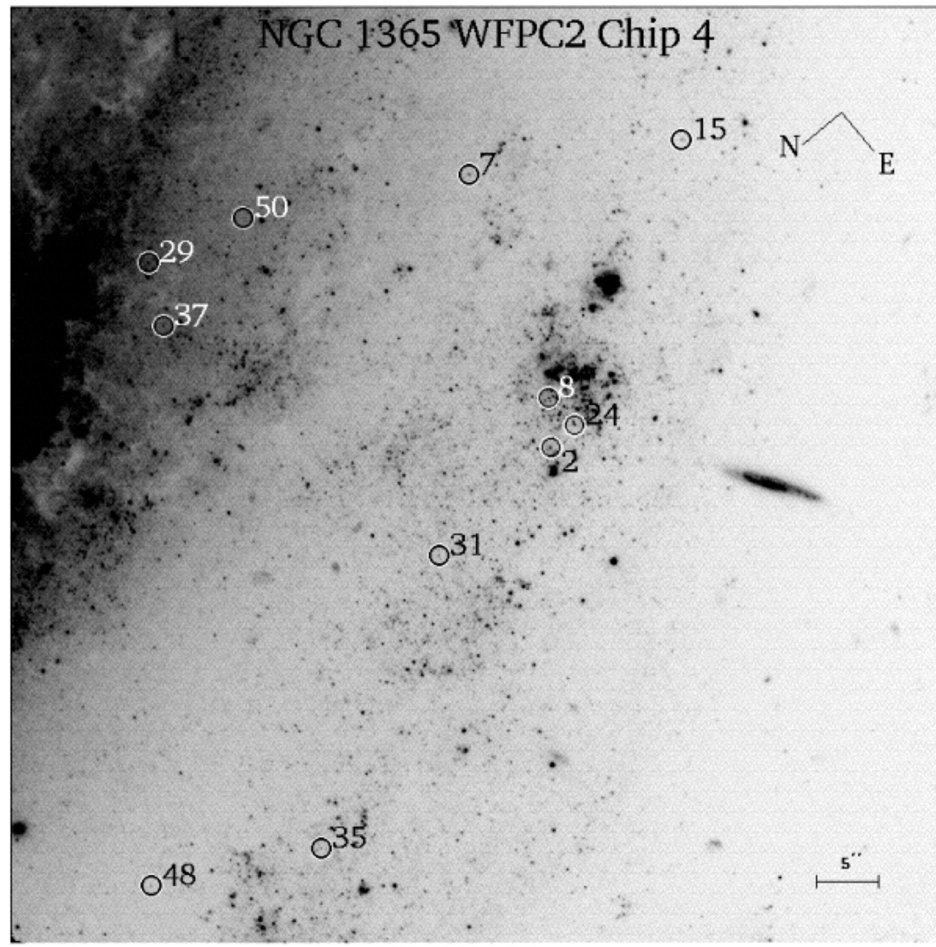


- RR Lyrae are their Pop II analogs

The HST H_0 Key Project

- Started in 1990, final results in 2001! Leaders include W. Freedman, R. Kennicutt, J. Mould, J. Huchra, and many others
- Observe Cepheids in ~18 spirals and improve calibration of other distance indicators





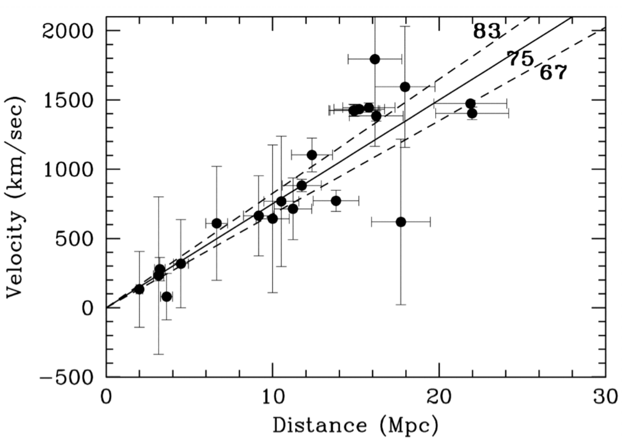
Sample HST images for discovery of Cepheids

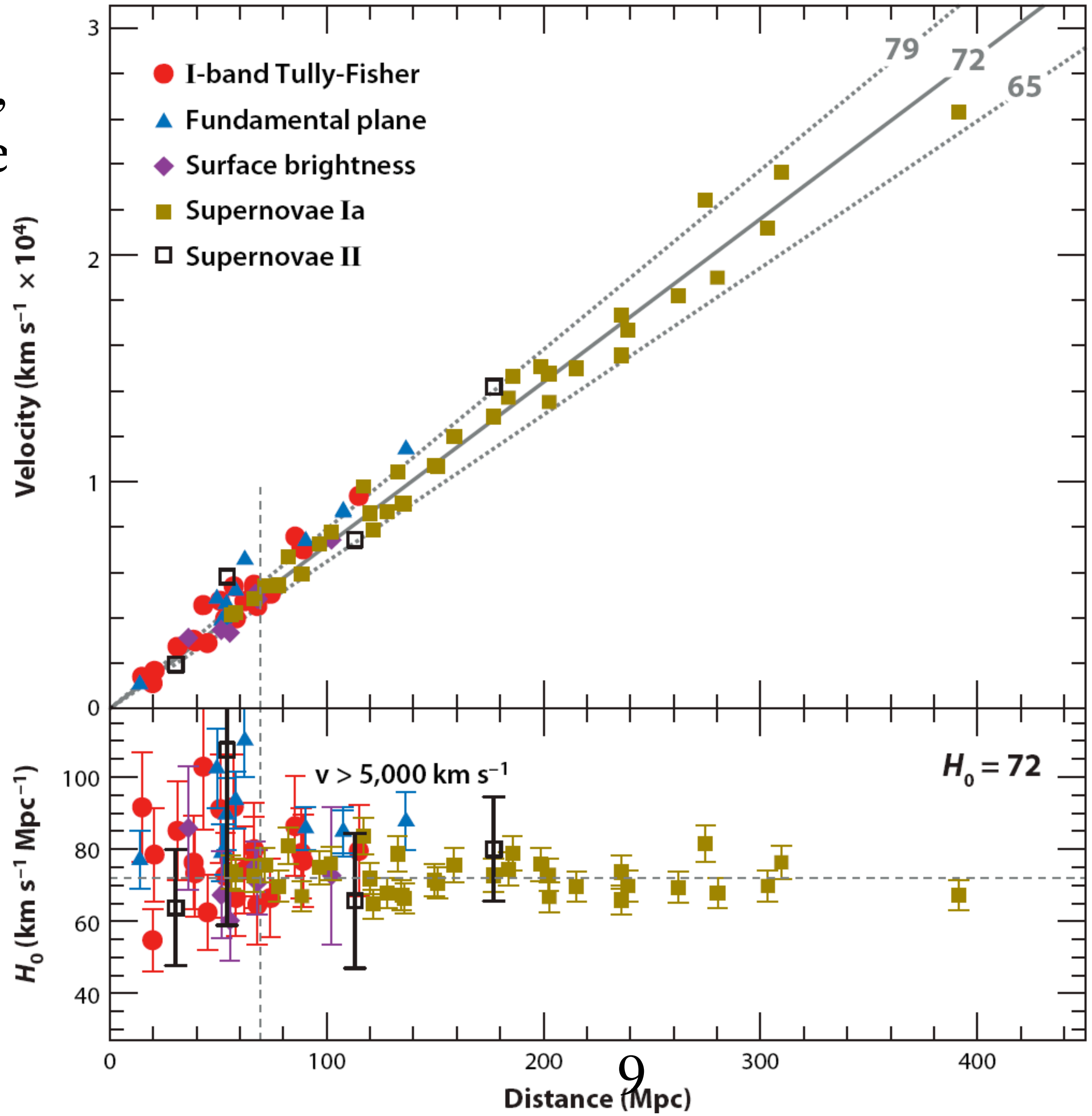
The HST H_0 Key Project Results

Overall Hubble diagram, from all types of distance indicators ➞

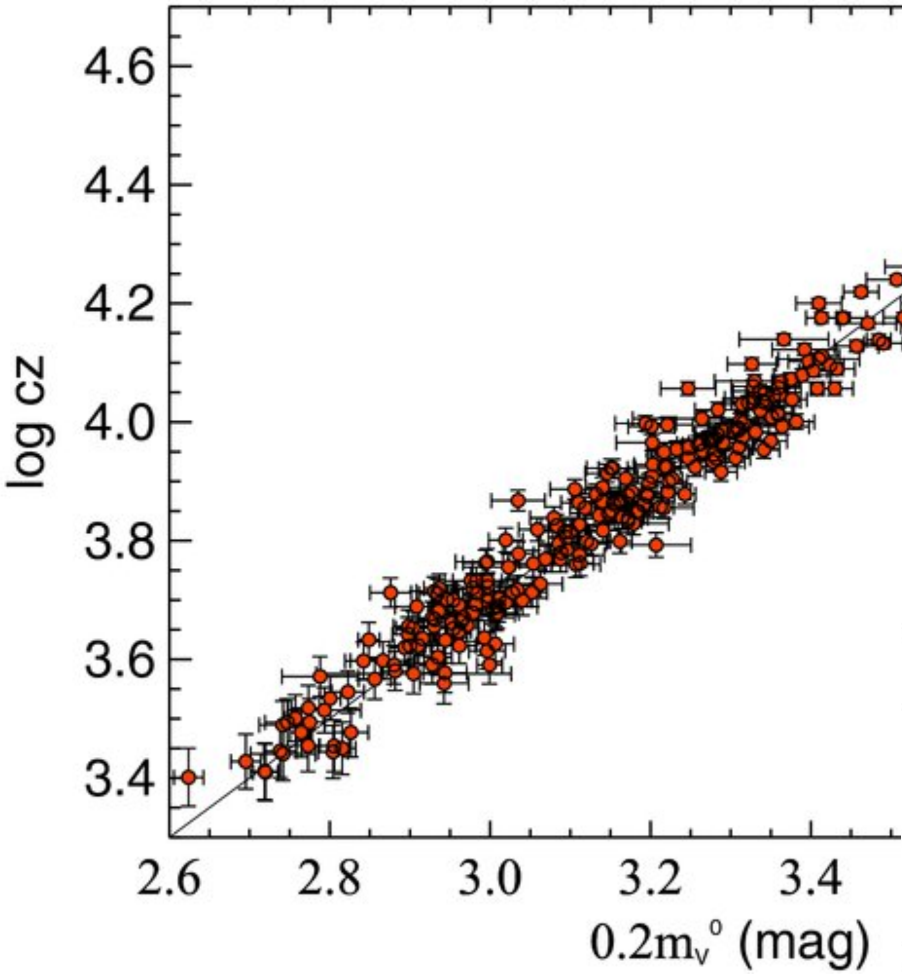
$$H_o = 72 \pm (3)_r \pm [7]_s$$

From Cepheid distances alone ➘



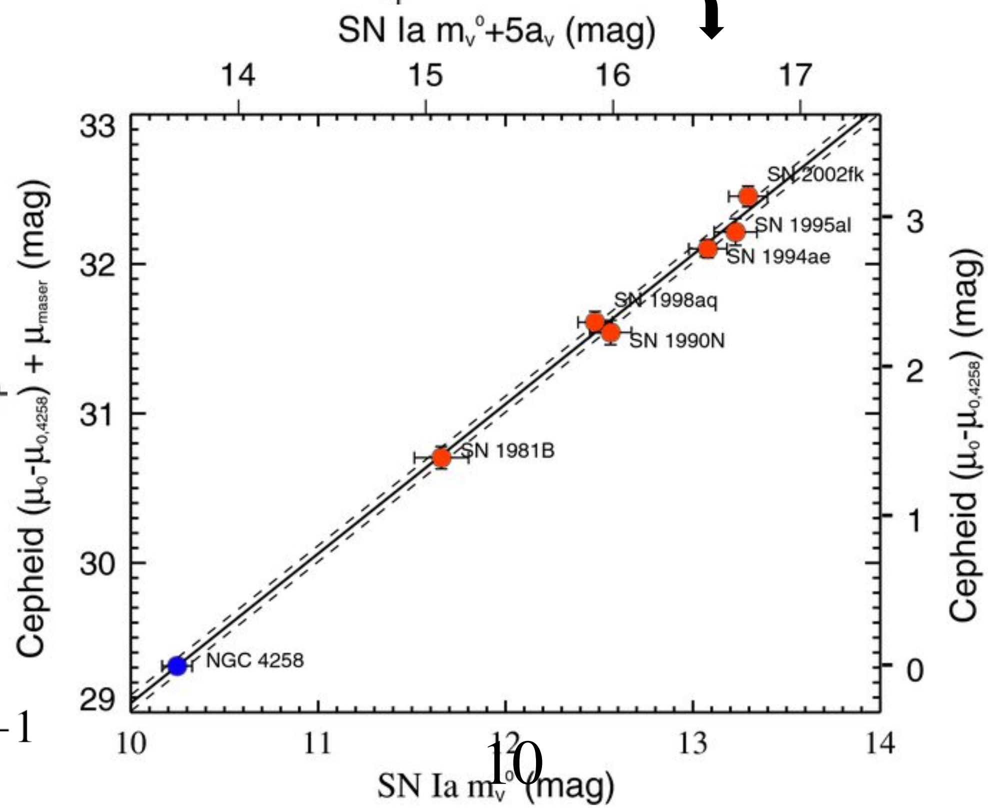


The Low-Redshift SN Ia Hubble Diagram



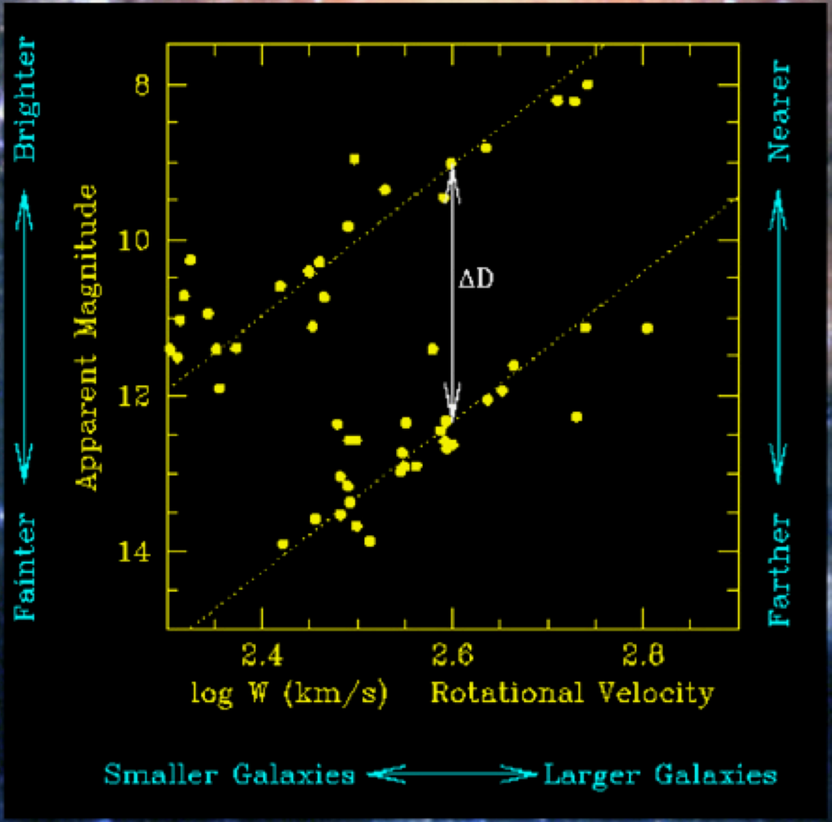
Riess et al. 2009
(also Riess et al. 2011 – slight changes)

Cepheid calibration



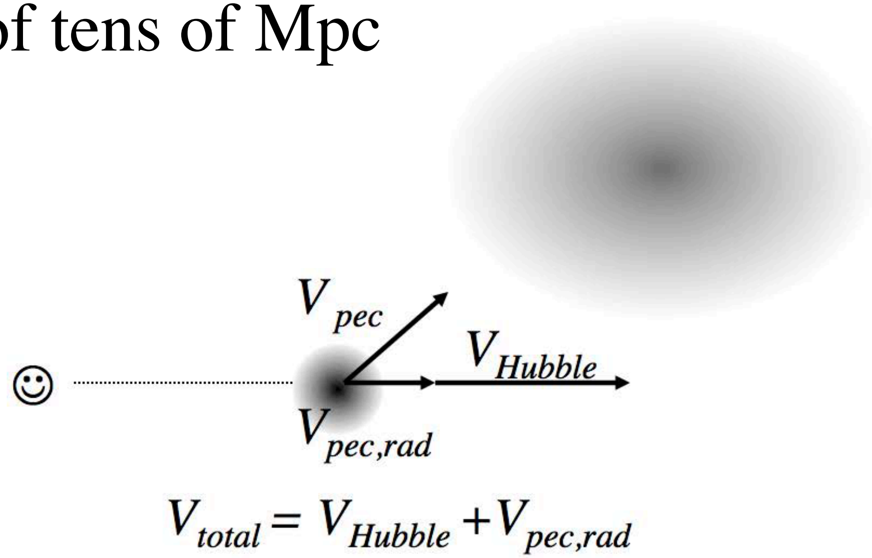
$$H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

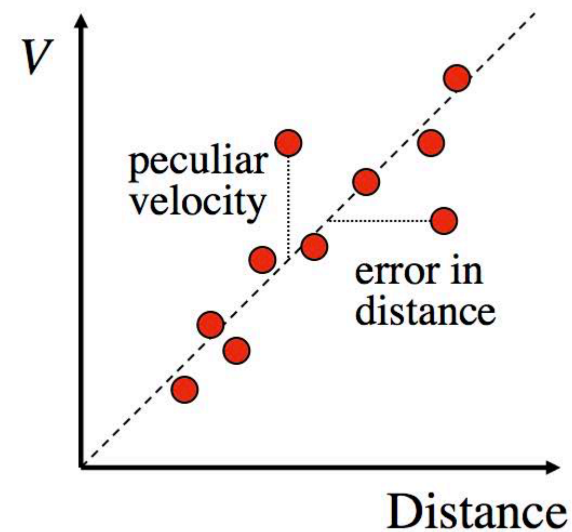
19.2 Distance Indicator Relations



Pushing Into the Hubble Flow

- Hubble's law: $\boldsymbol{D = H_0\, v}$ – but the – total observed velocity $\boldsymbol{v}$ is a combination of the cosmological expansion, and the *peculiar velocity* of any given galaxy, $\boldsymbol{v = v_{cosmo} + v_{pec}}$
- Typically $v_{pec} \sim$ a few hundred km/s, due to a gravitational infall into the local large scale structures, with characteristic scales of tens of Mpc

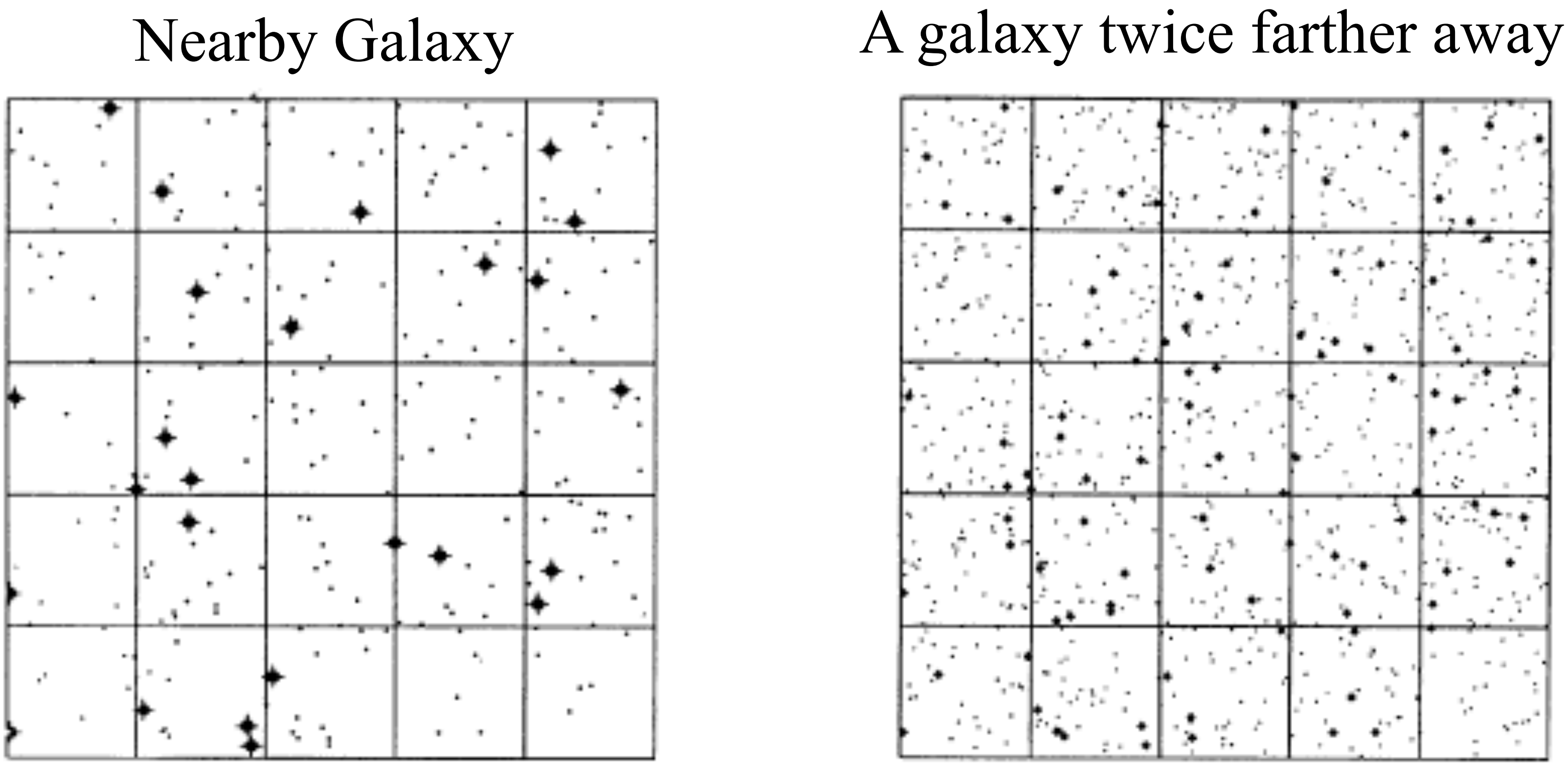




- Thus, we need to measure H_0 on scales greater than tens of Mpc, and where $\boldsymbol{v_{cosmo} >> v_{pec}}$. This requires *luminous standard candles* - galaxies or Supernovae

Surface Brightness Fluctuations

Consider stars projected onto a pixel grid of your detector:

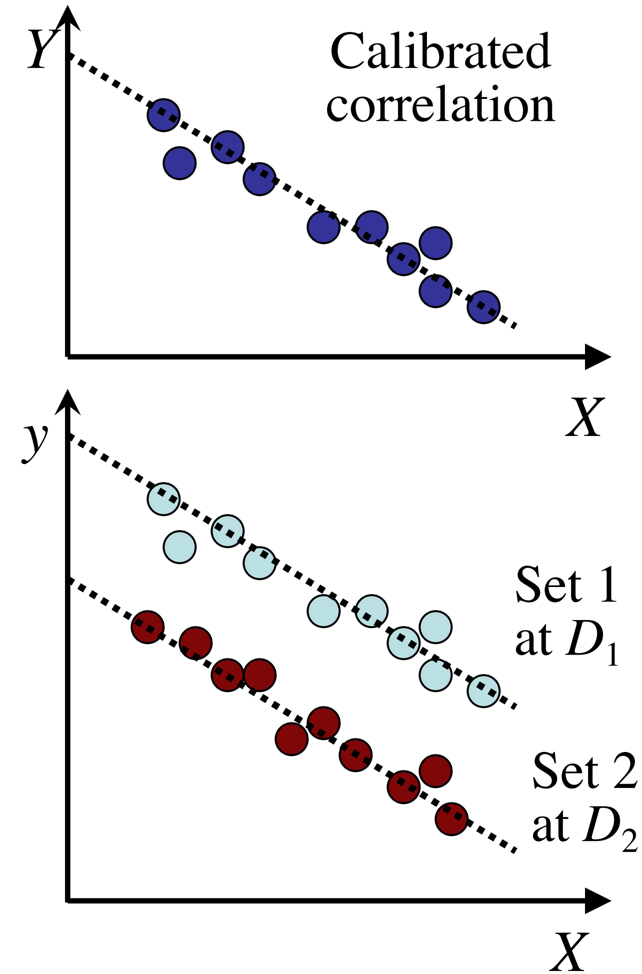


- Average flux per star = $\langle f \rangle$, average flux per pixel = $N\langle f \rangle$, Poissonian variations per pixel = $N^{1/2} \langle f \rangle$
- $N \sim D^{2}$, the flux per star $\sim D^{-2}$ and the RMS $\sim D^{-1}$. Thus a galaxy twice as far away appears twice as smooth

Distance Indicator Relations

- Need a correlation between a distance-independent quantity, "X", (e.g., temperature or color for stars in the H-R diagram, or the period for Cepheids), and a distance-dependent one, "Y", (e.g., stellar absolute magnitude, M)

- Two sets of objects at different distances will have a systematic shift in the *apparent* versions of "y" (e.g., stellar apparent magnitude, m), from which we can deduce their ***relative distance***

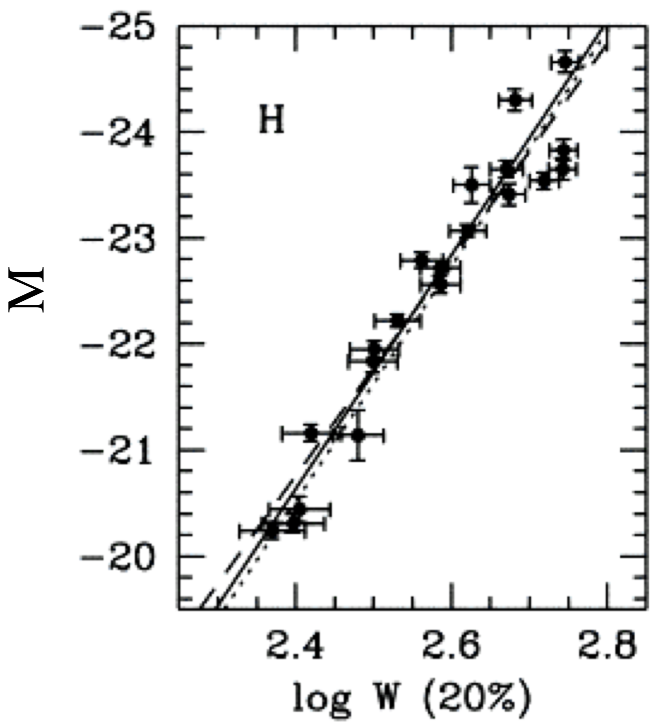


- This obviously works for stars (main sequence fitting, period-luminosity relations), but can we find such relations for galaxies?

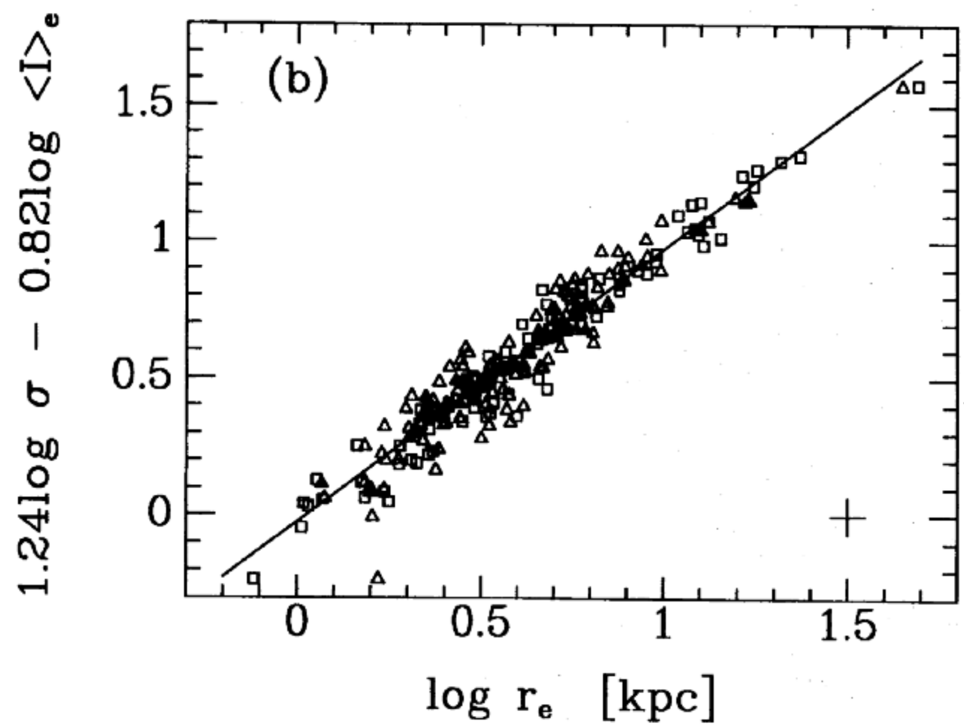
Galaxy Scaling Relations

- Correlations between distance-dependent quantities (luminosity, radius) and distance-independent ones (e.g., rotational speeds for disks, or velocity dispersions, surface brightness, etc.)
- Calibrated locally using other distance indicators, e.g. Cepheids or surface brightness fluctuations

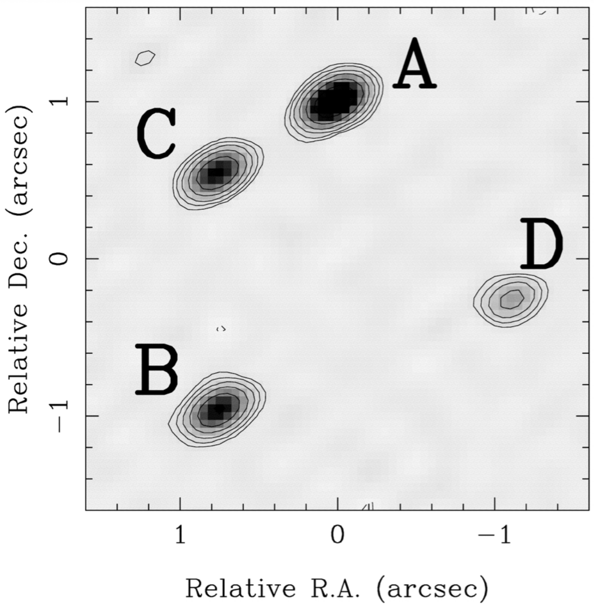
Tully-Fisher for spirals



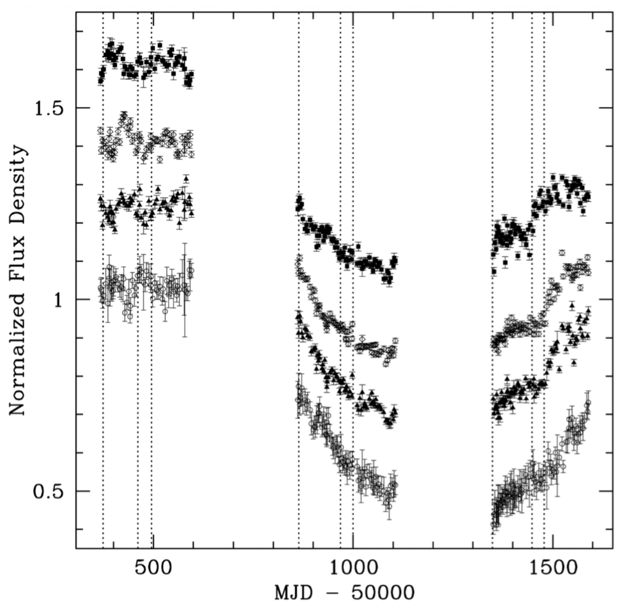
Fundamental Plane for Ellipticals

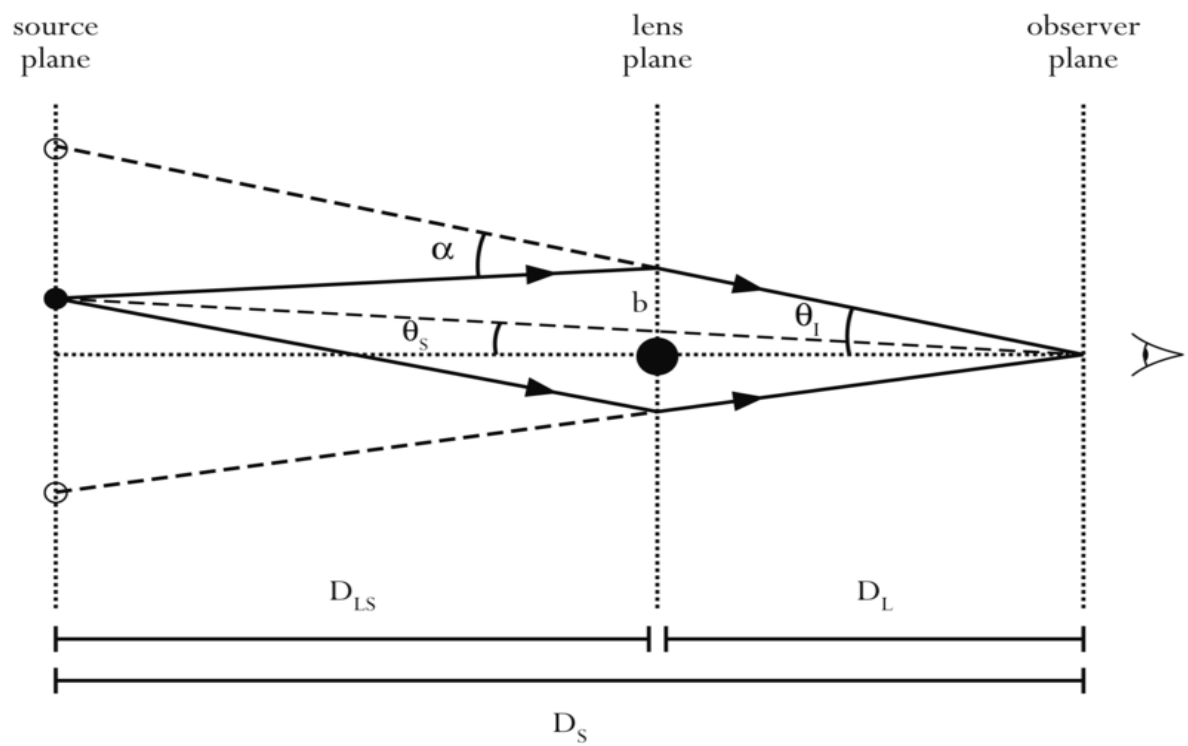


Gravitational Lens Time Delays



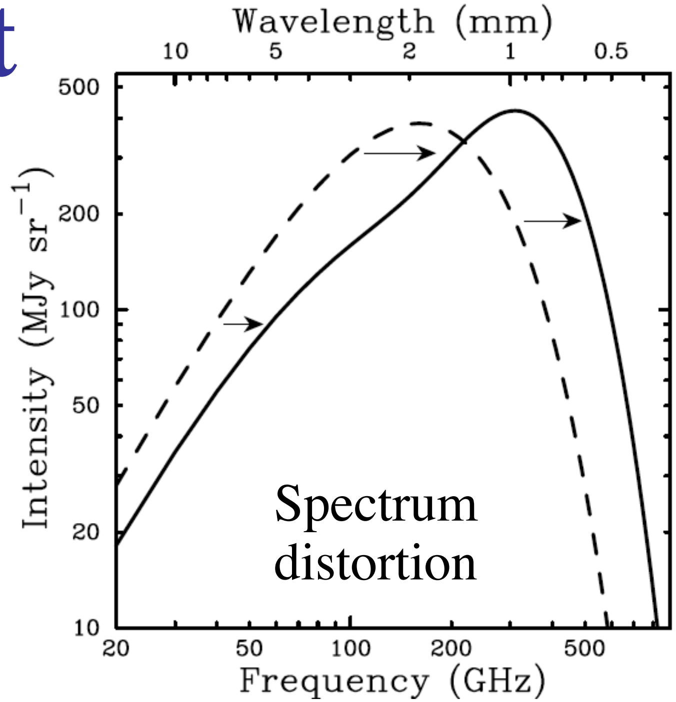
Assuming the mass model for the lensing galaxy of a gravitationally lensed quasar is well-known, the different light paths taken by various images of the quasar will lead to time delays in the arrival time of the light to us. The modeling is complex!

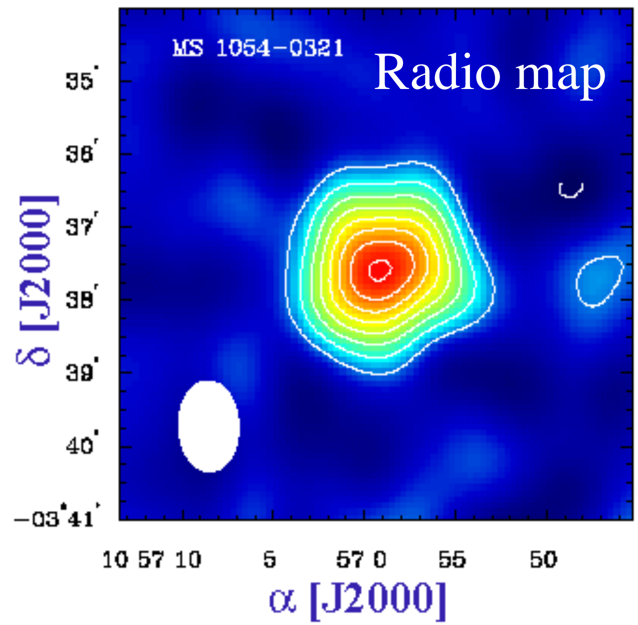




Synyaev-Zeldovich Effect

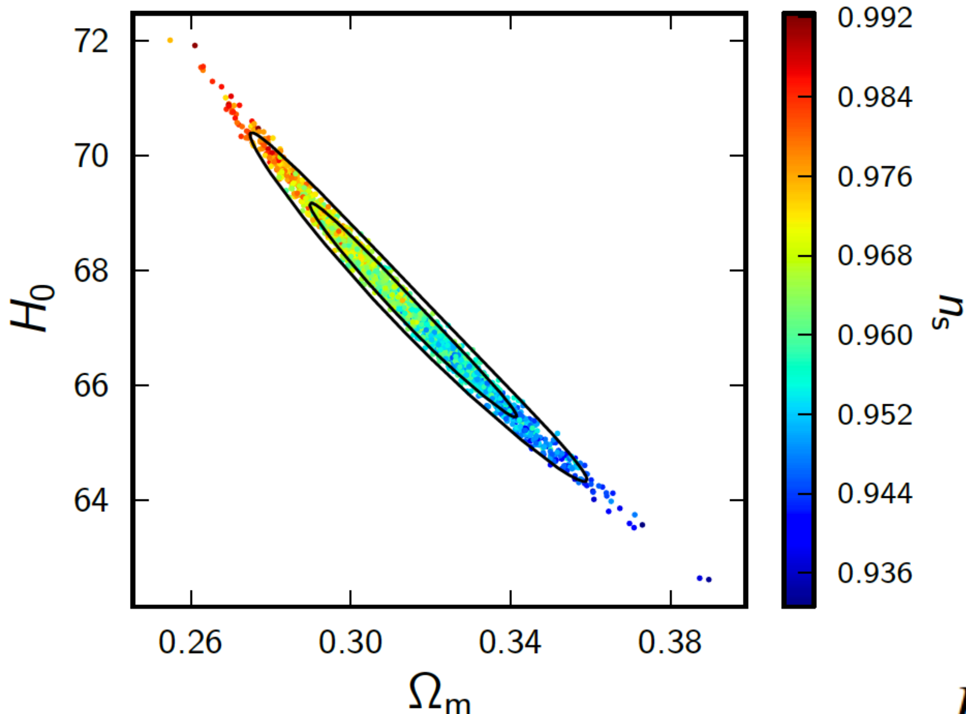
- If we can measure the electron density and temperature of the X-ray emitting gas along the line of sight from X-ray measurements, we can estimate the path length (~ cluster diameter) along the line of sight
- If we assume the cluster is spherical (??), from its angular diameter (projected on the sky) we can determine the distance to the cluster
- Potential uncertainties include cluster substructure or shape (e.g., non-spherical). It is also non-trivial to measure the X-ray temperature to derive the density at high redshifts.

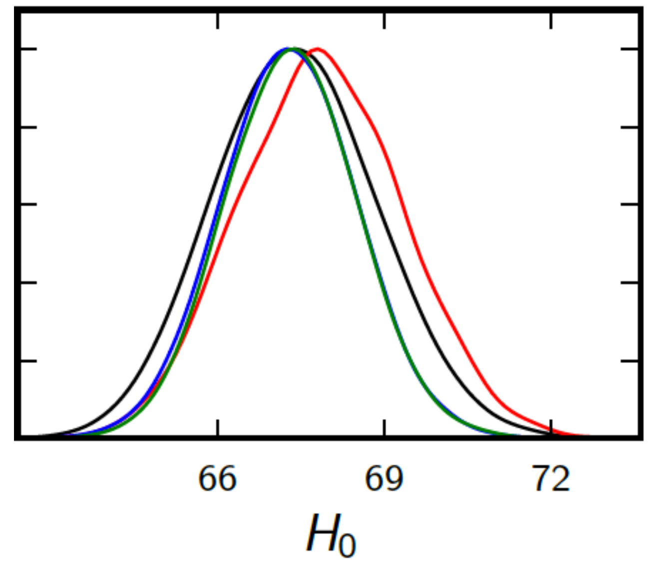




H_0 From the CMB

- Bayesian solutions from model fits to CMB fluctuations – cosmological parameters are coupled



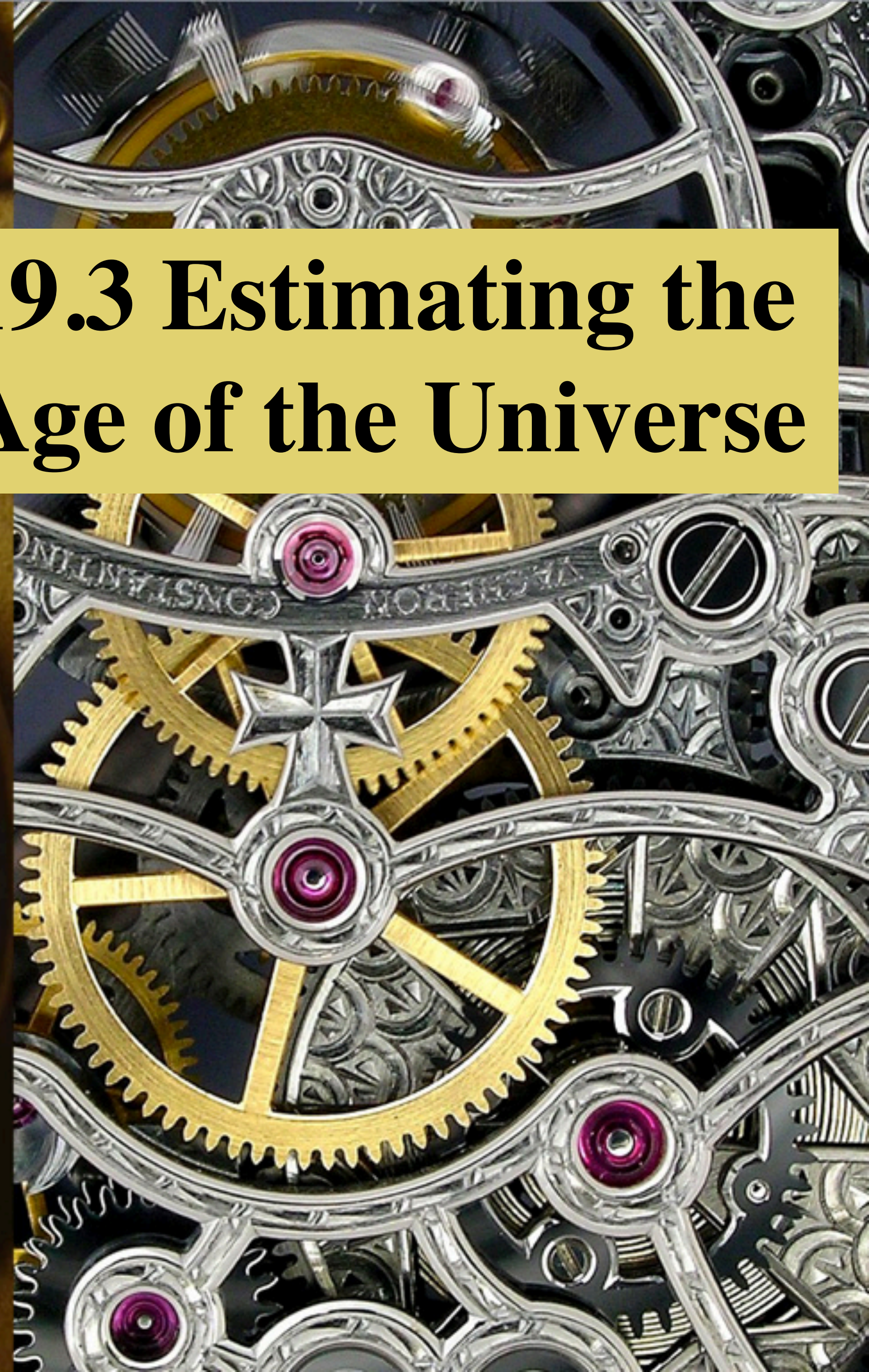


- Planck (2013) results:

$$H_0 = (67.3 \pm 1.2)\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

	Planck+WP		*Planck*+WP+highL		*Planck*+lensing+WP+highL		*Planck*+WP+highL+BAO	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
H_0	67.04	67.3 ± 1.2	67.15	67.3 ± 1.2	67.94	67.9 ± 1.0	67.77	67.80 ± 0.77
Age/Gyr	13.8242	13.817 ± 0.048	13.8170	13.813 ± 0.047	13.7914	13.794 ± 0.044	13.7965	13.798 ± 0.037

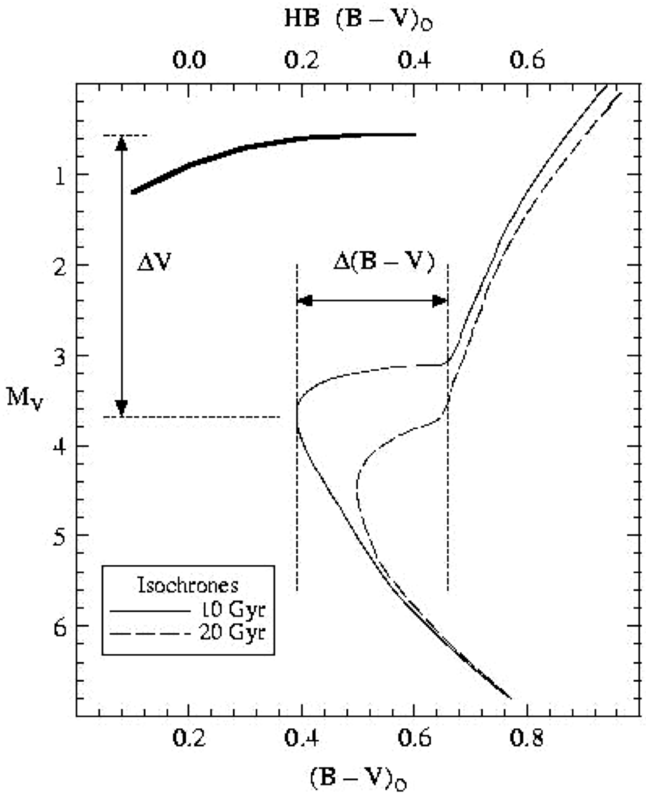
19.3 Estimating the Age of the Universe



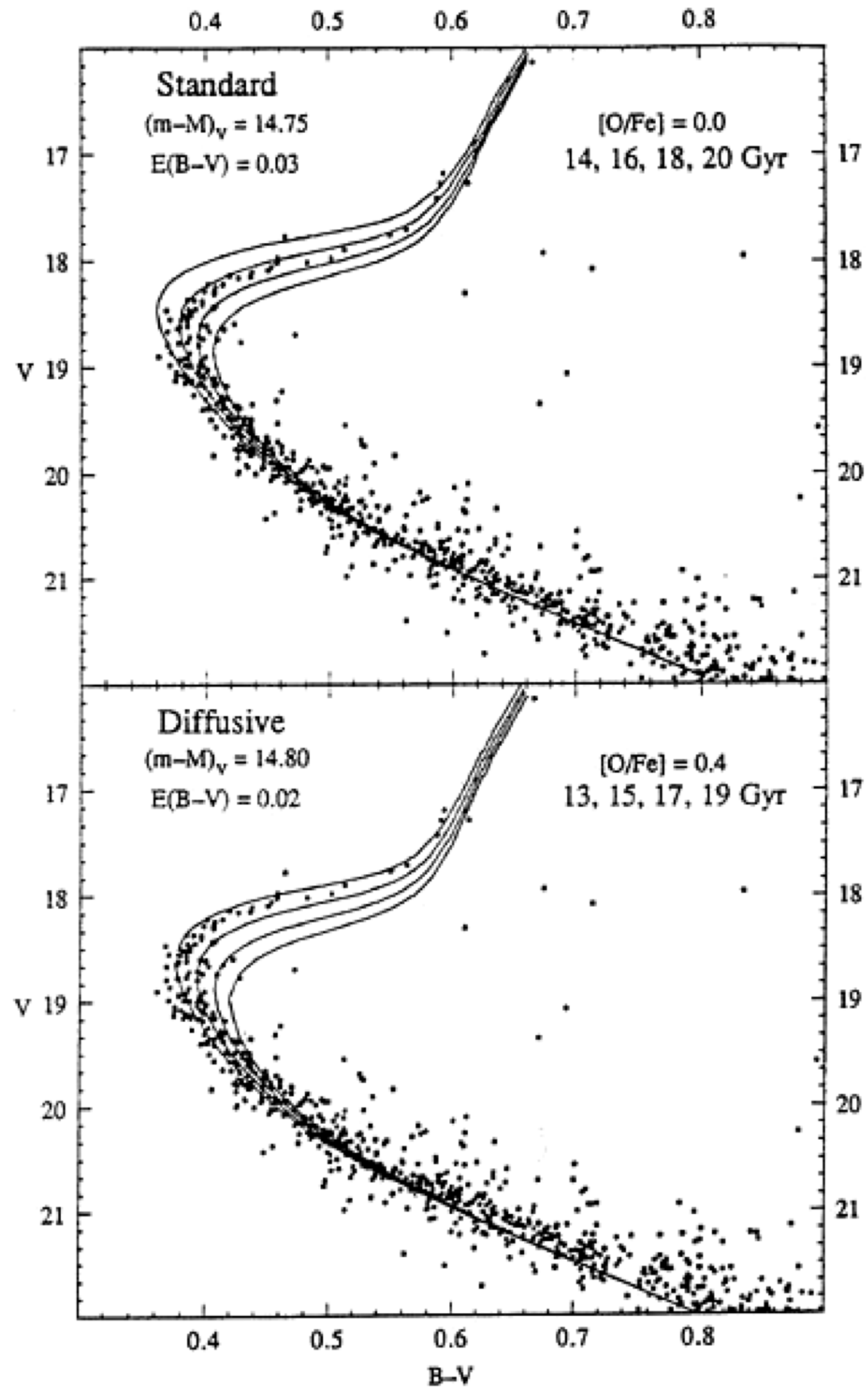
Measuring the Age of the Universe

- Related to the Hubble time $t_H = 1/H_0$, but the exact value depends on the cosmological parameters
- Could place a ***lower limit*** from the ages of astrophysical objects (pref. the oldest you can find), e.g.,
  - **Globular clusters** in our Galaxy; known to be very old. Need stellar evolution isochrones to fit to color-magnitude diagrams
  - **White dwarfs**, from their observed luminosity function, cooling theory, and assumed star formation rate
  - **Heavy elements**, produced in the first Supernovae; somewhat model-dependent
  - Age-dating **stellar populations** in distant galaxies; this is very tricky and requires modeling of stellar population evolution, with many uncertain parameters

Globular Cluster Ages

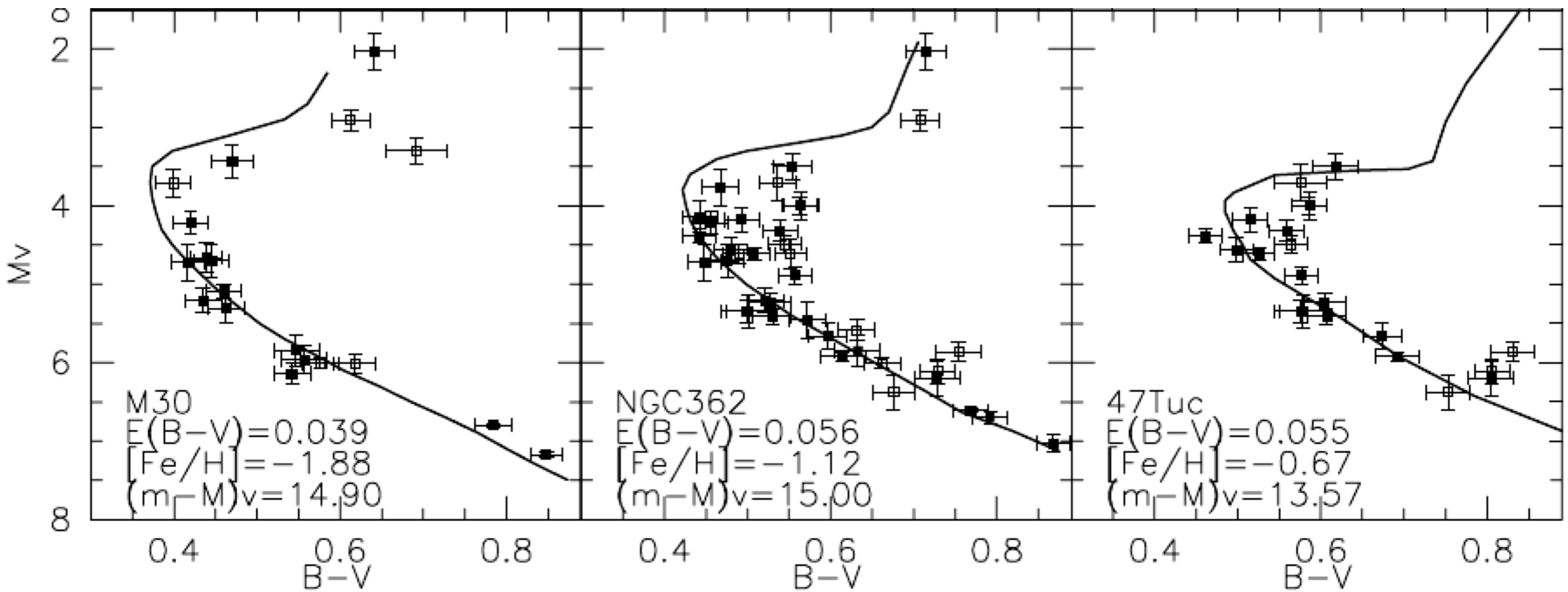


Schematic CMD and isochrones



Examples of actual model isochrones fits

Globular Cluster Ages From Hipparcos Calibrations of Their Main Sequences



Examples of g.c. main sequence isochrone fits, for clusters of a different metallicity (Graton et al.)

The same group has published two slightly different estimates of the mean age of the oldest clusters:

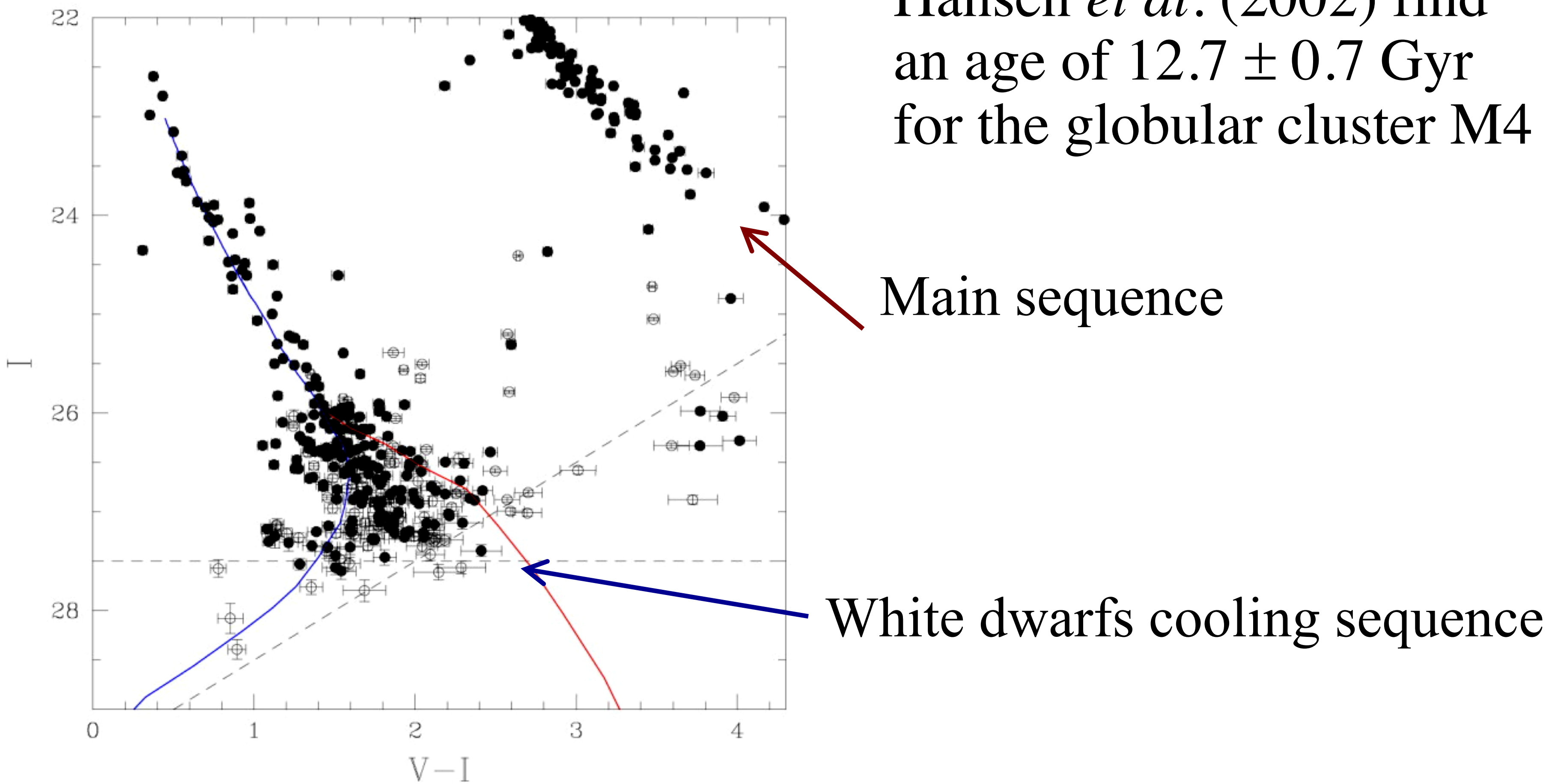
$$\mathrm{Age} = 11.8^{+2.1}_{-2.5}\mathrm{Gyr}$$

$$\mathrm{Age} = 12.3^{+2.1}_{-2.5}\ \mathrm{Gyr}$$

White Dwarf Cooling Curves

- Use the luminosity of the faintest WDs in a cluster to estimate the cluster age by comparing the observed luminosities to theoretical cooling curves
- Need deep HST observations

Hansen *et al*. (2002) find an age of 12.7 ± 0.7 Gyr for the globular cluster M4



Nucleocosmochronology

- Can use the radioactive decay of elements to age date the oldest stars in the galaxy. It has been done with ^{232}Th (half-life = 14 Gyr) and ^{238}U (half-life = 4.5 Gyr) and other elements
- Measuring the ratios of various elements from stellar spectroscopy of the oldest stars

CHRONOMETRIC AGE ESTIMATES FOR BD +17°3248

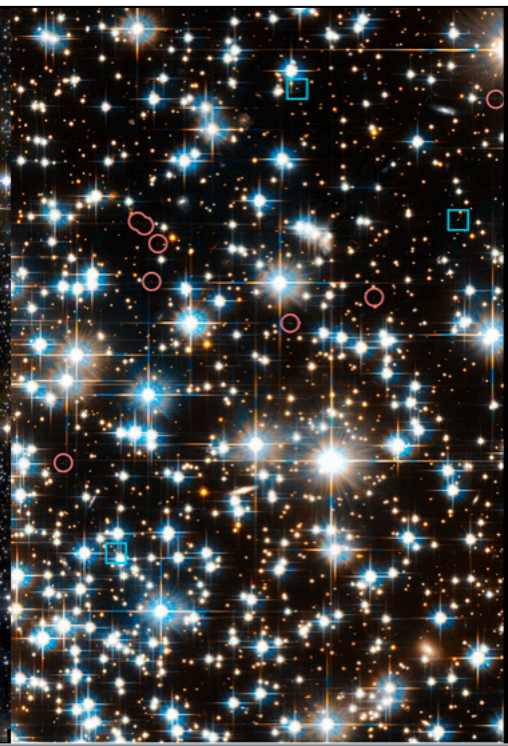
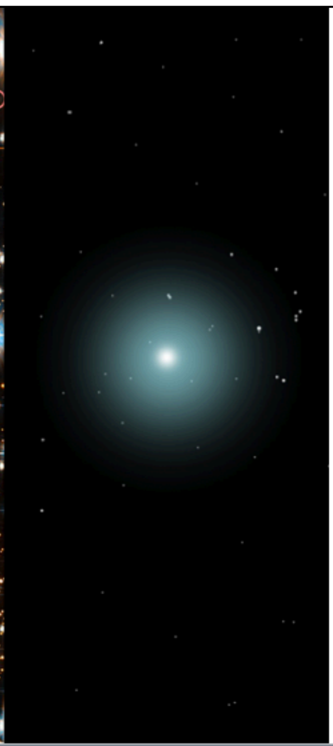
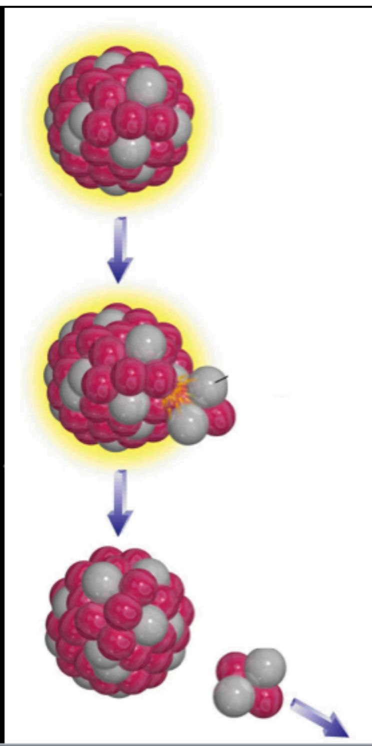
Chronometer Pair	Predicted	Observed	Age (Gyr)	Solar[a]	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	≥13.4	2.32	11.0
U/Ir	0.05036	0.0045	≥15.5	0.0369	13.5
U/Pt	0.013	0.0019	≥12.4	0.01846	14.6

[a] From Burris et al. 2001.

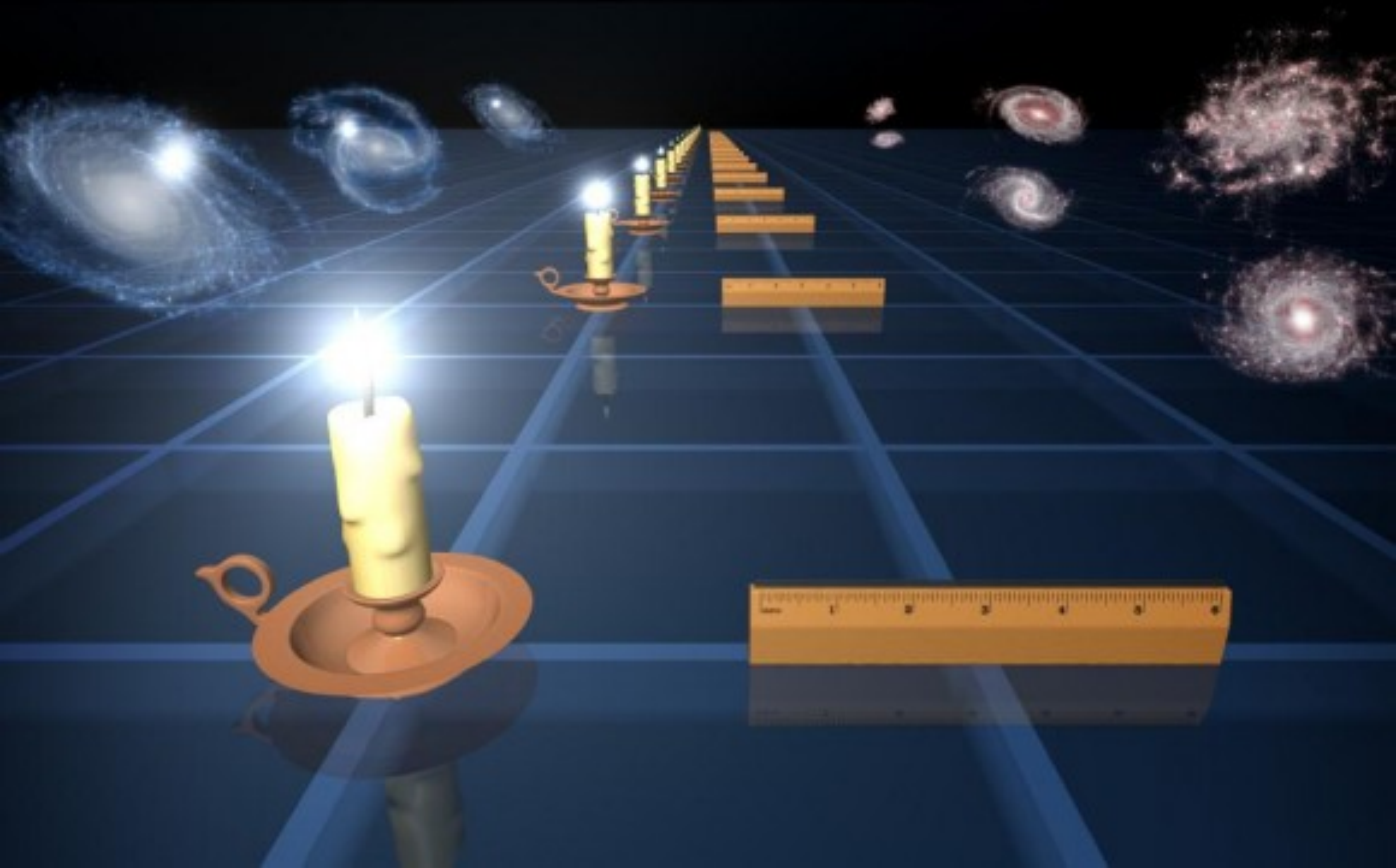
Mean = 13.8 +/- 4, but note the spread!

The Age of the Universe

- Several different methods (different physics, different measurements) agree that the lower limit to the age of the universe is ~ 12 – 13 Gyr
- This is in an excellent agreement with the age determined form the cosmological tests (~ 13.7 Gyr)

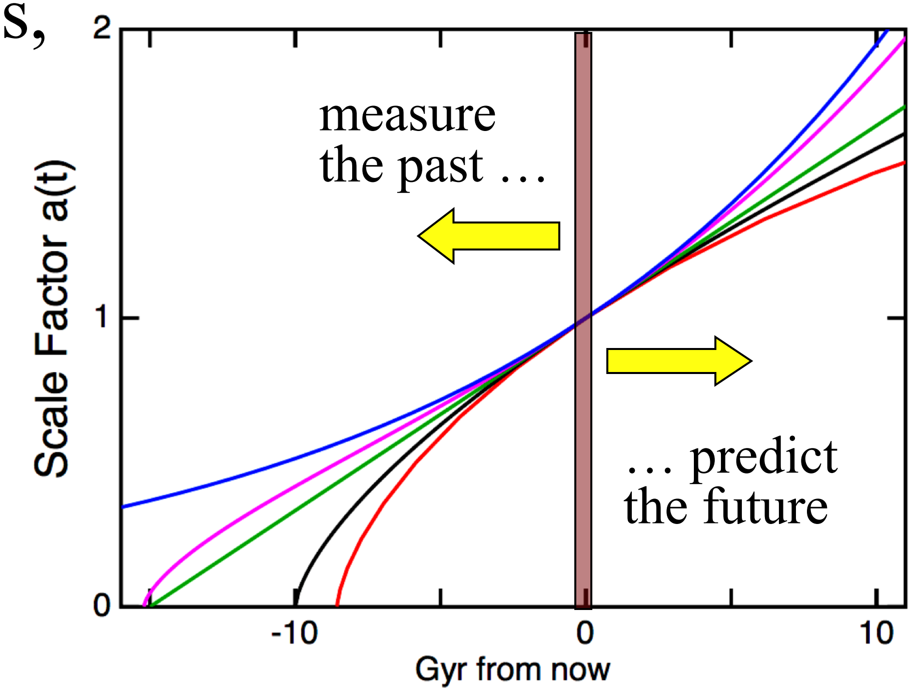





19.4 Cosmological Tests: An Introduction

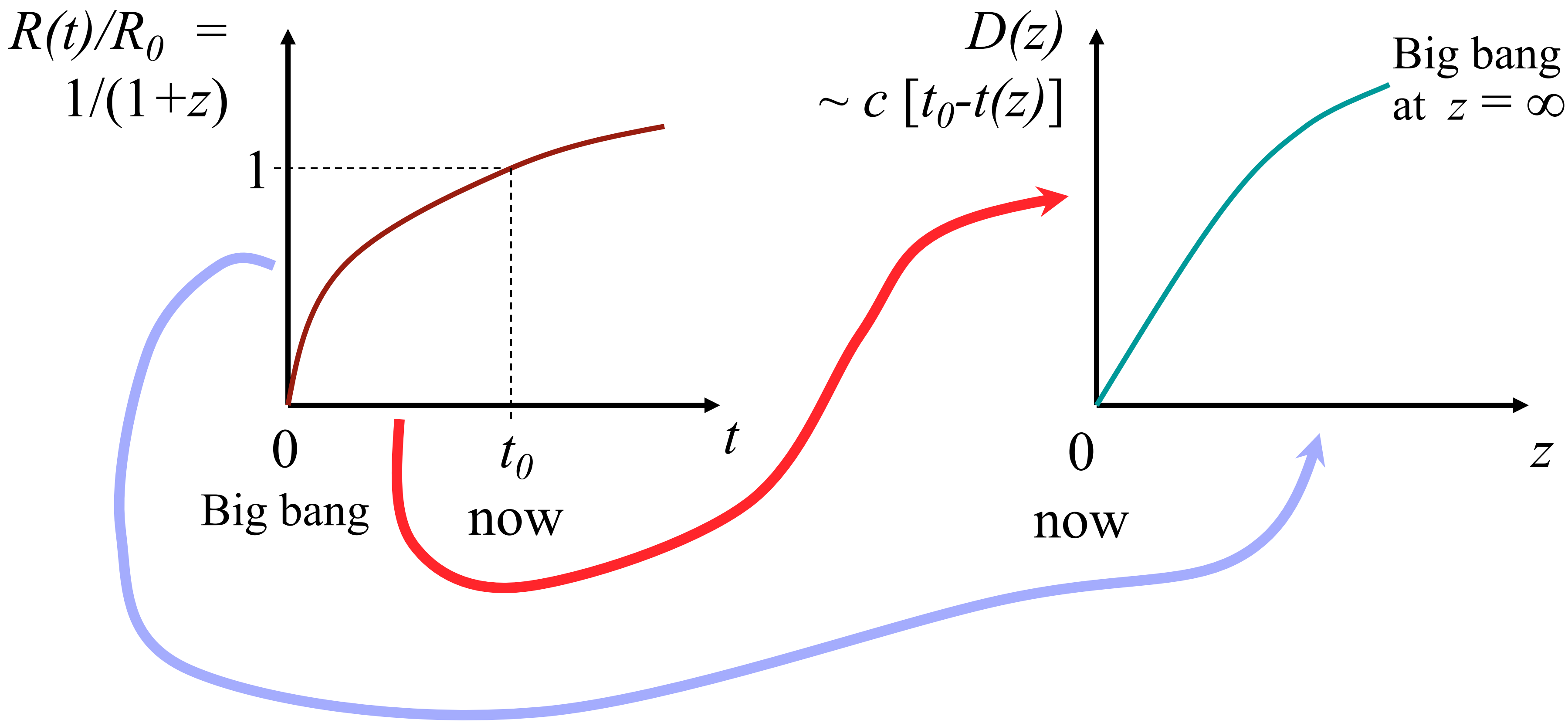


Cosmological Tests: The Why and How

- The goal is to determine the global geometry and the dynamics of the universe, and its ultimate fate
- The basic method is to somehow map the history of the expansion, and compare it with model predictions
- A model (or a family of models) is assumed, e.g., the Friedmann-Lemaitre models, typically defined by a set of parameters, e.g., H_0, $\Omega_{0,m}$, $\Omega_{0,\Lambda}$, q_0, etc.
- Model equations are integrated, and compared with the observations

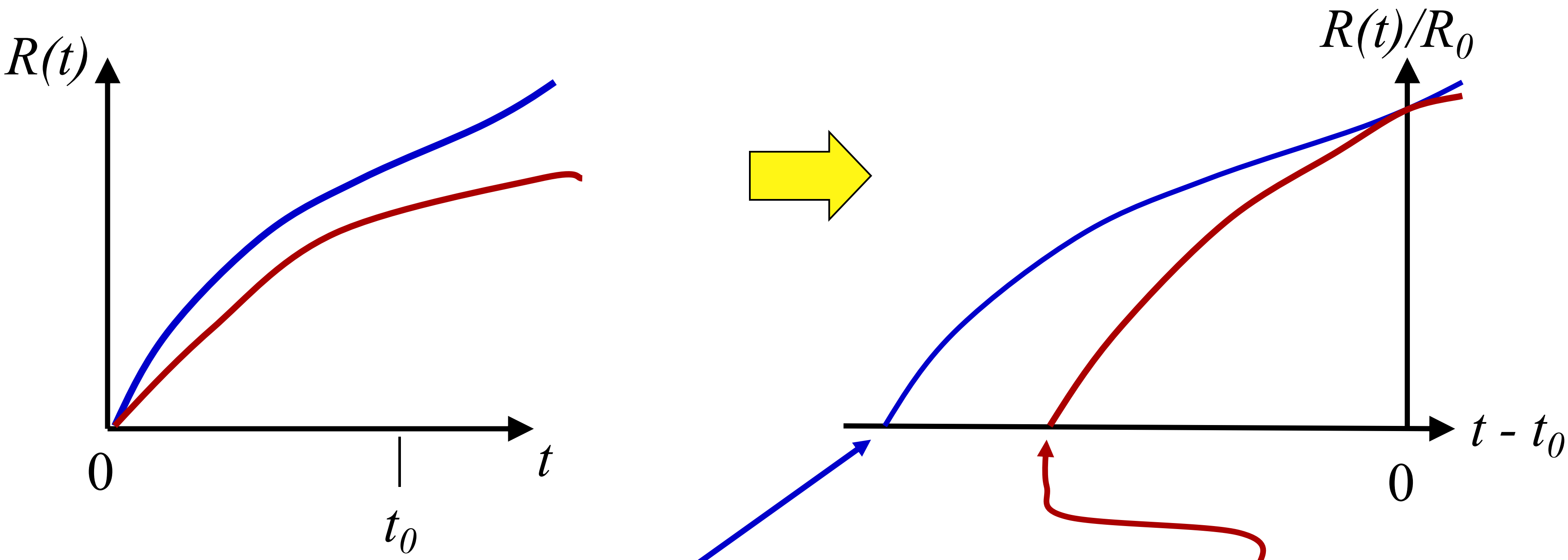


The Basis of Cosmological Tests



All cosmological tests essentially consist of comparing some measure of (relative) distance (or look-back time) to redshift. Absolute distance scaling is given by the H_0.

Cosmological Tests: Expected Generic Behavior of Various Models



Models with a lower density and/or positive Λ expand faster, are thus larger, older today, have more volume and thus higher source counts, at a given *z* sources are further away and thus appear fainter and smaller

Models with a higher density and lower Λ behave exactly the opposite

The Basic Concept

- If two sources have the same intrinsic luminosity ("standard candles"), from the ratio of their apparent brightness we can derive the ratio of their luminosity distances
- If two sources have the same physical size ("standard rulers"), from the ration of their apparent angular sizes we can derive the ratio of their angular diameter distances

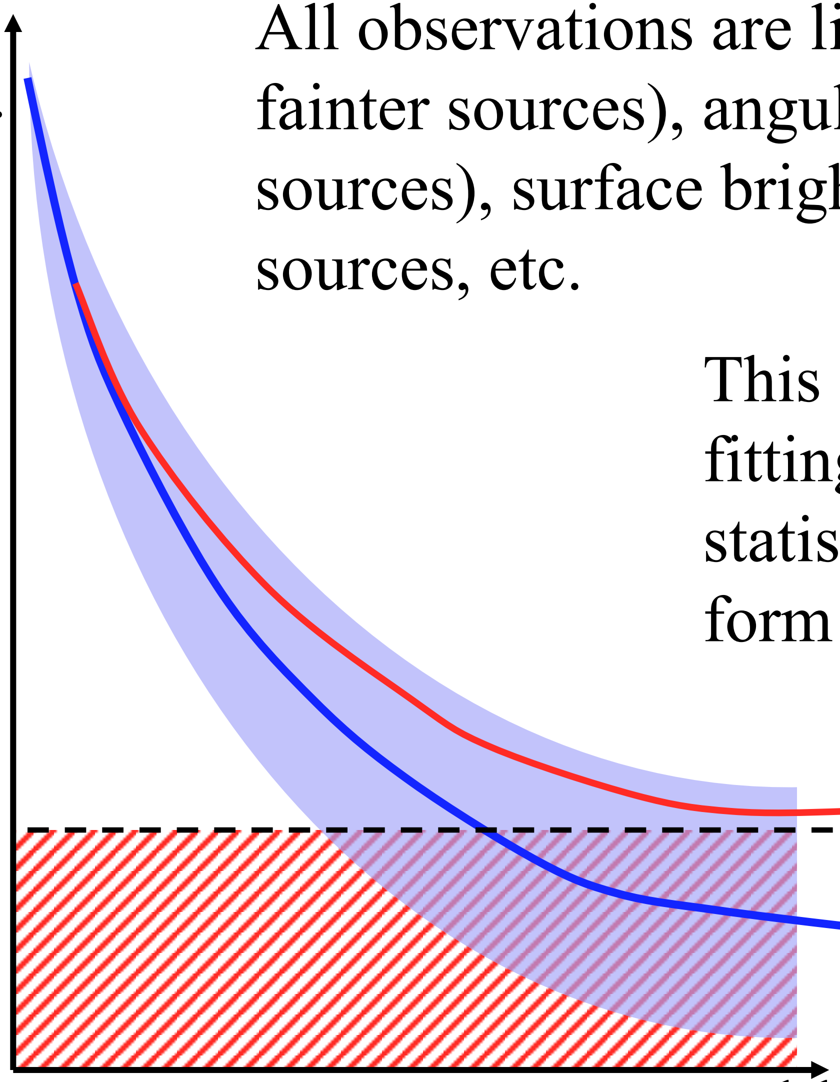
The Types of Cosmological Tests

- **The Hubble diagram:** flux (or magnitude) as a proxy for the luminosity distance, vs. redshift - requires ***"standard candles"***
- **Angular diameter** as a proxy for the angular distance, vs. redshift - requires ***"standard rulers"***
- **Source counts** as a function of redshift or flux (or magnitude), probing the evolution of a volume element - requires a population of sources with a constant comoving density - ***"standard populations"***
- Indirect tests of age vs. redshift, usually highly model-dependent - ***"standard clocks"***
- Local dynamical measurements of the mass density, Ω_{m0}
- If you measure H_0 and t_0 independently, you can constrain a combination of Ω_{m0} and Ω_Λ

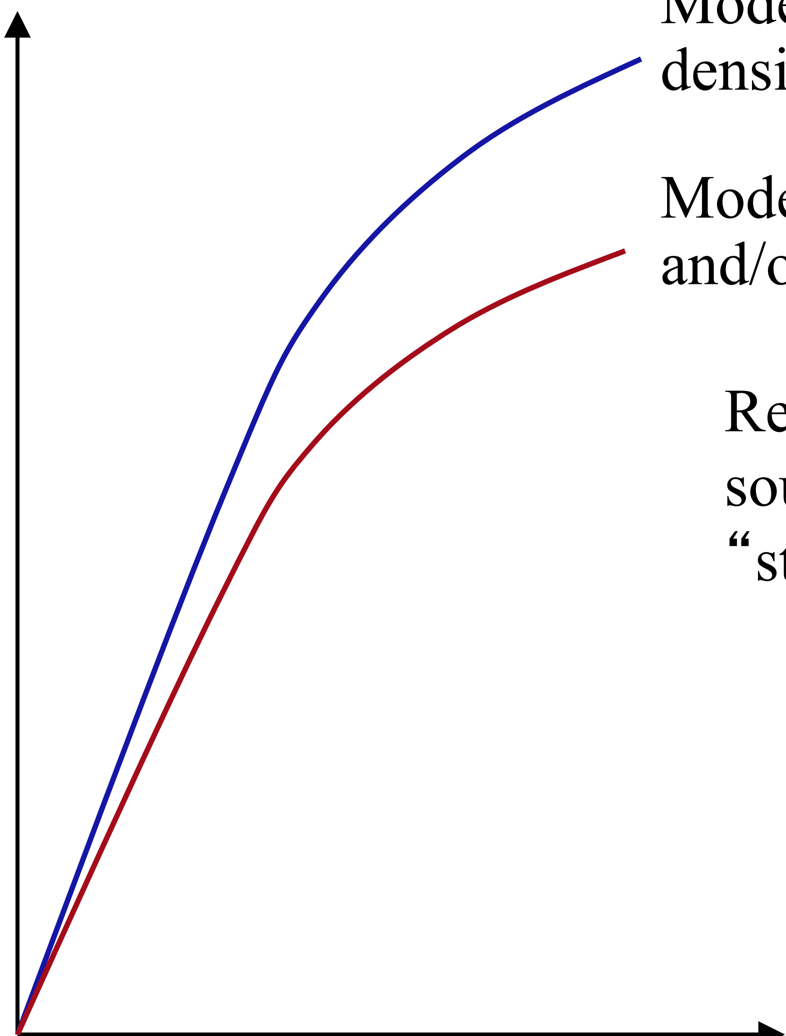
Selection Effects and Biases

All observations are limited in sensitivity (we miss fainter sources), angular resolution (we miss smaller sources), surface brightness (we miss very diffuse sources, etc.

This inevitably introduces a bias in fitting the data, unless a suitable statistical correction is made - but its form may not be always known!



The Hubble Diagram

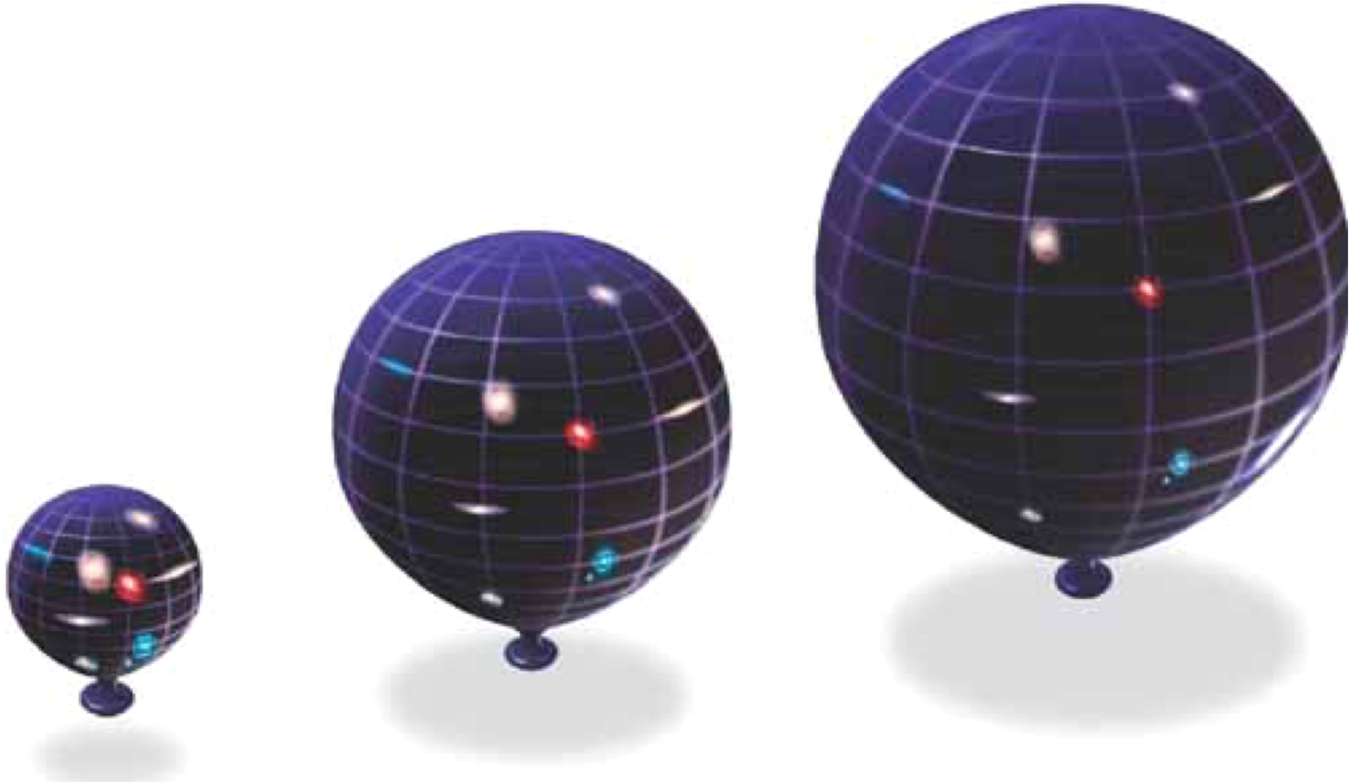


Requires a population on non-evolving sources with a fixed luminosity - “standard candles”. Some candidates:

- Brightest cluster ellipticals
- Supernovae of type Ia
- Luminosity functions in clusters
- GRB afterglows ??
- …

Tests for the Expansion of the Universe

- Tolman surface brightness (SB) test
  - In a stationary, Euclidean universe $SB = const.$, but in an expanding, relativistic universe it scales as $SB \sim (1+z)^{-4}$
- Time dilation of Supernova light curves
  - Time stretches by a factor of $(1+v/c) = (1+z)$



The Tolman Test

Surface brightness is flux per unit solid angle:
$$SB = \frac{f}{d\omega}$$

This is the same as the luminosity per unit area, at some distance D. In cosmology,
$$SB = \frac{L}{D_L^2}\frac{D_A^2}{dl^2}$$

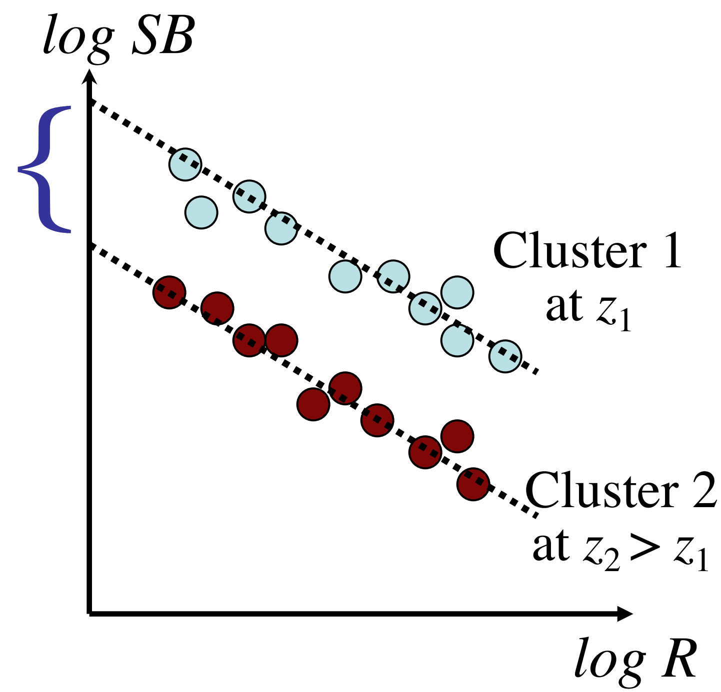
In a stationary, Euclidean case, $D = D_L = D_A$, so the distances cancel, and $SB = const$. But in an expanding universe, $D_L = D\,(1+z)$, and $D_A = D\,/\,(1+z)$, so:
$$SB = \frac{L}{dl^2}\frac{D_A^2}{D_L^2} = \frac{L}{dl^2}(1+z)^{-4}$$

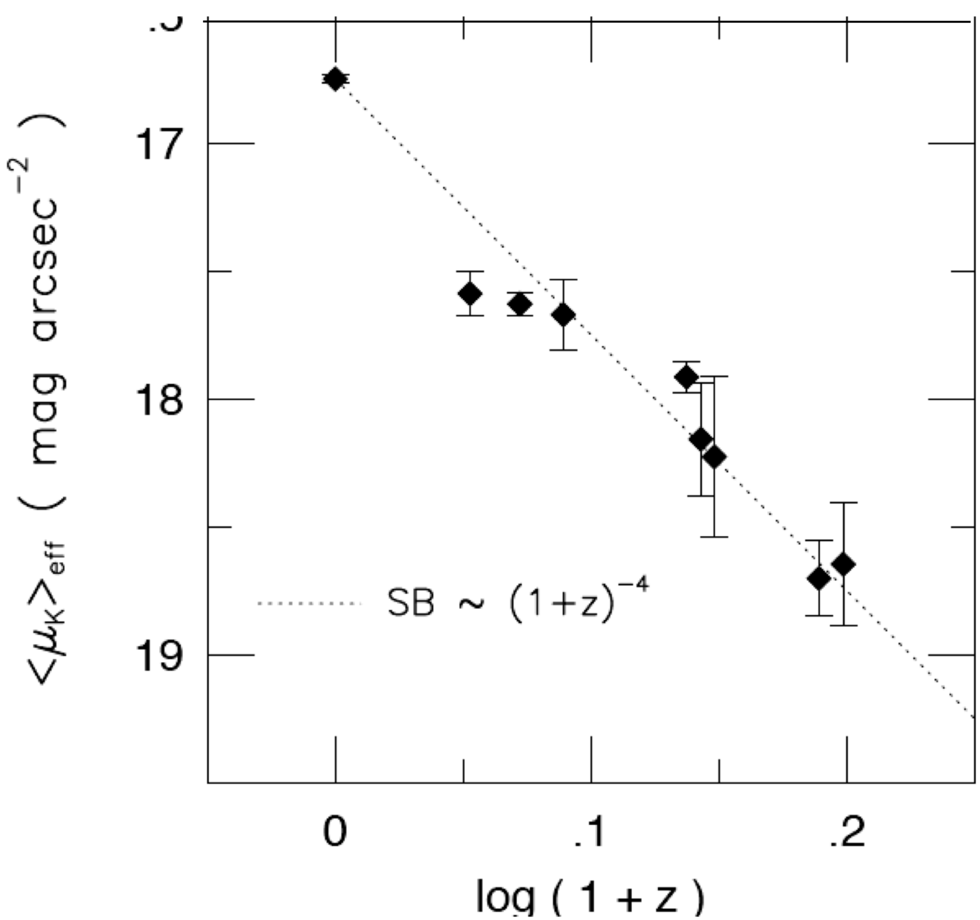
We need a standard (constant) unit of *SB*, to observe at a range of redshifts (a “standard fuzz”?)

A good choice is the ***intercept of SB scaling relations*** for elliptical galaxies in clusters

Performing the The Tolman Test

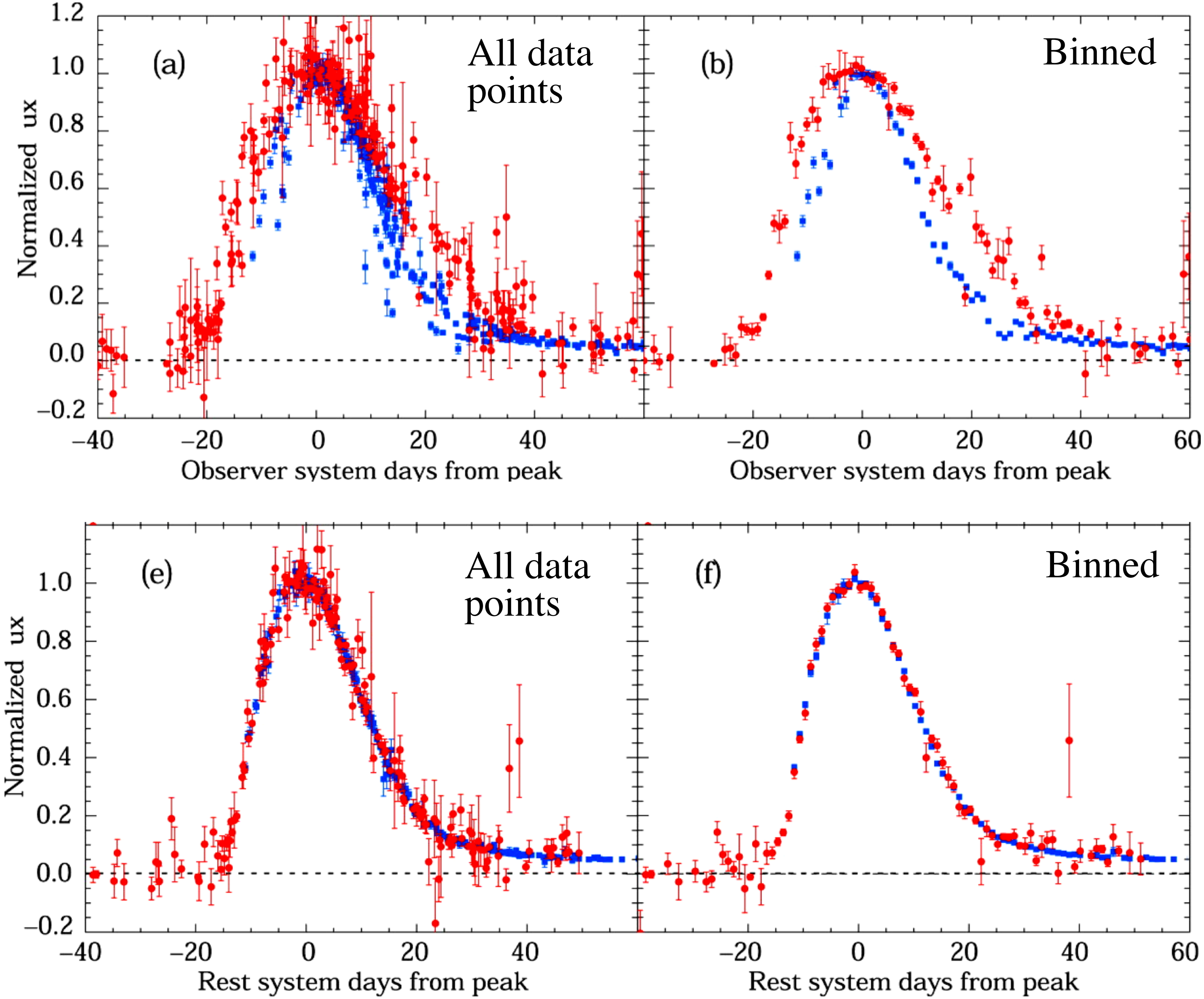
Use the SB-Radius and the Fundamental Plane correlations, with SB on the Y axis:





After a mild evolution correction, the results confirm the prediction of the relativistic expansion

Time Dilation of Supernova Lightcurves



Blue dots: a low-z dataset

Red dots: a high-z dataset

After applying the proper stretch factor

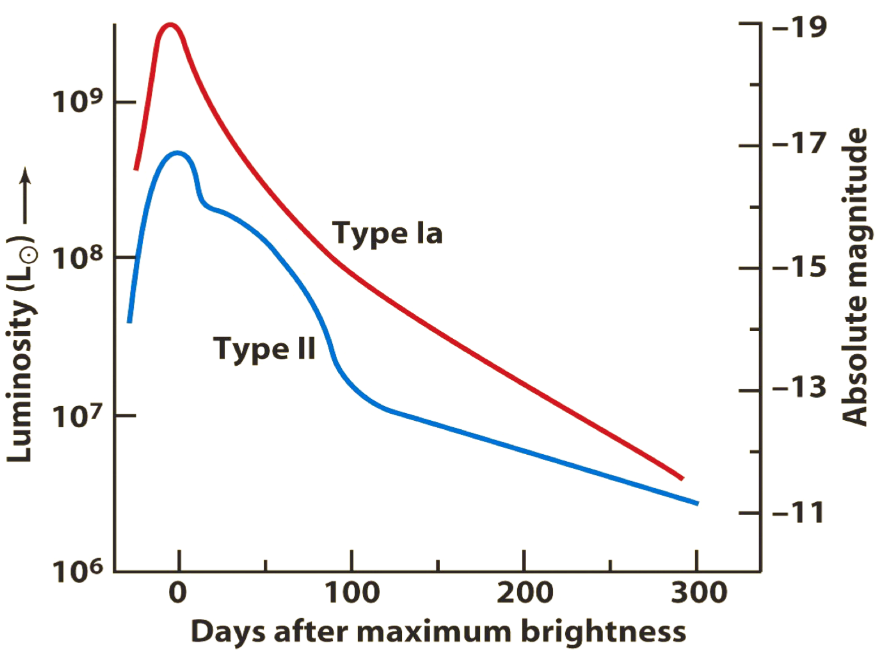
(Goldhaber et al.)

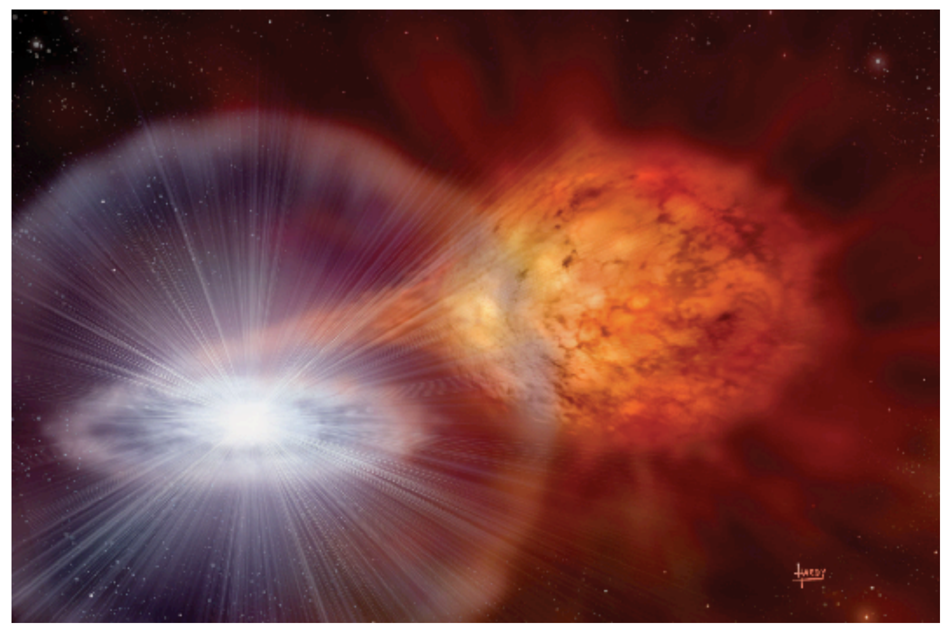
19.5 Supernova Standard Candles

and the Hubble Diagram

Supernovae (SNe) as Standard Candles

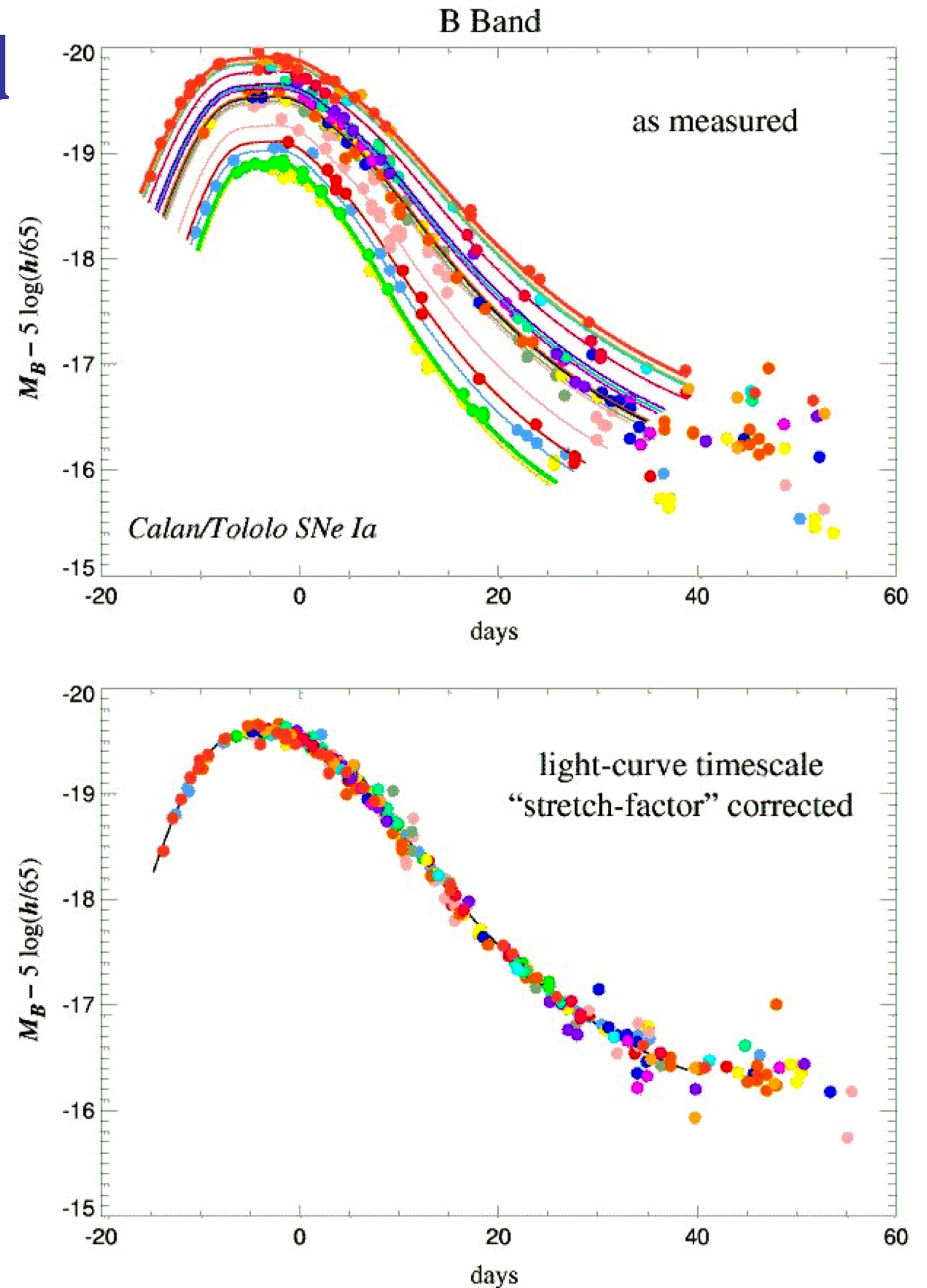
- Bright and thus visible far away
- **Type Ia** SNe are used as standard candles:
  - Binary white dwarfs accreting material and detonating
  - Pretty good standard candles, peak $M_V \sim -19.3$
  - There scatter can be removed by using a light curve shape stretch factor to a peak magnitude scatter of ~ 10%





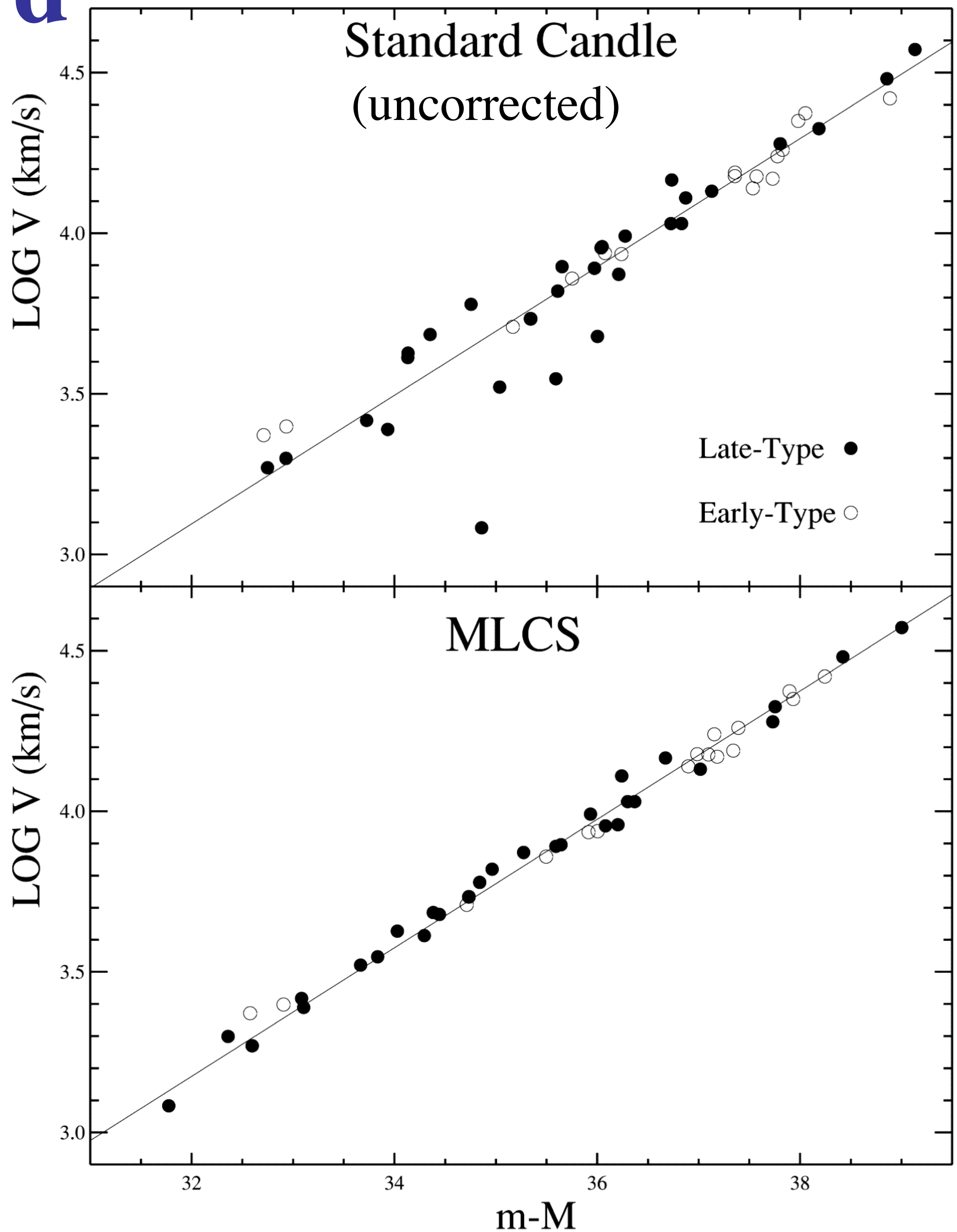
SNe Ia as Standard Candles

- The peak brightness of a SN Ia correlates with the shape of its light curve (steeper → fainter)
- Correcting for this effect standardizes the peak luminosity to ~10% or better
- However, the absolute zero-point of the SN Ia distance scale has to be calibrated externally, e.g., with Cepheids

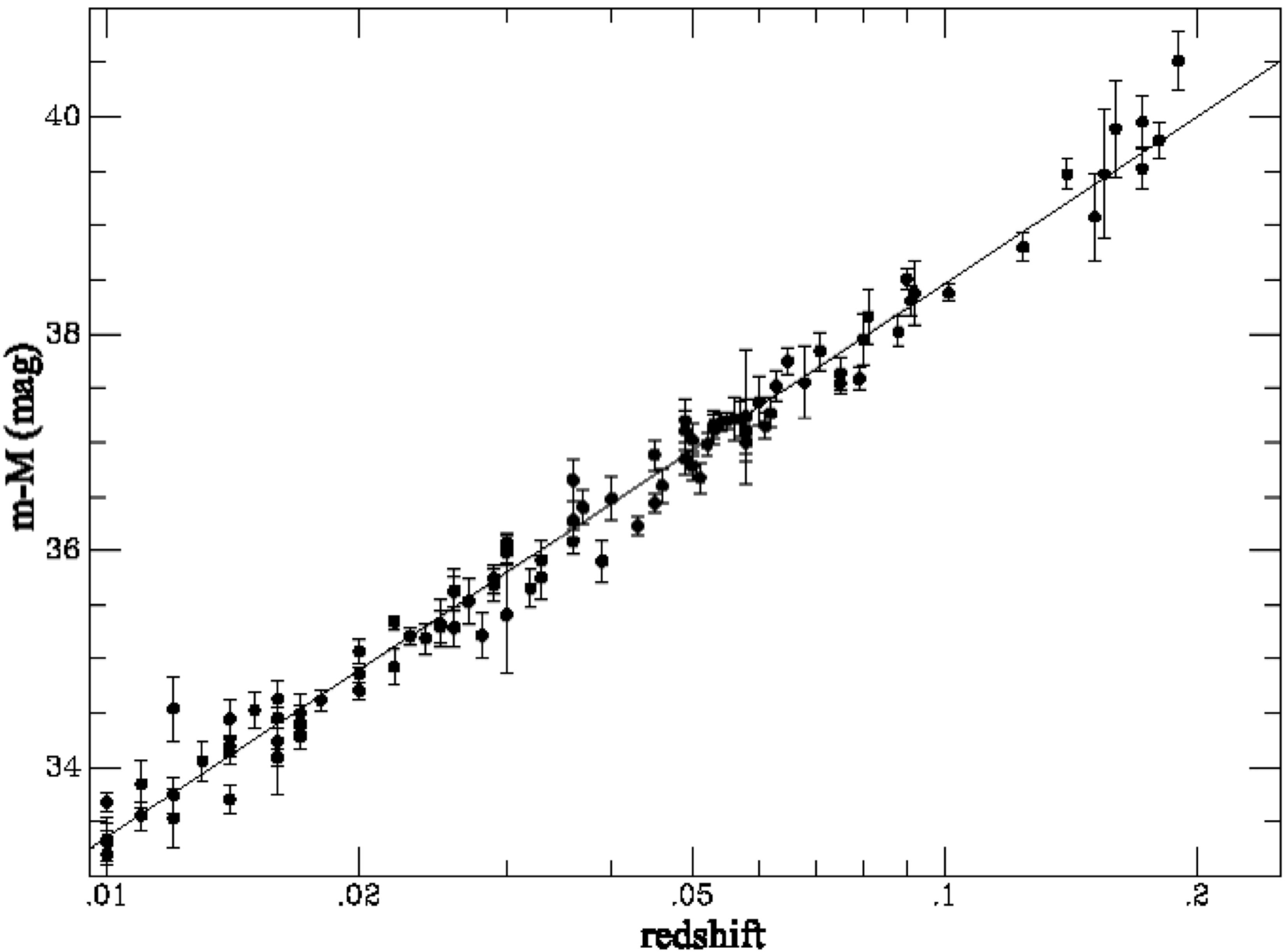


SNe Ia as Standard Candles

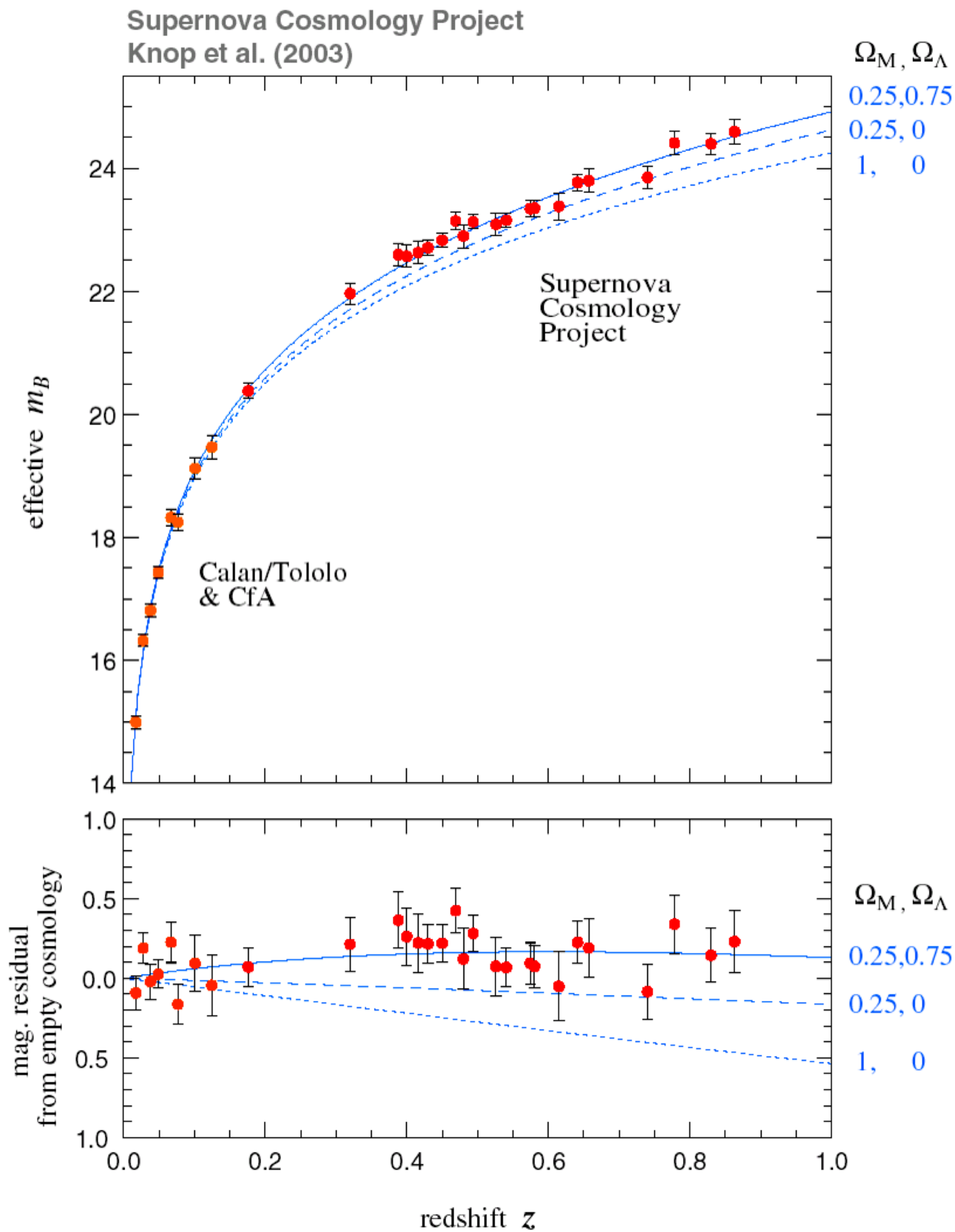
- A comparable or better correction also uses the color information (the Multicolor Light Curve method)
- This makes SNe Ia a superb cosmological tool (note: you only need relative distances to test cosmological models; absolute distances are only needed for the H_0)



The Low-Redshift SN Ia Hubble Diagram

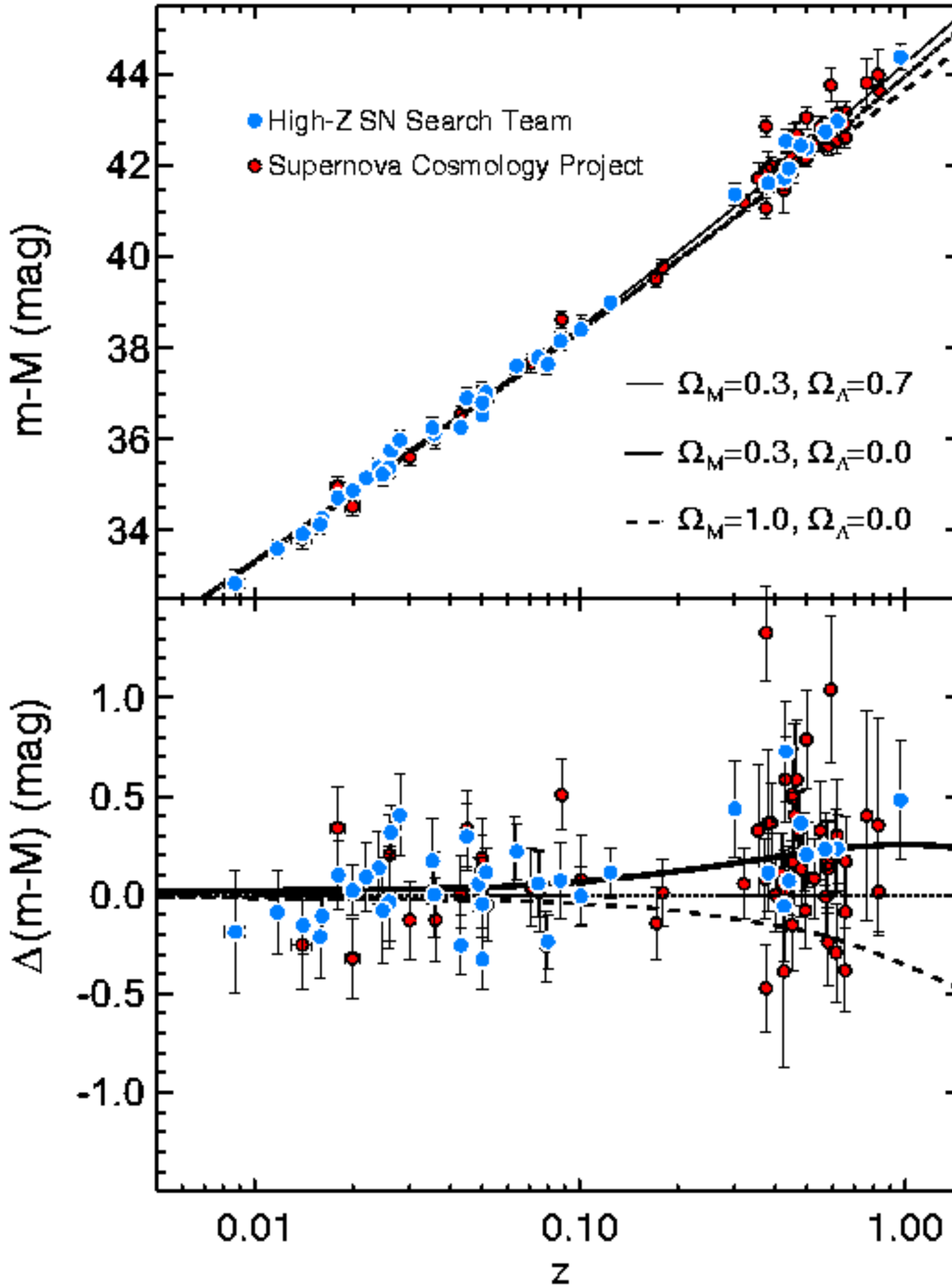


This yielded the **evidence for an accelerating universe and the positive cosmological constant,** independently and simultaneously by two groups:
The Supernova Cosmology Project at LBL (Perlmutter et al.), and …



… and by the High-Z Supernova Team
(B. Schmidt, A. Riess, et al.)

Both teams found very similar results …

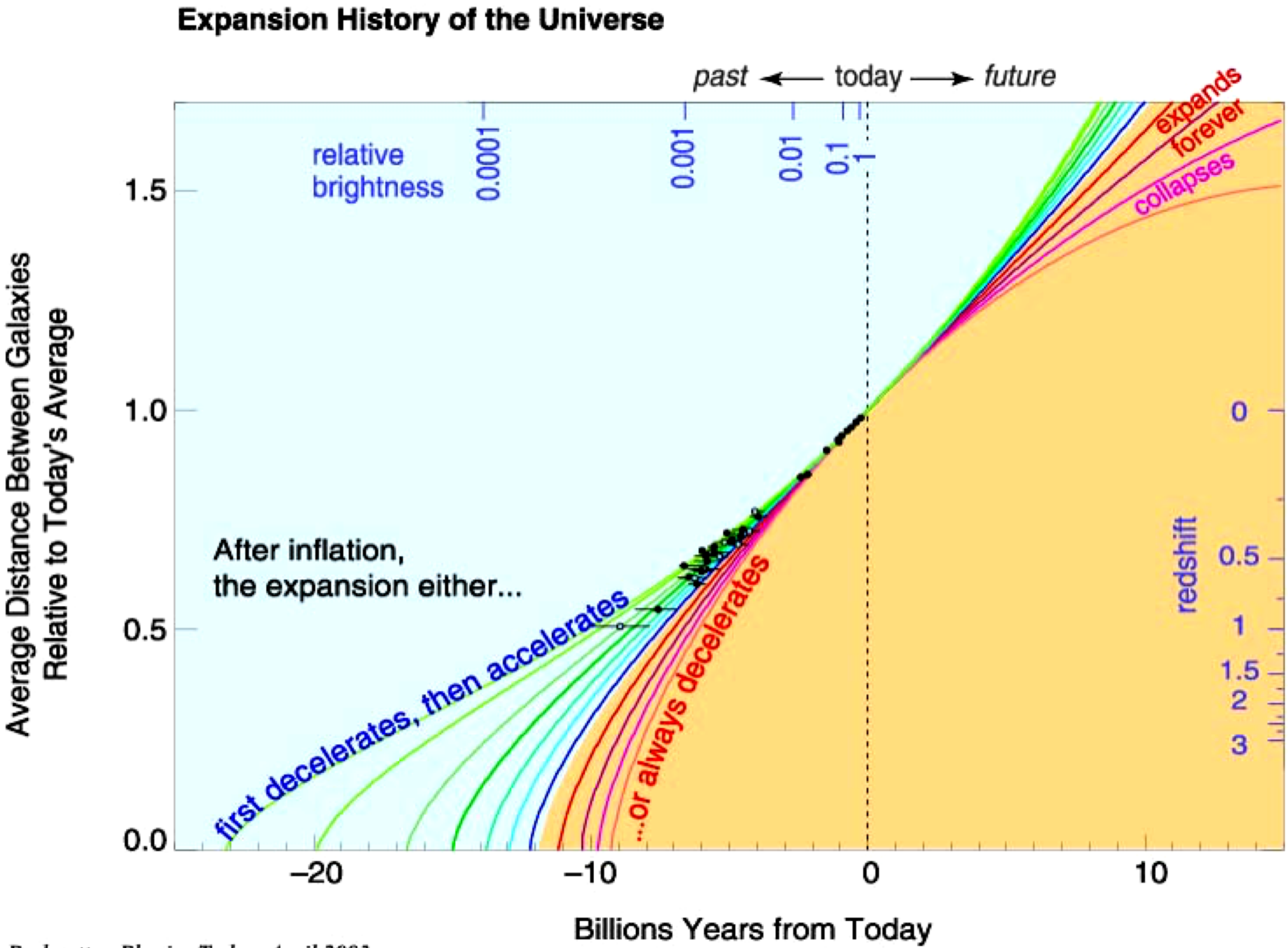


… So They Got a Nobel Prize

A. Riess

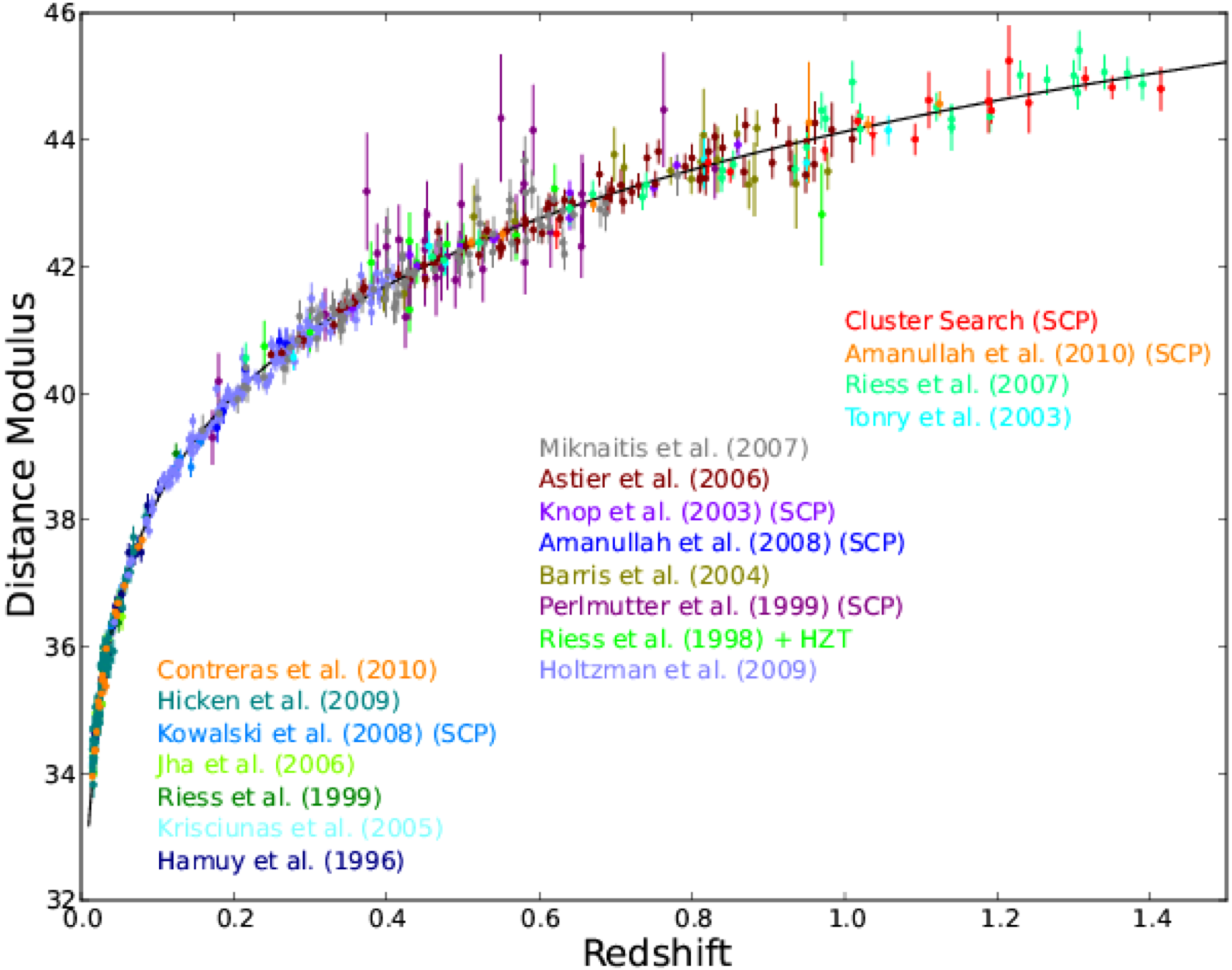
S. Perlmutter

B. Schmidt

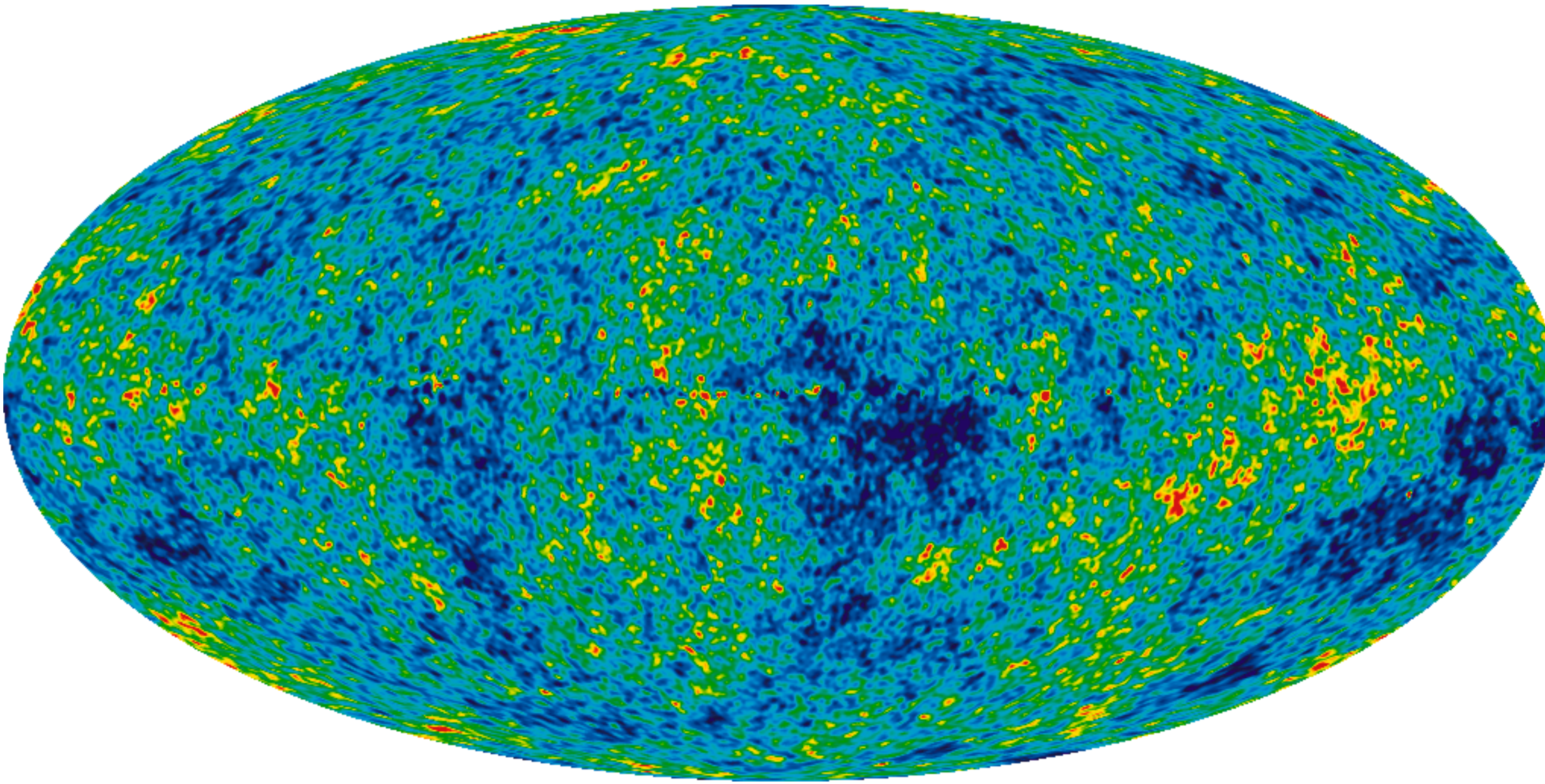


Perlmutter, Physics Today, April 2003

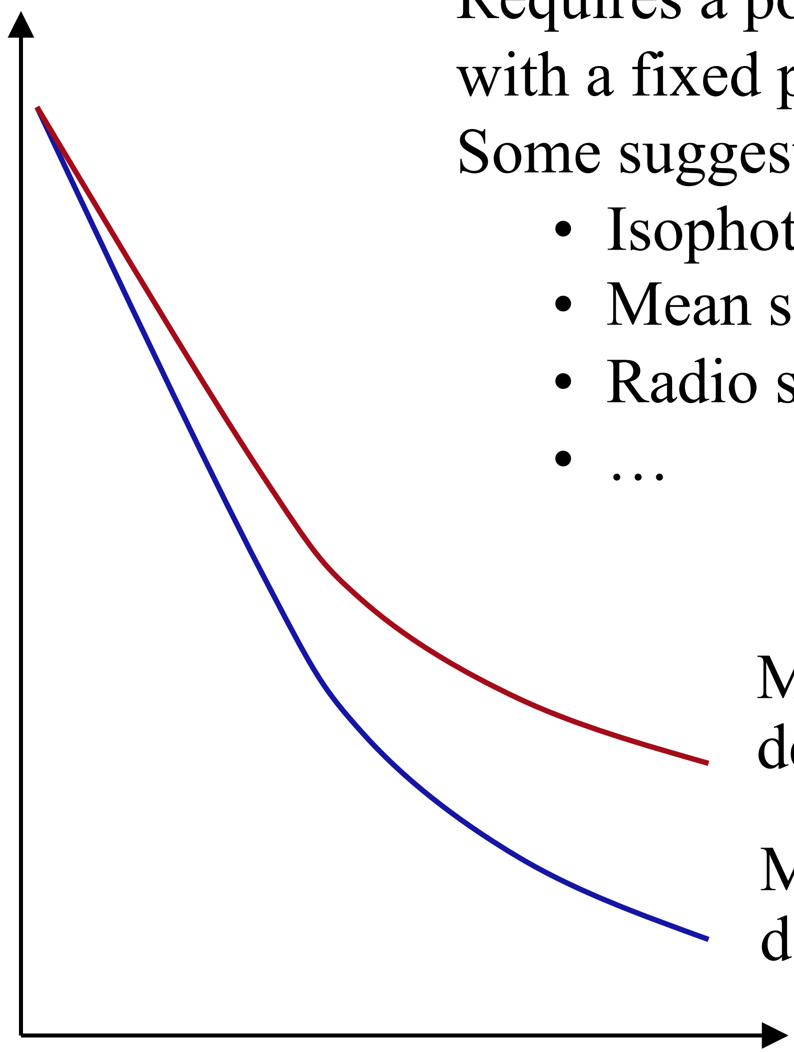
A Modern Version of the SN Hubble Diagram



19.6 Cosmology With the Cosmic Microwave Background



The Angular Diameter Test



Requires a population on non-evolving sources with a fixed proper size - "standard rulers".

Some suggested candidates:

- Isophotal diameters of brightest cluster gal.
- Mean separation of galaxies in clusters
- Radio source lobe separations
- …

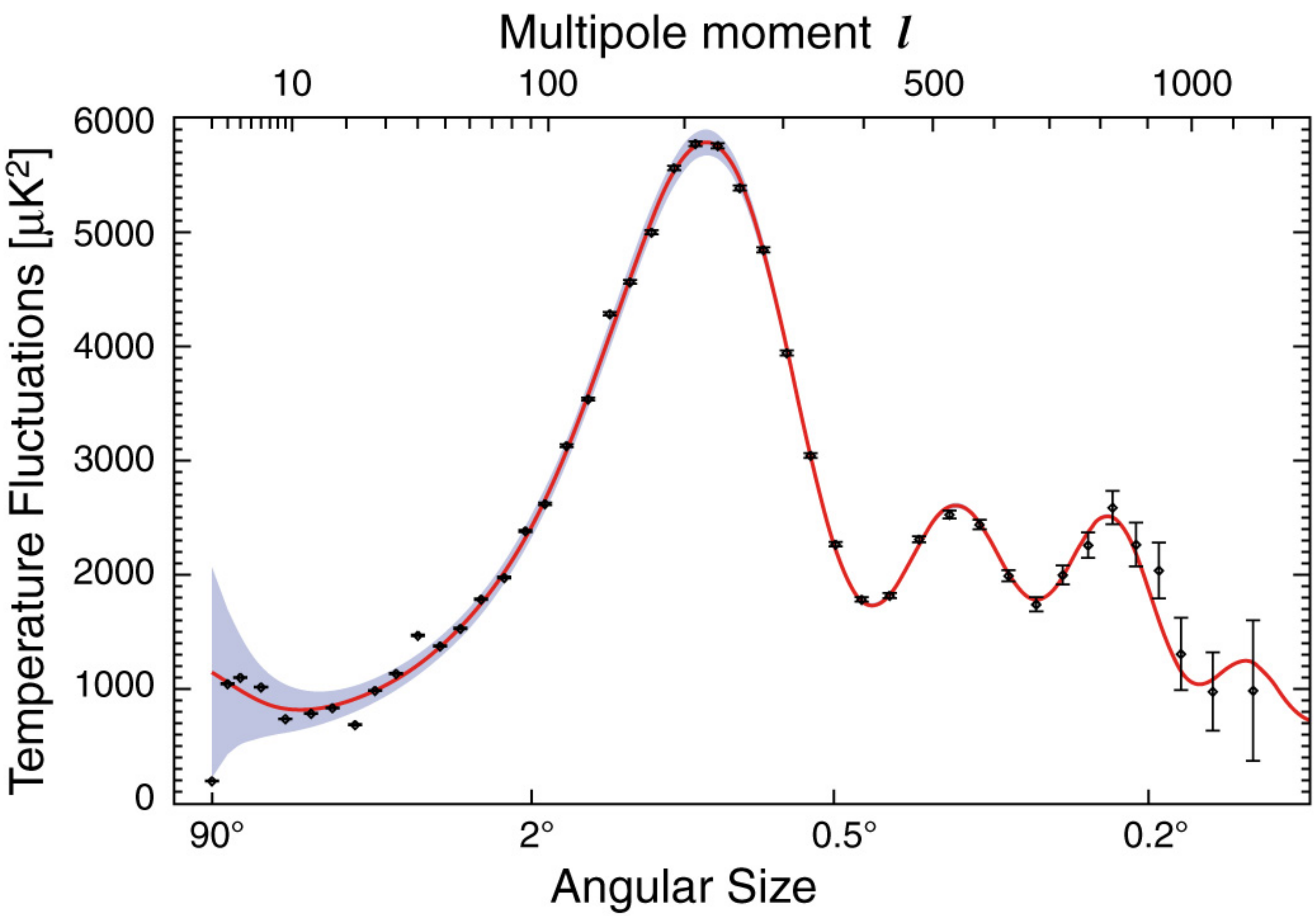
The Modern Angular Diameter Test: CMBR Fluctuations

- Uses the size of the particle horizon at the time of the recombination (the release of the CMBR) as a standard ruler
- This governs the largest wavelength of the sound waves produced in the universe then, due to the infall of baryons into the large-scale density fluctuations
- These sound waves cause small fluctuations in the temperature of the CMB ($\Delta T/T \sim 10^{-5}$ - 10^{-6}) at the appropriate angular scales (~ a degree and less)
- They are measured as the angular power spectra of temperature fluctuations of the CMBR

Is the Universe Flat, Open, or Closed?

Doppler peaks define a physical scale of the particle horizon at recombination. The corresponding angular size depends on the geometry of the universe

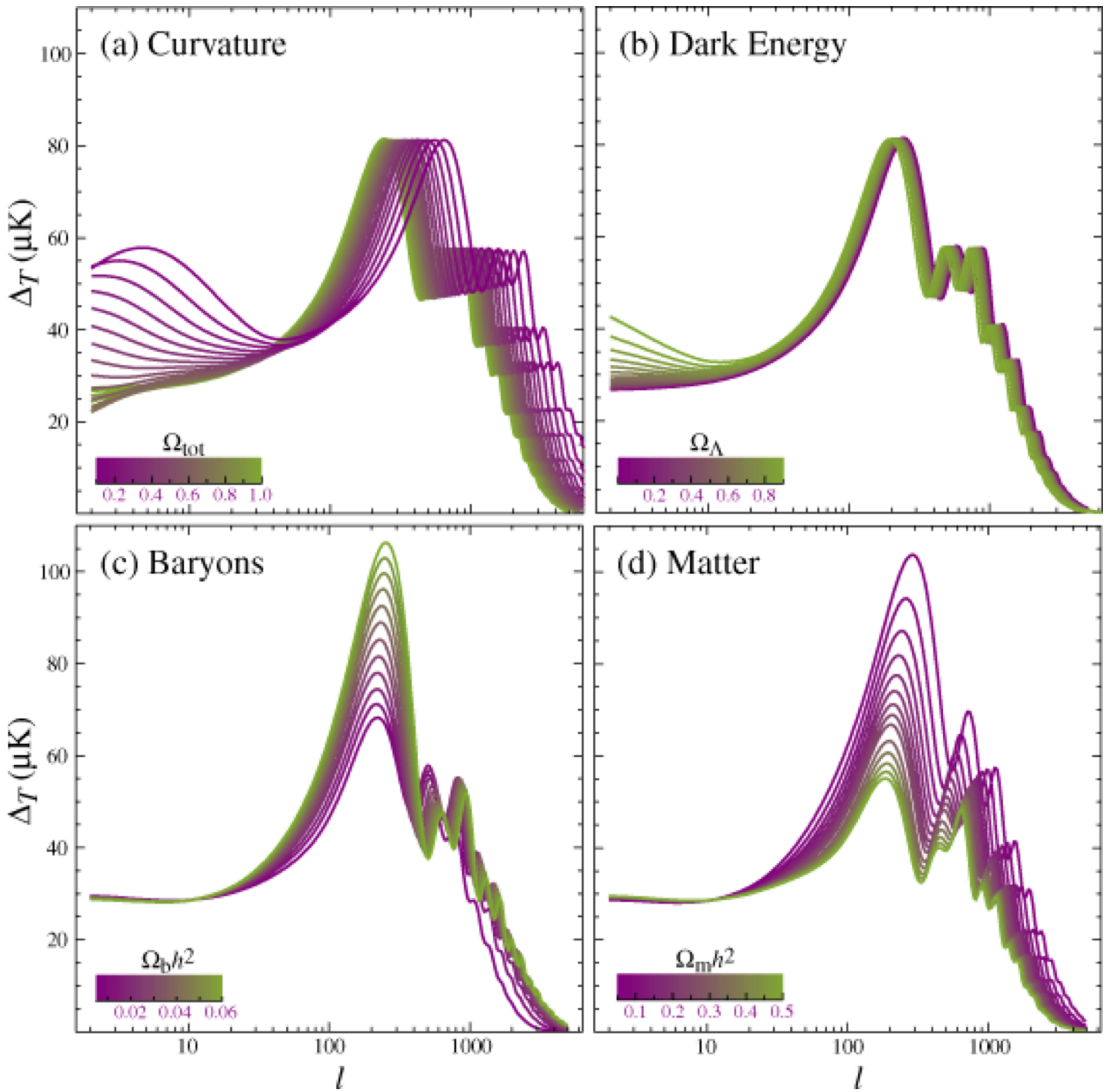
$$l = 220 \Rightarrow \Omega_{total} = 1.02 \pm 0.02$$



The universe is flat (or very close to flat)

WMAP 7 yr data

Positions and amplitudes of peaks depend on a variety of cosmological parameters in a complex fashion



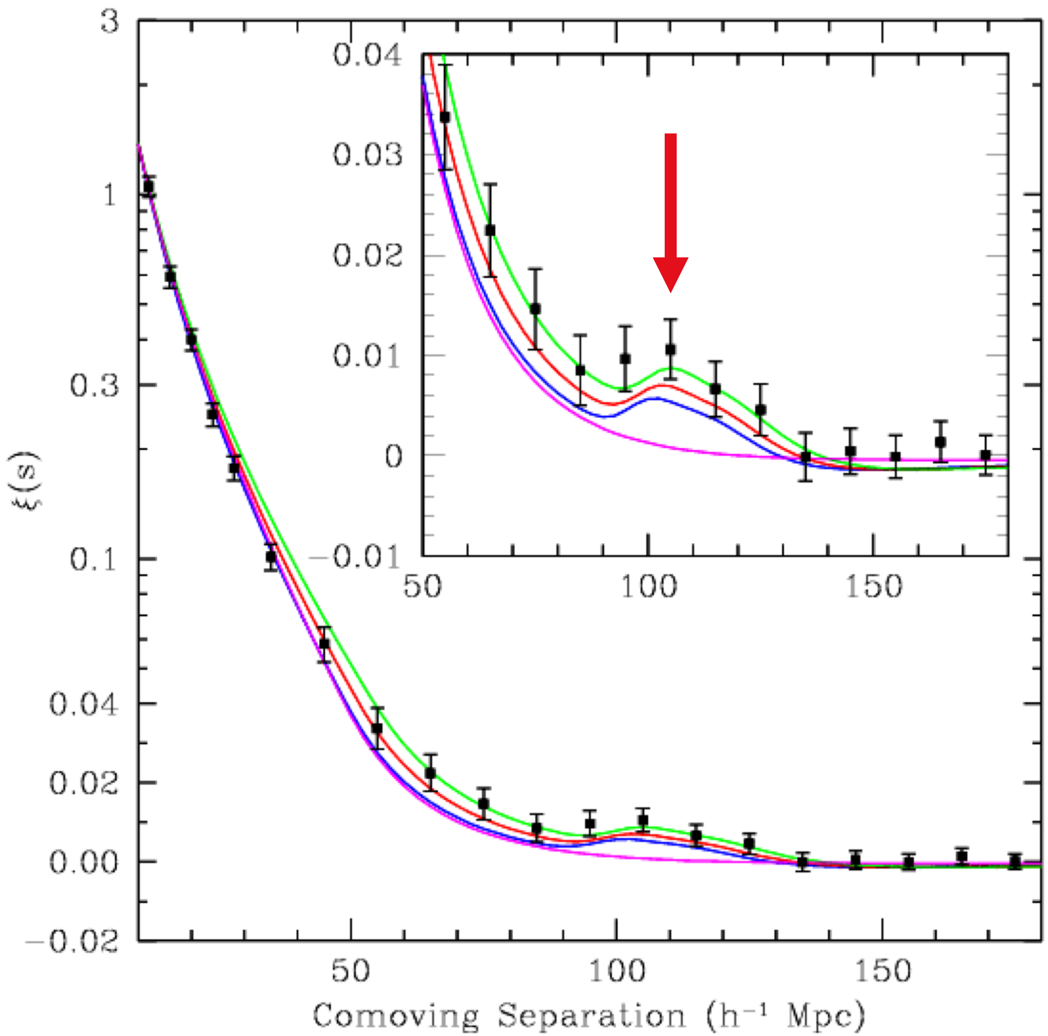
Thus, CMB fluctuations can also constrain other cosmological parameters, including the H_0, the age of the universe, relative contributions of the baryons, the dark matter, the dark energy, etc.

Planck results, 2015

Parameter	TT+lowP 68% limits	TT+lowP+lensing 68% limits	TT+lowP+lensing+ext 68% limits	TT,TE,EE+lowP 68% limits	TT,TE,EE+lowP+lensing 68% limits	TT,TE,EE+lowP+lensing+ext 68% limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023

Baryon Acoustic Oscillations (BAO)

Eisenstein et al. 2005 (using SDSS red galaxies); also seen by the 2dF redshift survey



The 1st Doppler peak seen in the CMBR imprints a preferred scale for clustering of galaxies.

Detection of this feature in galaxy clustering at $z \sim 0.3$ gives us another instance of a "standard ruler" for an angular diameter test, at redshifts $z < 1100$

Future redshift surveys can do much better yet