Basic Cosmology

Expanding Universe, Cosmological Models, Parameters, and Tests

Space, Time, Matter, and Gravity

In Einstein's ***Special Theory of Relativity*** [SR] (1905), *space and time are no longer a passive stage* on which physics happens. They are a part of the process.

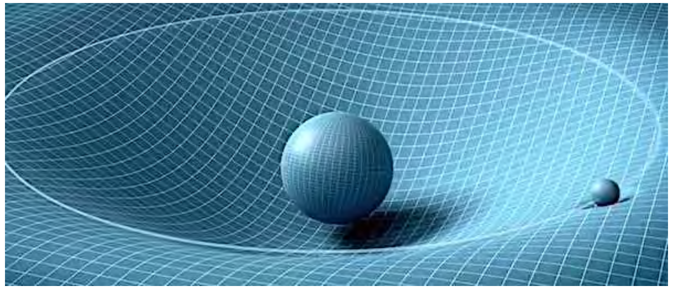
SR postulates equivalence among all *unaccelerated frames of reference* (inertial observers). This has several consequences:

- Space and time are deeply interconnected in the *spacetime*, which is a *4-dimensional* continuum (3 for space, 1 for time)
- The *speed of light c* is the maximum velocity. It is not a property of light, but a *property of spacetime*. Only massless particle (photons and gravitons) can move with that speed, any mass would require an *infinite energy* to reach it
- Lorentz contraction and time dilation; *time runs slower* in a moving frame of reference; thus the twins paradox
- *Equivalence* of mass and energy ($E=mc^2$)

But what about the ***accelerated*** frames of reference? You need …

General Theory of Relativity [GR] (1915)

Postulates equivalence among **all** frames of reference, and is thus the **theory of gravity**, and thus the theoretical basis of cosmology

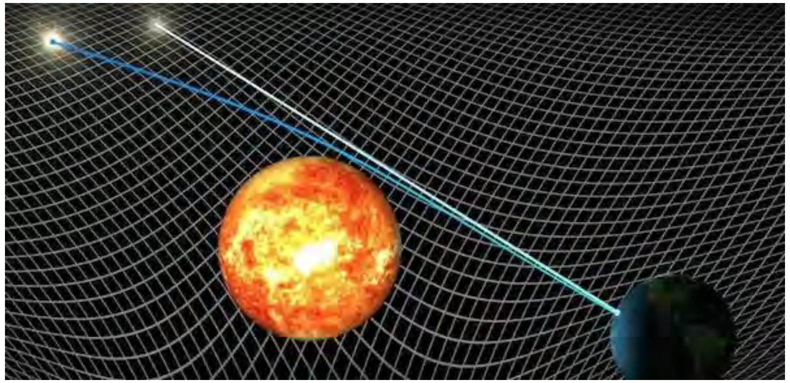


It connects space and time with the presence of matter or energy, and introduces the *curvature of space*

Presence of mass/energy determines the geometry of space
Geometry of space determines the motion of mass/energy

A consistent expression of this are the ***Einstein Equations*** (which you don't need to know)

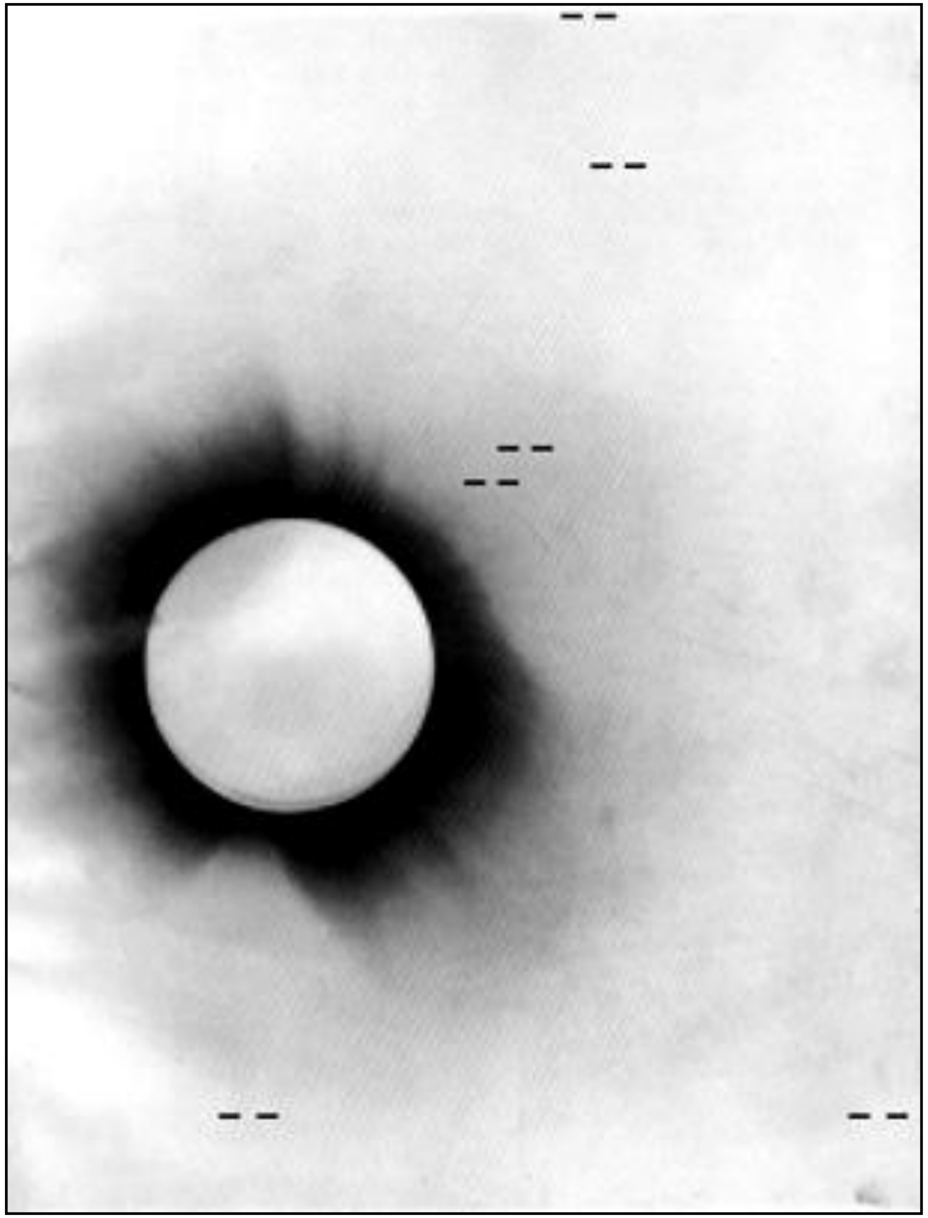
GR makes number of *testable* predictions:



- Geometry of spacetime can change in real time
- Light should be blue/redshifted in a gravitational field
- Light paths in a gravitational field should be curved
- Gravitational waves

… etc.

Experimental Confirmation of the GR



Eddington's 1919 eclipse observations confirmed Einstein's relativistic prediction of positions of stars off by α = 1.78 arcsec due to the bending of light by the Sun's gravity

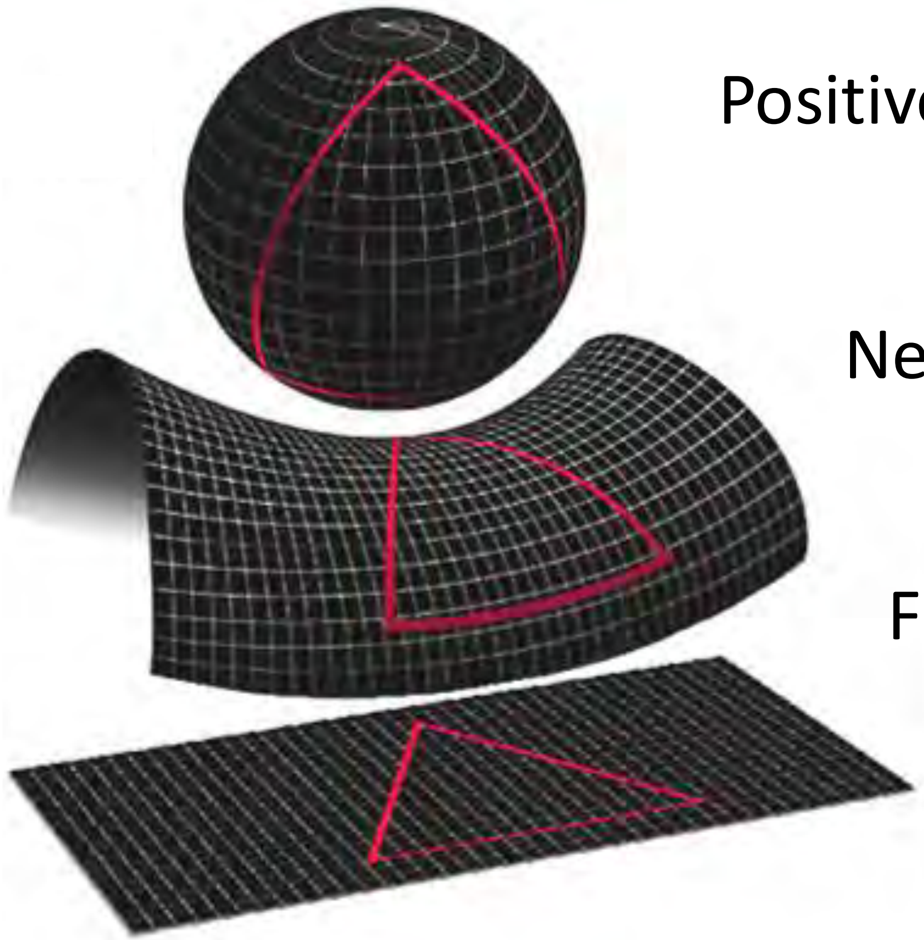
GR has passed *all* experimental tests to date, on the scales ranging from a laboratory to the entire universe. It is *the best tested physical theory ever*

Note:

- Newtonian dynamics is still an excellent approximation to SR for the velocities << *c*
- Newtonian gravity is still an excellent approximation to GR for weak gravitational fields and on the scales << universe

What is the Global Geometry of the Universe?

Generally, space geometry can be curved (Riemann geometry), and the Euclidean (flat) geometry is just a special case. It can be …



Positive (a closed surface, like a sphere, with a finite area and volume)

Negative (like a hyperboloid saddle, and infinite)

Flat (Euclidean, zero curvature, and infinite)

Which one will apply to the universe depends on the amount of matter/energy in it; its presence makes the curvature more positive

The Cosmological Principle

Relativistic cosmology uses some symmetry assumptions or principles in order to make the problem of "solving the universe" viable. The **Cosmological Principle** states that:

At each epoch, the universe is the same at all locations and in all directions, except for local irregularities

Then the only coordinate that matters is the radial displacement. That reduces 16 Einstein equations to one, called the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

(You do not have to remember it)

It essentially means:

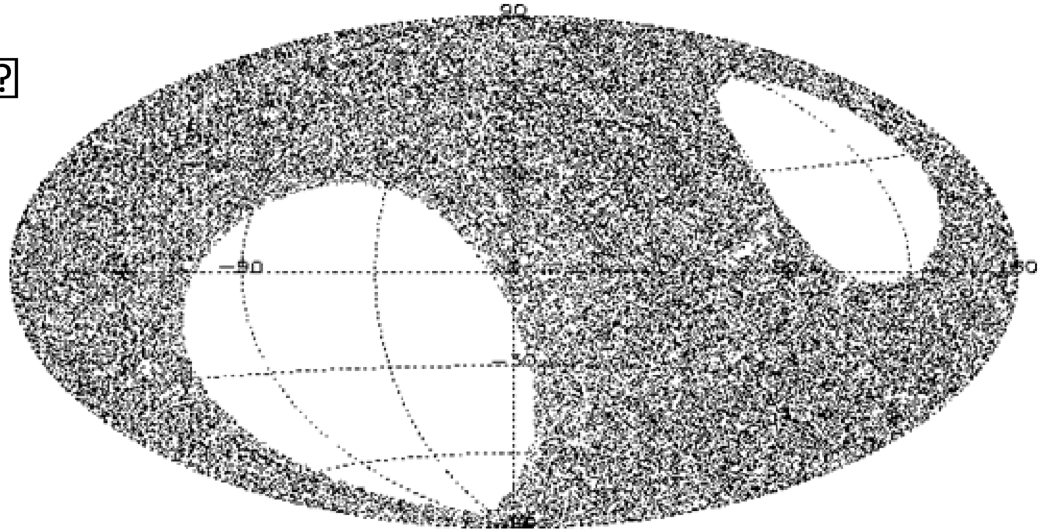
Kinematics of the universe = Matter/energy density term + Space curvature term + Cosmological constant term

More about this later

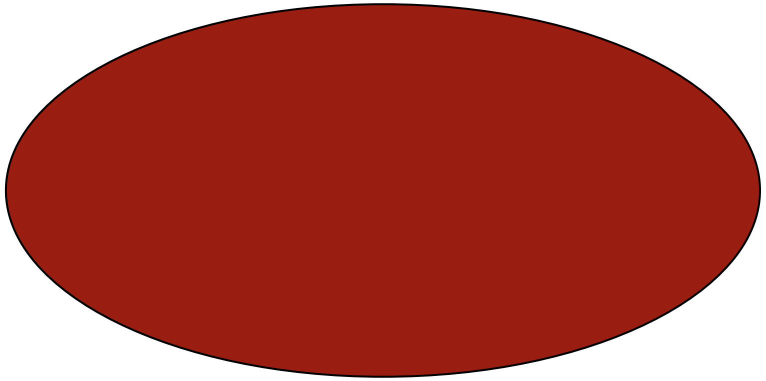
Is the Universe Really Homogeneous and Isotropic?

On scales larger than ~ 100 Mpc, say, it is - so the cosmological principle is valid

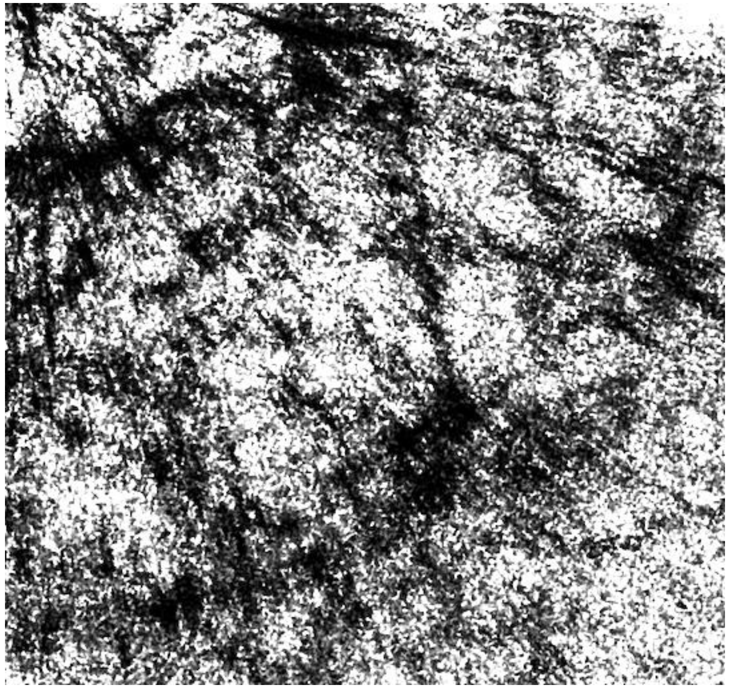
Distribution on the sky of 65000 distant radio sources from the Texas survey



CMBR sky at z = 1100 is uniform to better than $\Delta T/T < 10^{-5}$, after taking the dipole out ↓



But what about the Large Scale Structure?
↓



LSS represents 10^{-2} density nonuniformities on scales of 10-100 Mpc

It causes peculiar velocities but does no affect much the overall expansion

Discovery of the Expanding Universe

In 1929, Hubble made another great discovery: galaxies are receding away from the Milky Way with the speeds proportional to their distance:

$$V = H\ D$$

Velocity

Distance

Hubble constant ~ 70 km/s/Mpc

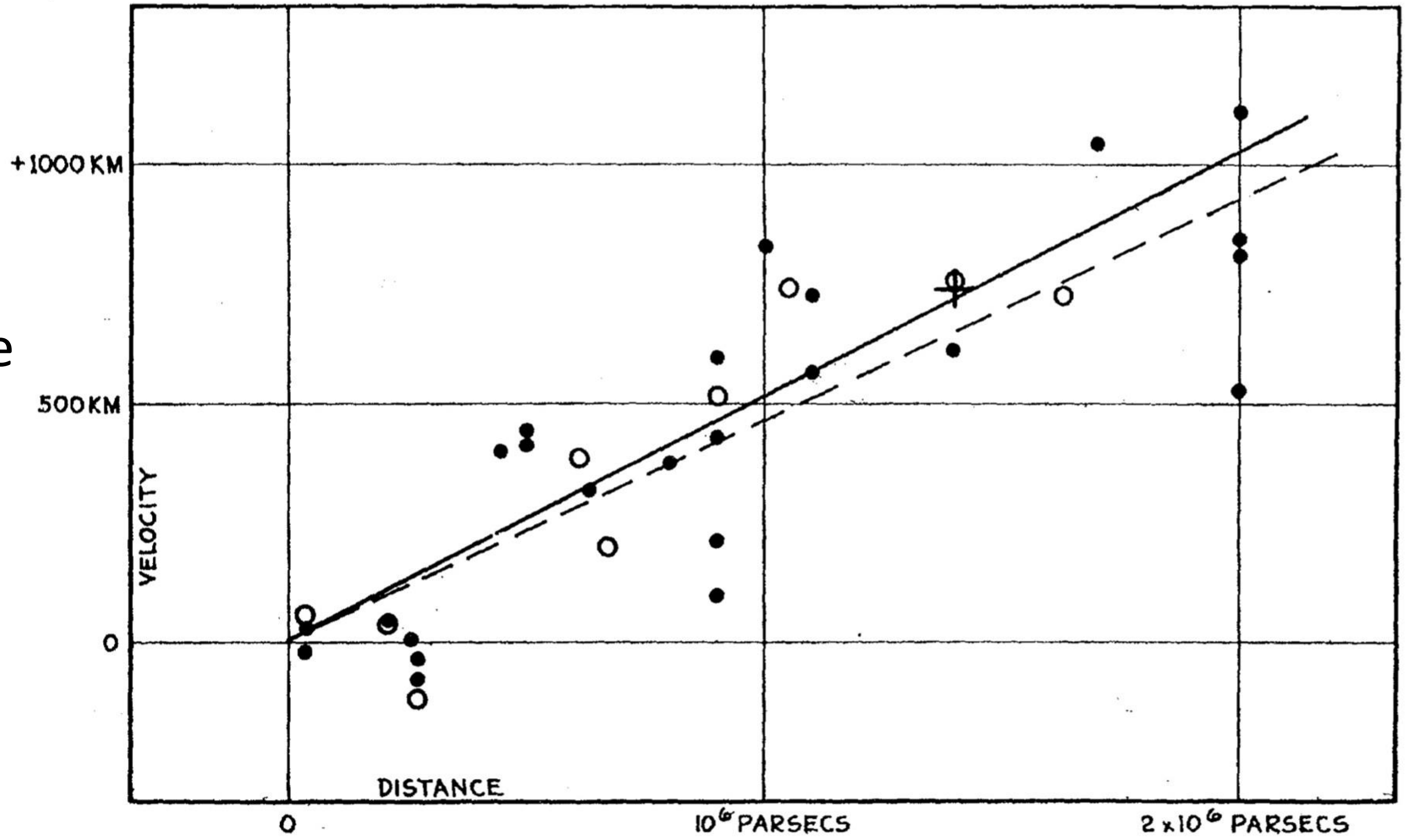


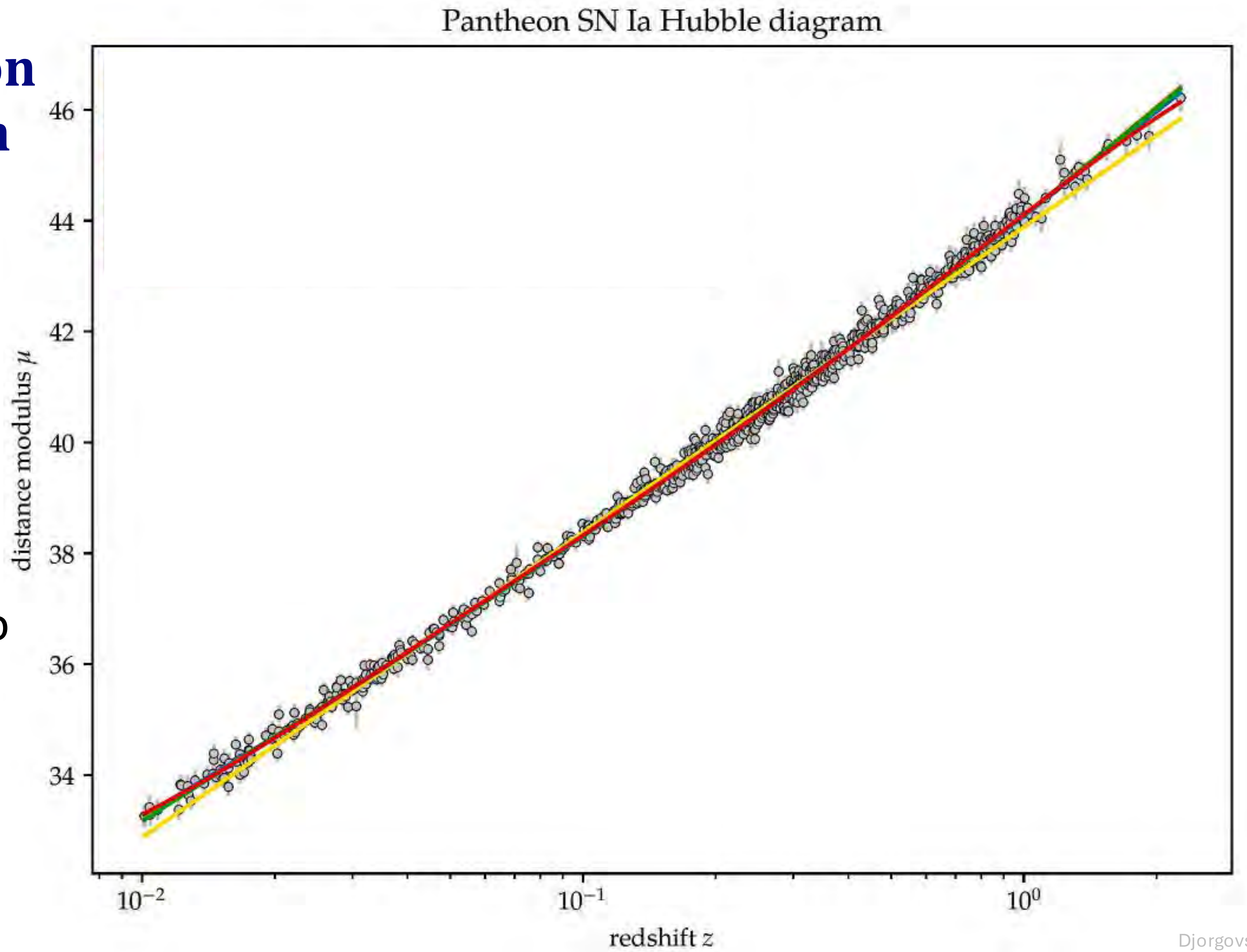
FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

This is the ***Hubble diagram***, and it plays a major role in cosmology

This implies that ***the entire universe is expanding***

A modern version of the Supernova Hubble diagram

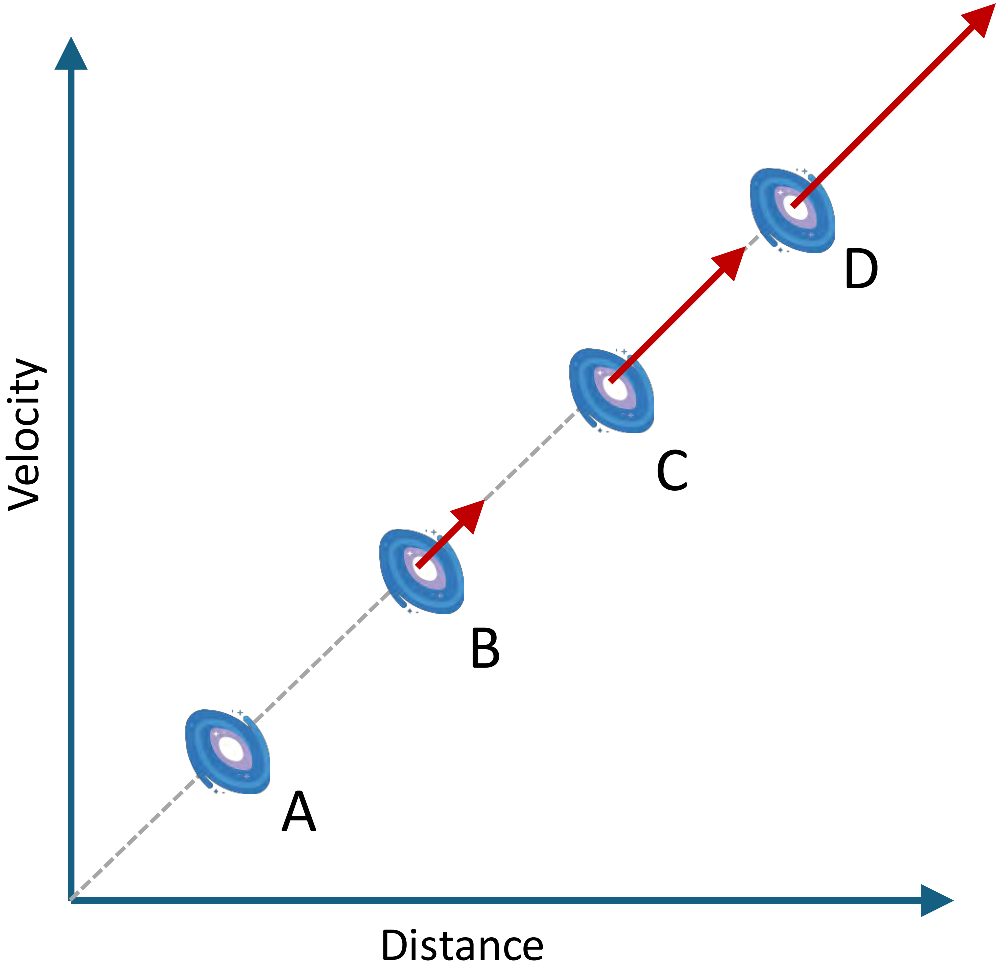


Now reaching out to many Gigaparsecs, and speeds comparable to the speed of light

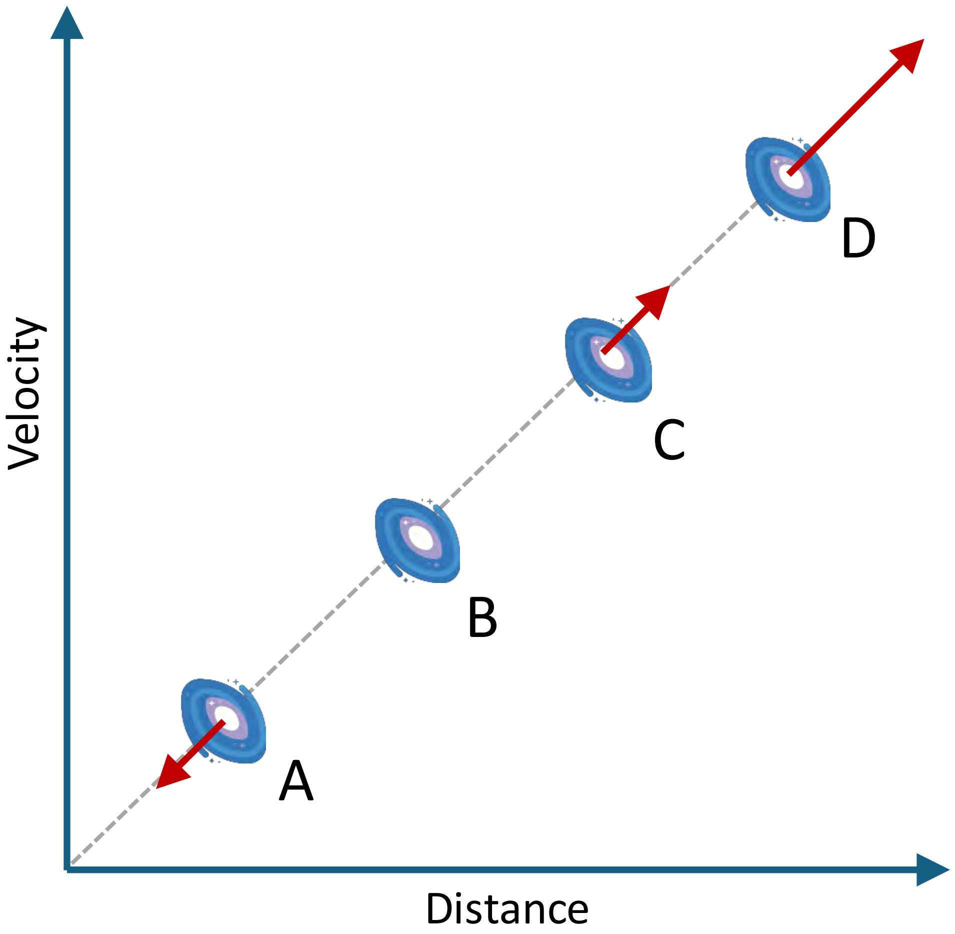
Djorgovski

Why Hubble Diagram Implies an Expansion?

From the viewpoint of galaxy A



From the viewpoint of galaxy B



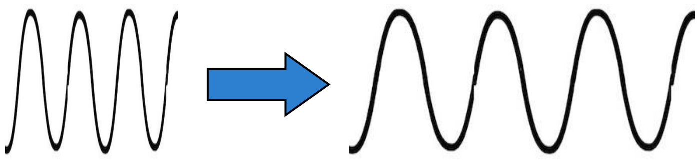
And this applies in all directions. *Every galaxy seems like the center* of expansion

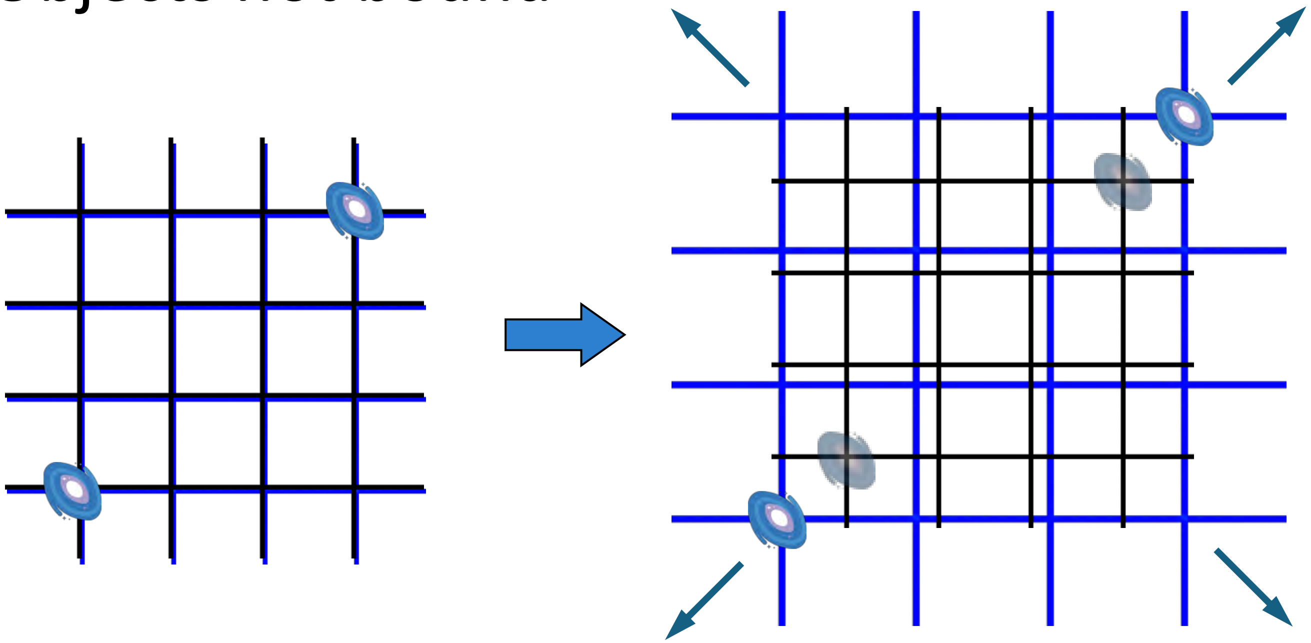
Expansion Relative to What?

There are **two kinds of coordinates**:

Proper coordinates that stay fixed. Objects bound by gravity or other forces (e.g., atoms, rulers, rocks, planets, galaxies…) ***stay fixed*** in the proper coordinates.

Comoving coordinates that expand with the expanding space, relative to the proper coordinates. Objects not bound by gravity (e.g., galaxies far apart) or other forces get ***carried by the expanding space***.



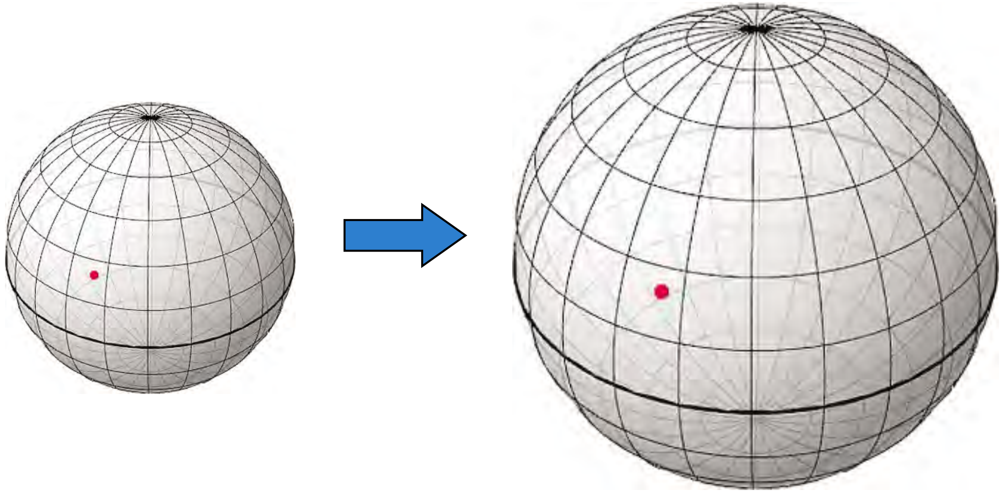


Expansion into What?

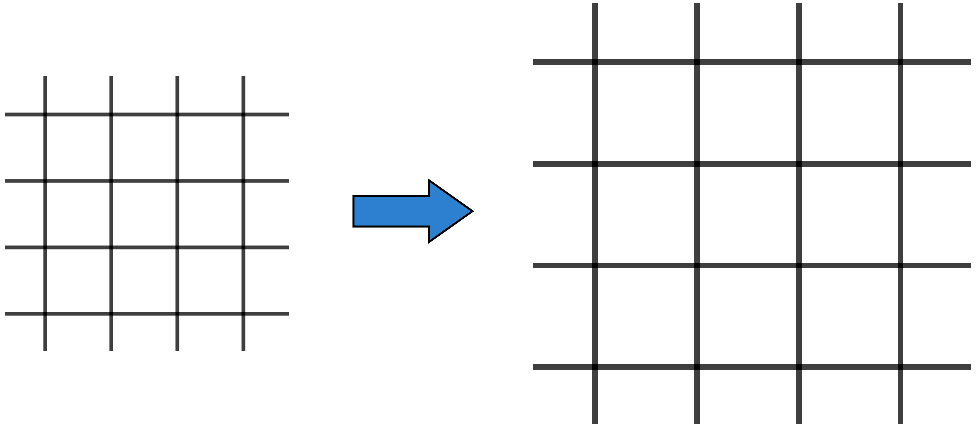
Into itself. Or into time. There is nothing "outside" the universe

Two-dimensional analogy:

A positive curvature universe is like the surface of a sphere, but in one extra dimension. Its ***volume is finite***, but changes with the expansion of space.



A **flat or a negative curvature universe** is ***infinite in all directions***; the comoving coordinate grid stretches relative to the proper coordinates



In either case, there is no "edge", and there is no center at that surface

The Cosmological Redshift

The expanding balloon analogy:

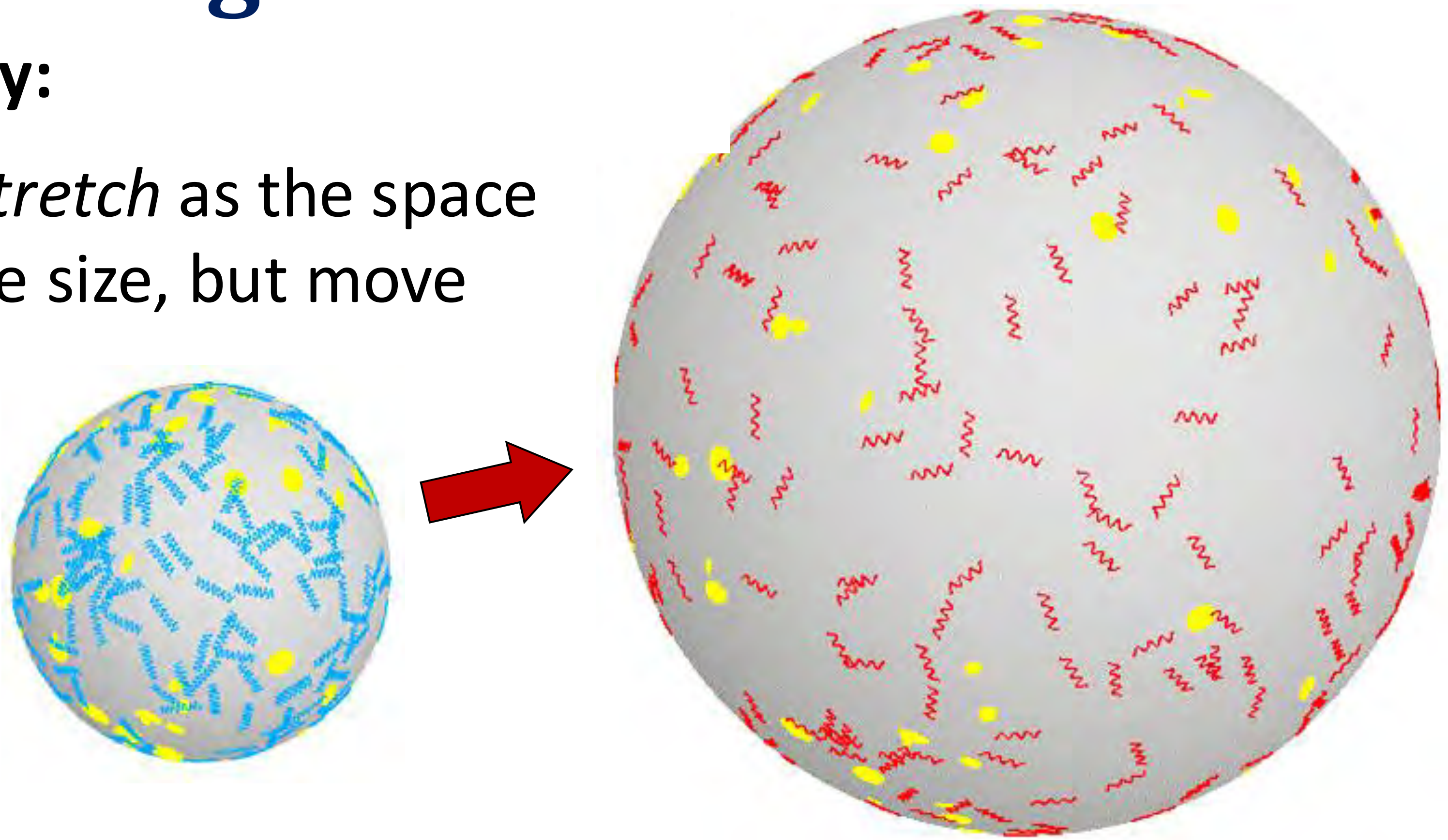
Wavelengths of photons also stretch as the space expands; galaxies stay the same size, but move further apart.

The expansion factor is

$\lambda_{obs}/\lambda_{em} = 1+z$

λ is the photon wavelength

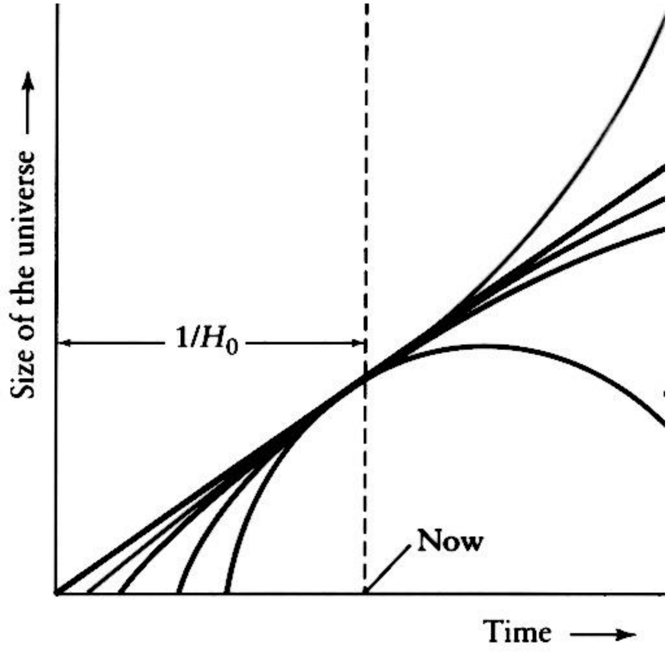
z is the **redshift**



As the universe expands, photon wavelengths become longer, i.e., **redder**

Thanks to the Hubble law (velocity ~ distance), the cosmological redshift is equivalent to the *Doppler shift*, $z = v/c$, where v is the apparent recession velocity of the galaxy and c is the speed of light.

Cosmological Models



Given a set of cosmological parameters, one can solve the Friedmann equation to map the expansion history of the universe, and extrapolate it into the future. Those are the **cosmological models**, and they can be tested against the observations

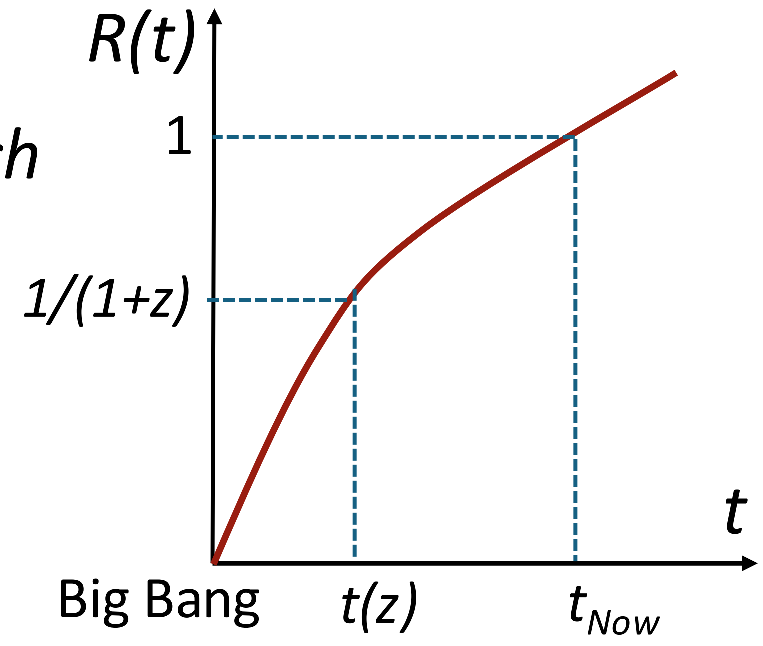
Alexander Friedmann in 1922 developed a GR-based, expanding universe model, but the expansion of the universe has not yet been established

Georges Lemaître in 1927 independently developed cosmological models like Friedmann's. In 1933, he "ran the film backwards" to a hot, dense, early state of the universe. This early prediction of the Big Bang was largely ignored

These models are for a ***homogeneous, isotropic universe***, and so far work great

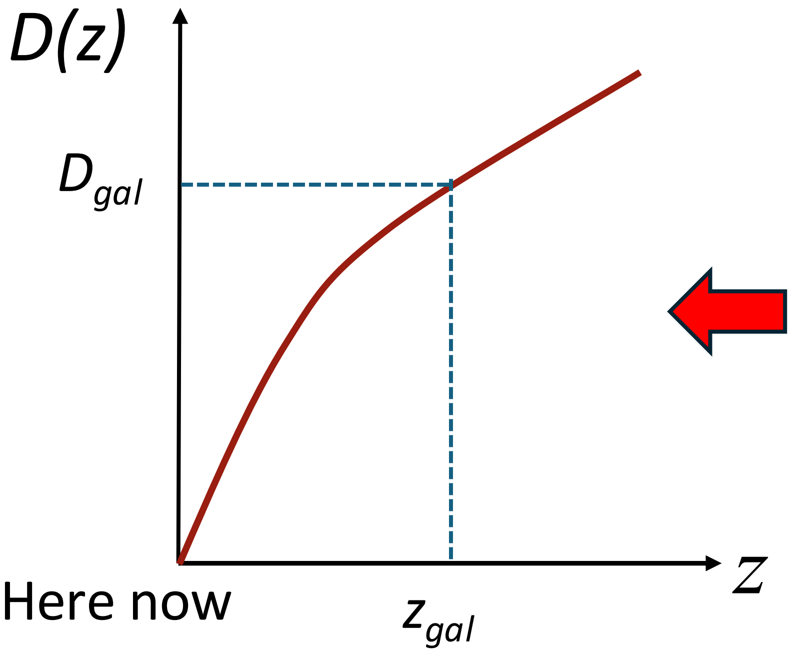
Quantifying the Expansion of the Universe

We introduce a **scale factor**, commonly denoted as ***R(t)*** or ***a(t)***: *a spatial distance between any two objects which move with their comoving coordinates*



Size (now) = (1+*z*) Size (then) *z* = redshift

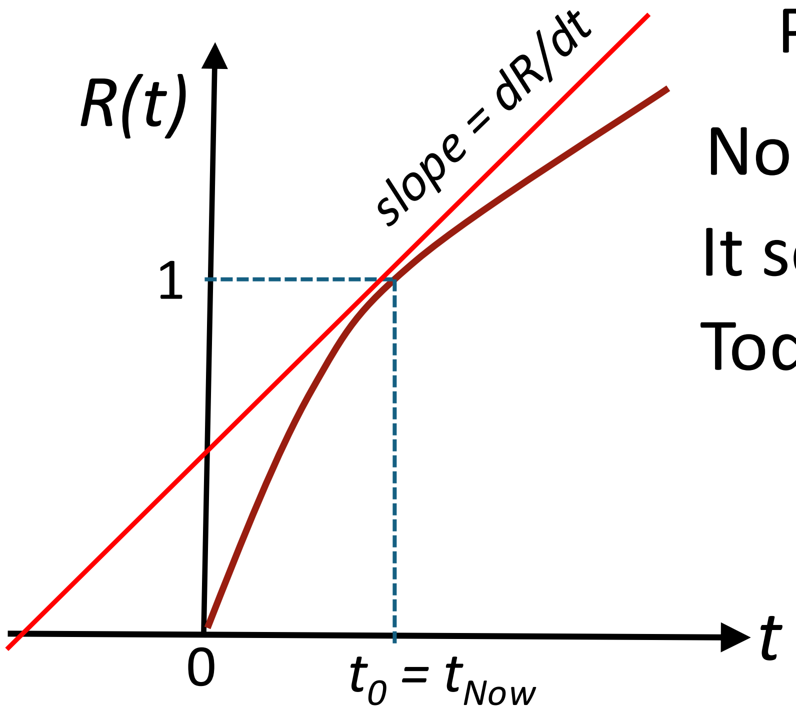
Scale factor $R(t) = R(z) = 1 / (1+z)$



Since *R(t)* is not directly observable, we use redshift *z* as the independent variable. Likewise, we cannot directly observe the lookback time, $t_{Now} - t(z)$, so we multiply it by *c* to get the distance, D(z) = c Δt, which is in principle observable (but hard)

Cosmological Parameters

They determine a particular cosmological model



Present day values have the subscript 0, e.g., t_0, H_0, etc.

Normalized expansion rate $E(z) = \dot{R}/R =$ **Hubble constant**

It sets the overall scale of the universe

Today's value: $H_0 \approx 70$ km/s/Mpc

Define **critical density** $\rho_{crit} = \dfrac{3H^2}{8\pi G}$

Today's value: $\rho_{crit,0} = 0.921 \times 10^{-29}\, h_{70}^2$ g cm^{-3}

$h_{70} = H_0$ in units of 70 km/s/Mpc

Define **density parameter** $\Omega = \rho / \rho_{crit}$ It can be for a given component of mass/energy, e.g., Ω_m for matter, Ω_r for radiation, Ω_Λ for dark energy, etc.

For a **flat** universe, $\Omega = \Omega_m + \Omega_r + \Omega_\Lambda = 1$

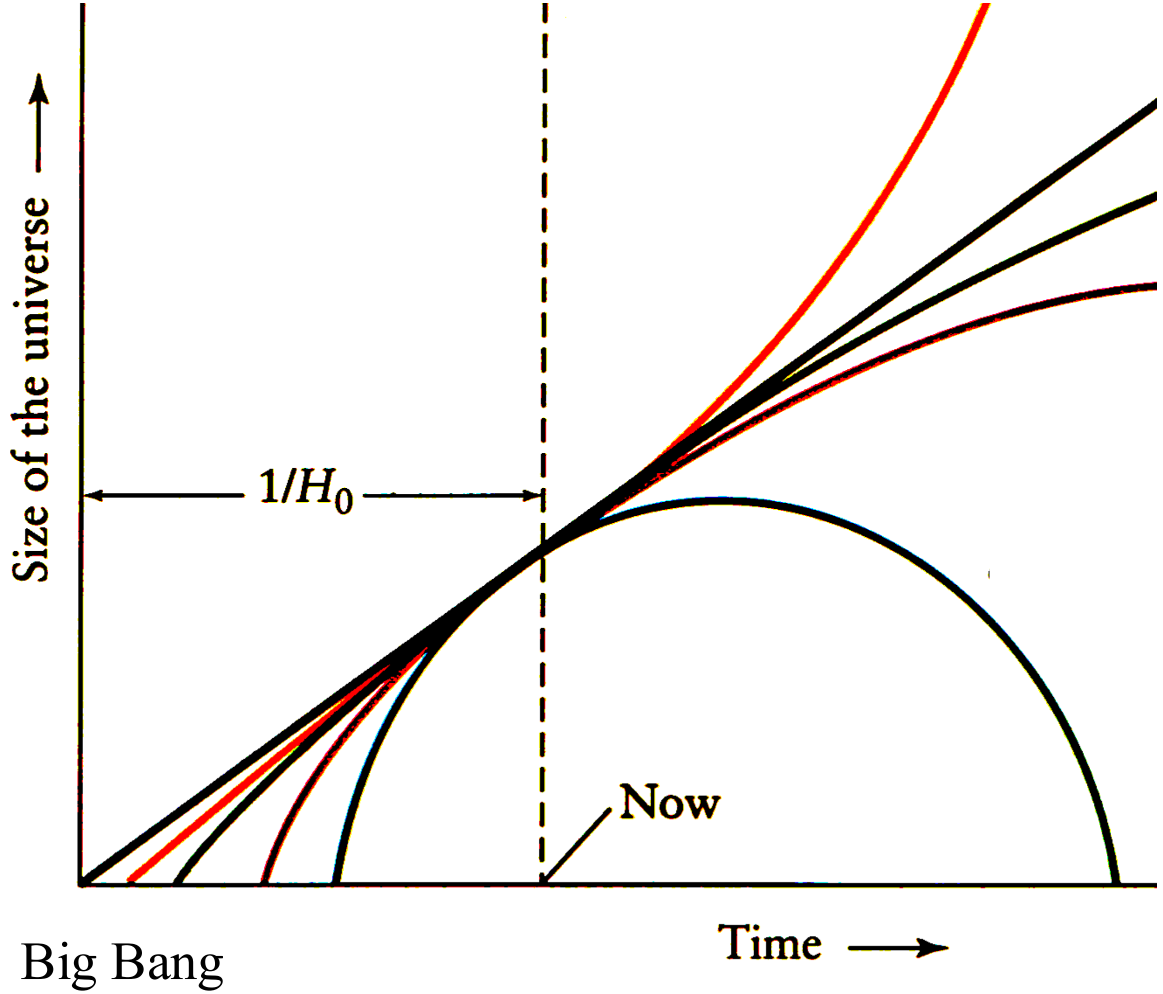
Cosmological Models

Hubble constant H_0 sets the **scale** of the universe, but not the shape of the model curves

The **shapes** are determined by the **mass/energy contents** of the universe, i.e., a combination of Ω_m, Ω_r, and Ω_Λ

That also determines the global geometry of the universe and its future. If $\Omega < 1$ the universe is **open**, for $\Omega = 1$ it is **flat**, and for $\Omega > 1$ it is **closed**

Current measurements indicate that the universe is very close to **flat**, $\Omega = 1$



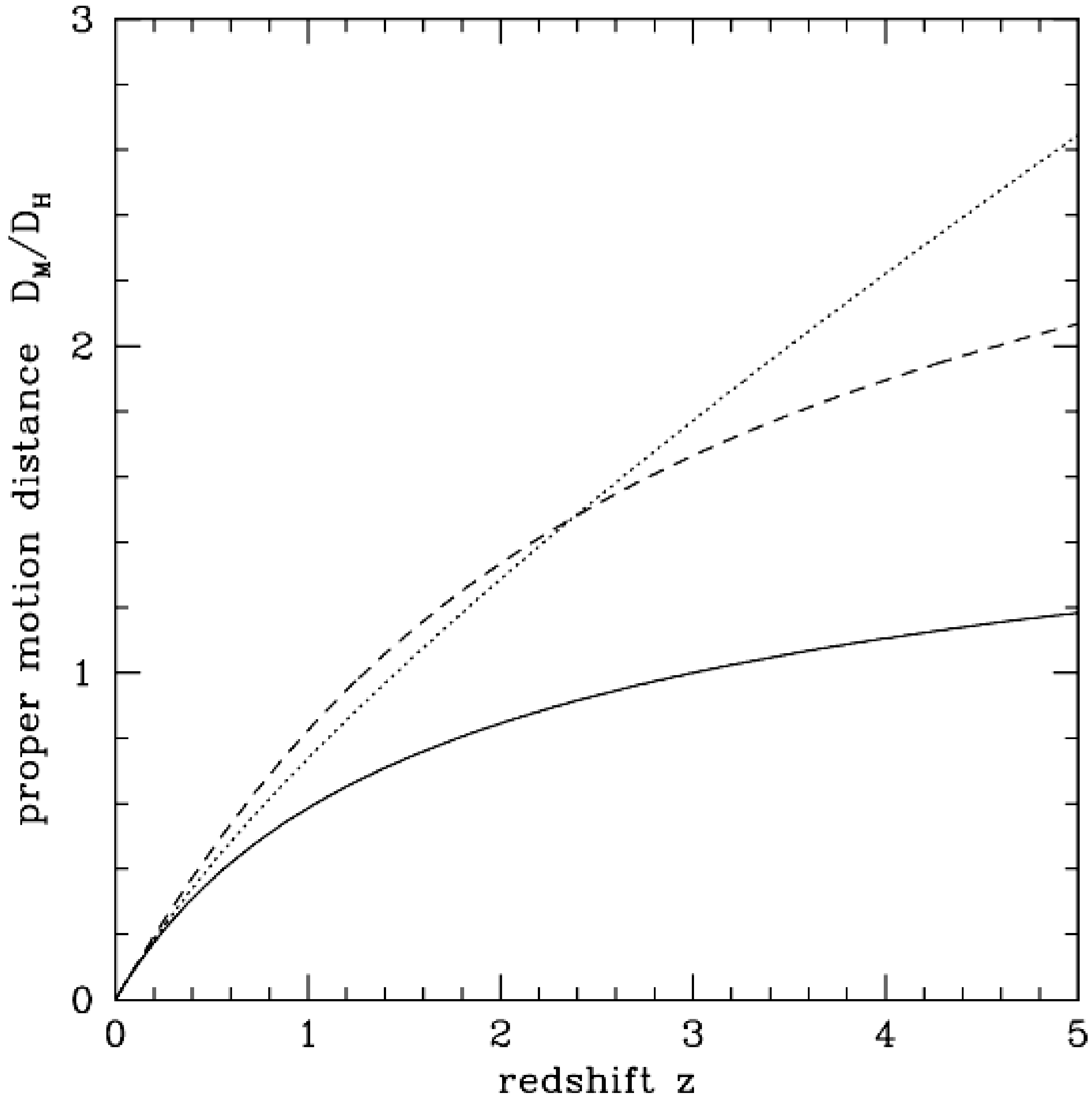
Flat and open universes **expand for ever**, closed universes **recollapse**

Distances in Cosmology

We need the distances to convert observed quantities (e.g., magnitudes) into physical ones (e.g., luminosity)

They depend on the cosmological parameters, and can be computed from cosmological models

Thus we can compare the observations with the models, and find out what is the actual model of the universe



Measuring the Hubble Constant

It is the slope of the distance-velocity relation on the Hubble diagram, so we need velocities (easy) and distances (hard), and ideally far enough so that the peculiar velocities are unimportant – "the pure Hubble flow"

There are **many methods**, but it has now converged to two independent ones:

1. Local, using a "distance ladder" with parallaxes, Cepheids, and Supernovae

Use **parallaxes** to calibrate the distances to Cepheids

Use **Cepheids** and the period-luminosity (Leavitt) relation to measure the distances to nearby galaxies which hosted Type Ia Supernovae

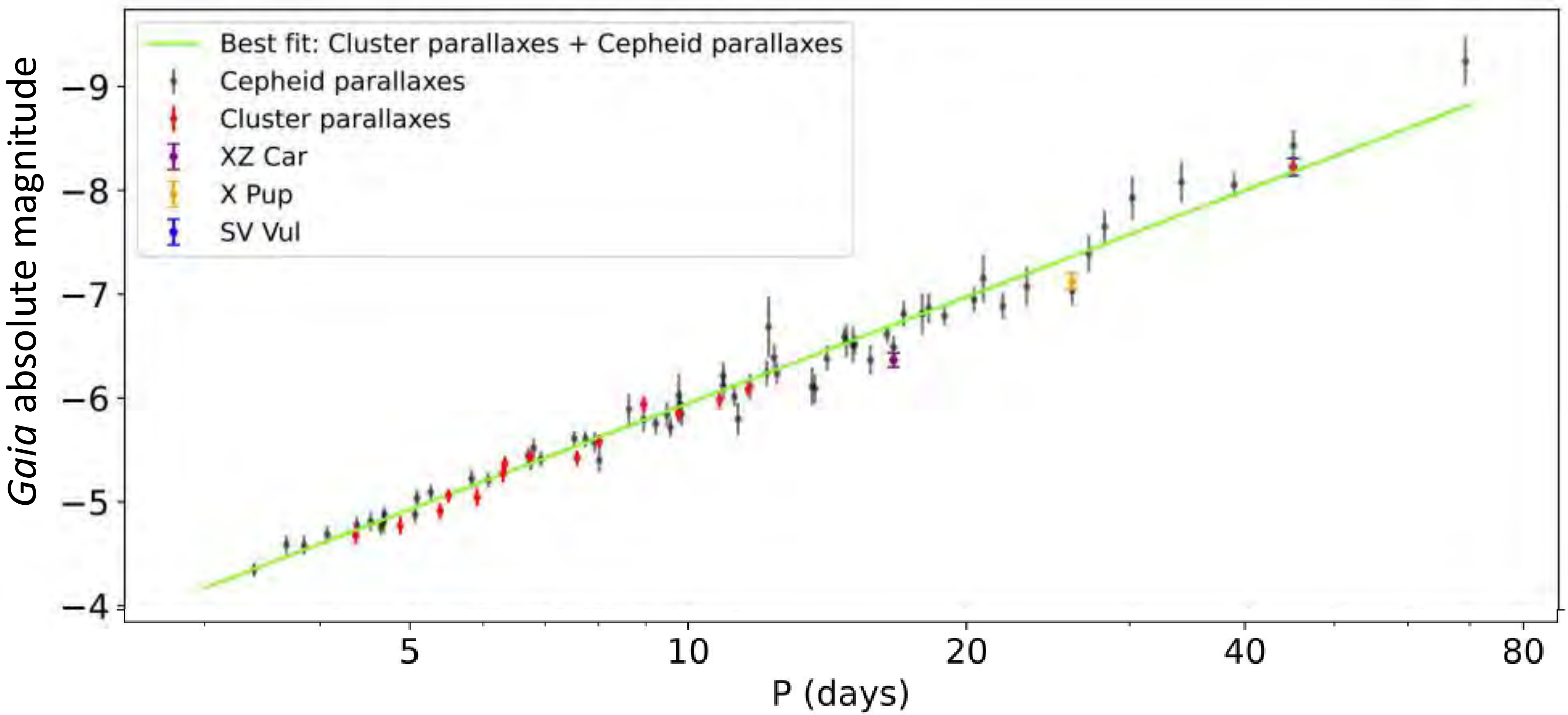
Use **Supernovae** (bright, so observable far away) as "standard candles" to reach into the pure Hubble flow

2. Cosmic Microwave Background fluctuations and theoretical modeling

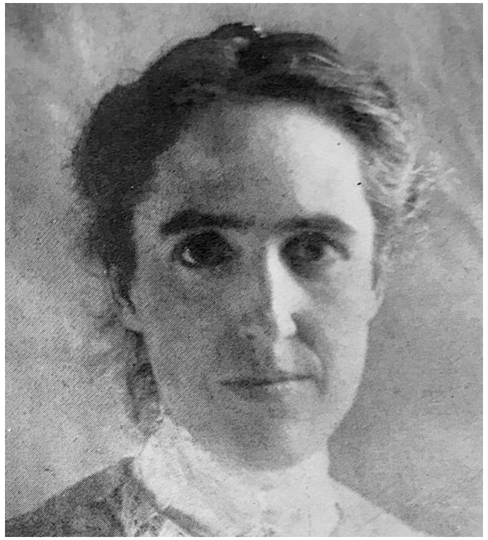
They agree within a few %, $H_0 \approx 70$ km/s/Mpc, but there are significant differences

Gaia Calibration of the Cepheid Period-Luminosity Relation

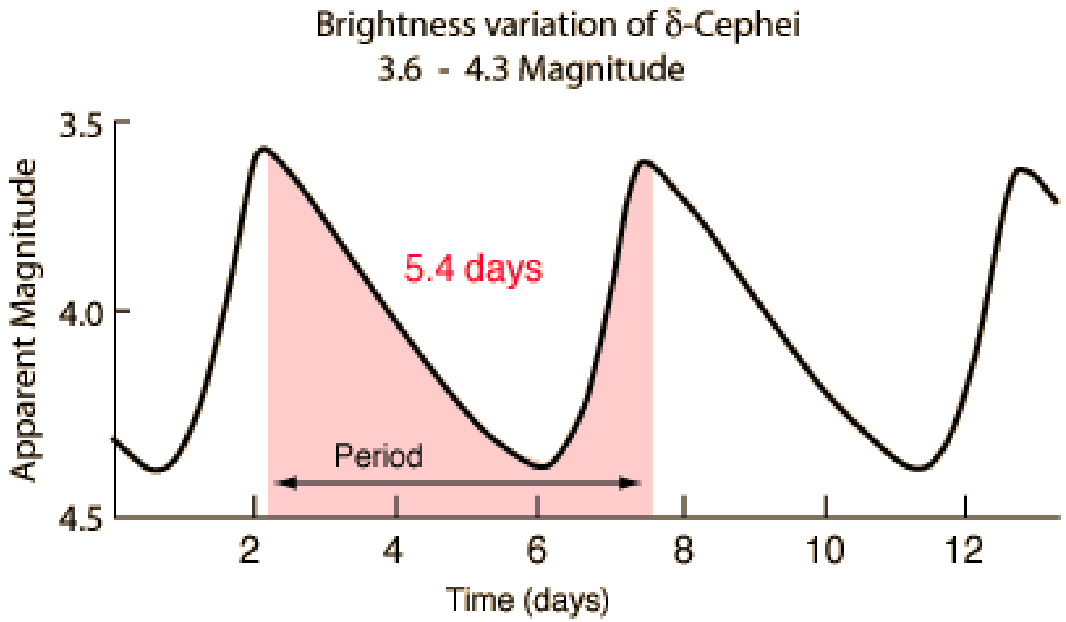
The basic calibration of the extragalactic distance scale

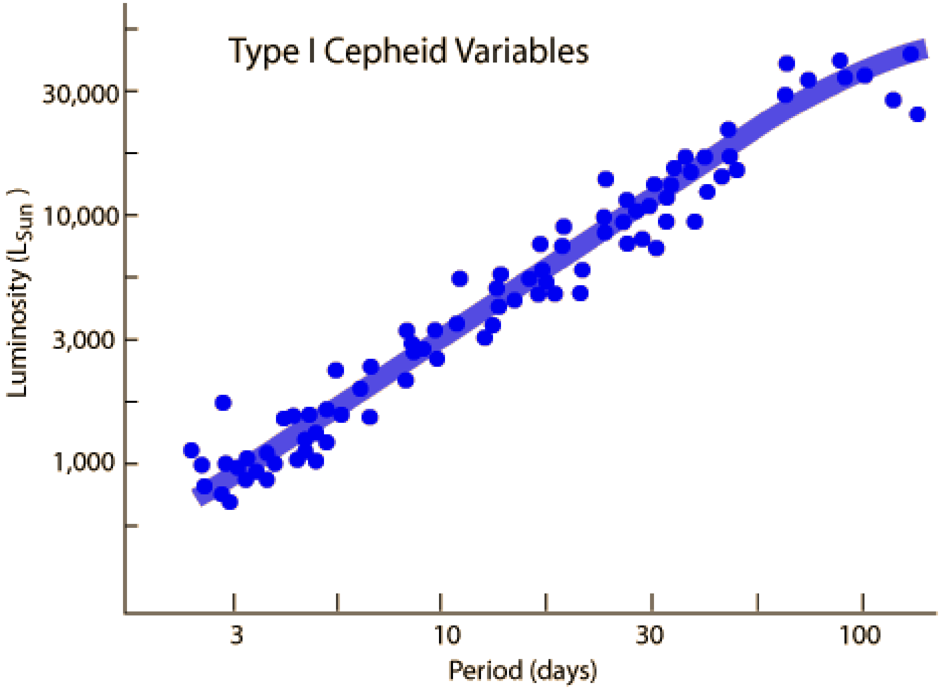


Measuring Distances using Cepheids



In 1912, **Henrietta Leavitt** at Harvard was studying pulsating variable stars, **Cepheids**, in the Magelanic Clouds



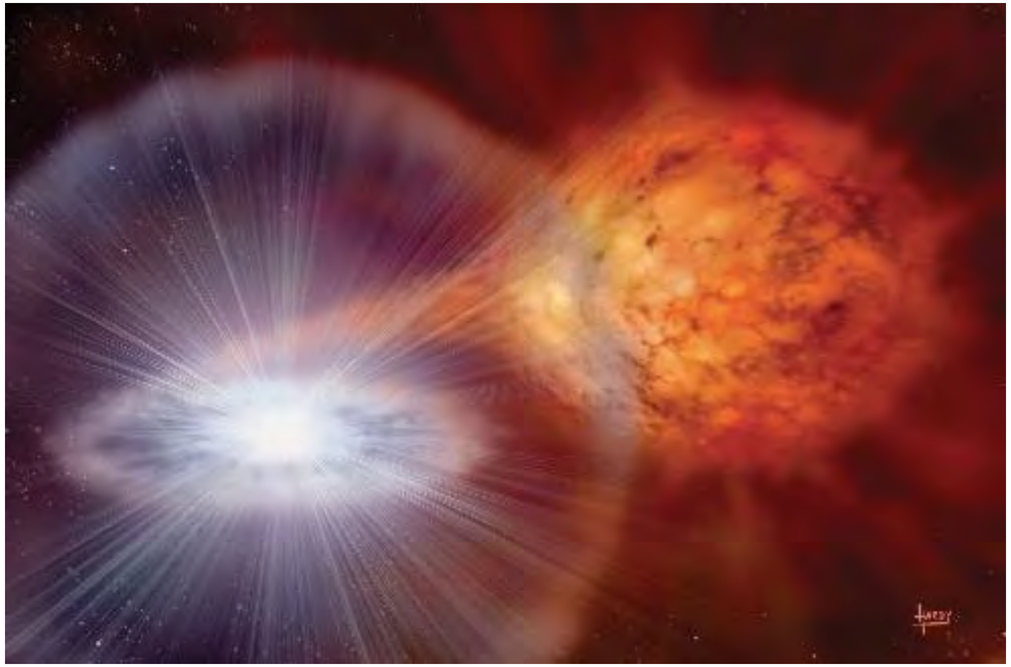


She discovered that the more luminous ones have longer periods. That made them good **distance indicators**. Periods do not depend on the distance. They can be used to deduce the absolute luminosity, and from that and the apparent brightness, the distance.

Supernovae (SNe) as Standard Candles

Bright and thus visible far away

Type Ia from a binary white dwarfs merging or accreting material, going over the Chandrasekhar limit, and detonating



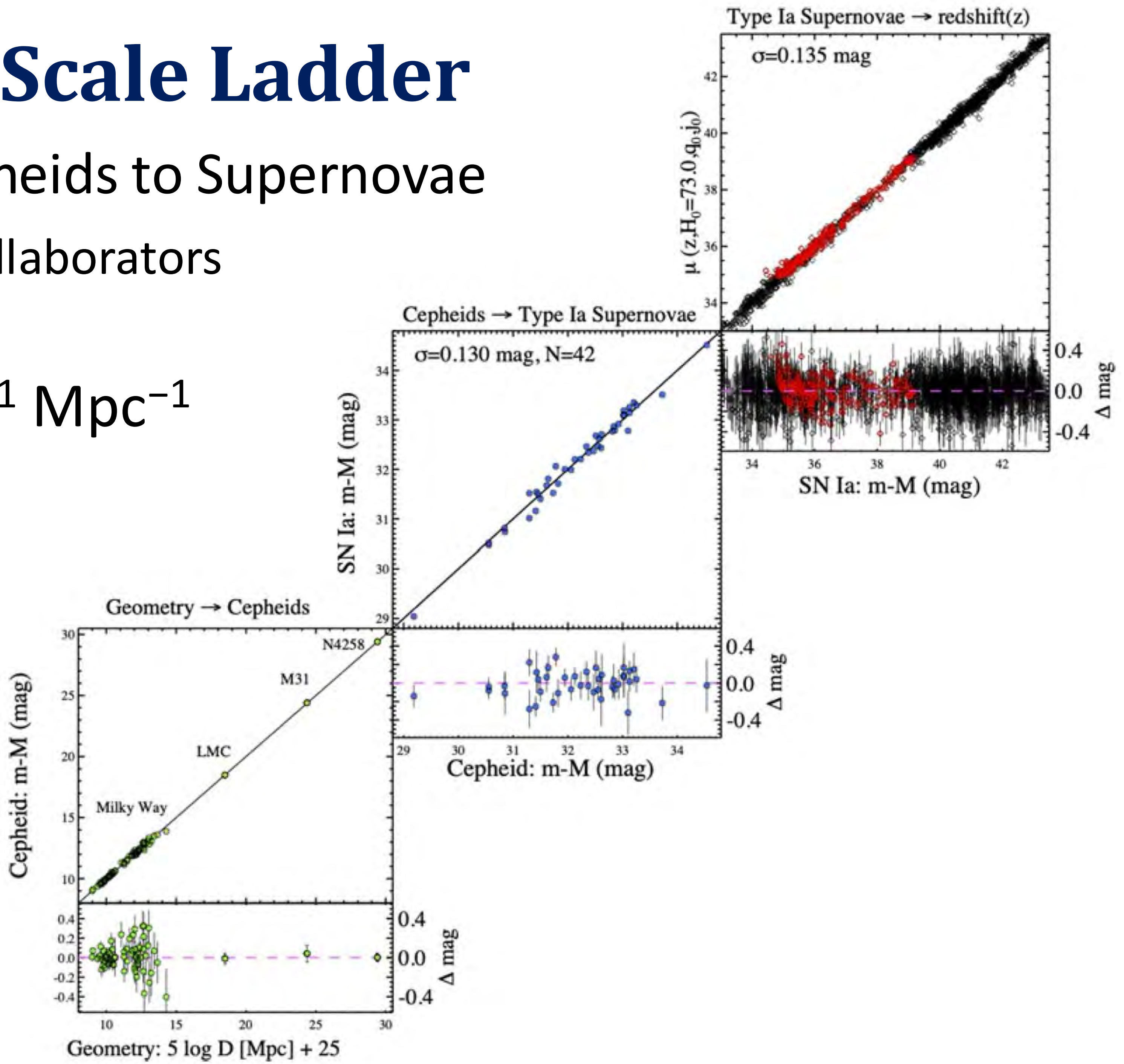
- **Remarkably homogenous properties**: Same type of an object exploding in each case; explains the uniformity of their light curves
- **Good standard candles**, peak $M_V \sim -19.3$
- Light curves can be standardized to a peak magnitude scatter of ~ 10%
- Old stellar population: observed in elliptical galaxies as well as spirals
- Light curve fit by radioactive decay of about a Solar mass of ^{56}Ni

Local Distance Scale Ladder

From Parallaxes to Cepheids to Supernovae

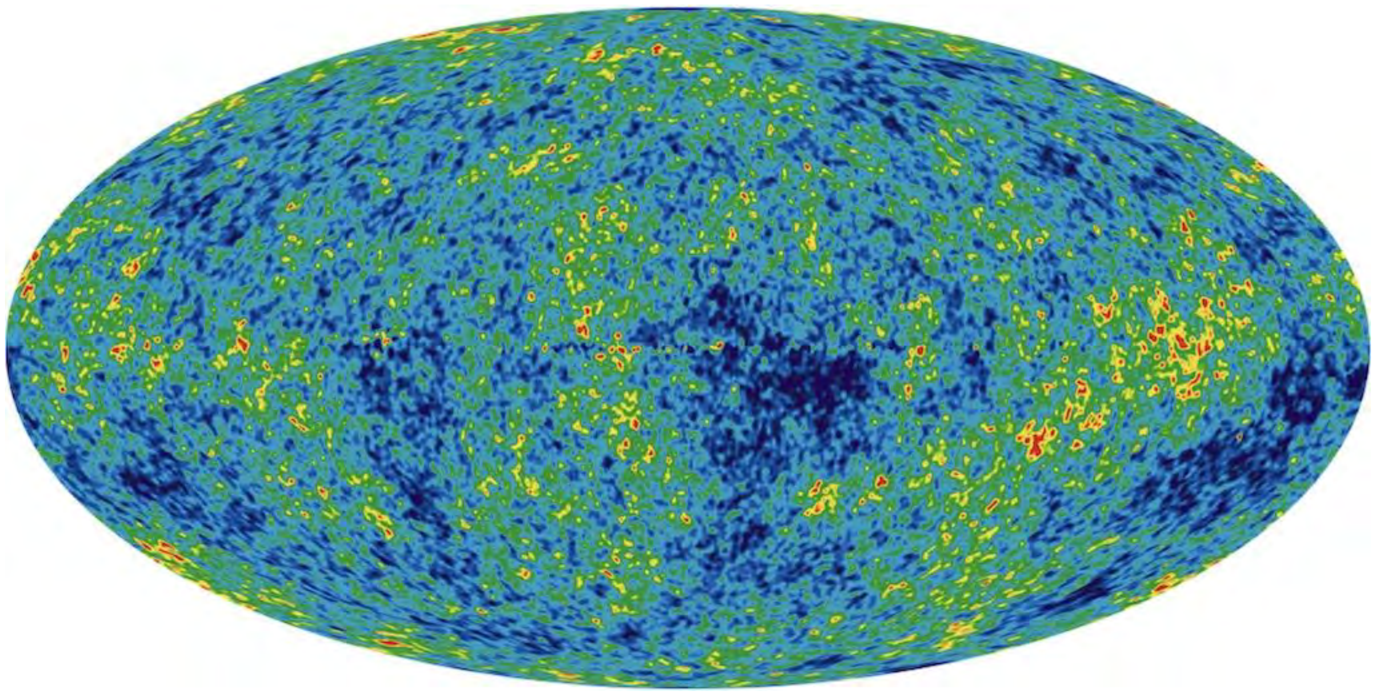
A. Riess and collaborators

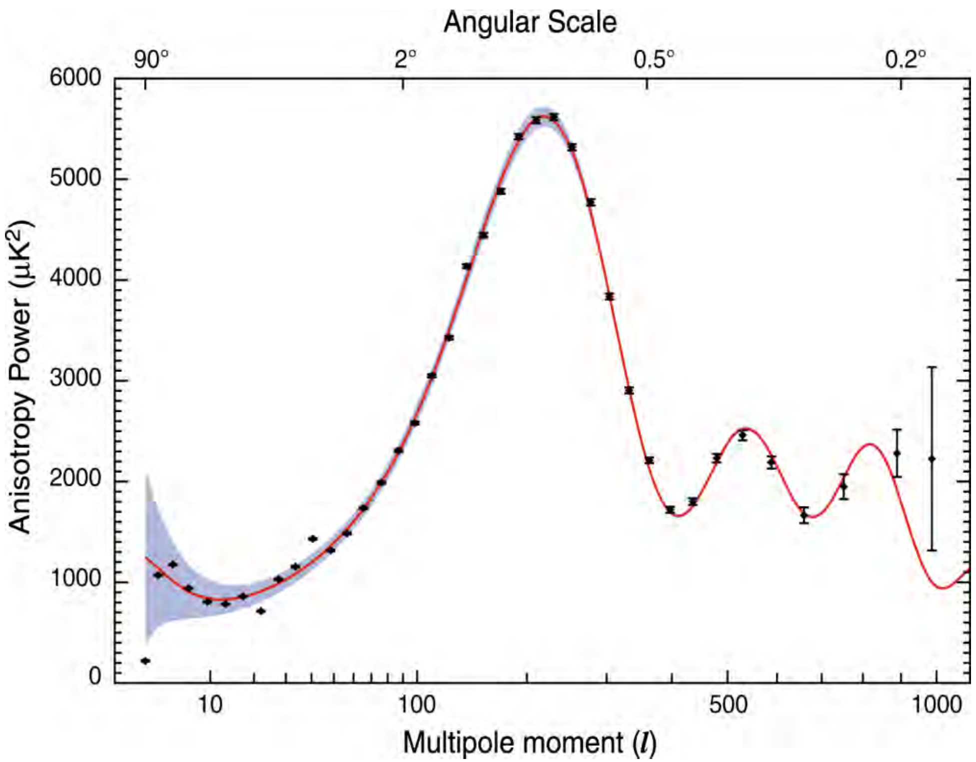
$H_0 = 73.50 \pm 0.81$ km s^{-1} Mpc^{-1}



Hubble Constant from CMB

CMB sky shows fluctuations on a preferred scale of ~ 0.9 degrees





The corresponding physical scale depends on many parameters, and after a complex analysis, it yields the best fit value of H_0:

$$H_0 = 67.62 \pm 0.50 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

The Hubble Tension

Between the **Cepheid-based** and **CMB-based** measurements of the Hubble Constant

Cepheids:

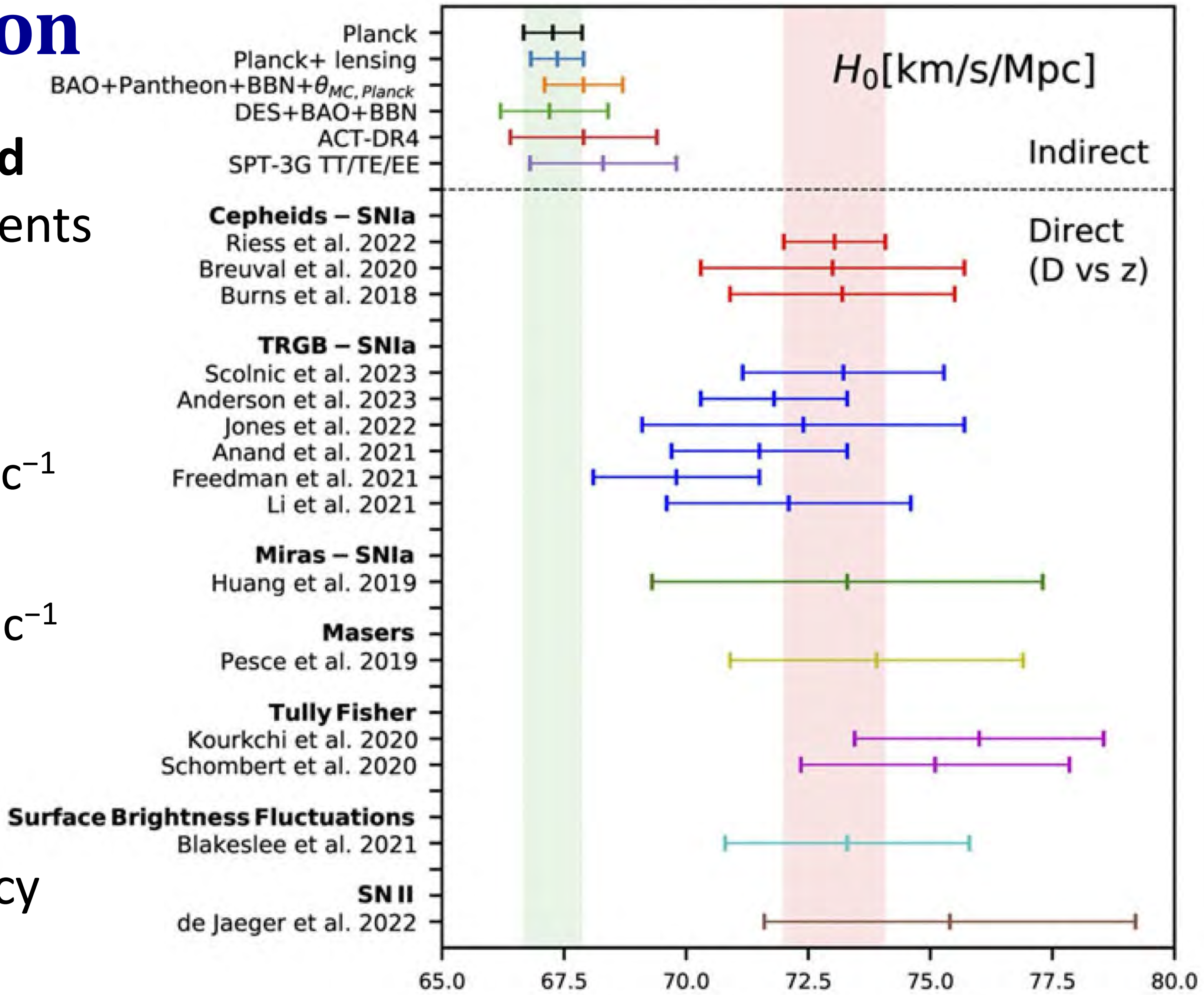
$H_0 = 73.50 \pm 0.81 \text{ km s}^{-1} \text{ Mpc}^{-1}$

CMB:

$H_0 = 67.62 \pm 0.50 \text{ km s}^{-1} \text{ Mpc}^{-1}$

4.5 σ diference

The cause of this discrepancy is currently unknown

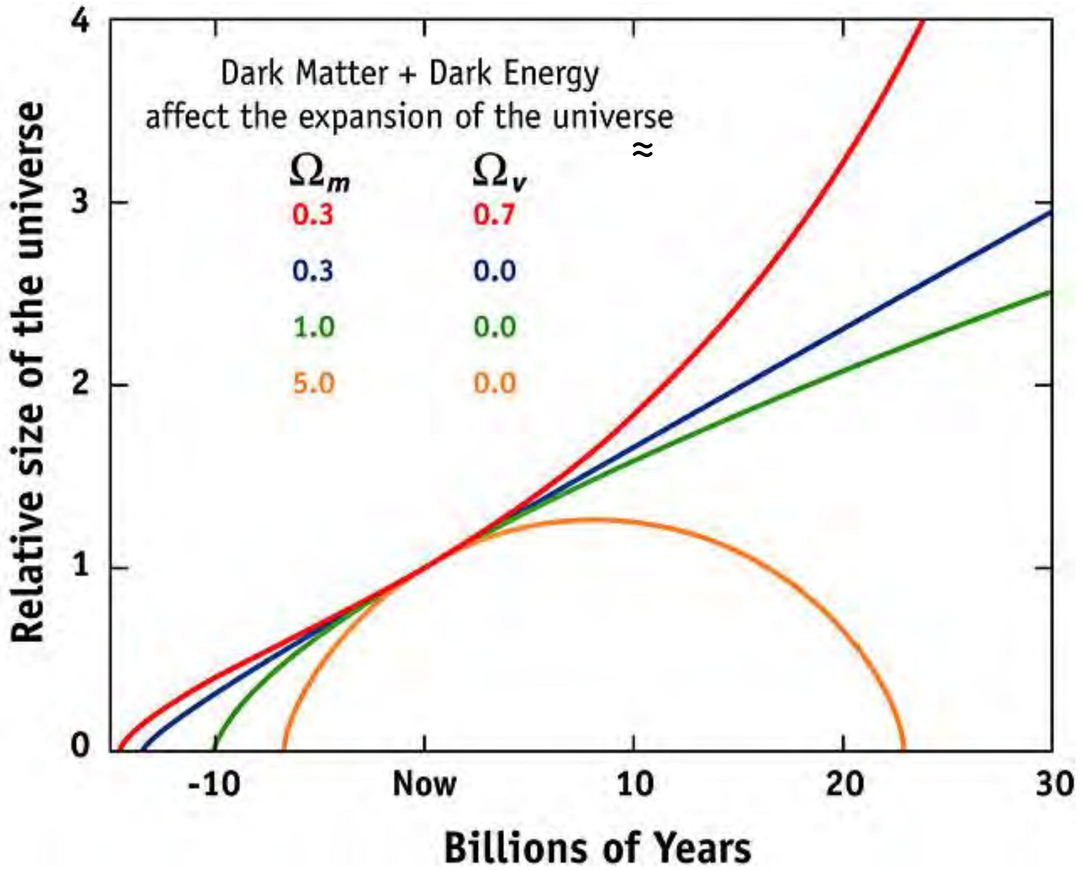


Cosmological Models: Expected Behaviour

Expansion rate is governed by the matter/energy density, curvature (≈ 0), and cosmological constant (dark energy), which acts like repulsive force

Higher density ⟶ higher gravity ⟶ slower expansion ⟶ less volume

Low density and/or larger cosmological constant ⟶ faster expansion ⟶ more volume



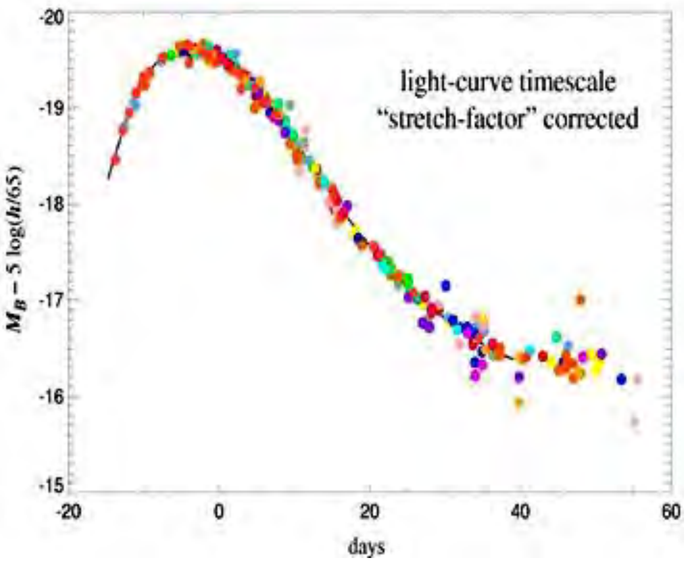
Higher density models: smaller distances, smaller ages, brighter and larger sources

Lower density and/o cosmological constant models: larger distances, larger ages, dimmer and smaller sources

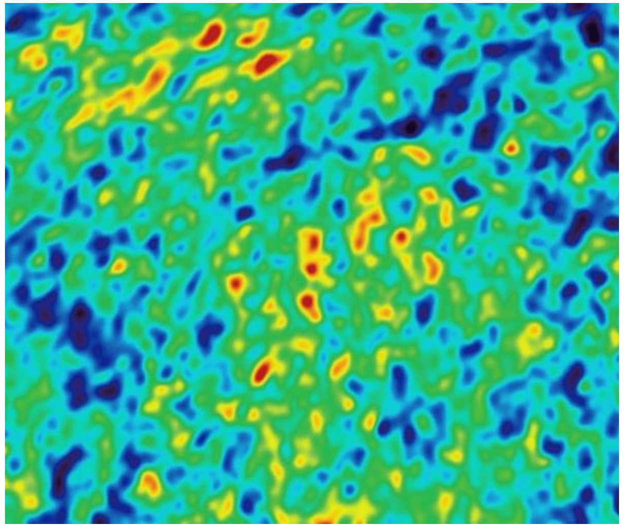
These differences enable cosmological tests, by observing "standard" sources at different redshifts

Cosmological Tests

There are many, and they all compare some measure of relative distances vs. redshift, but the most important are:



Hubble diagram: Observed flux from some population of sources with the same intrinsic luminosity (**"standard candles"**), using a relativistic inverse square law. By far the best of those are Supernovae of Type Ia, typical absolute magnitude $M_V = -19.3$. Requires calibration of SN luminosities, e.g., through Cepheids

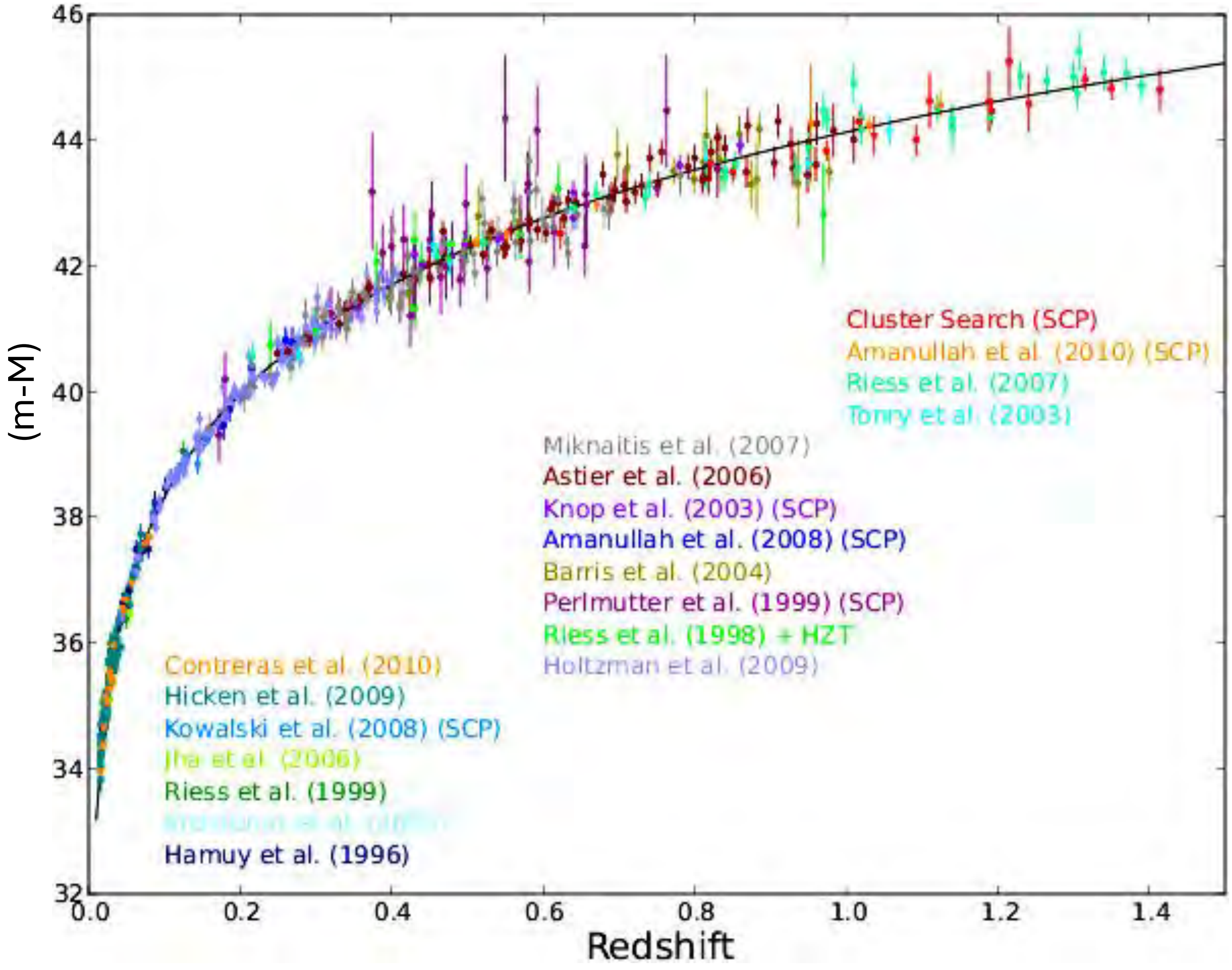


Angular Diameter Test: Apparent angular size of some sources with the same physical size. By far the best are the CMB fluctuations at $z \approx 1100$, corresponding to the regions of the size ~ 100 Mpc. Requires a lor of theoretical and statistical modeling

A Modern Version of the SN Hubble Diagram

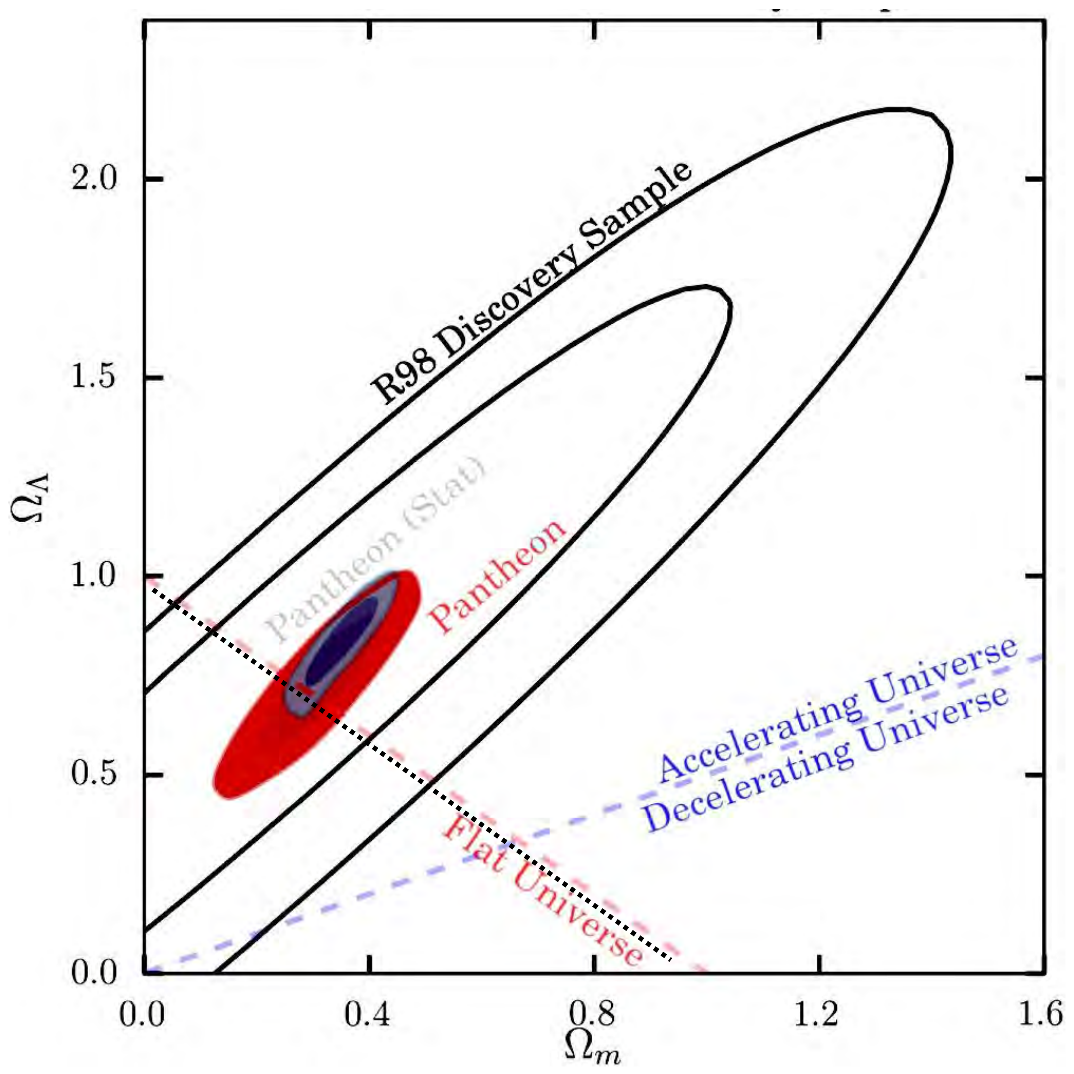
The plot combines data from number of different groups, carefully corrected for the observational and redshift effects

The solid line is the best fit cosmological model, ΛCDM with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$

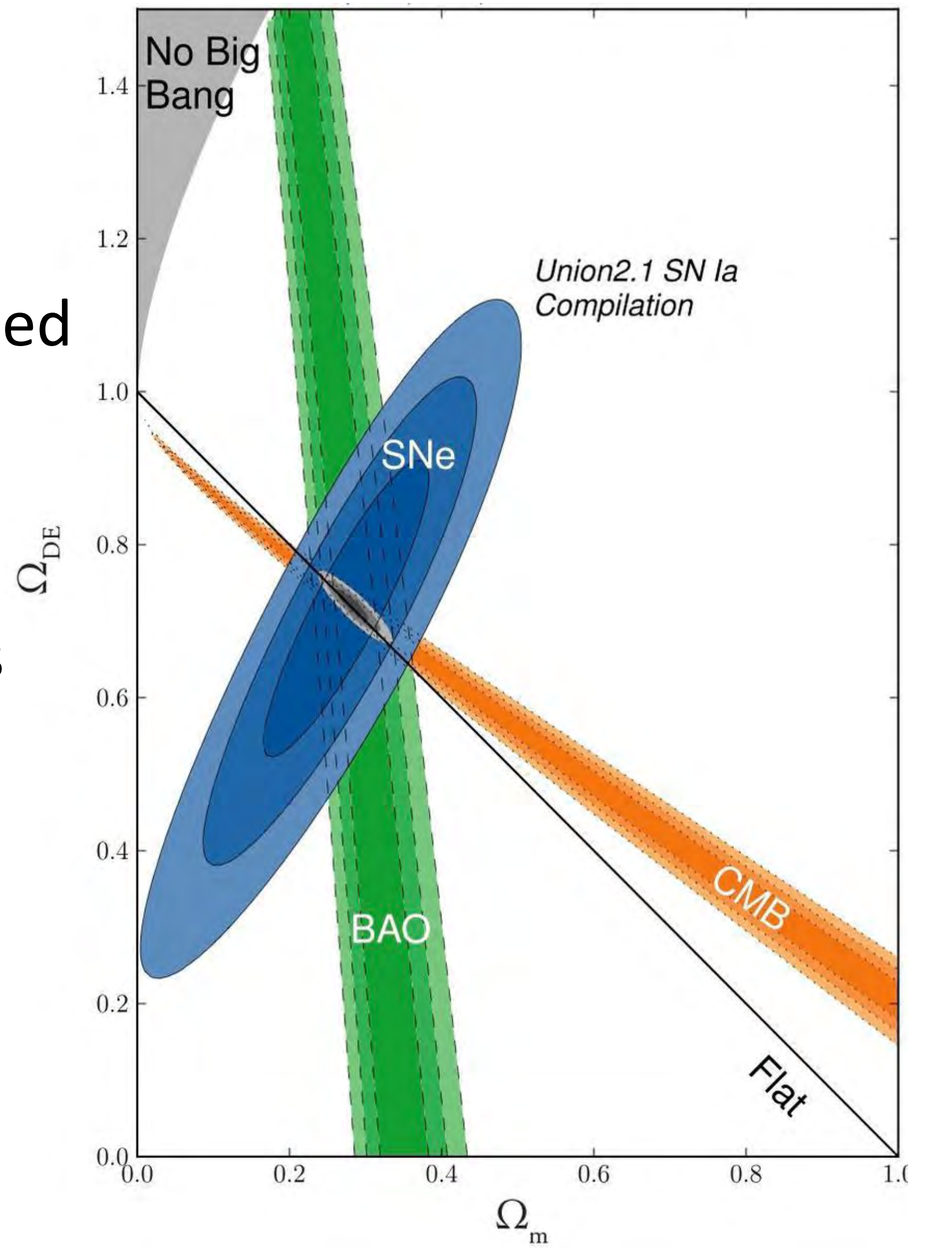


SN Hubble Diagram Results

Usually shown on an Ω_m-Ω_Λ diagram. The dotted line represents the flat universe with $\Omega_m+\Omega_\Lambda = 1$

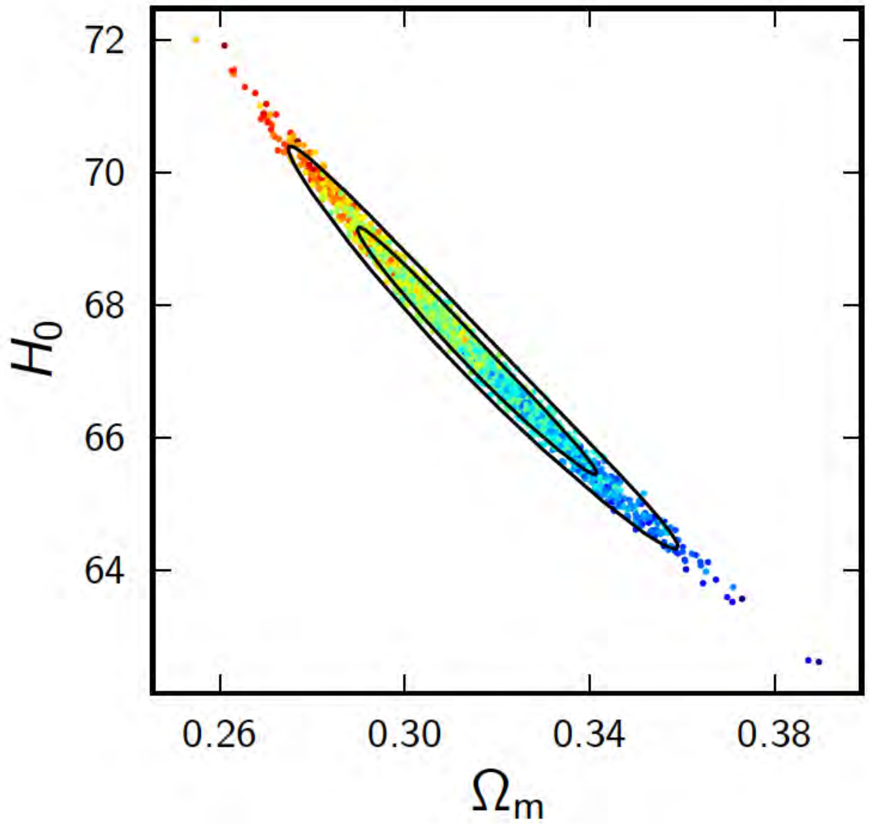


Here it is combined with the CMB results (orange) and galaxy clustering results (BAO, green)



Angular Diameter Test: CMB Fluctuations

- Uses the *size of the particle horizon* at the time of the recombination (the release of the CMB at z ~ 1100) as a ***standard ruler***, ~ 100 Mpc h^{-1} (comoving)
- This governs the *largest wavelength of the sound waves produced in the universe* then, which *cause small fluctuations in the temperature of the CMB* ($\Delta T/T \sim 10^{-5} - 10^{-6}$) at the appropriate angular scales (~ a degree and less)
- They persist in galaxy clustering at lower redshift as Baryonic Acoustic Oscillations (BAO)
- CMB data are often analyzed jointly with the data from other methods, e.g., SNe, galaxy clustering, etc.
- Many different parameters are measured at once



Parameters can be correlated, producing elongated error ellipses. Better constraints are obtained CMB with other data

Some *Planck* Results

The best results to date were from the *Planck* mission (2009-2013)

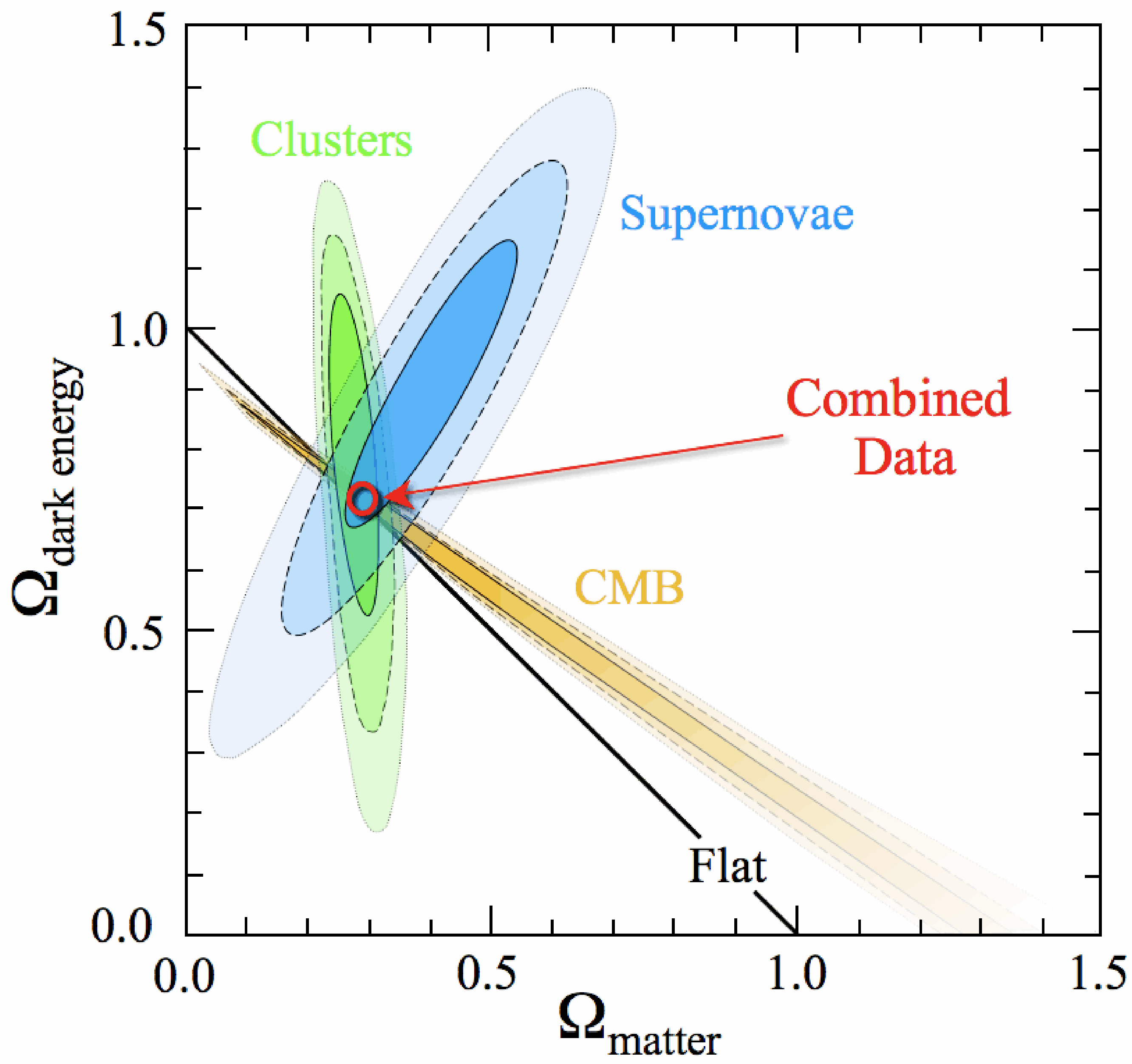
Notice the precision! ⟶

CMB opened the era of "precision cosmology", but independent checks are needed

Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.022383	0.022447
$\Omega_c h^2$	0.12011	0.11923
$100\theta_{MC}$	1.040909	1.041010
τ	0.0543	0.0568
$\ln(10^{10} A_s)$	3.0448	3.0480
n_s	0.96605	0.96824
H_0 [km s^{-1}Mpc^{-1}]	67.32	67.70
Ω_Λ	0.6842	0.6894
Ω_m	0.3158	0.3106
$\Omega_m h^2$	0.1431	0.1424
$\Omega_m h^3$	0.0964	0.0964
σ_8	0.8120	0.8110
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.8253
z_{re}	7.68	7.90
Age [Gyr]	13.7971	13.7839

A spatially flat universe $\Omega_K = 0.0007 \pm 0.0019$

The Cosmic Concordance



Multiple types of *different*, *independent* measurements and different cosmological probes all agree, intersecting in a small overlapping region of the parameter space

(Not all are shown here, e.g., local Ω_m measurements, BAO, BBNS, ages of globular clusters, etc.)

This defines the standard (but perhaps not yet entirely accurate or final) cosmological model**, ΛCDM** with $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$

Key Concepts From This Lecture (1)

- General Relativity (GR), as the theory of gravity, is the **theoretical basis of the modern cosmology**, connecting matter/energy, gravity, and geometry of space
- The universe expands, and the expansion rate is given by the **Hubble constant**. Two principal ways of measuring it are the "**distance ladder**" (parallaxes to Cepheids to Supernovae), and CMB fluctuations. They disagree slightly, but the difference is significant, and it is the cause of the "Hubble tension"
- While the full GR has 16 differential equations, symmetry assumptions (homogeneity and isotropy) reduce it to one, the **Friedmann equation**. Its solutions are **cosmological models** that give the past and the future expansion rates, and that are determined by the **cosmological parameters**, e.g., the densities of the various mass/energy components
- If the sum of all densities is equal to the critical density, the **geometry of the universe** is **flat** (Euclidean), and expands for ever. If it is smaller, it is **open**, infinite, and also expands for ever. If it is larger, the universe is **closed** and will recollapse

Key Concepts From This Lecture (2)

- **Cosmological parameters**, mainly the **matter density** and the **dark energy**, determine the overall geometry of the universe, and whether it will expand for ever
- There are many way of measuring them, but the principal ones are:
  1. **Supernova Hubble Diagram**, which uses the brightness of the Type Ia Supernovae as the "**standard candles**" to map the distance vs. redshift, which constrains the cosmological parameters
  2. **CMB fluctuations**, which serve as a "**standard ruler**" in the angular diameter test, based on the largest sound waves in the universe at redshift z ~ 1100, and require a theoretical and statistical analysis. These fluctuations are detectable in the galaxy clustering at lower redshifts, as **Baryon Acoustic Oscillations** (BAO)
- All current measurements intersect and agree in a small patch of the parameter space, which defines the **standard cosmological model**, **ΛCDM,** which is spatially flat, with matter contributing $\sim$ 30% of the total density, and dark energy $\sim$ 70% of it, and is 13.8 Gyr old

?