Kepler's Laws, Orbits, Planets, Their Properties, and the Solar System

Ay 1 – Lecture 6

Kepler’ s Laws:

1. The orbits of planets are elliptical, with the Sun at a focus
2. Radius vectors of planets sweep out equal areas per unit time
3. Squares of orbital periods are proportional to cubes of semimajor axes:

$$P^{2}\,[\mathrm{yr}] = a_{pl}^{3}\,[\mathrm{au}]$$

- Derived empirically from Tycho Brahe’s data
- Explained by the Newton’ s theory of gravity

Newton' s Laws

1. Inertia (object at rest...)
2. Force: $F = m\,a$
3. $F_{action} = F_{reaction}$

→ Conservation laws (E, ***p***, ***L***)

e.g., for a circular motion in grav. field:
centifugal force = centripetal force

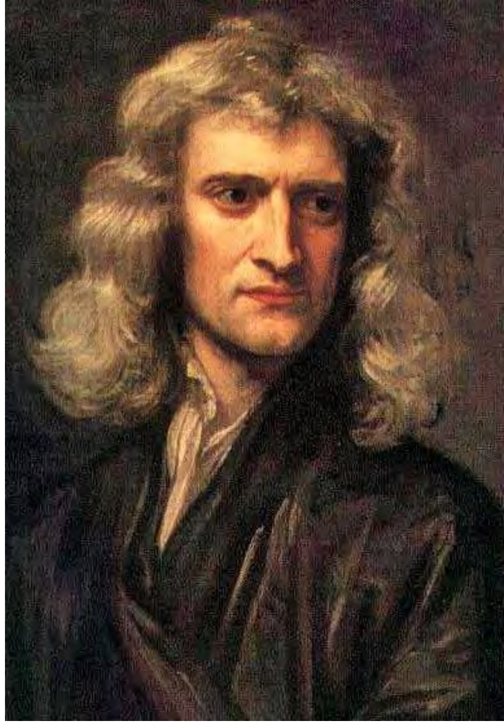
$$\frac{m\,V^2}{R} = G\,\frac{m\,M}{R^2}$$

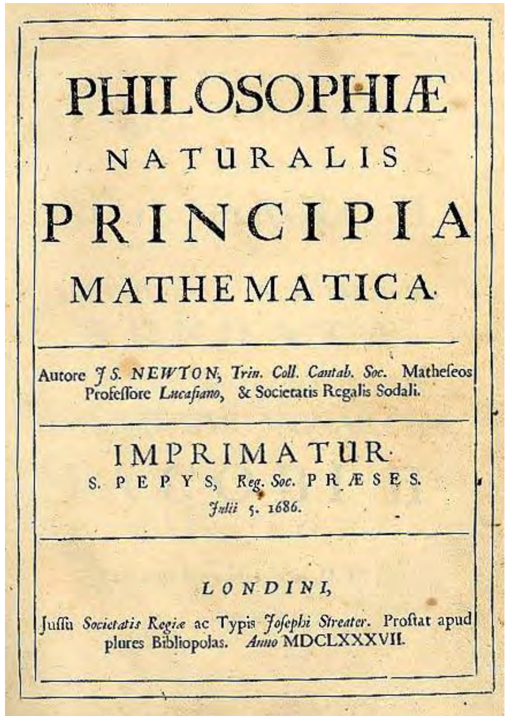
- The law of gravity: $F = G\,\dfrac{m_1\,m_2}{r^2}$ (G = Newton's gravitational constant)

- Energy: $E_{total} = E_{kinetic} + E_{potential}$

$$E_{kinetic} = \frac{mV^2}{2} \qquad E_{potential} = \frac{G\,m\,M}{R}$$ (gravitational)

- Angular momentum: $L = m\,V\,R$ (for a point mass)





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Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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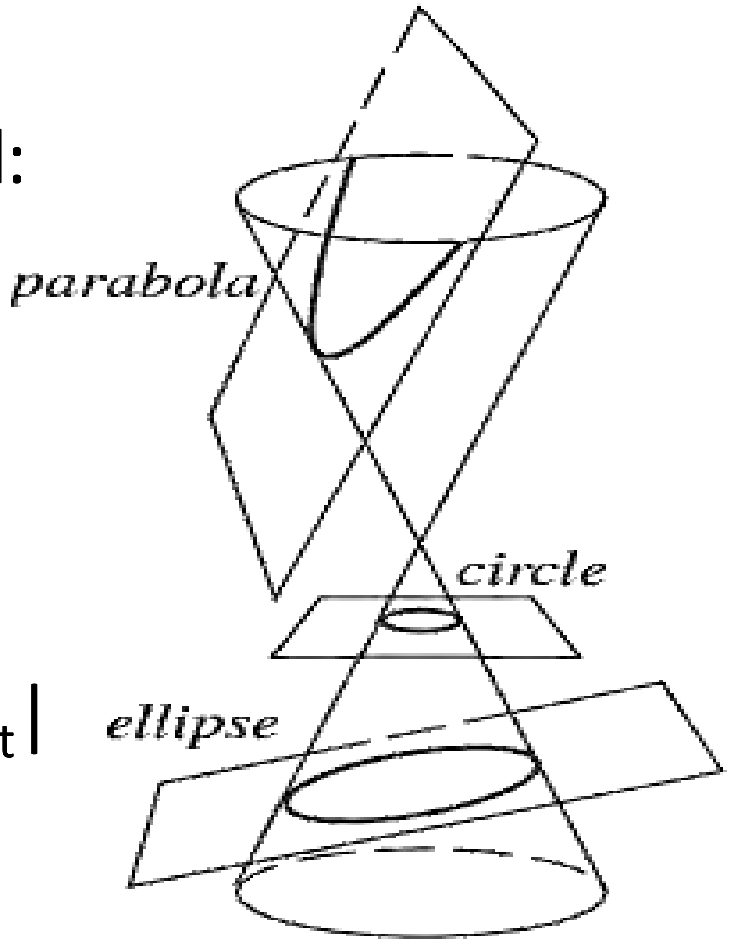
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Motions in a Gravitational Field

It can be shown that motions of two particles interacting according to the inverse square law are **conic sections**

Marginally bound:

$E_{kin} = |E_{pot}|$

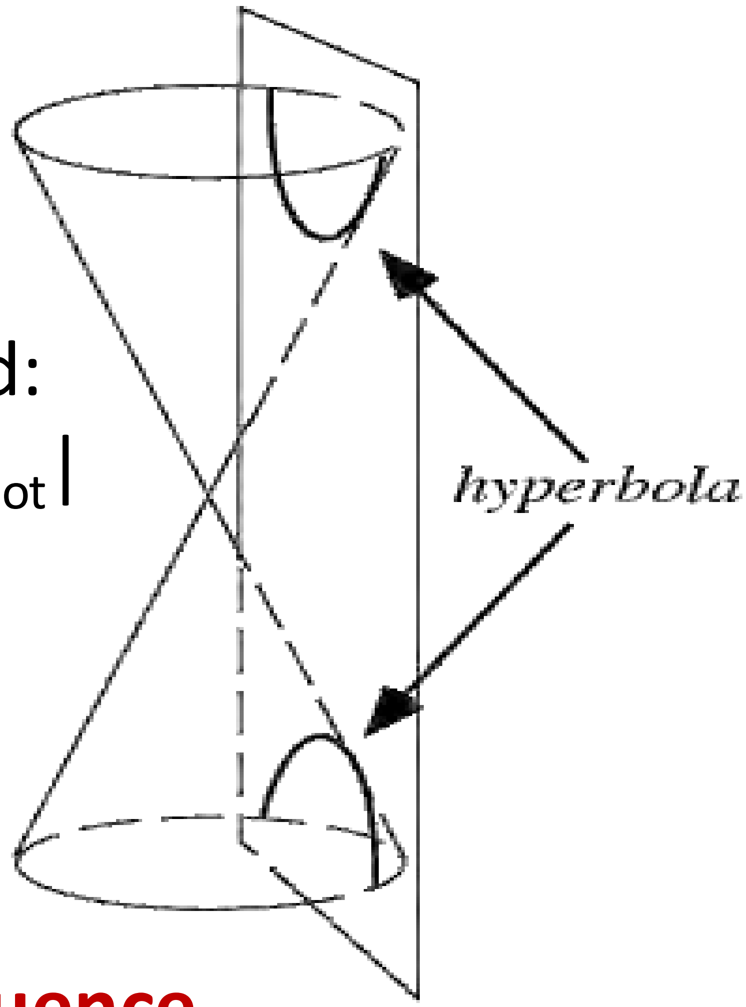


Bound:

$E_{kin} < |E_{pot}|$

Unbound:

$E_{kin} > |E_{pot}|$

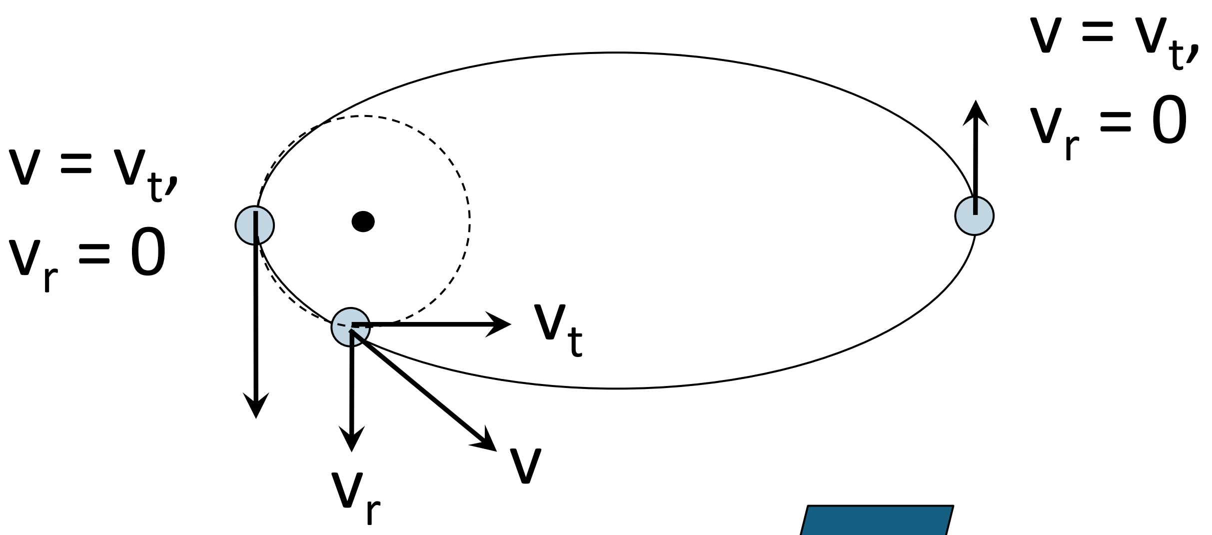


Kepler's 1st law is a direct consequence

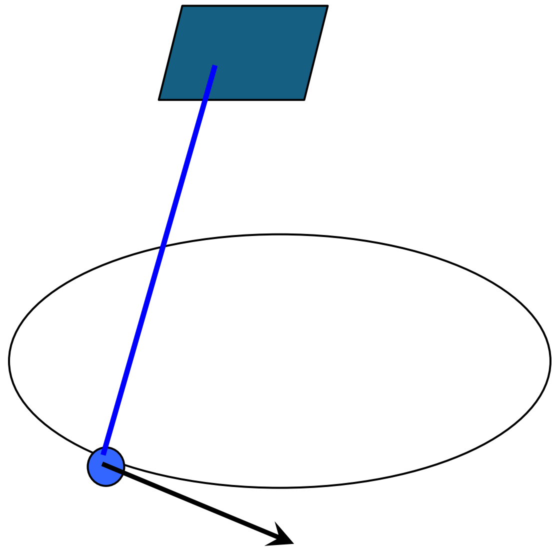
Why Ellipses?

A rigorous derivation (in polar coordinates) is a bit tedious, but we can have a simple intuitive hint:

Decompose the total velocity v into the radial (v_r) and tangential (v_t) components

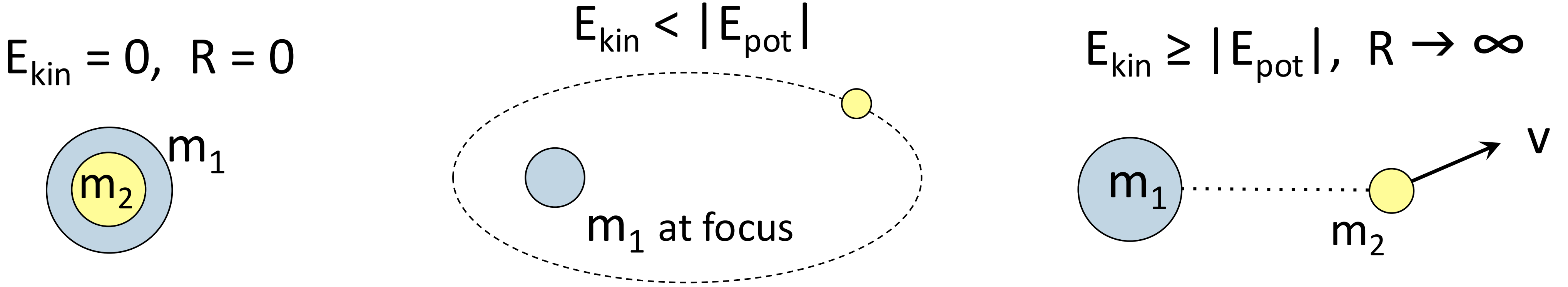


Consider the total motion as a synchronous combination of a radial and circular harmonic oscillator with the same period (recall that the period does not depend on the amplitude)



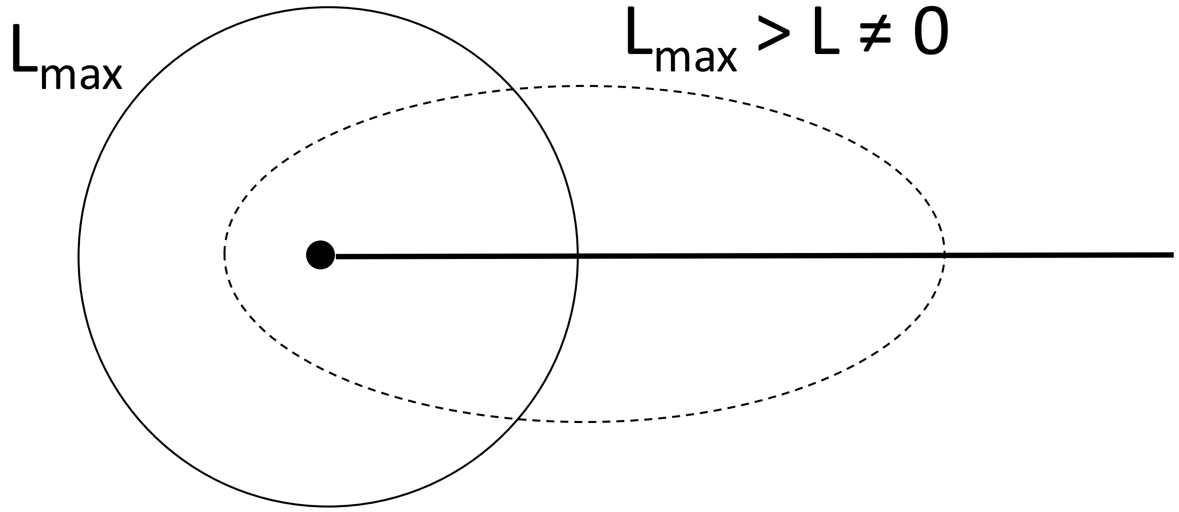
Orbit Sizes and Shapes

For bound (elliptical) orbits, the ***size*** (semimajor axis) depends on the total ***energy***:



The ***shape*** (eccentricity) of the orbit depends on the ***angular momentum:***

Circular orbit: maximum angular momentum for a given energy



Radial orbit: zero angular momentum

$L = 0$

Kepler's 2nd Law: A quick and simple derivation

Angular momentum, at any time: $L = M_{pl}\, V\, r = const.$ (conservation)

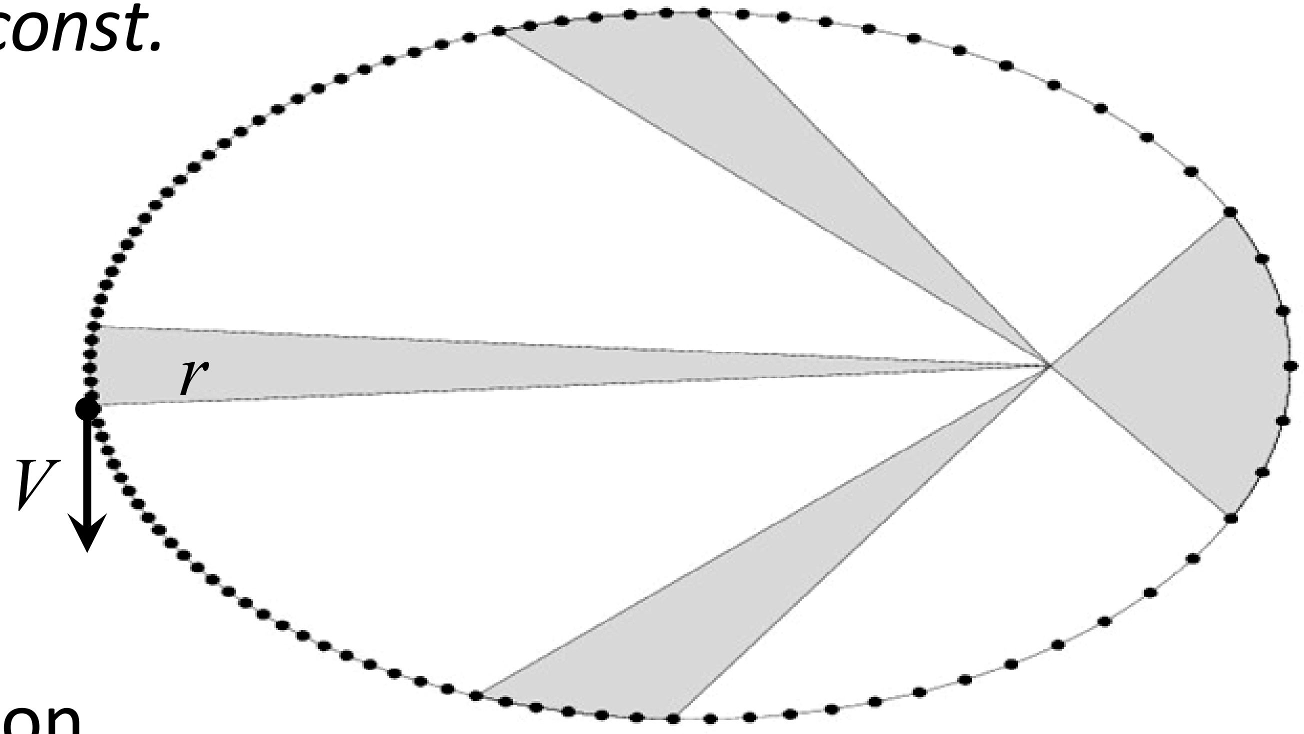
Thus, since M_{pl} = const., $V\, r = const.$ (this is also an "adiabatic invariant")

Element of area swept by the radius vector: $dA = V\, r\, dt$

Sectorial velocity: $dA/dt = V\, r = const.$

Independent of M_{pl} !

It is ***a consequence of the conservation of the angular momentum***



Planets move slower at the aphelion and faster at the perihelion

Kepler' s 3rd Law: A quick and simple derivation

Centripetal and centrifugal force:

$F_{cp} = G\, M_{pl}\, M_{\odot} / (a_{pl} + a_{\odot})^{2} \approx G\, M_{pl}\, M_{\odot} / a_{pl}^{\;2}$

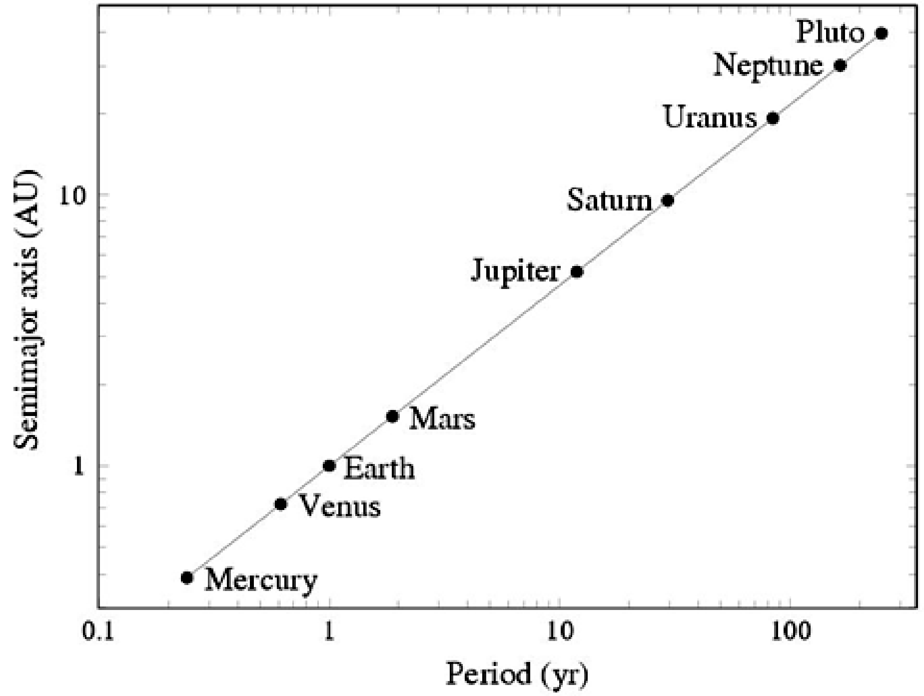
(since $M_{pl} << M_{\odot}$, $a_{pl} >> a_{\odot}$)

$F_{cf} = M_{pl}\, V_{pl}^{2} / a_{pl} = 4\,\pi^{2}\, M_{pl}\, a_{pl} / P^{2}$

(since $V_{pl} = 2\,\pi\, a_{pl} / P$)

(a is the orbit semimajor axis)

Since $F_{cp} = F_{cf}$ □ $\boxed{4\,\pi^{2}\, a_{pl}^{\;3} = G\, M_{\odot}\, P^{2}}$



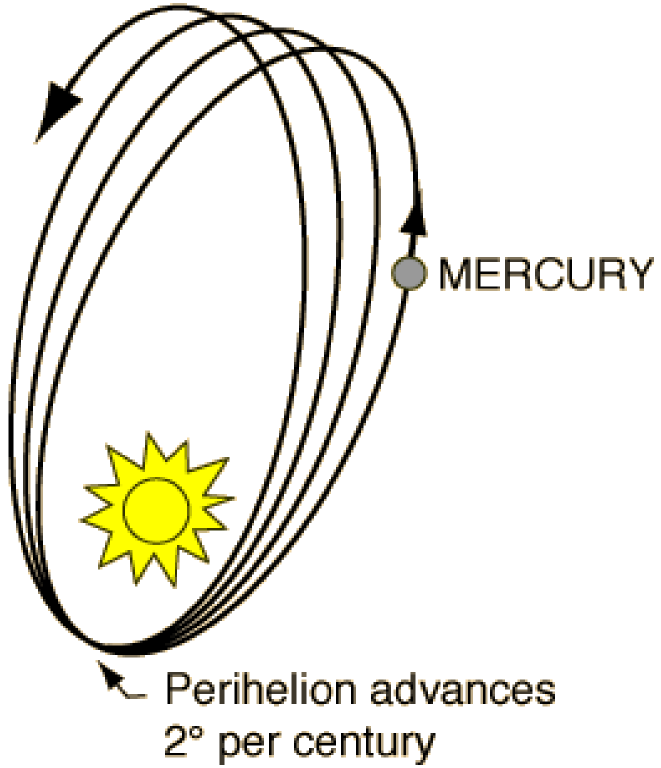
Another way: $E_{kin} = M_{pl} V_{pl}^{2} / 2 = E_{pot} \approx G\, M_{pl} M_{\odot} / a_{pl}$, $V_{pl} = 2\pi\, a_{pl}/P$

Substitute for V_{pl}^{2}: $4\,\pi^{2}\, a_{pl}^{\;3} = G\, M_{\odot}\, P^{2}$

□ It is ***a consequence of the conservation of energy***

It Is Actually A Bit More Complex …

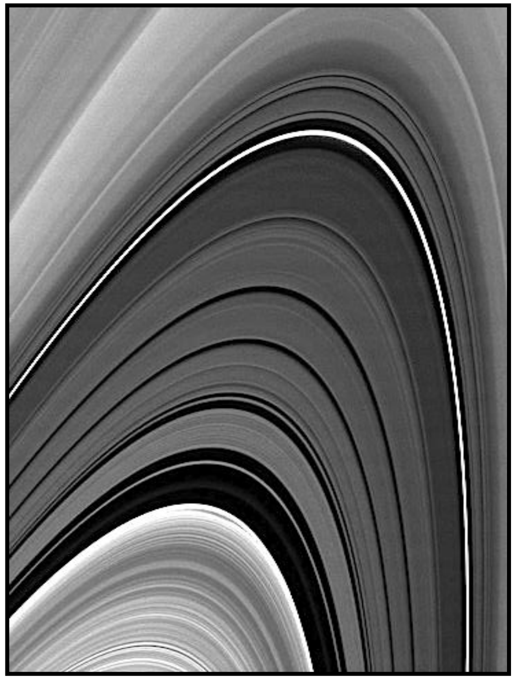
- Kepler's laws are just an approximation: we are treating the whole system as a collection of isolated 2-body problems
- There are **no analytical solutions for a general problem** with > 2 bodies! But there is a good ***perturbation theory,*** which can produce very precise, but always approximate solutions. Examples:
    - Discovery of Neptune (1846)
    - Comet impacts on Jupiter
    - Planet 9?
- Newtonian theory is an approximation of the theory of relativity. Relativistic effects can be used to test it, e.g., precession of Mercury's orbit

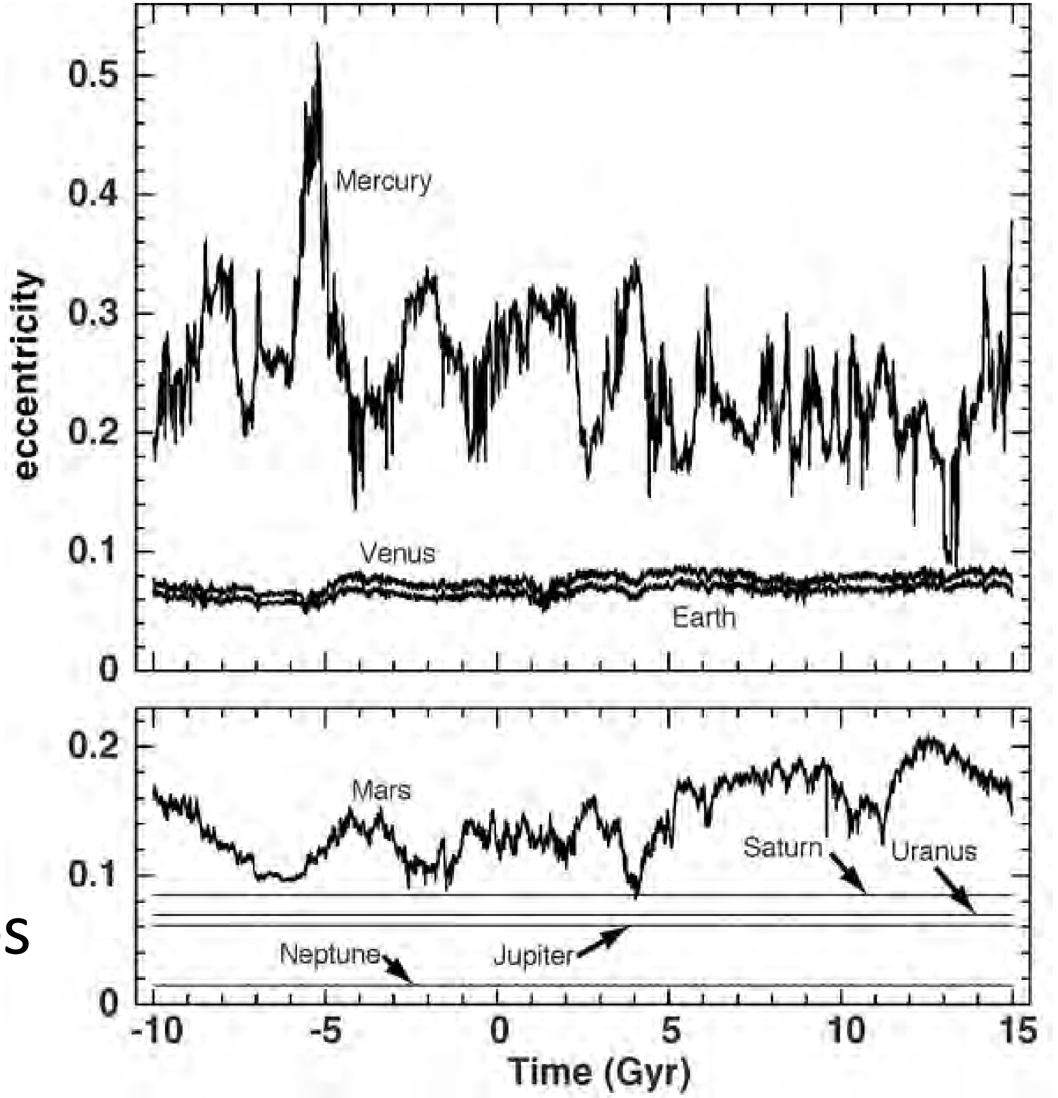


And Even More Complex: Dynamical Chaos

- Systems of many gravitationally interacting bodies can develop some very complex dynamical behavior

- Dynamical resonances can develop between rotation and revolution periods, Kirkwood gaps in Saturn's rings; etc.





- If you wait long enough, more complex dynamics can occur, including dynamical chaos

(Is Solar System stable?)

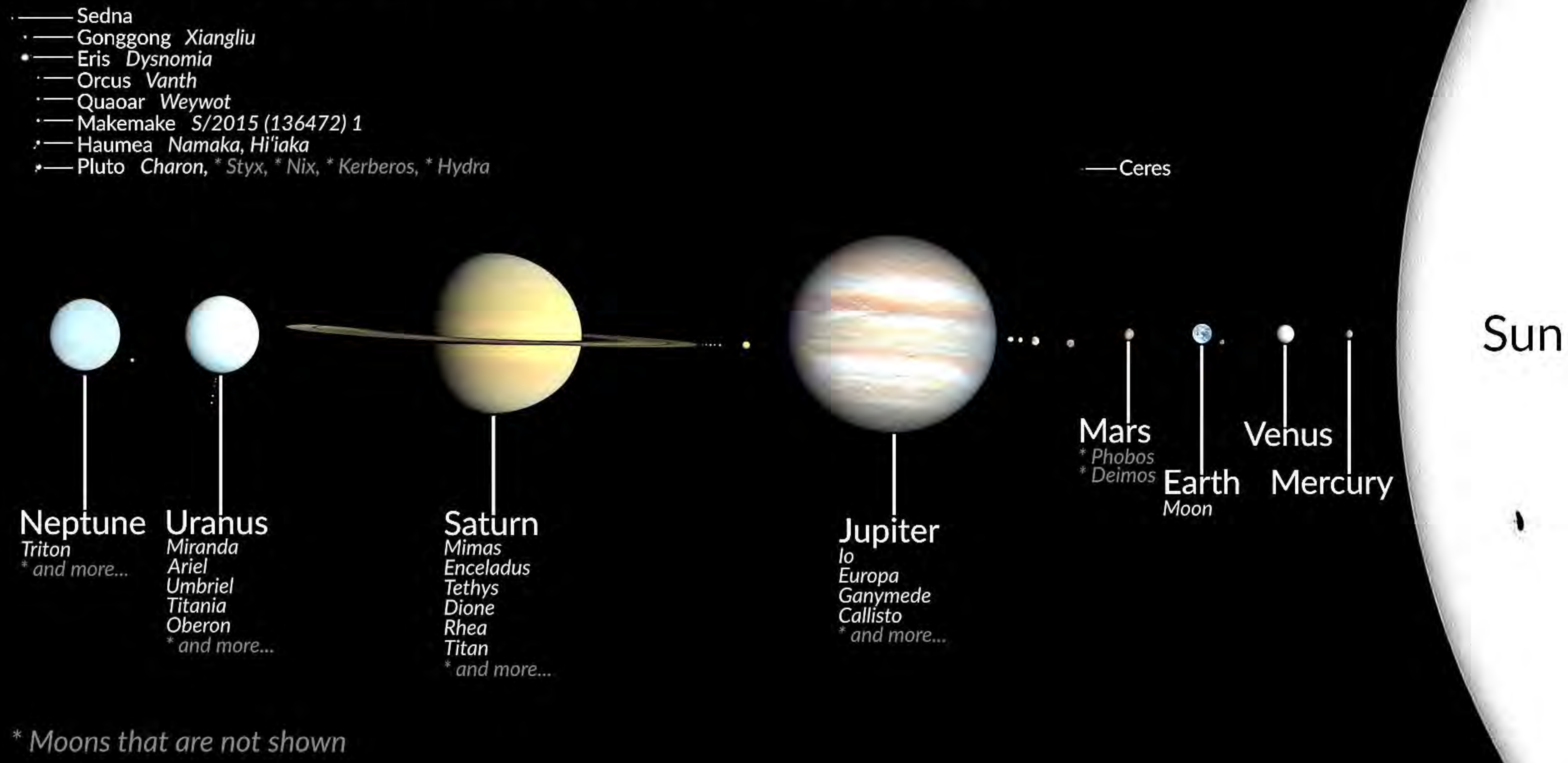
Kepler's 3rd law is…

A. Cubes of orbit sizes ~ squares of orbital periods
B. Squares of orbit sizes ~ cubes of orbital periods
C. A consequence of the conservation of energy
D. A consequence of the conservation of angular momentum

The shape of a closed orbit depends on

A. Total energy
B. Total angular momentum
C. Angular momentum for a given energy
D. None of the above

Solar System in true imagery, color and size



Three Kinds of Planets

- Rocky: inner Solar system, smaller, high density, composed mostly of heavier elements
  - Mercury, Venus, Earth, Mars

- Gas giants: Outer Solar system, large, massive, lower densities, lighter elements are abundant
  - Jupiter, Saturn, Uranus, Neptune

- Dwarf/Icy planets: Very Outer Solar system, low mass, small, icy, but large enough to achieve a spherical shape
  - Pluto, Sedna, Eris, Makemake, Ceres, etc.

Planetary Demographics

Name	Distance from Sun (AU)[2]	Revolution Period (y)	Diameter (km)	Mass (10^{23} kg)	Density (g/cm^3)[3]
Mercury	0.39	0.24	4,878	3.3	5.4
Venus	0.72	0.62	12,120	48.7	5.2
Earth	1.00	1.00	12,756	59.8	5.5
Mars	1.52	1.88	6,787	6.4	3.9
Jupiter	5.20	11.86	142,984	18,991	1.3
Saturn	9.54	29.46	120,536	5686	0.7
Uranus	19.18	84.07	51,118	866	1.3
Neptune	30.06	164.82	49,660	1030	1.6

Mostly rocks

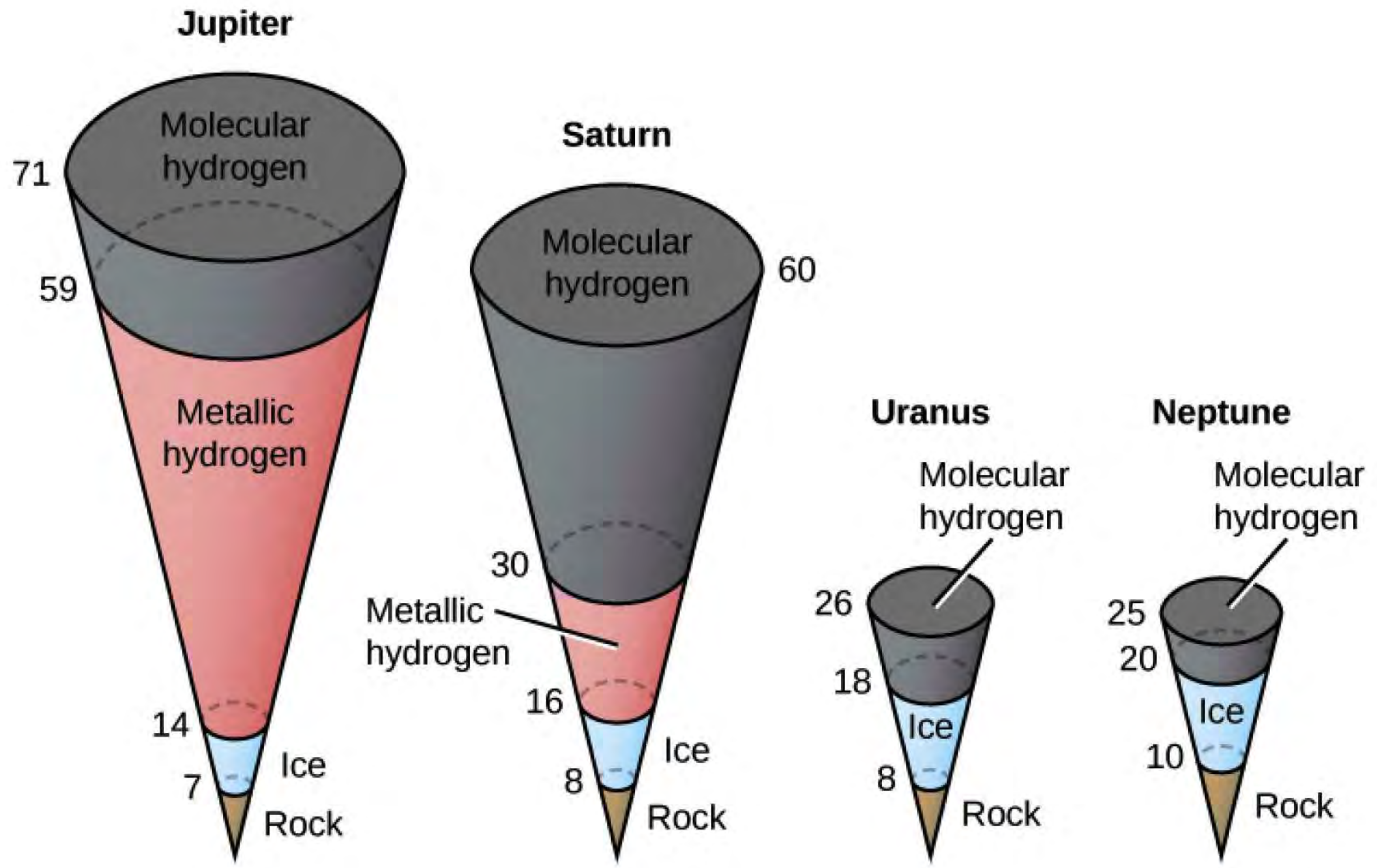
Jump in size and mass

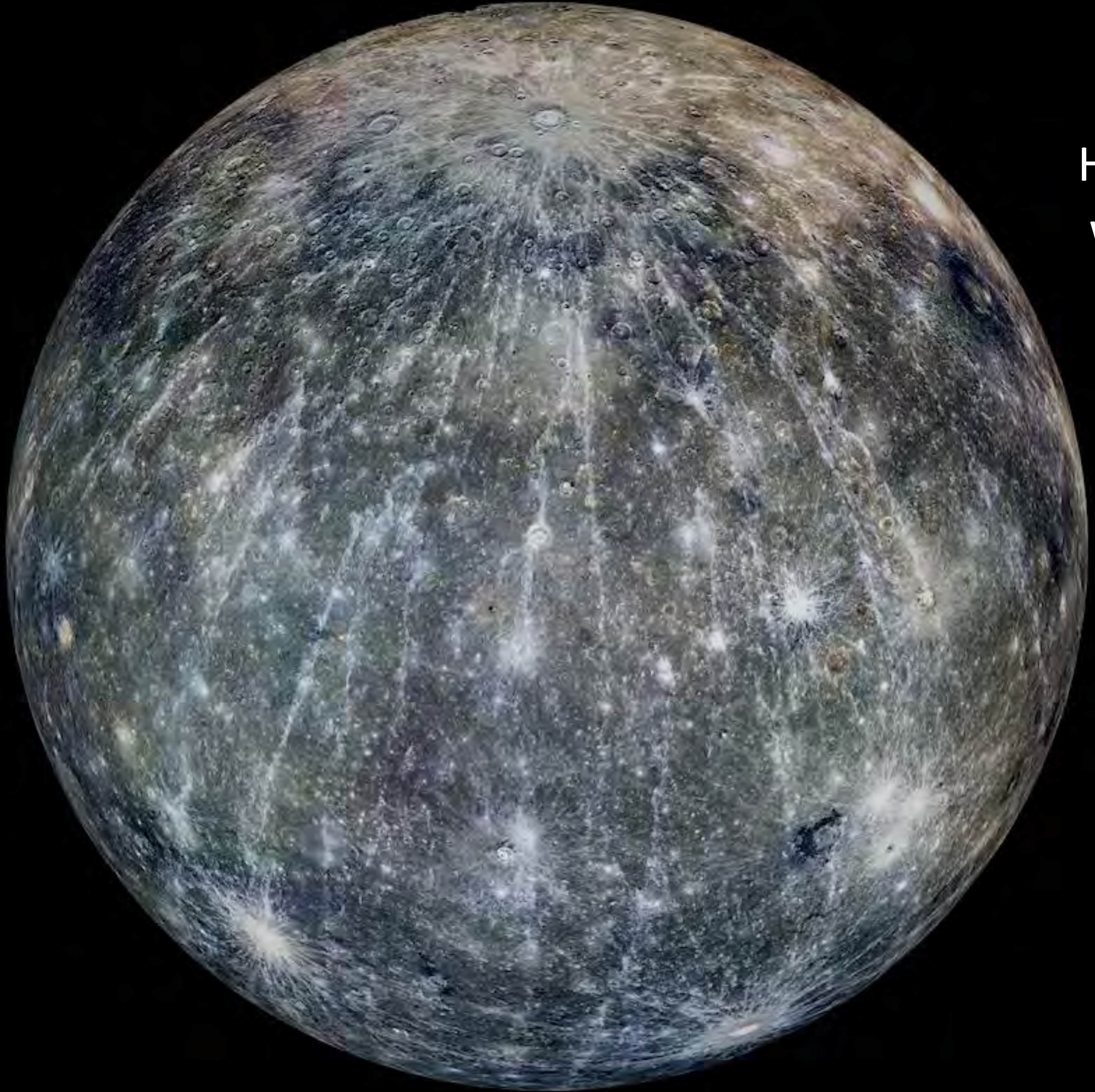
Mostly gas

Masses and Compositions of the Major Planets

- At the location of the terrestrial planets, there was not much mass in the planetesimals, since they were formed of heavier, non-abundant elements
- In the outer solar system, there was more mass in the planetesimals, since they were formed of abundant, hydrogen-bearing compounds. They produced more massive planetesimals that incorporated the hydrogen and helium gas that makes up most of Jupiter and Saturn
- At the position of the Earth, only silicates and other more "refractory" substances would have precipitated from the vapor state. At Jupiter and beyond, ices of water, ammonia, methane, would have condensed
- Beyond Neptune, there density of the protoplanetary disk was too low for the massive planets to form, but icy dwarf planets did form

Composition and Structure of the Gas Giants





Mercury

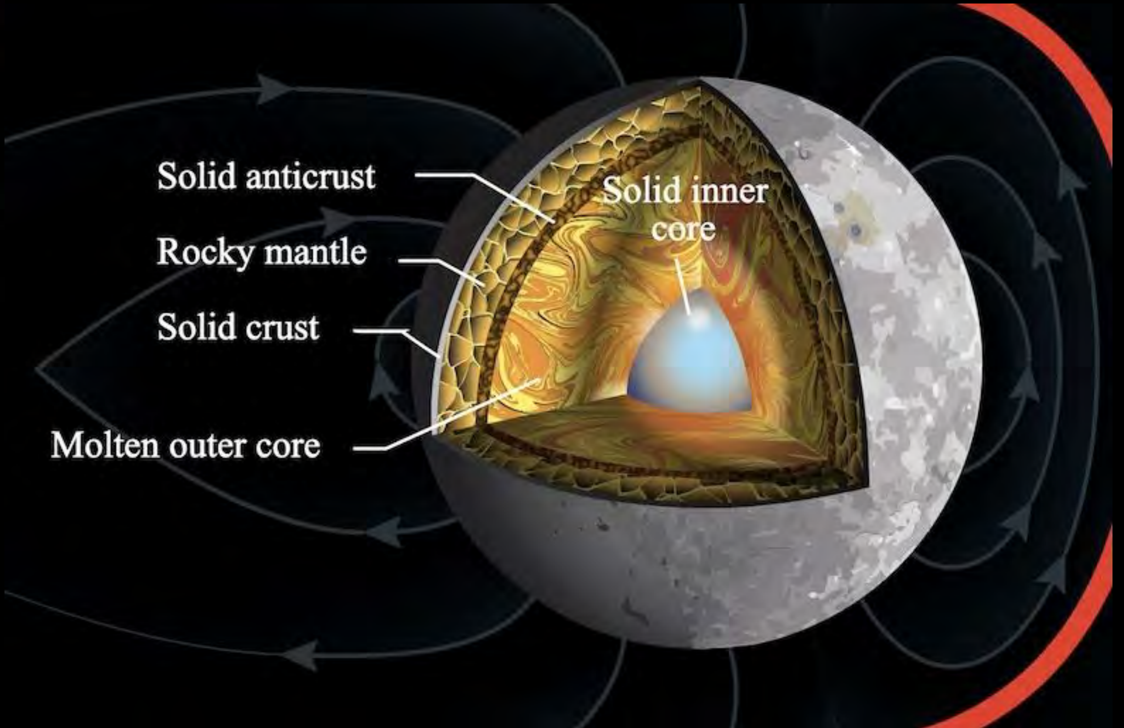
High density due to a large metallic core

Weak magnetic field, thin atmosphere

Surface temperature 100 – 700 K

Precessing elliptical orbit

Mapped by *Mariner 10*, *MESSENGER*



Venus

Comparable in size, density to Earth

Thick dense atmosphere, 96.5% CO_2

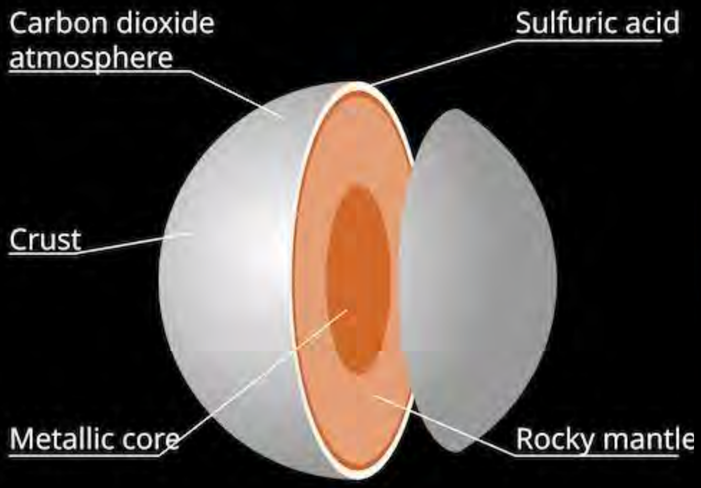
→ Runaway greenhouse effect

Atmospheric surface pressure 92 atm

High altitude sulphuric acid clouds

Ongoing volcanic activity

← Radar mapped by *Magellan* probe



Clouds in UV

Earth's Moon

Tidally locked rotation

Negligible atmosphere → no erosion

Composition and density like the Earth's mantle

Maria = former magma oceans

Surface gravity ~ 1/6 *g*

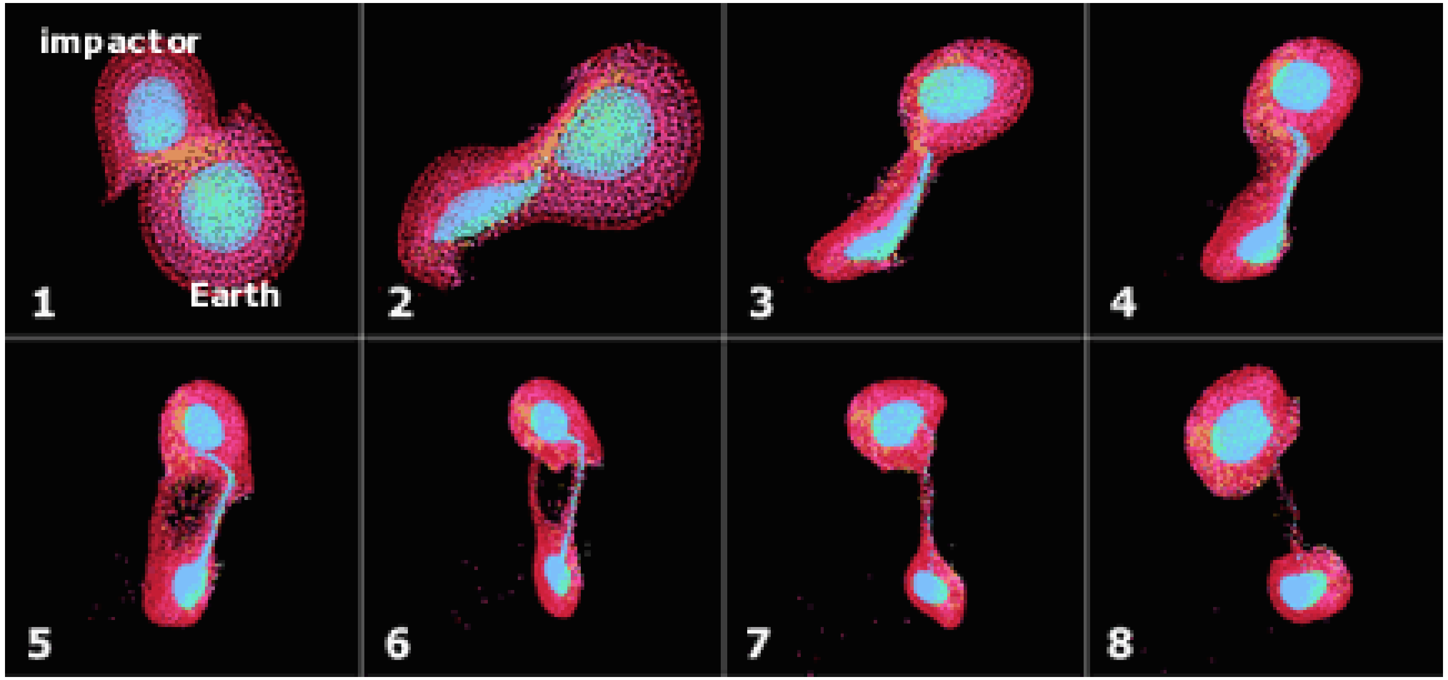
Surface temperature 100 – 390 K

Some water ice near the poles

Extensively studied and mapped by many missions, e.g., Lunar Reconnaissance Orbiter with ~1 m resolution

The Origin of the Moon

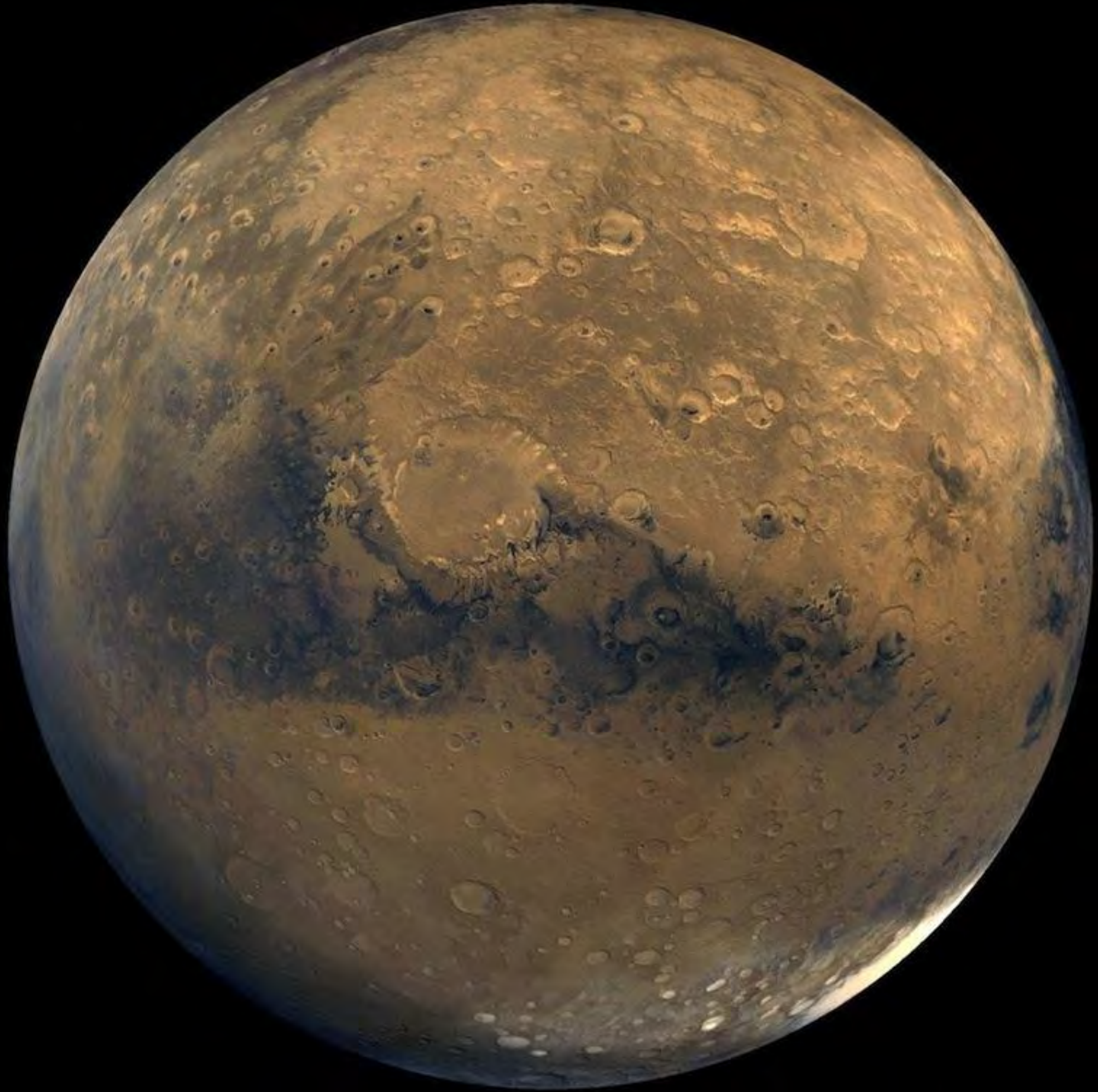
While the panets were still forming 4.5 Gigayears ago, a Mars-sized protoplanet (named Theia) collided with the proto-Earth, which ejected large quantities of melted rocks. The moon was formed from that debris



(Courtesy of A. G. W. Cameron, Harvard College Observatory.)

This explains why Moon has the same composition as the Earth's mantle

It is also probably the reason for the tilt of the Earth's axis



Mars

~ ½ size and 11% of Earth mass

Surface gravity 0.38 *g*

Thin atmosphere, 0.006 atm, mostly CO_2, but strong winds, dust storms

Surface temperature 160 – 310 K

Polar CO_2 ice caps

Axis tilt 25° ~ Earth, has seasons

Satellines Phobos and Deimos

Lots of evidence for liquid water in the past, and even an ocean

Explored by numerous space missions, orbiters, landers, rovers

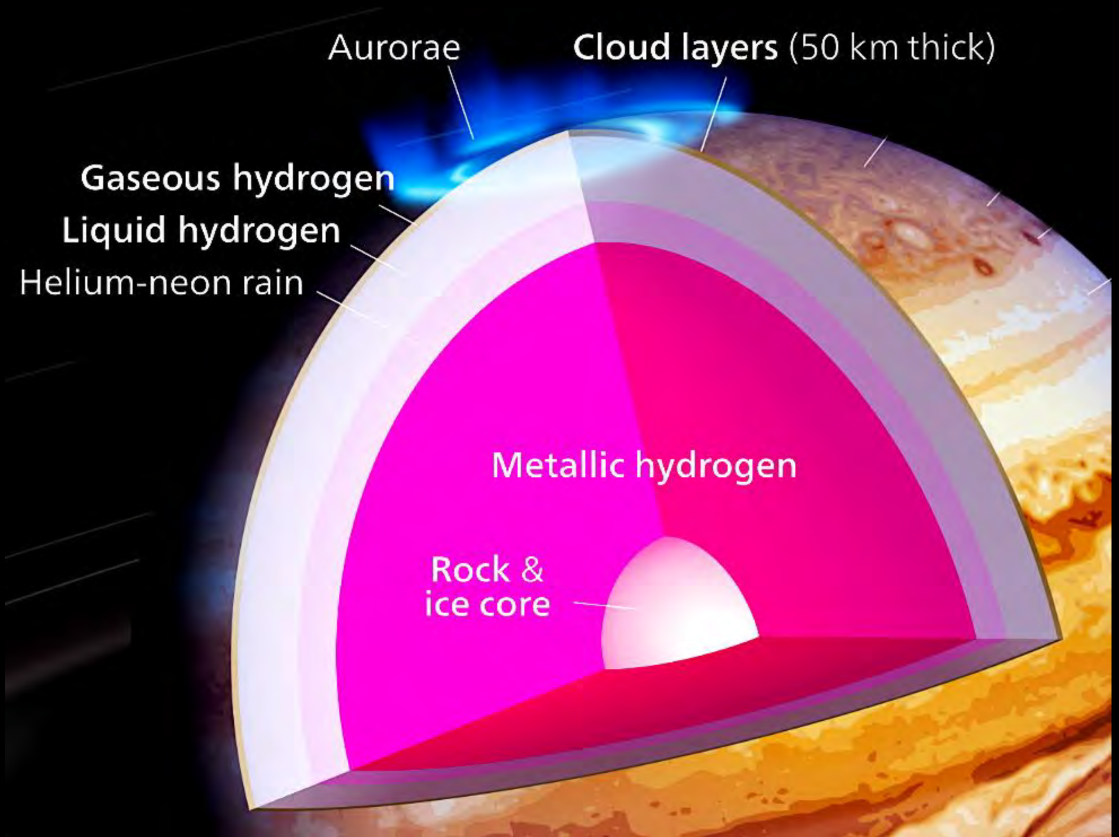
Jupiter

11 × larger and 320 × mass of the Earth

Composition 89% H, 10% He

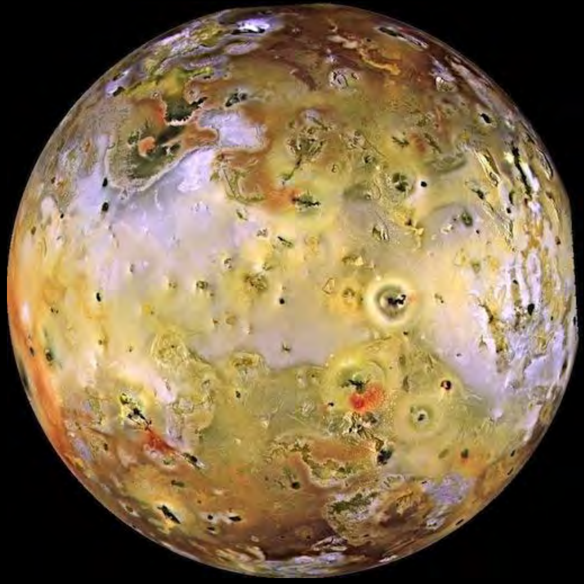
Blackbody temperature 88 K

Strong magnetic field, radiation belts

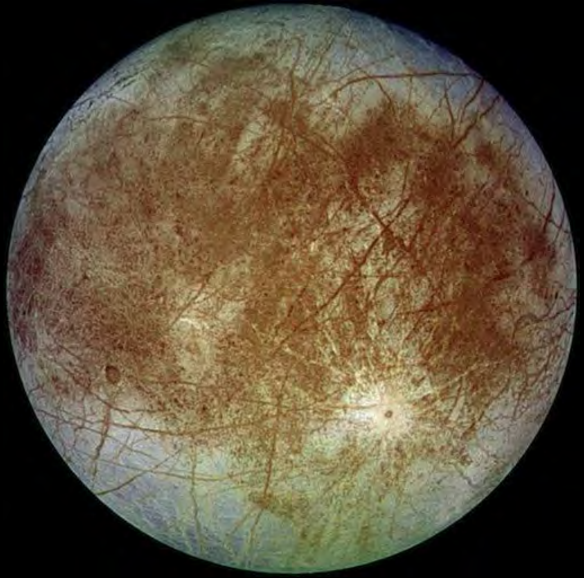


Jupiter's Galileian Moons

Io



Europa



Ganymede

Calisto

Over 400 active volcanoes, due to tidal heating

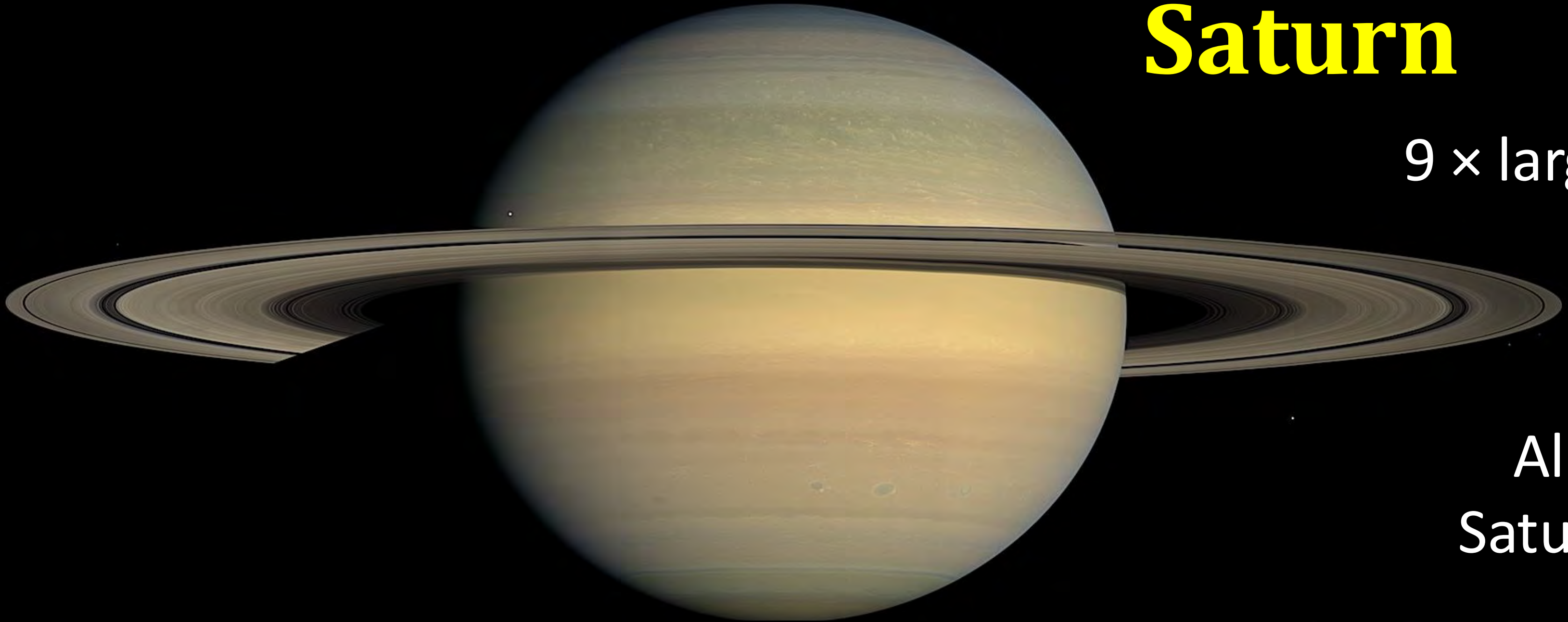
Subsurface ocean, with more water than on the Earth

1^{st} and 3^{rd} largest moons > Mercury

May also have subsurface oceans

In addition, Jupiter also has > 100 small moons, likely captured asteroids, and a system of rings similar, but fainter than Saturn's

Saturn



9 × larger and 95 × mass of the Earth

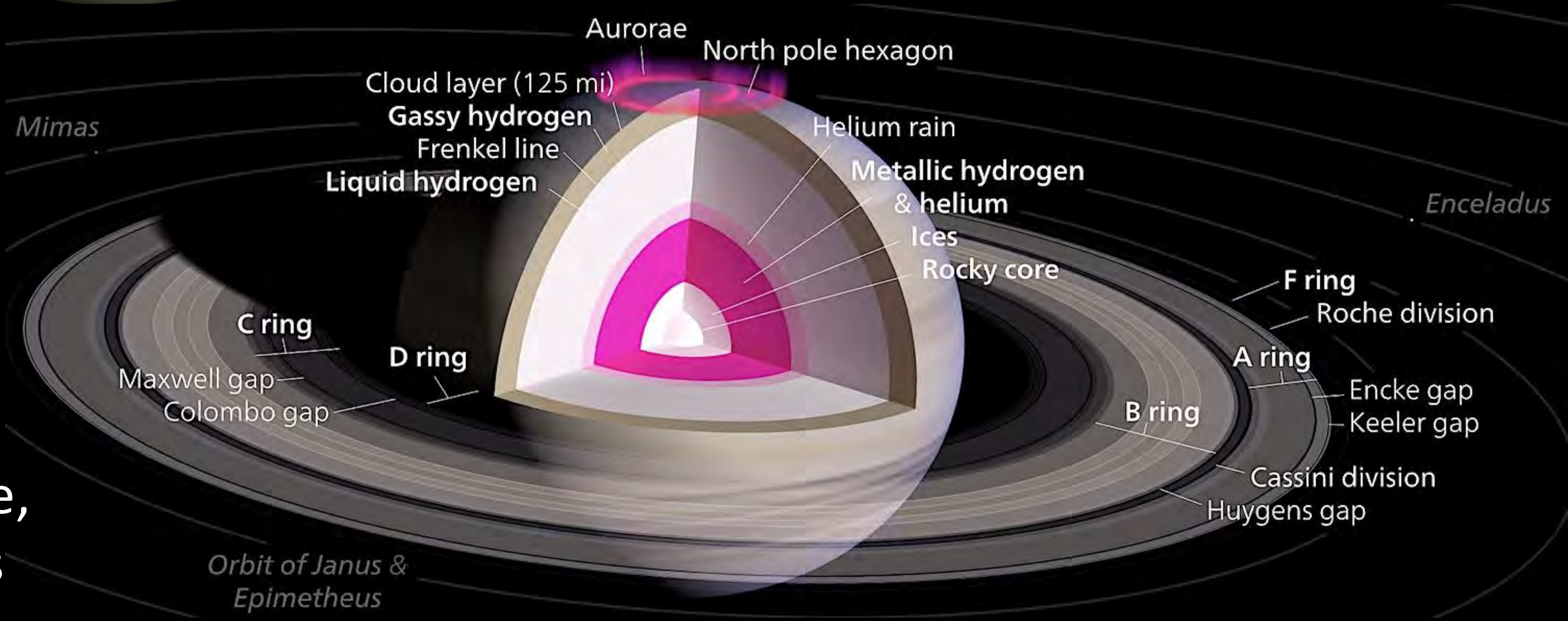
Similar to Jupiter

~ 300 known moons

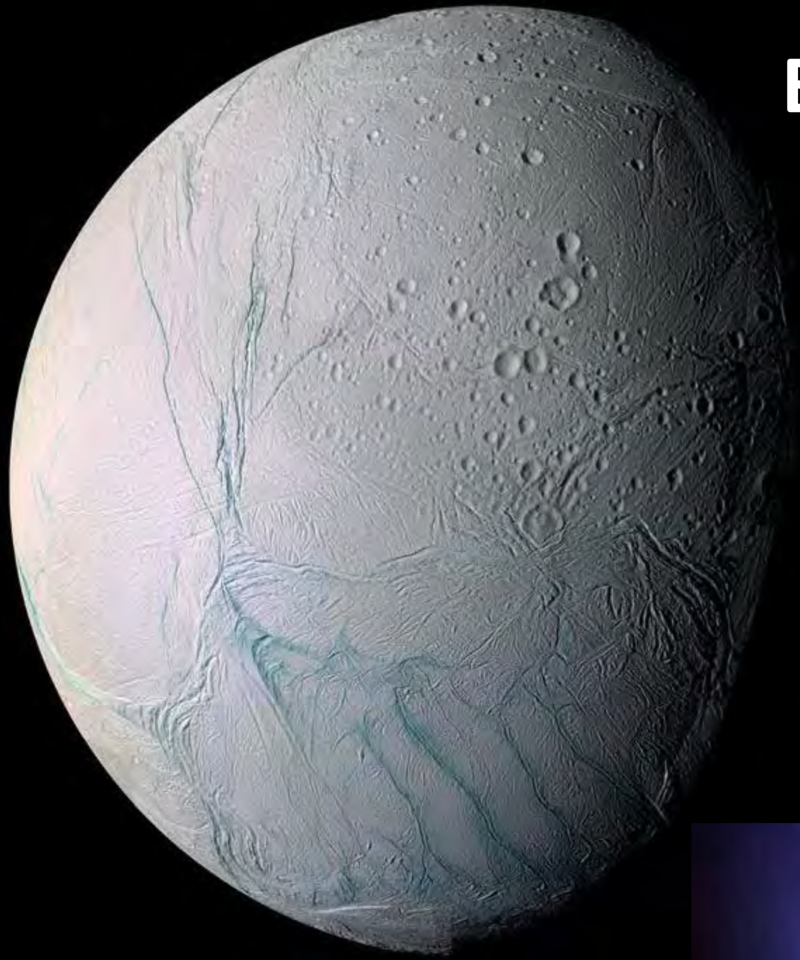
All gas giants have rings, but Saturn's are the most prominent

The rings are 120,000 km wide and only 20 m thick, composed mainly of ice particles ranging in size from dust to 10 m

They have a fine structure, due to orbital resonances



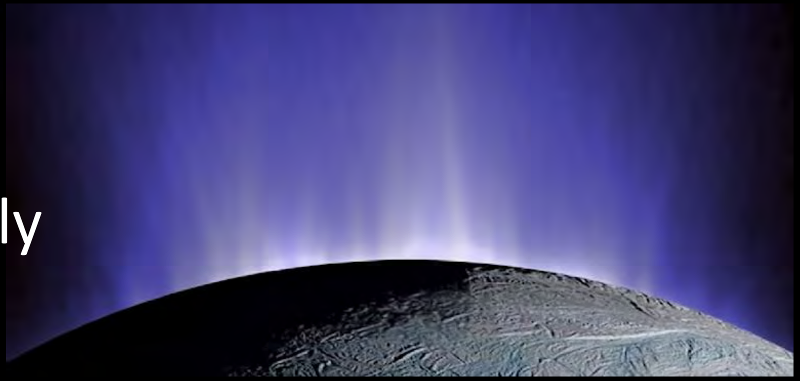
Saturn's Large Moons



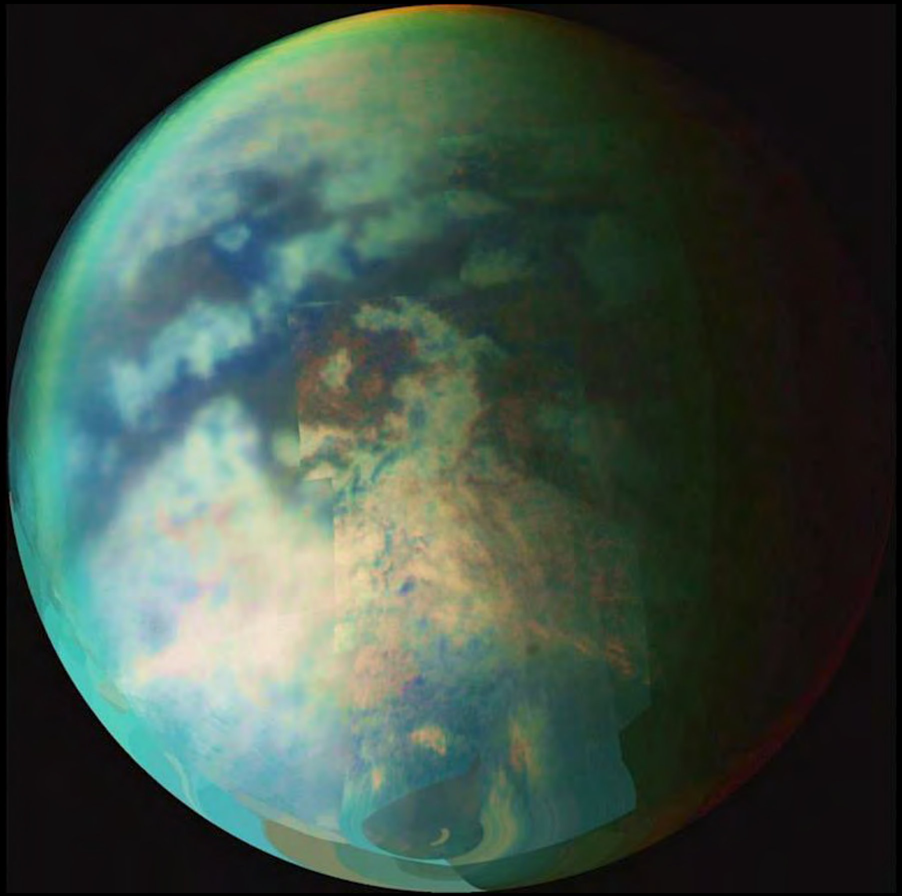
Enceladus

Subsurface
ocean

Plumes/gaysers, mostly
water, other volatiles



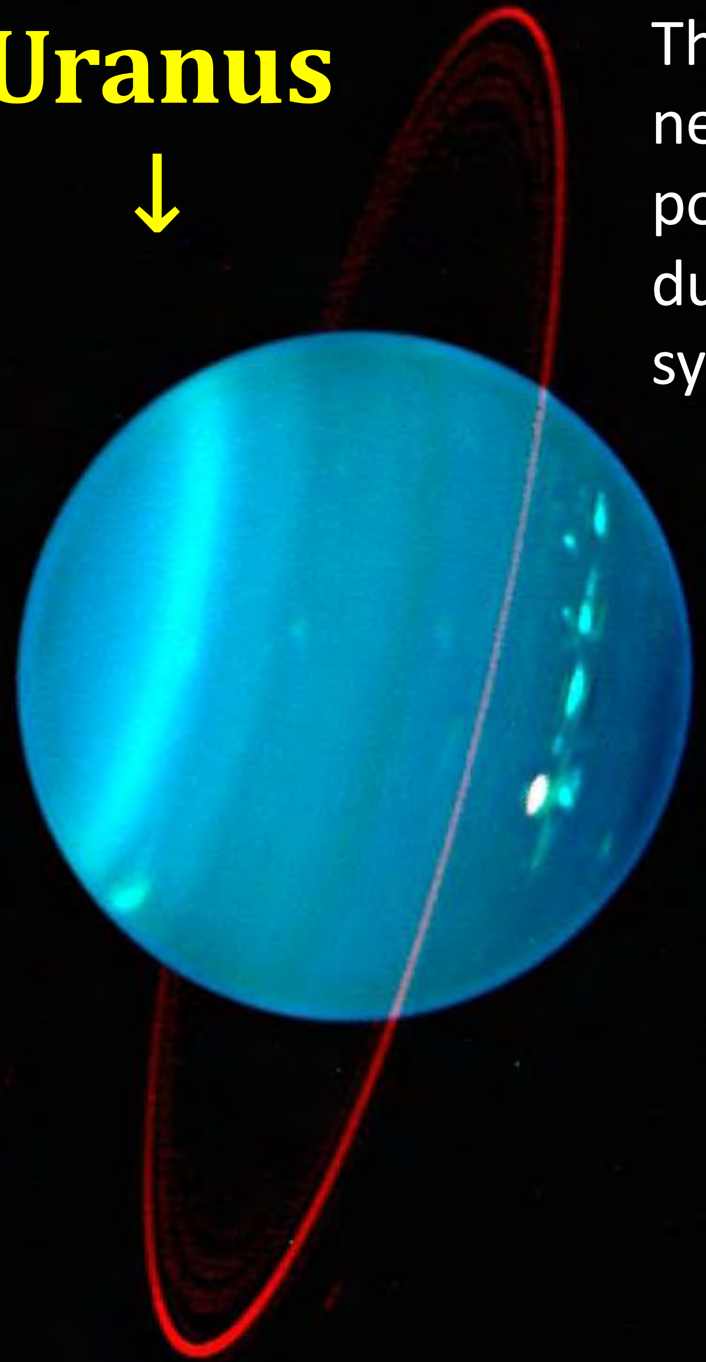
Titan



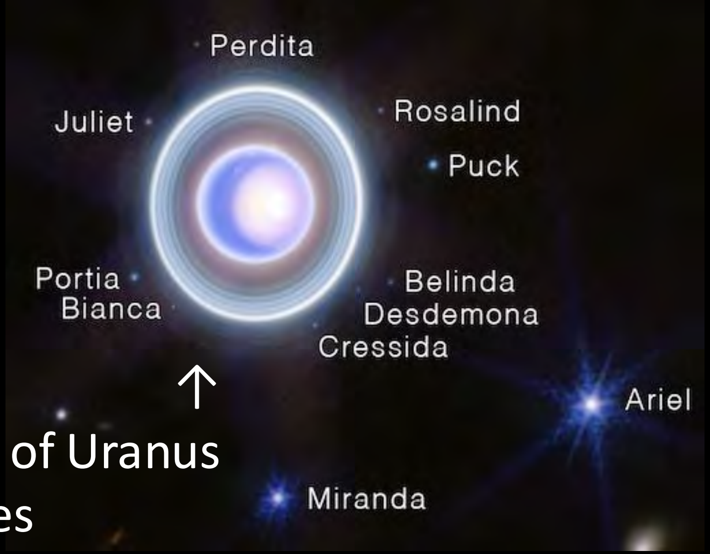
Dense atmosphere of nitrogen,
methane, and hydrogen

Lakes and seas of liquid
hydrocarbons on the surface,
and a subsurface water ocean

Uranus ↓



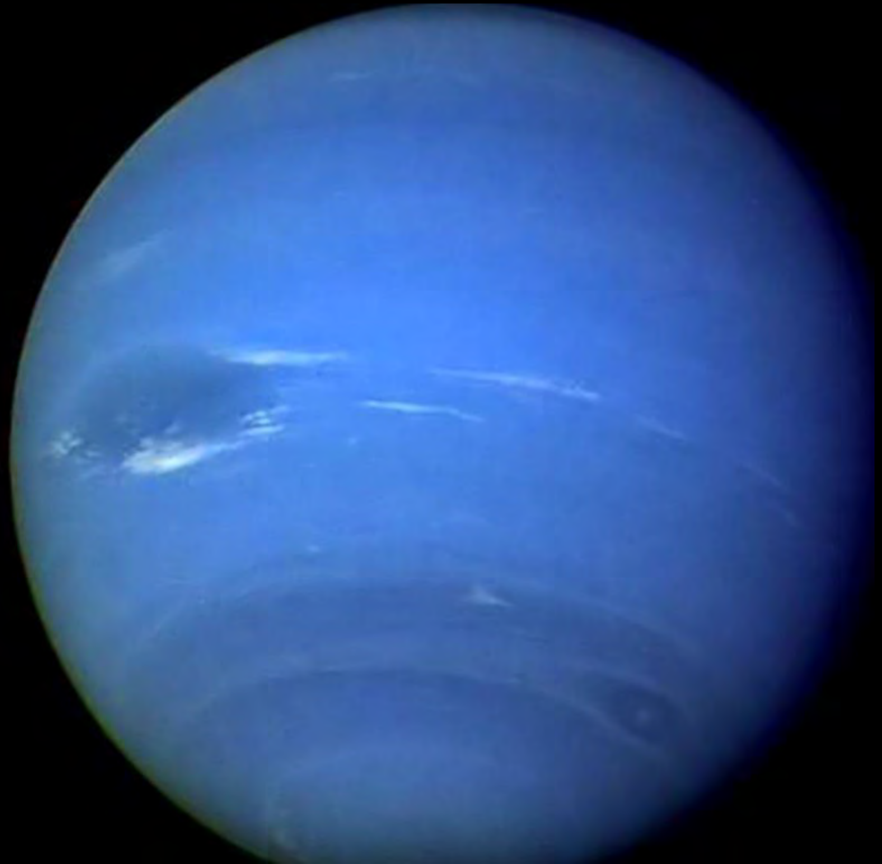
The only planet with the rotation axis nearly parallel to its orbital plane, possibly caused by major impacts during the formation of the Solar system



↑
JWST IR image of Uranus and its satellites

Neptune →

Discovered in 1846 by Galle, based on the predictions by Adams and Le Verrier, from perturbations of the orbit of Uranus



Pluto

The first Trans-Neptunian Object (TNO) and the first dwarf planet

Discovered in 1930 by Clyde Tombaugh, demoted to a dwarf planet in 2005 by Mike Brown and the IAU

The surface is mostly nitrogen ice, the mountains are water ice

Pluto's moon Charon →

Images from the *New Horizons* mission flyby in 2015

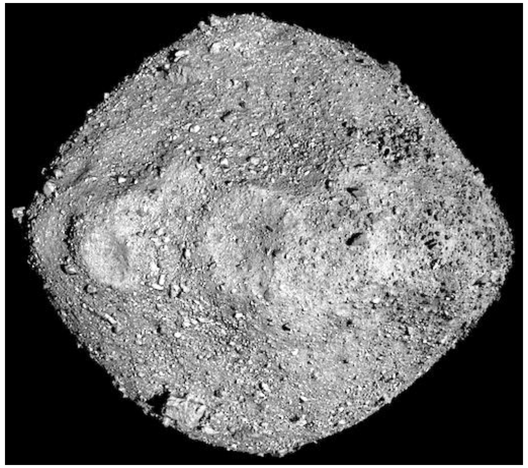
The Asteroids

Leftover rocky planetesimals

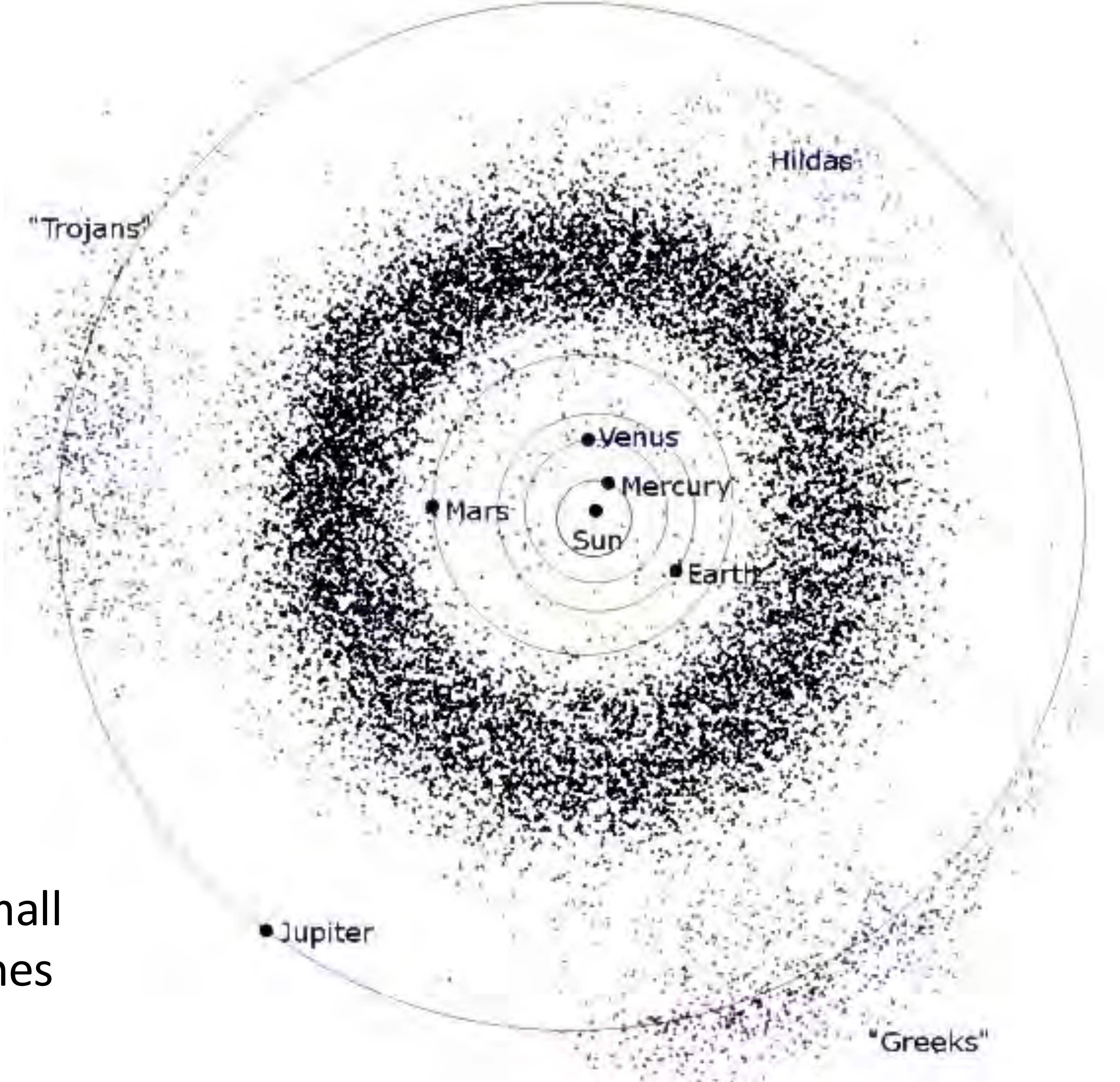
Most, but not all of them are in the **asteroid belt,** between Mars and Jupiter

Eros from *NEAR*

Bennu from *Osiris-REx*

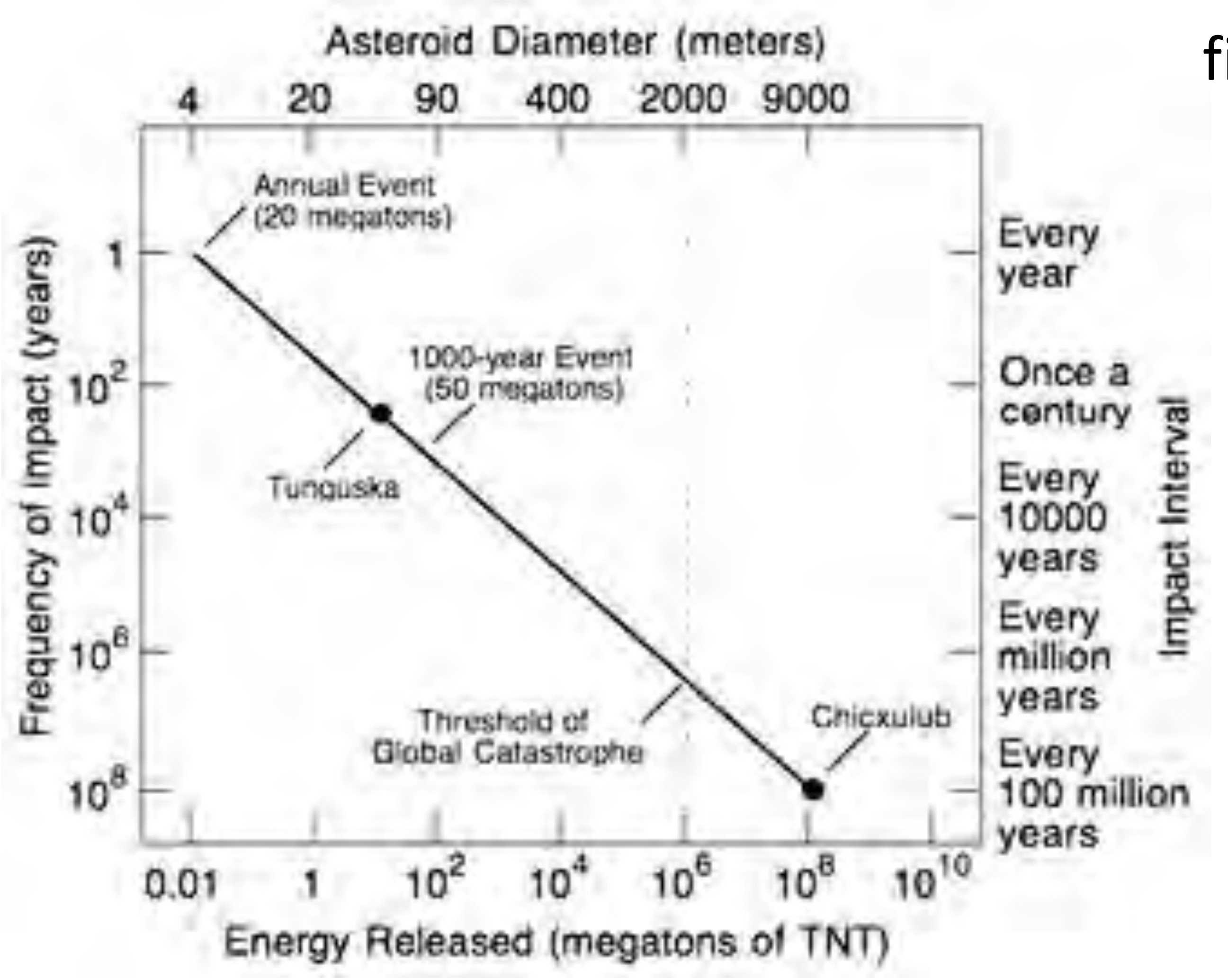


If their orbit crosses the Earth's, the small ones are the meteors, and the large ones impacts can me catastrophic

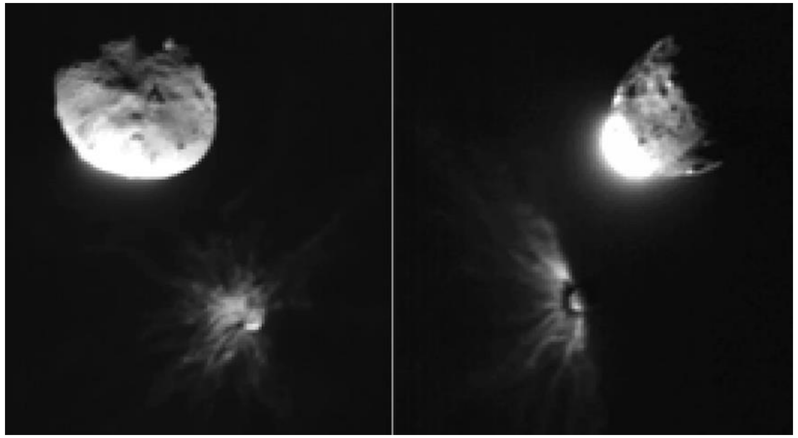


Asteroid/Comet Impacts on the Earth

There are now multiple sky surveys looking for the potential hazard asteroids (PHAs)



The goal is to find them early enough to deflect them



DART mission post-impact images

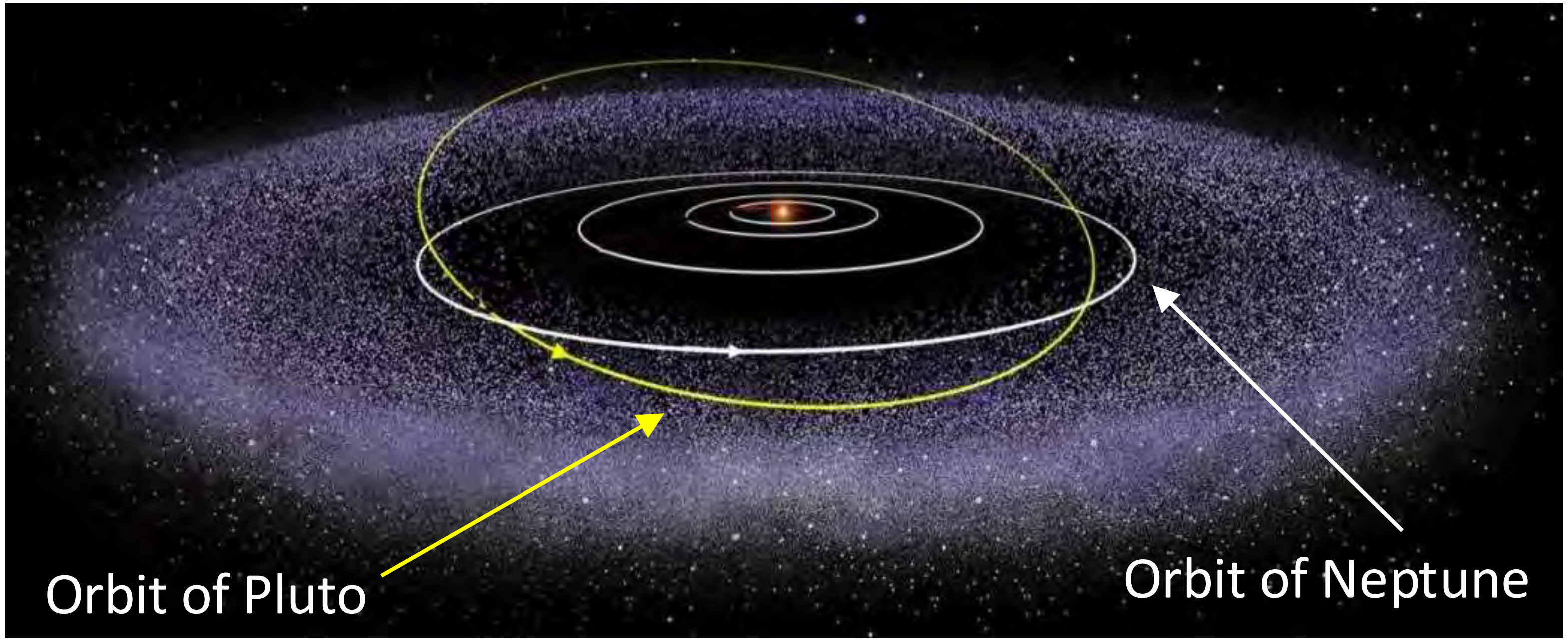
Barringer crater, Arizona, ~11.5 km wide

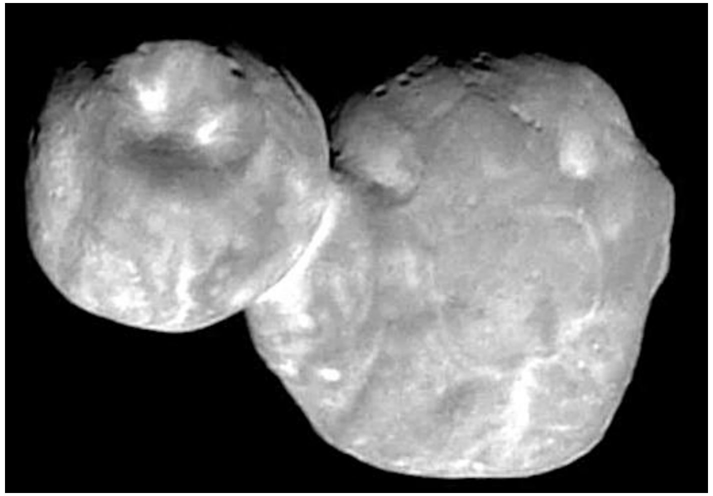
The Kuiper Belt: Leftover Icy/Rocky Planetesimals

~30 to ~55 au from the Sun

Kuiper Belt Objects (KBOs), also called Trans-Neptunian Objects (TNOs)

Orbits are often inclined relative to the Ecliptic plane



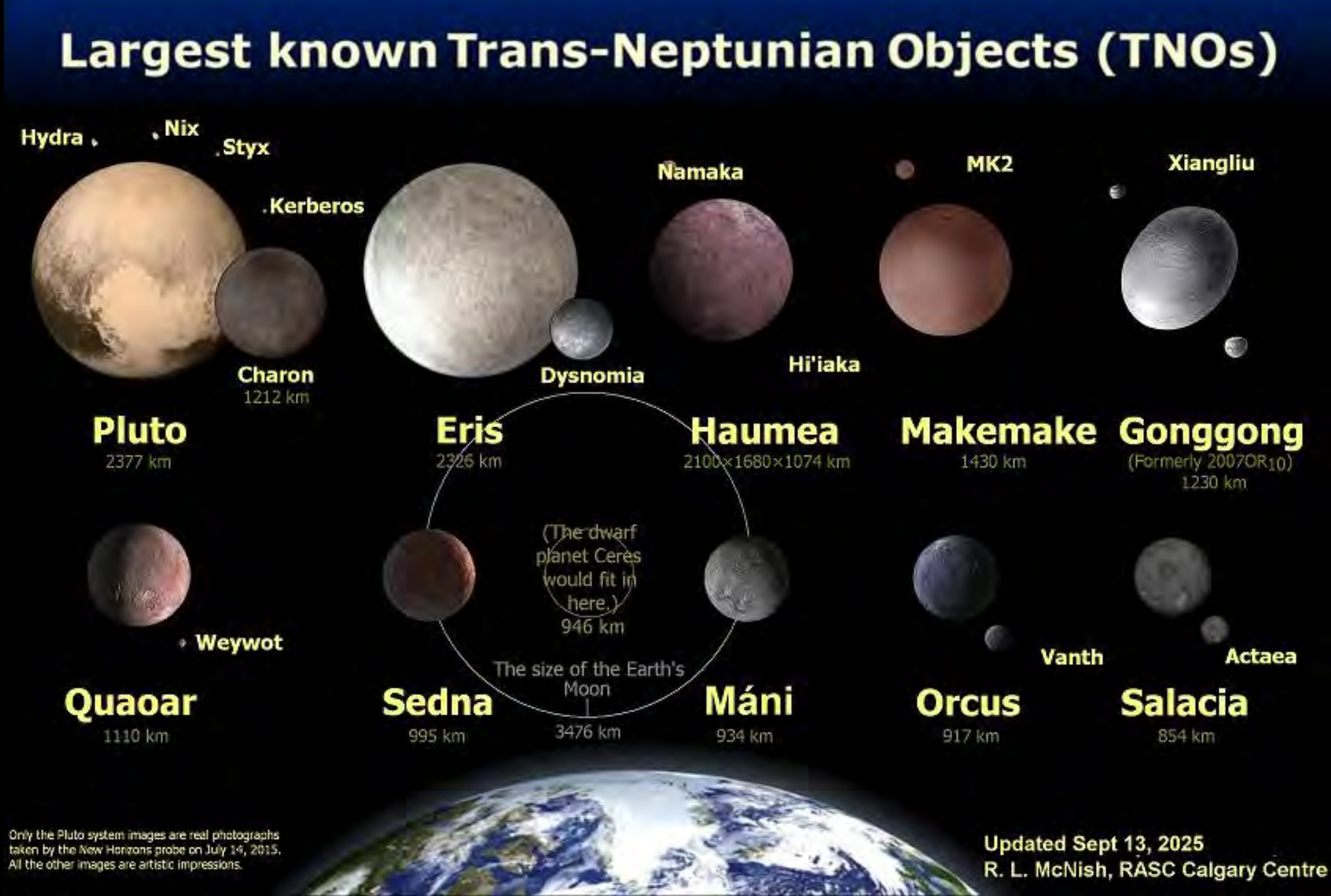


KBO Arrokoth from *New Horizons*

Some KBOs have coalesced to form **dwarf planets**

Some comets (most periodic ones, with elliptical orbits) come from the Kuiper Belt, and some (on the parabolic orbits) from the **Oort Cloud**, ~2,000 to ~100,000 au from the Sun

Dwarf planets and their satellites

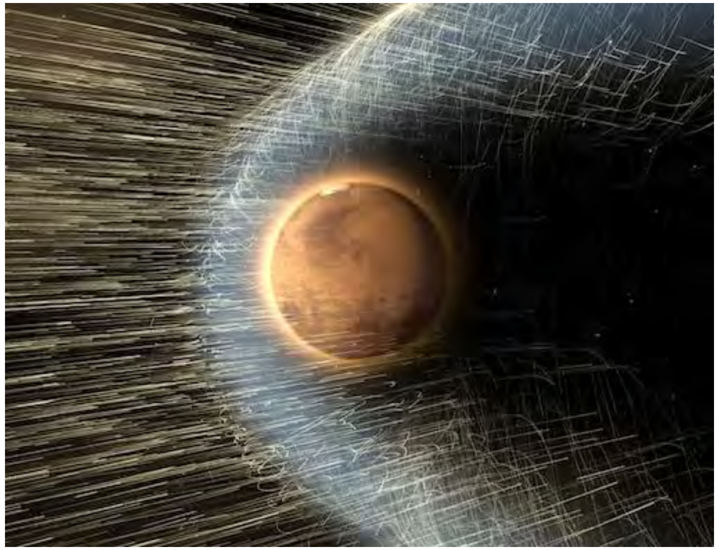


How Planets Obtain an Atmosphere?

- Accreted gas from the protoplanetary disk
- Gain volatiles by comet impacts mainly during the final stages of planet formation
- Outgassing during differentiation of planets
- Ongoing outgassing by volcanoes

Atmosphere is ***kept*** by the planet's gravity:

- Low mass worlds = almost no atmosphere
- High mass worlds = thick atmosphere

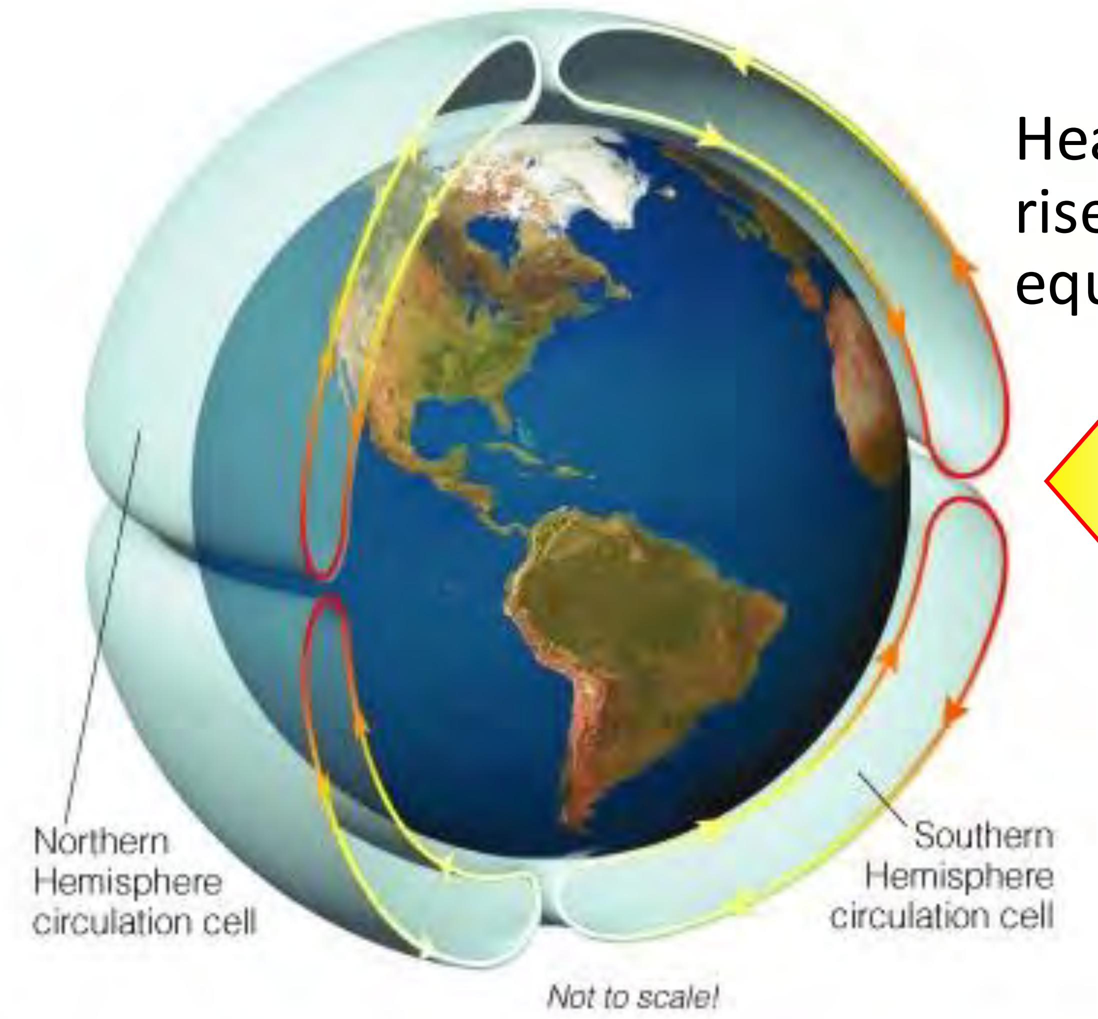


However, it can be also ***lost*** by being swept by the Solar wind, especially if the planet does not have a magnetic field to shield it from the charged particles. That probably happened with **Mars**

Atmospheric Dynamics: Why are the winds blowing?

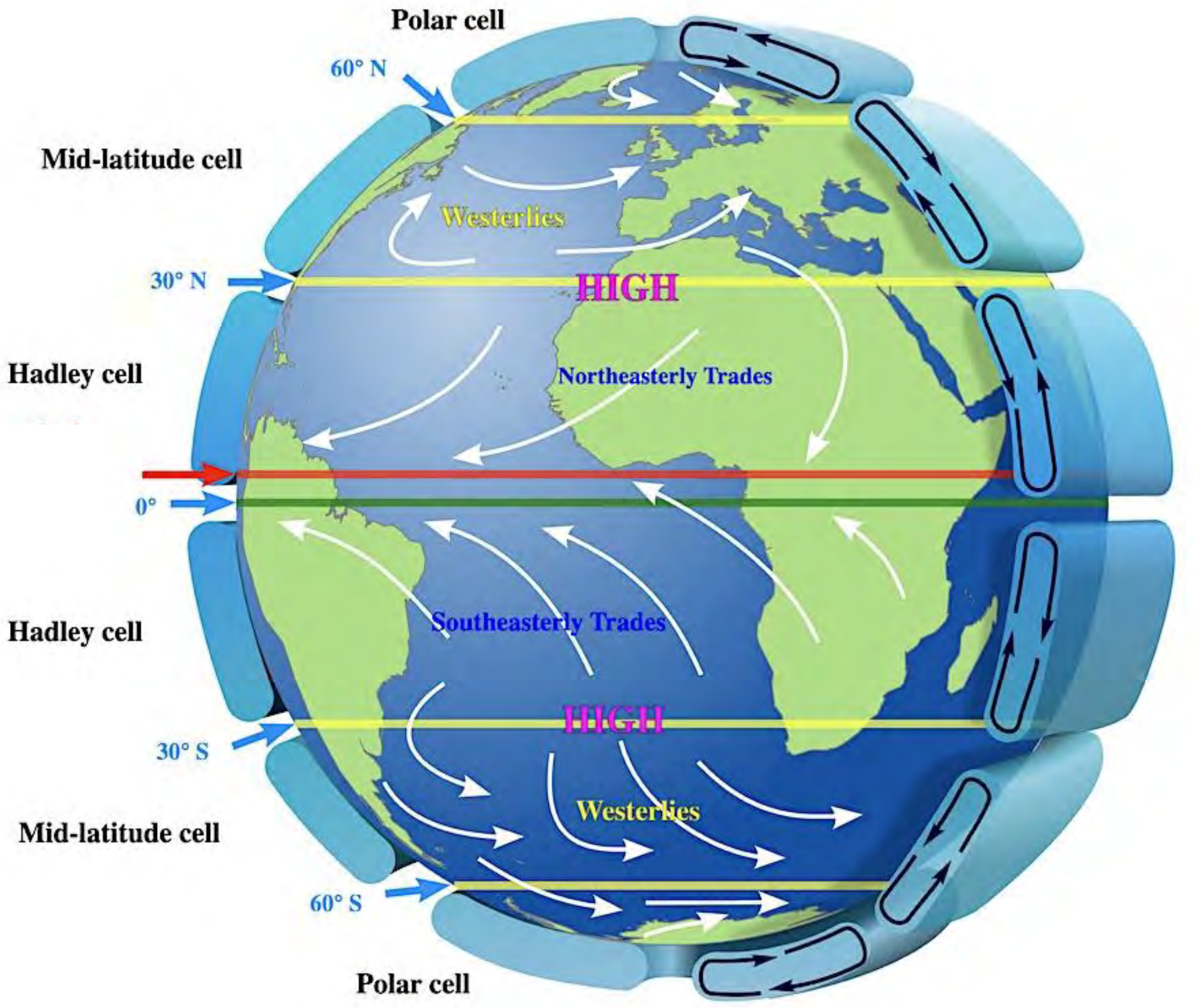
The answer, my friends, is they redistribute the thermal energy from sunlight

Cooler air descends at poles

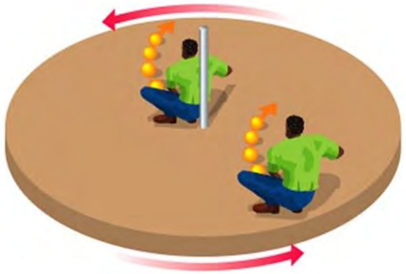


Heated air rises at equator

Sun

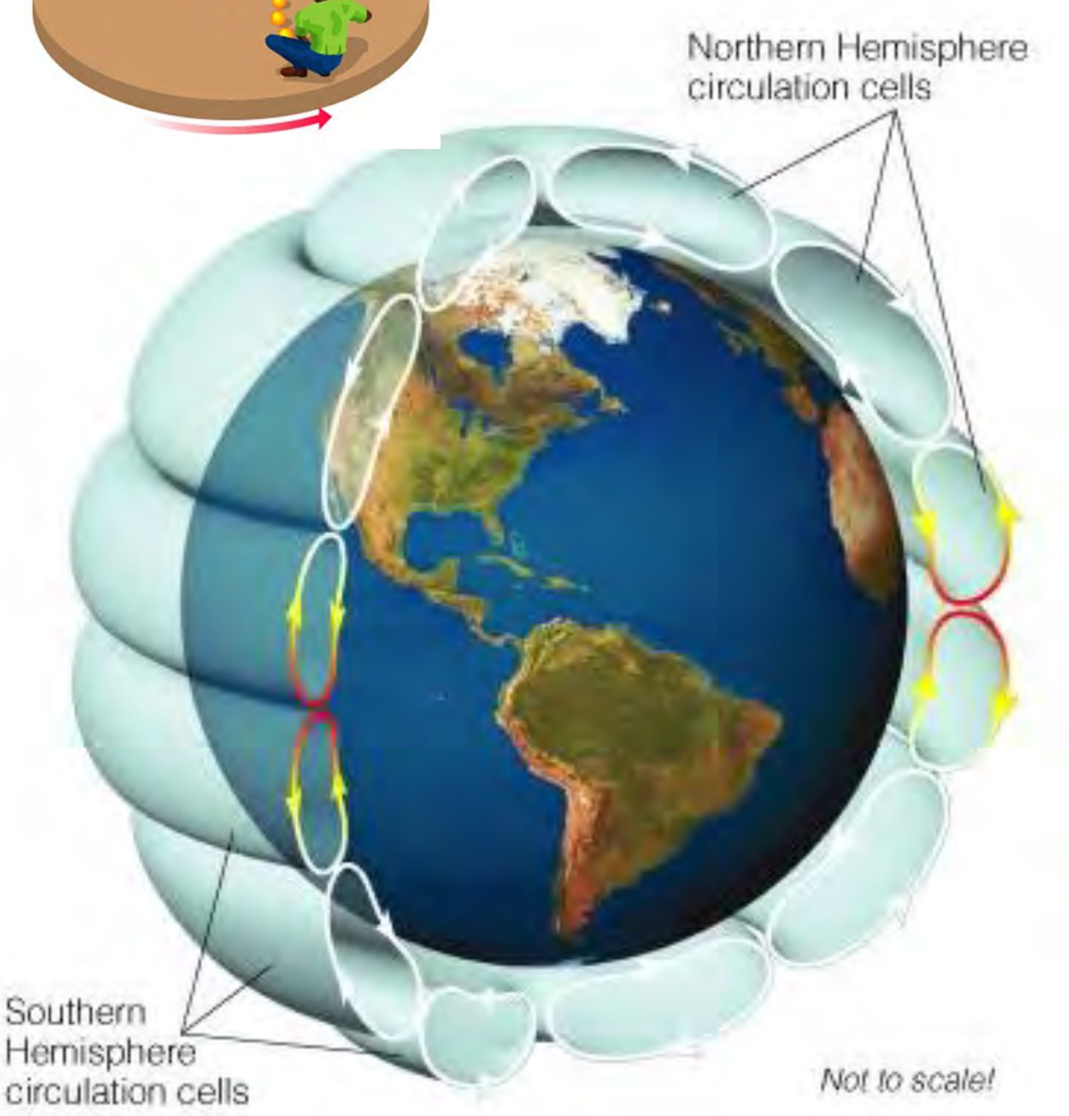


Planetary Rotation Also Plays a Role



Cells break due to the Coriolis force

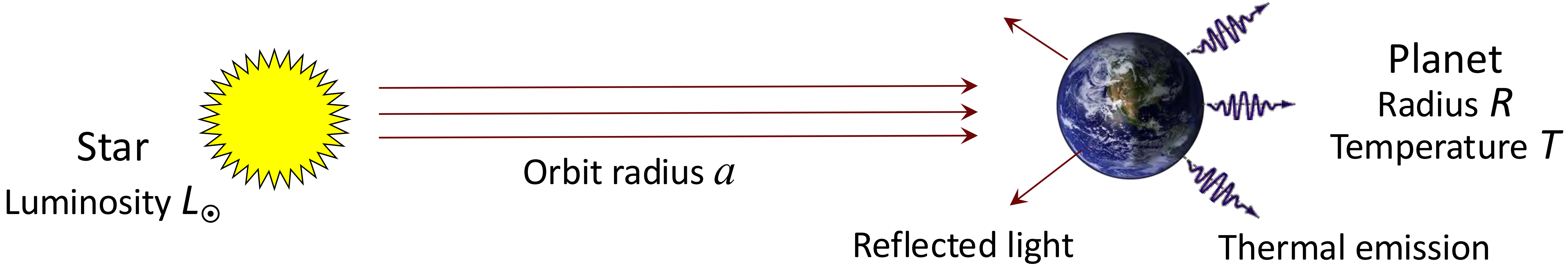
- On Earth the large circulation cell breaks up into 3 smaller ones, moving diagonally
- Other worlds have more or fewer circulation cells depending on their rotation rate



e.g., Jupiter:
Rotation period ~ 10 h
Radius ~ 70,000 km
Equatorial rotation speed ~ 12 km/s
(Earth: ~0.46 km/s)

Turbulence is atop of the zonal motions

Planets in a Thermal Balance



α = albedo, fraction of the reflected light (for the Earth, $\alpha \sim 0.3 - 0.35$)

Fraction of the intercepted luminosity: $\dfrac{\pi R^2}{4 \pi a^2}$

$$\text{Absorbed luminosity} = \frac{R^2}{4 a^2} (1 - \alpha) L_\odot = 4 \pi R^2 \sigma T^4 = \text{Emitted luminosity}$$

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}

Planet's effective blackbody temperature: $T^4 = L_\odot \dfrac{(1 - \alpha)}{16 \pi \sigma a^2}$

Planet's Temperature

For a given stellar luminosity, it depends *only* on the orbit radius and the albedo:

$$T^4 = L_\odot \frac{(1-\alpha)}{16\,\pi\,\sigma\,a^2}$$

This works well for the planets with no atmosphere

For the Earth: $\alpha \sim 0.33,\ a \sim 1.5 \times 10^{13}$ cm

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}

$L_\odot = 3.85 \times 10^{33}$ erg/s

Estimate $T \sim 253$ K

Yet, the actual value is more like $T \sim 287$ K. Why?

The answer: **the Greenhouse Effect**

Various gases in the Earth's atmosphere (mainly CO_2) trap some of the thermal infrared emission. That effectively acts as an additional incoming luminosity

Temperature increases until a new equilibrium is reached

Venus is an extreme case of this

Habitable Zones and Goldilock Planets

Recall that for a given stellar luminosity, planet's temperature depends *only* on the orbit radius and the albedo

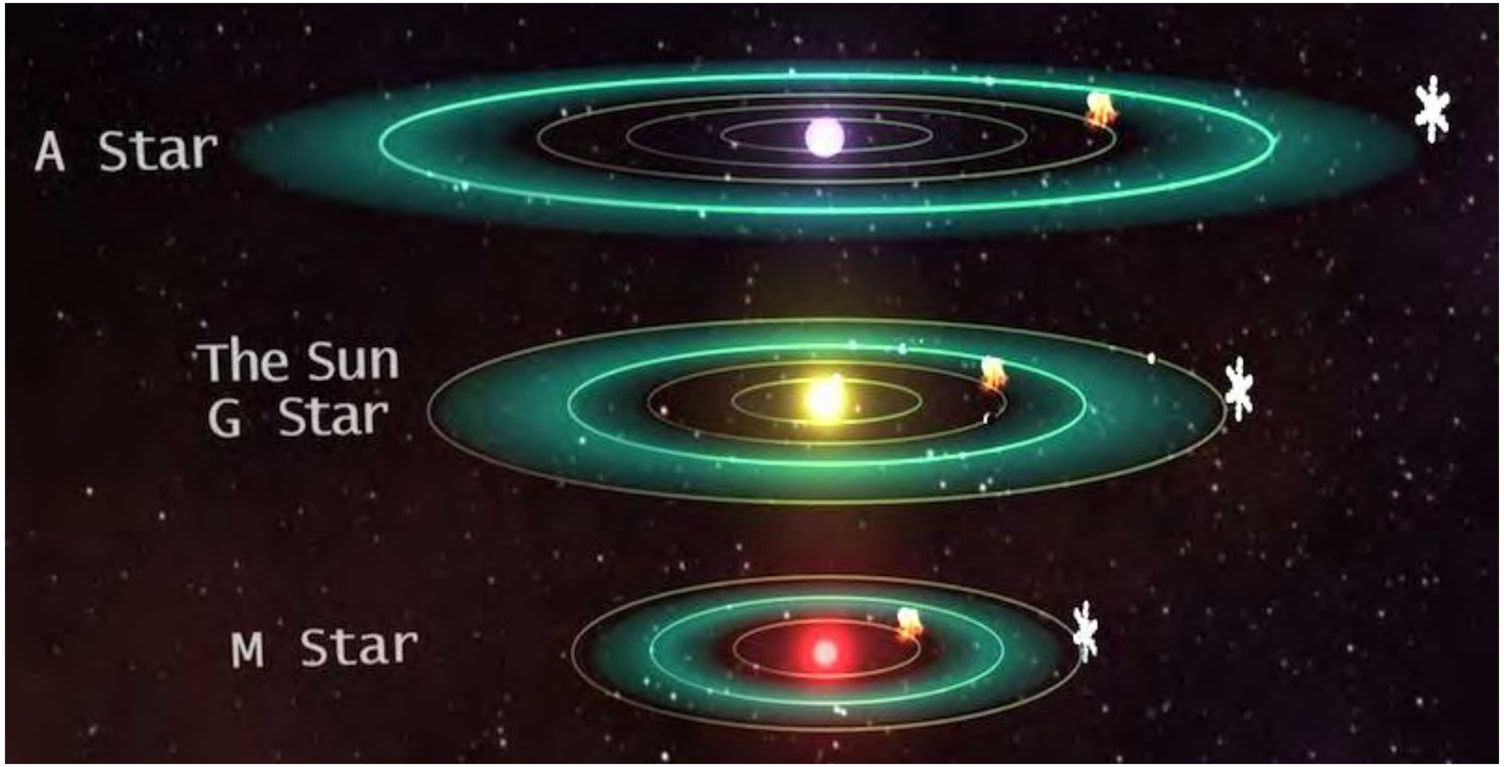
$$T^4 = L_{\odot} \frac{(1-\alpha)}{16\,\pi\,\sigma\,a^2}$$

Liquid water can exist if 273 K < T < 373 K

Frost line

Steam line

Since the albedo dependence is relatively weak, this defines a range of planetary orbit radii where liquid water (and thus life?) can exist on the surface



This is known as the "Goldilocks zone". We now know many exoplanets that satisfy this

Key Concepts in This Lecture (1)

- Planetary motions and orbits follow the **Kepler's laws**, and are **direct consequences** of **Newton's gravity**:
  1. Planetary orbits are ellipses, with the Sun at the focus
  2. Planet velocities vary along the orbit, but they sweep the same area per unit time
     - It is a consequence of the **conservation of angular momentum**
  3. Cubes of the semimajor axis are proportional to the squares of the periods
     - It is a consequence of the **conservation of energy**
- More generally, the **orbits are conic sections**, and **depend on the energy**: ellipses for the gravitationally bound planets ($E_{kin} < E_{pot}$), parabolas when $E_{kin} = E_{pot}$, and hyperbolas for the unbound ones ($E_{kin} > E_{pot}$)
- Orbit's **size** depends on the **energy**: the larger the energy, the larger orbit
- Orbit's **shape (eccentricity)** depends on the **angular momentum**; circular orbits have the maximum angular momentum for a given energy

Key Concepts in This Lecture (2)

- In our Solar system, there are 3 **kinds of planets**; **rocky**, closer to the Sun; **gas giants**, further out, and **dwarf** planets, even further out ant into the Kuiper belt. **Asteroids** and **comets** are leftover rocky and icy **planetesimals** from the formation era
- **The Moon** probably formed from the material ejected due to a collision of a smaller protoplanet with the young Earth, and it condensed in its orbit
- The larger **moons of Jupiter and Saturn** can have **oceans under the icy crust**, in particular Europa and Enceladus, and likely most, if not all of the rest
- **Planetary atmospheres** develop belt-like circulation cells due to the Coriolis force
- Planets reflect some of their star's flux (**albedo**), absorb the rest, and **re-emit the energy in the thermal IR**. In the absence of an atmosphere, planet's temperature **depends only on its albedo and the distance** from the parent star
- **Atmospheric opacity** prevents some of that energy escaping, creating a **greenhouse effect**, that leads to **global warming**

?