

Ay1 – Lectures 17 and 18 summary



Measuring The Universe: Cosmological Distance Scale and Cosmological Tests

Dark Matter, Dark Energy,
and the Concordance Cosmology

The Scale of the Universe

- The **Hubble length**, $D_H = c/H_0$, and the **Hubble time**, $t_H = 1/H_0$ give the approximate spatial and temporal scales of the universe
- H_0 is independent of the “shape parameters” (expressed as density parameters) Ω_m , Ω_Λ , Ω_k , w , etc., which govern the global geometry and dynamics of the universe
- Distances to galaxies, quasars, etc., scale linearly with H_0 , $D \approx cz / H_0$. They are necessary in order to convert observable quantities (e.g., fluxes, angular sizes) into physical ones (luminosities, linear sizes, energies, masses, etc.)

Measuring the Scale of the Universe

- The only clean-cut distance measurements in astronomy are from trigonometric parallaxes. Everything else requires physical modeling and/or a set of calibration steps (the “*distance ladder*”), and always some statistics:
 - Use parallaxes to calibrate some set of distance indicators**
 - Use them to calibrate another distance indicator further away**
 - And then another, reaching even further**
 - etc. etc.**
 - Until you reach a “pure Hubble flow”**
- The age of the universe can be constrained independently from the H_0 , by estimating ages of the oldest things one can find around (e.g., globular clusters, heavy elements, white dwarfs)

Distance Ladder: Methods

Methods yielding absolute distances:

Parallax (trigonometric, secular, and statistical)

The moving cluster method - has some assumptions

Baade-Wesselink method for pulsating stars

Expanding photosphere method for Type II SNe

Sunyaev-Zeldovich effect

Gravitational lens time delays

} *Model dependent!*

kpc
Mpc
Gpc

Secondary distance indicators: “*standard candles*”, requiring a calibration from an absolute method applied to local objects - *the distance ladder*:

Pulsating variables: Cepheids, RR Lyrae, Miras

Main sequence fitting to star clusters

Brightest red giants

Planetary nebula luminosity function

Globular cluster luminosity function

Surface brightness fluctuations

Tully-Fisher, D_n - σ , FP scaling relations for galaxies

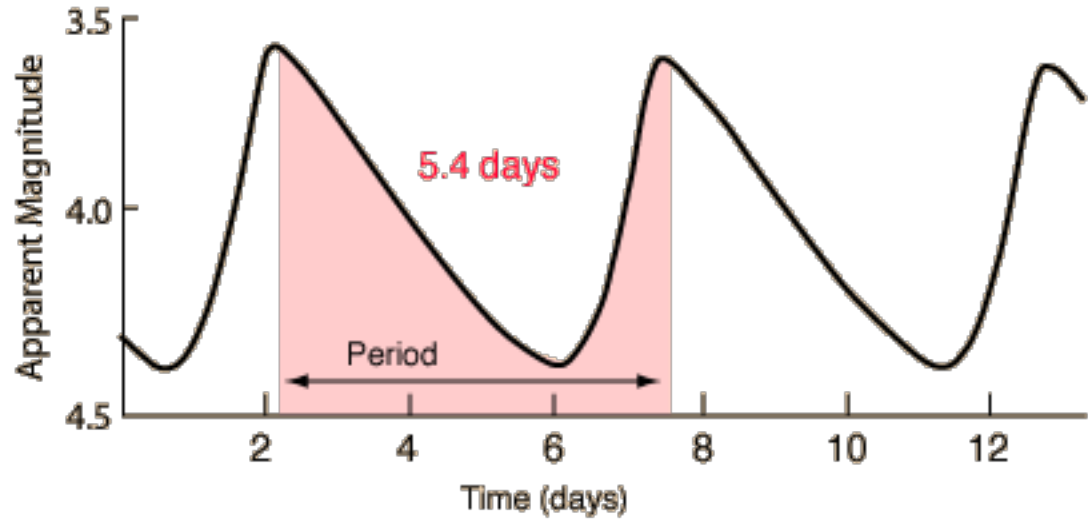
Type Ia Supernovae

... etc.

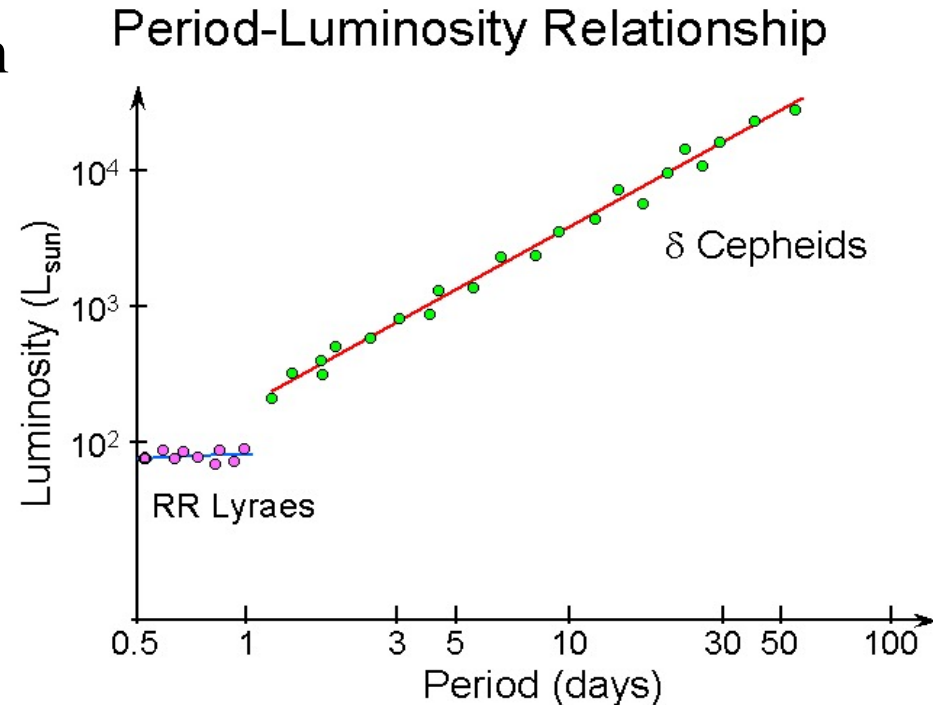
kpc
Mpc
Gpc

Cepheids

- Luminous ($M \sim -4$ to -7 mag), pulsating variables, evolved high-mass stars on the instability strip in the H-R diagram
 - Can be observed out to a few tens of Mpc



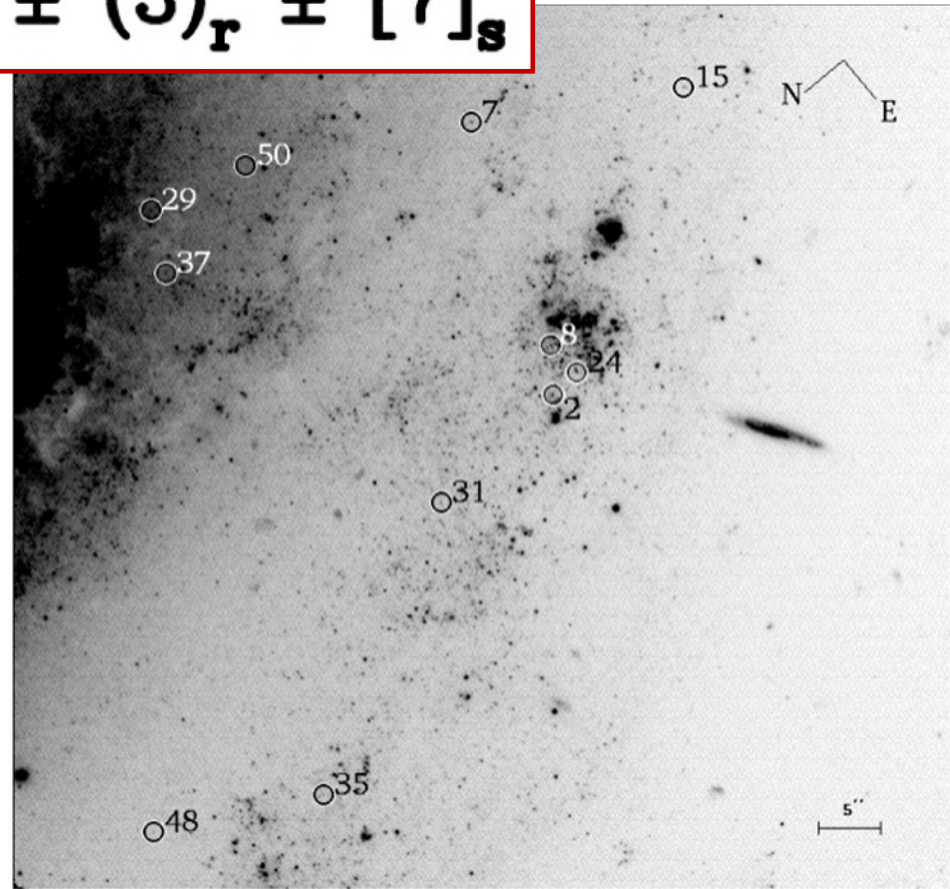
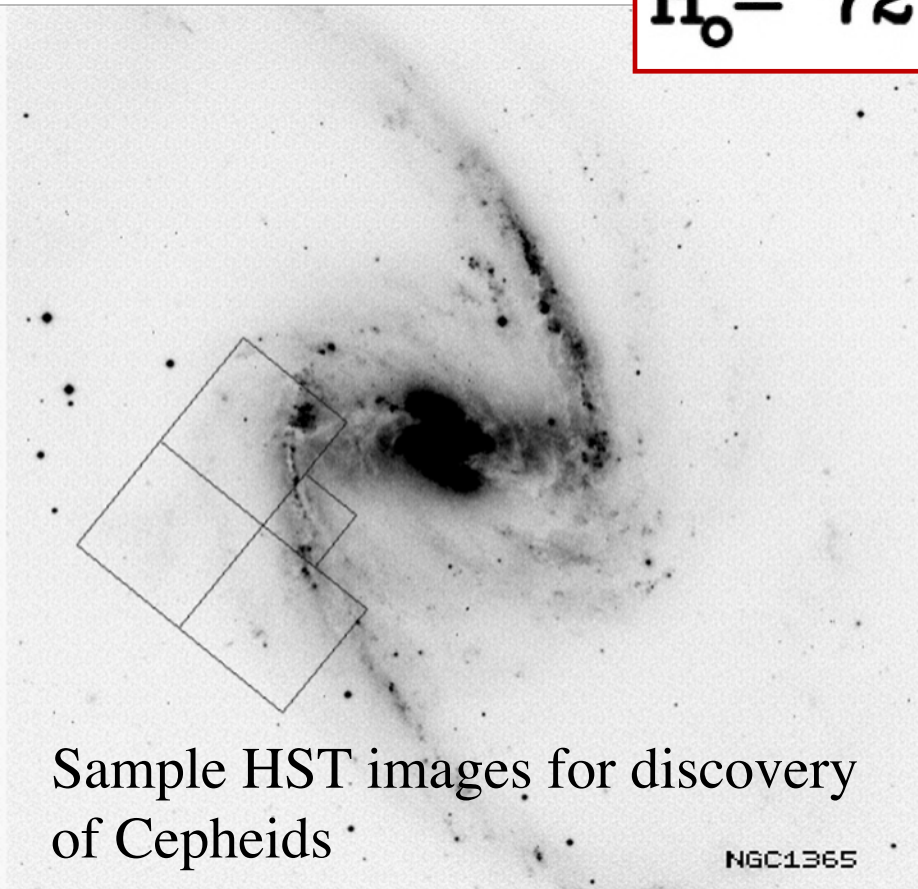
- Obey a period-luminosity relation (P-L): brighter Cepheids have longer periods than fainter ones
 - ✧ Calibrated using Hipparcos parallaxes
- RR Lyrae are their Pop II analogs



The HST H_0 Key Project

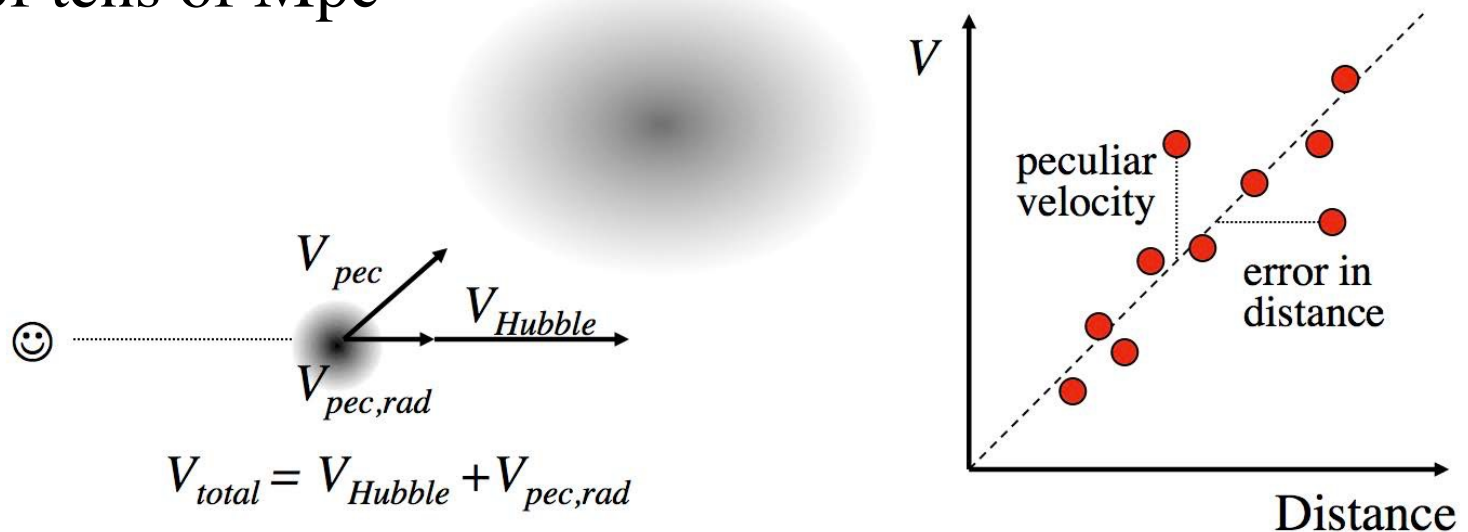
- Started in 1990, final results in 2001 (W. Freedman, R. Kennicutt, J. Mould, J. Huchra, and many others)
- Observe Cepheids in ~ 18 spirals and improve calibration of other distance indicators

$$H_0 = 72 \pm (3)_r \pm [7]_s$$



Pushing Into the Hubble Flow

- Hubble's law: $D = H_0 v$ – but the – total observed velocity v is a combination of the cosmological expansion, and the *peculiar velocity* of any given galaxy, $v = v_{cosmo} + v_{pec}$
- Typically $v_{pec} \sim$ a few hundred km/s, due to a gravitational infall into the local large scale structures, with characteristic scales of tens of Mpc

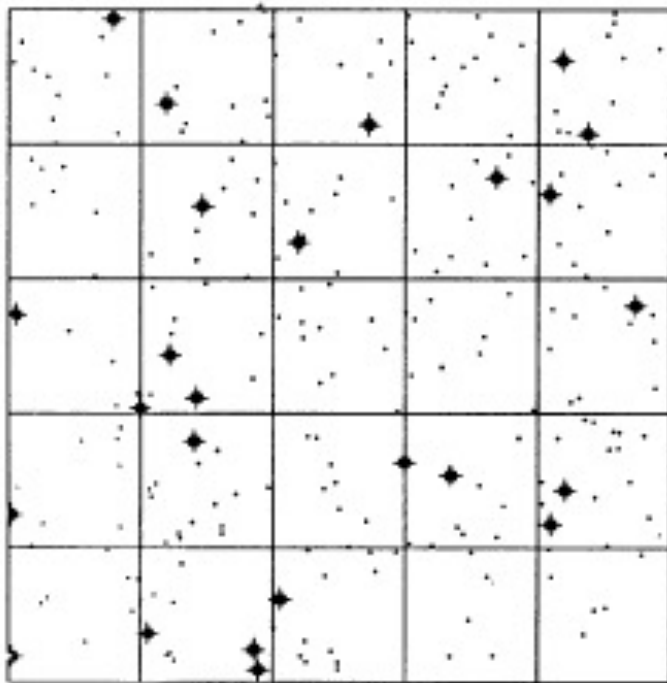


- Thus, we need to measure H_0 on scales greater than tens of Mpc, and where $v_{cosmo} \gg v_{pec}$. This requires *luminous standard candles* - galaxies or Supernovae

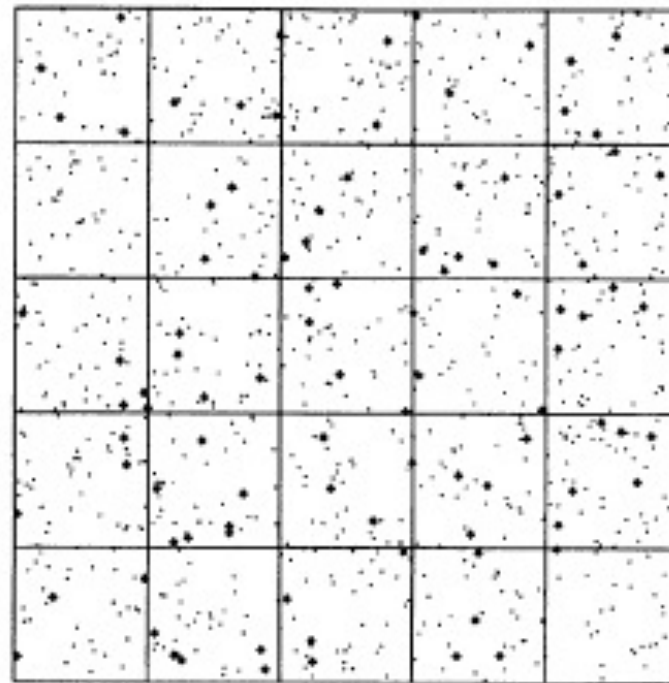
Surface Brightness Fluctuations

Consider stars projected onto a pixel grid of your detector:

Nearby Galaxy



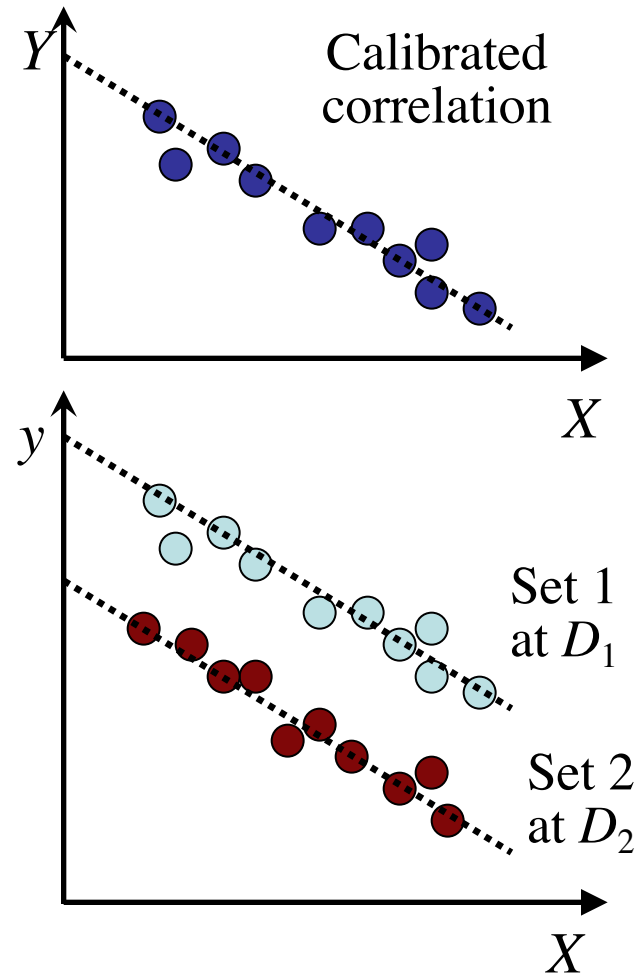
A galaxy twice farther away



- Average flux per star = $\langle f \rangle$, average flux per pixel = $N\langle f \rangle$, Poissonian variations per pixel = $N^{1/2} \langle f \rangle$
- $N \sim D^2$, the flux per star $\sim D^{-2}$ and the RMS $\sim D^{-1}$. Thus a galaxy twice as far away appears twice as smooth

Distance Indicator Relations

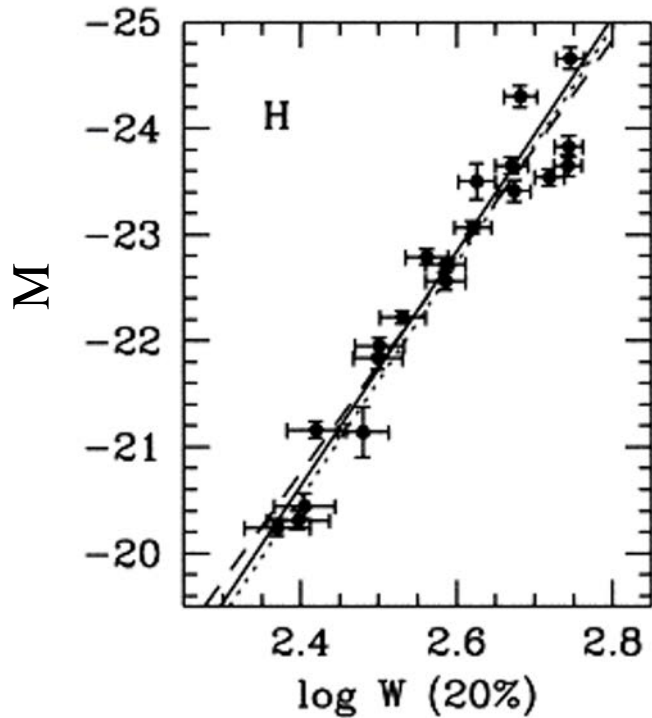
- Need a correlation between a distance-independent quantity, “ X ”, (e.g., temperature or color for stars in the H-R diagram, or the period for Cepheids), and a distance-dependent one, “ Y ”, (e.g., stellar absolute magnitude, M)
- Two sets of objects at different distances will have a systematic shift in the *apparent* versions of “ y ” (e.g., stellar apparent magnitude, m), from which we can deduce their *relative distance*
- This obviously works for stars (main sequence fitting, period-luminosity relations), but can we find such relations for galaxies?



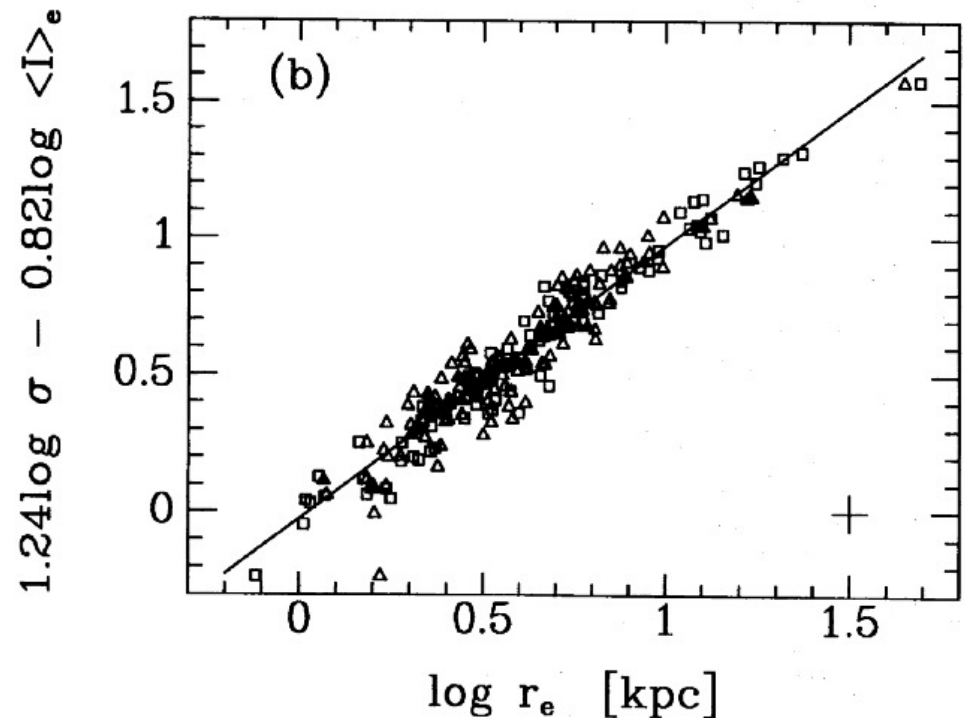
Galaxy Scaling Relations

- Correlations between distance-dependent quantities (luminosity, radius) and distance-independent ones (e.g., rotational speeds for disks, or velocity dispersions, surface brightness, etc.)
- Calibrated locally using other distance indicators, e.g. Cepheids or surface brightness fluctuations

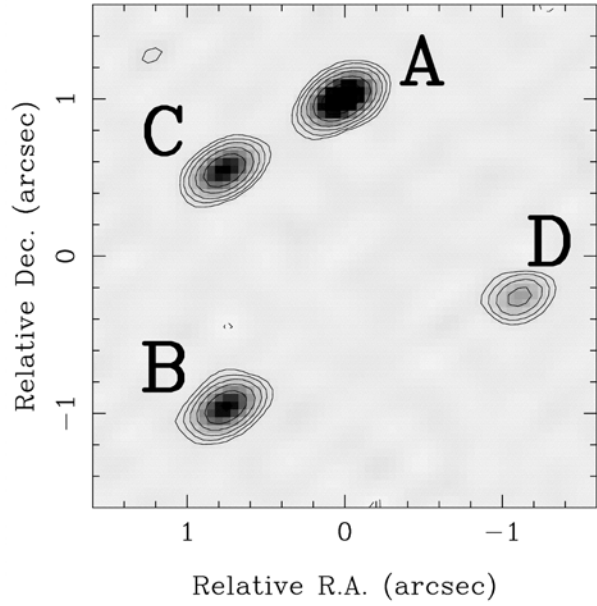
Tully-Fisher for spirals



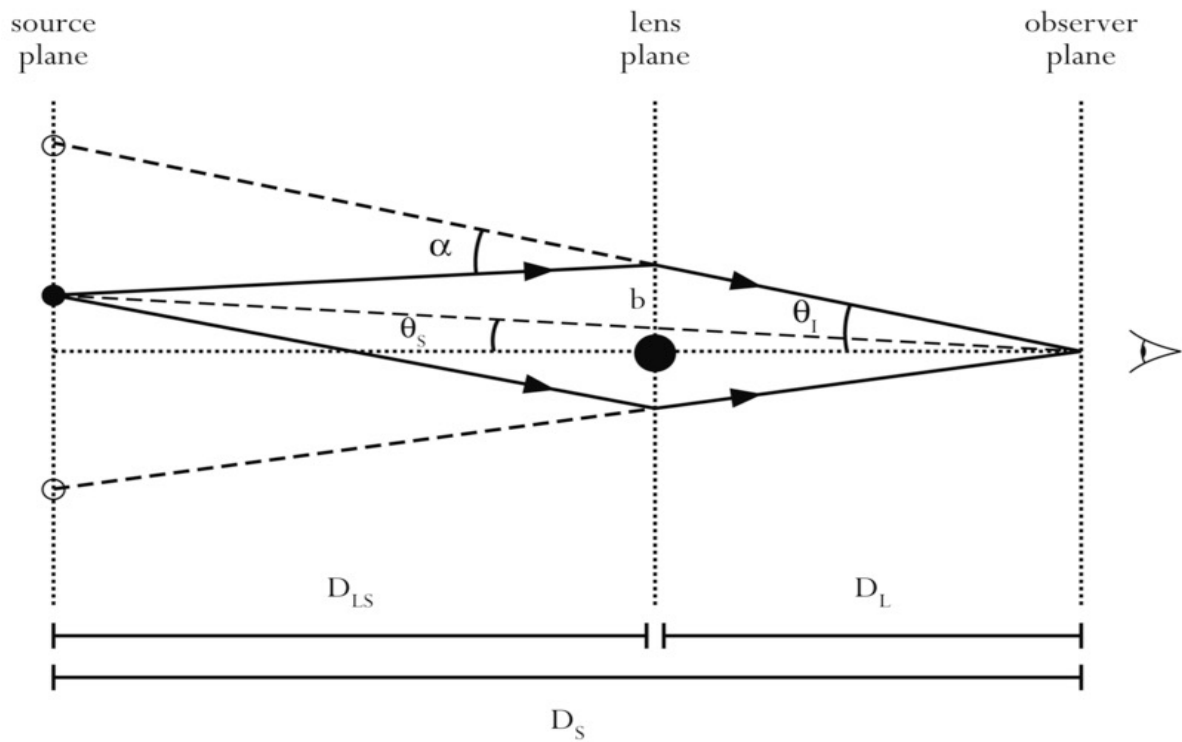
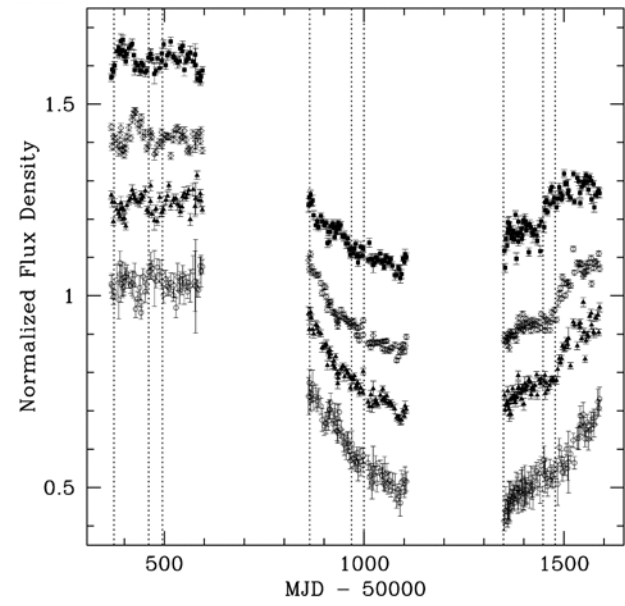
Fundamental Plane for Ellipticals



Gravitational Lens Time Delays

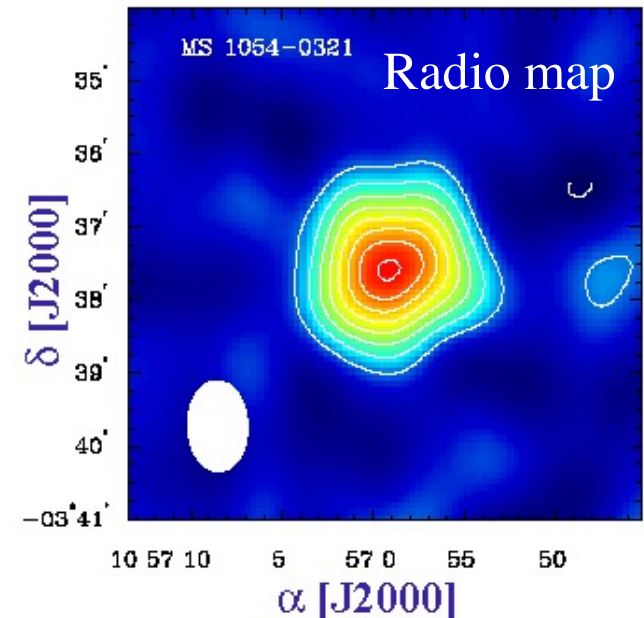
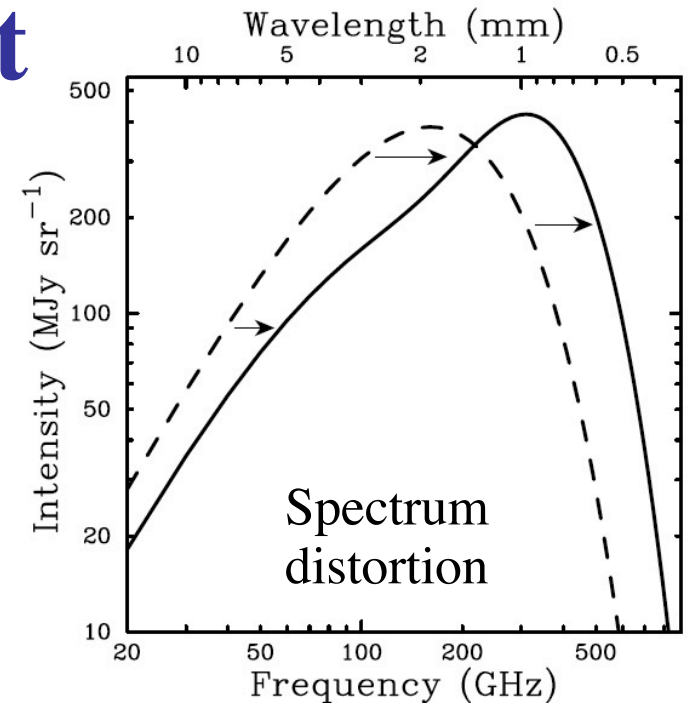


Assuming the mass model for the lensing galaxy of a gravitationally lensed quasar is well-known, the different light paths taken by various images of the quasar will lead to time delays in the arrival time of the light to us. The modeling is complex!



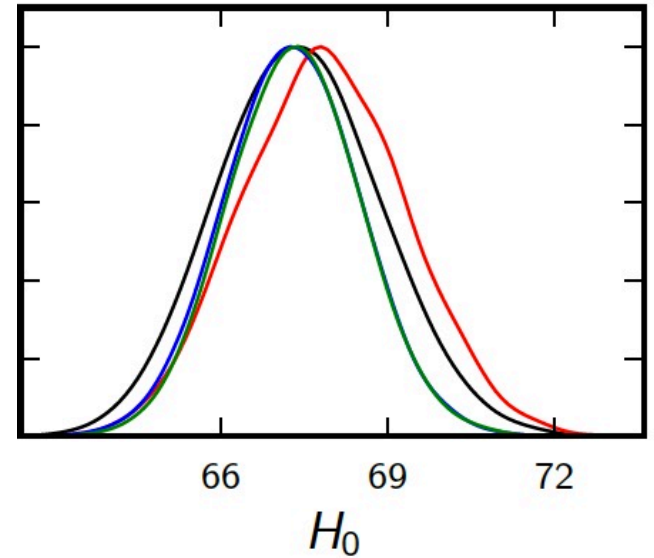
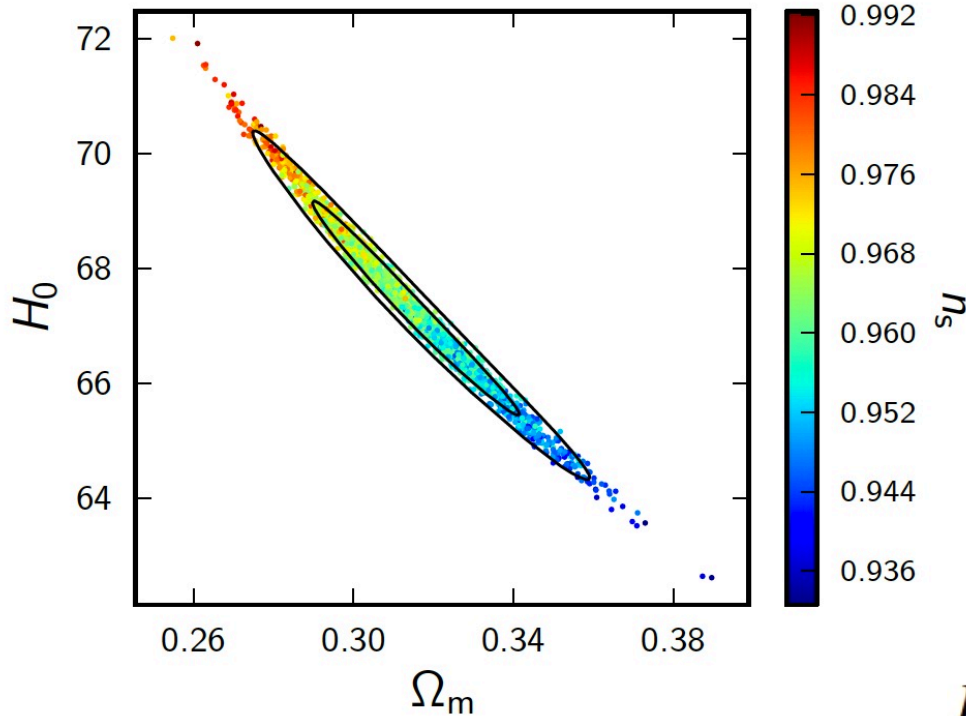
Synyaev-Zeldovich Effect

- If we can measure the electron density and temperature of the X-ray emitting gas along the line of sight from X-ray measurements, we can estimate the path length (\sim cluster diameter) along the line of sight
- If we assume the cluster is spherical (??), from its angular diameter (projected on the sky) we can determine the distance to the cluster
- Potential uncertainties include cluster substructure or shape (e.g., non-spherical). It is also non-trivial to measure the X-ray temperature to derive the density at high redshifts.



H_0 From the CMB

- Bayesian solutions from model fits to CMB fluctuations – cosmological parameters are coupled

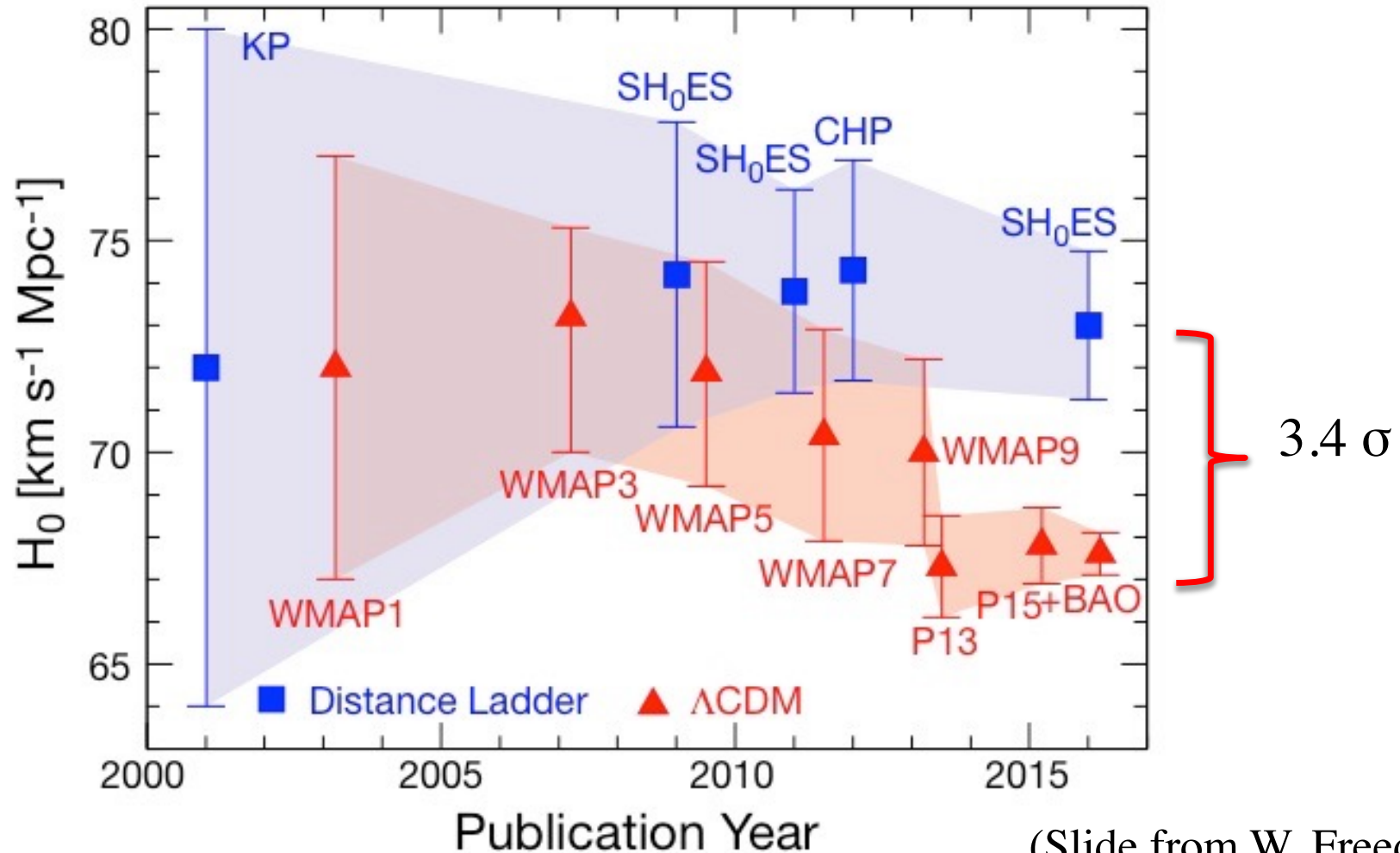


$$H_0 = (67.3 \pm 1.2) \text{ km s}^{-1} \text{ Mpc}^{-1}$$

- Planck (2013) results:

Parameter	<i>Planck</i> +WP		<i>Planck</i> +WP+highL		<i>Planck</i> +lensing+WP+highL		<i>Planck</i> +WP+highL+BAO	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
H_0	67.04	67.3 ± 1.2	67.15	67.3 ± 1.2	67.94	67.9 ± 1.0	67.77	67.80 ± 0.77
Age/Gyr	13.8242	13.817 ± 0.048	13.8170	13.813 ± 0.047	13.7914	13.794 ± 0.044	13.7965	13.798 ± 0.037

The Tension Between the Cepheid Based and CMB Based Measurements of the Hubble Constant

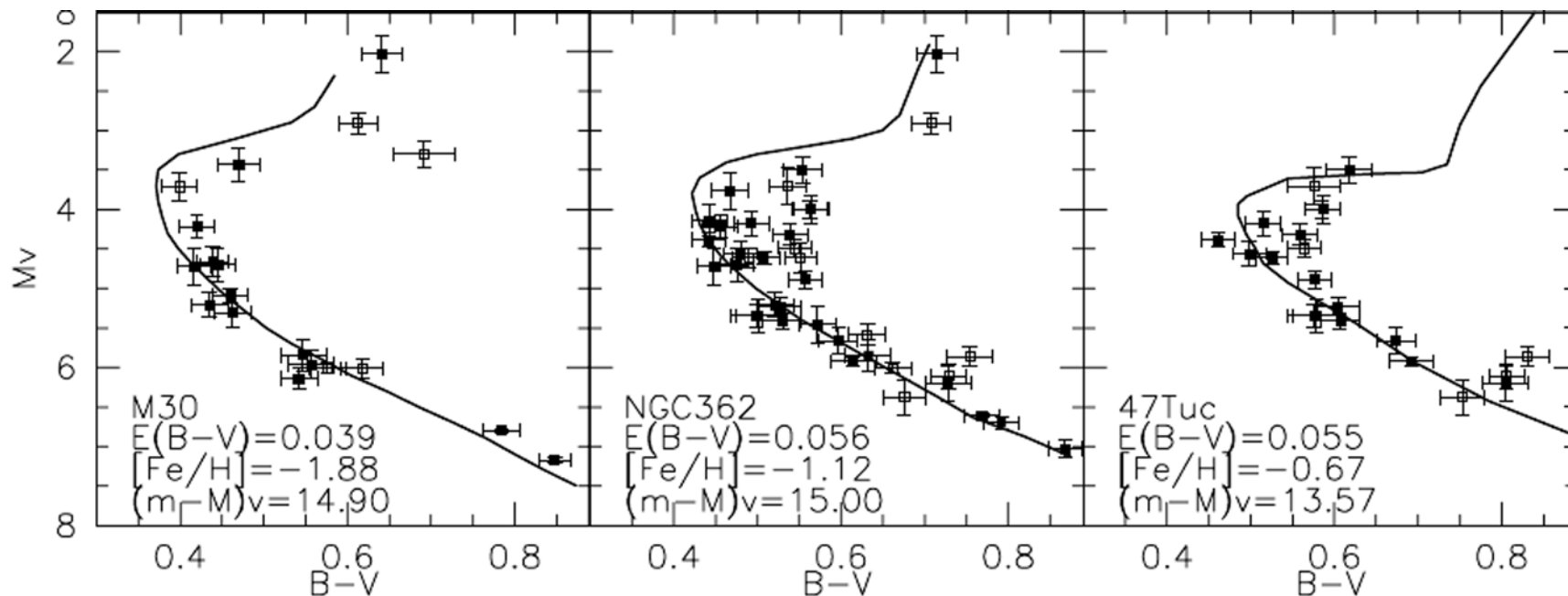


(Slide from W. Freedman)

Measuring the Age of the Universe

- Related to the Hubble time $t_H = 1/H_0$, but the exact value depends on the cosmological parameters
- Could place a *lower limit* from the ages of astrophysical objects (pref. the oldest you can find), e.g.,
 - **Globular clusters** in our Galaxy; known to be very old. Need stellar evolution isochrones to fit to color-magnitude diagrams
 - **White dwarfs**, from their observed luminosity function, cooling theory, and assumed star formation rate
 - **Heavy elements**, produced in the first Supernovae; somewhat model-dependent
 - Age-dating **stellar populations** in distant galaxies; this is very tricky and requires modeling of stellar population evolution, with many uncertain parameters

Globular Cluster Ages From Hipparcos Calibrations of Their Main Sequences



Examples of g.c. main sequence isochrone fits, for clusters of a different metallicity (Graton et al.)

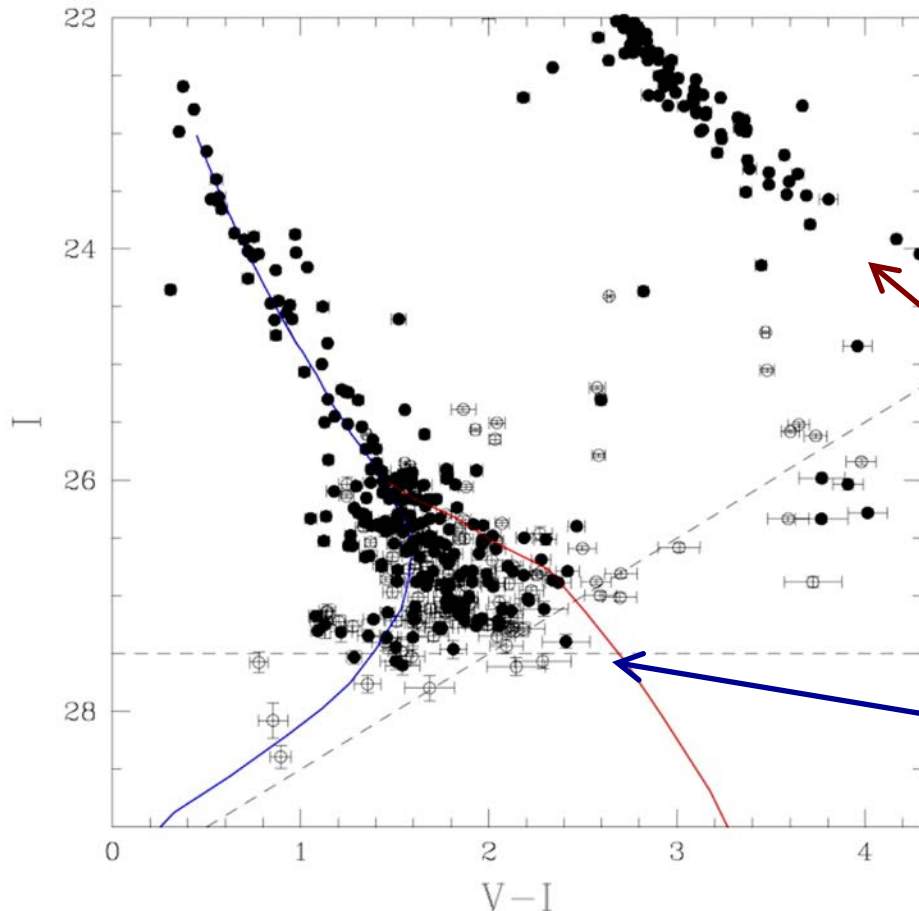
The same group has published two slightly different estimates of the mean age of the oldest clusters:

$$\text{Age} = 11.8_{-2.5}^{+2.1} \text{ Gyr}$$

$$\text{Age} = 12.3_{-2.5}^{+2.1} \text{ Gyr}$$

White Dwarf Cooling Curves

- Use the luminosity of the faintest WDs in a cluster to estimate the cluster age by comparing the observed luminosities to theoretical cooling curves
- Need deep HST observations



Hansen *et al.* (2002) find an age of 12.7 ± 0.7 Gyr for the globular cluster M4

Main sequence

White dwarfs cooling sequence

Nucleocosmochronology

- Can use the radioactive decay of elements to age date the oldest stars in the galaxy. It has been done with ^{232}Th (half-life = 14 Gyr) and ^{238}U (half-life = 4.5 Gyr) and other elements
- Measuring the ratios of various elements from stellar spectroscopy of the oldest stars

CHRONOMETRIC AGE ESTIMATES FOR BD +17°3248

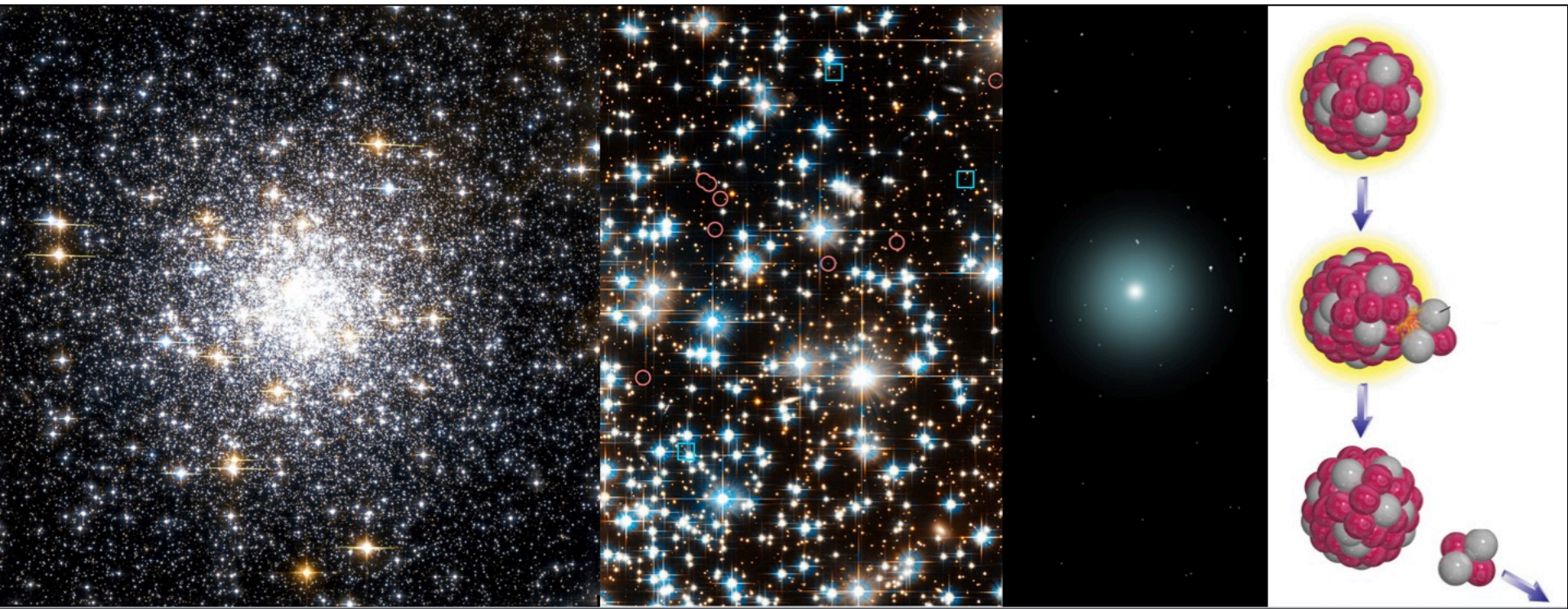
Chronometer Pair	Predicted	Observed	Age (Gyr)	Solar ^a	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	≥ 13.4	2.32	11.0
U/Ir	0.05036	0.0045	≥ 15.5	0.0369	13.5
U/Pt.....	0.013	0.0019	≥ 12.4	0.01846	14.6

^a From Burris et al. 2001.

Mean = 13.8 +/- 4, but note the spread!

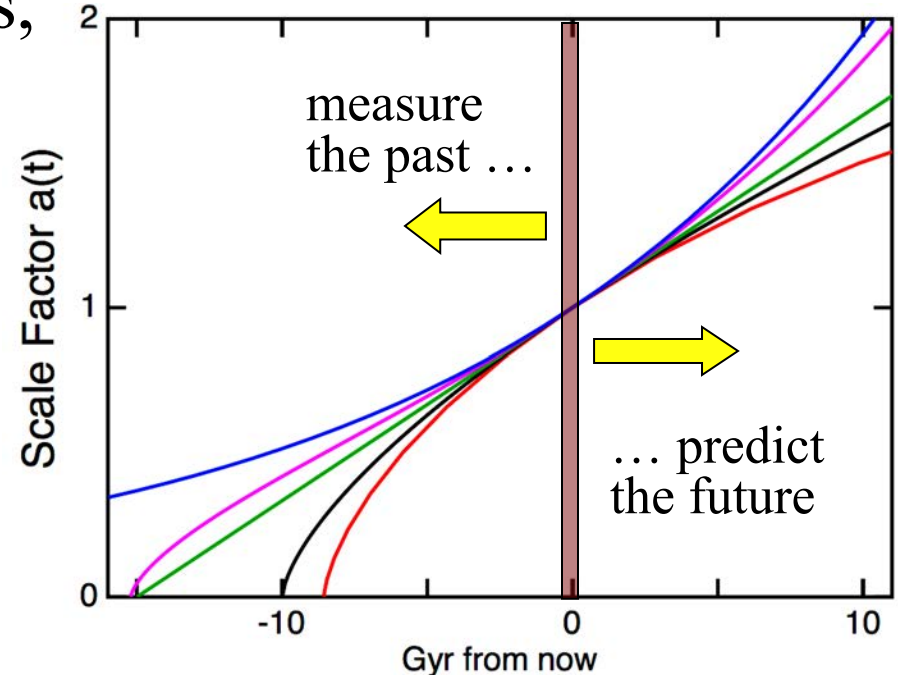
The Age of the Universe

- Several different methods (different physics, different measurements) agree that the lower limit to the age of the universe is $\sim 12 - 13$ Gyr
- This is in an excellent agreement with the age determined from the cosmological tests (~ 13.7 Gyr)

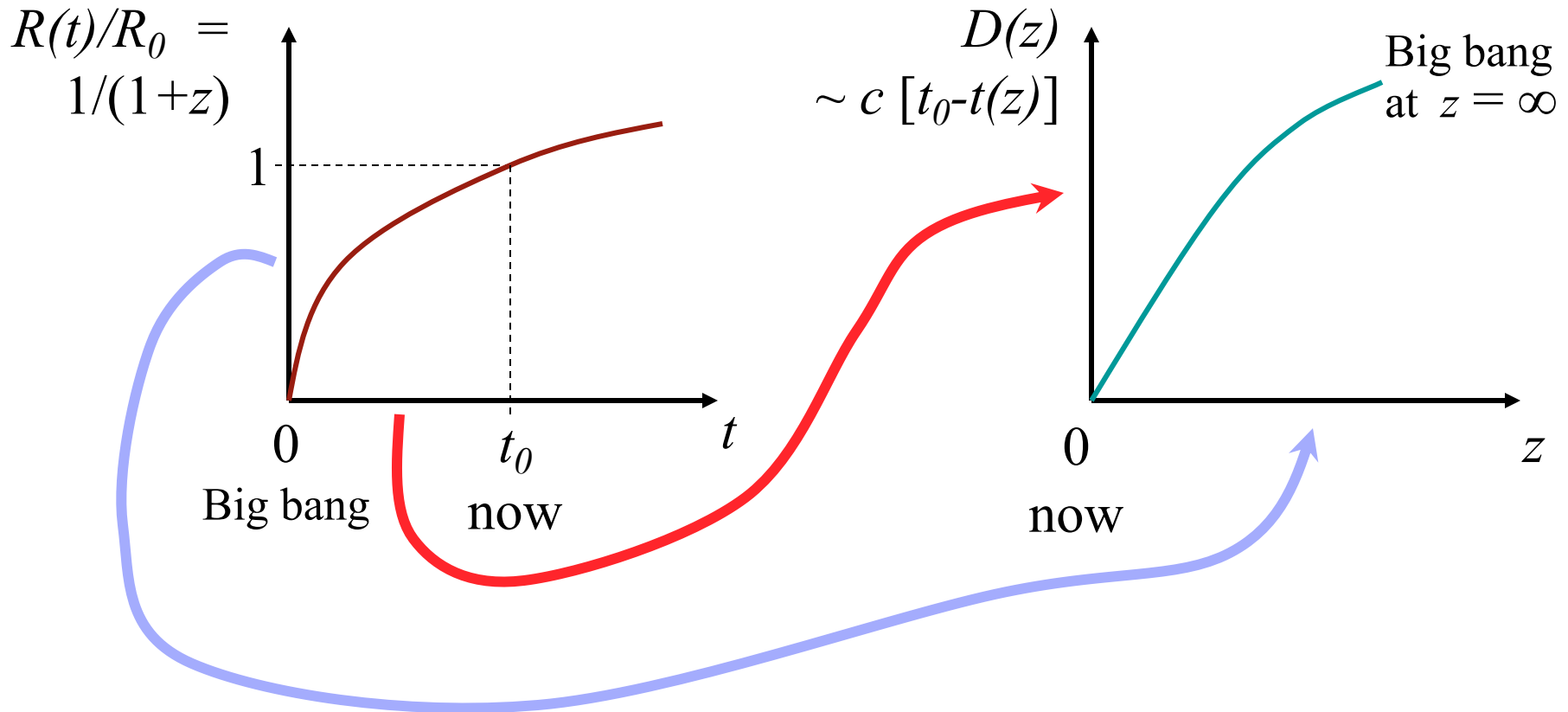


Cosmological Tests: The Why and How

- The goal is to determine the global geometry and the dynamics of the universe, and its ultimate fate
- The basic method is to somehow map the history of the expansion, and compare it with model predictions
- A model (or a family of models) is assumed, e.g., the Friedmann-Lemaitre models, typically defined by a set of parameters, e.g., H_0 , $\Omega_{0,m}$, $\Omega_{0,\Lambda}$, q_0 , etc.
- Model equations are integrated, and compared with the observations

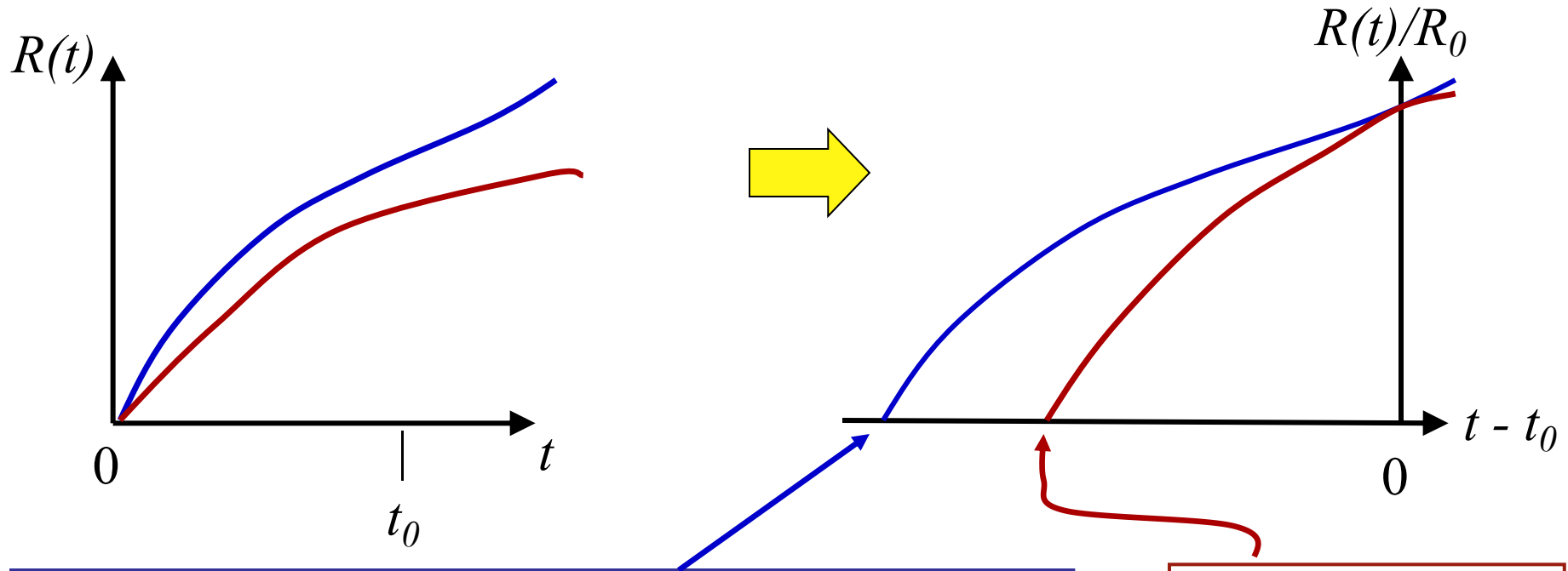


The Basis of Cosmological Tests



All cosmological tests essentially consist of comparing some measure of (relative) distance (or look-back time) to redshift. Absolute distance scaling is given by the H_0 .

Cosmological Tests: Expected Generic Behavior of Various Models



Models with a lower density and/or positive Λ expand faster, are thus larger, older today, have more volume and thus higher source counts, at a given z sources are further away and thus appear fainter and smaller

Models with a higher density and lower Λ behave exactly the opposite

The Basic Concept

- If two sources have the same intrinsic luminosity (“standard candles”), from the ratio of their apparent brightness we can derive the ratio of their luminosity distances
- If two sources have the same physical size (“standard rulers”), from the ratio of their apparent angular sizes we can derive the ratio of their angular diameter distances

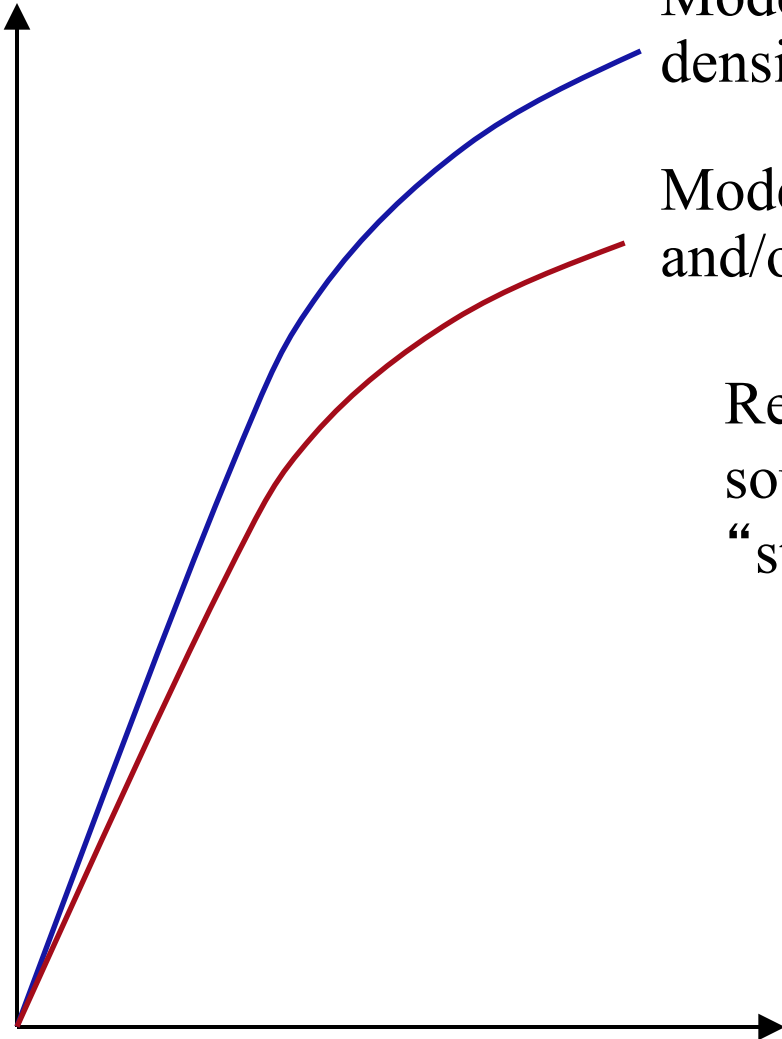


The Types of Cosmological Tests

- **The Hubble diagram:** flux (or magnitude) as a proxy for the luminosity distance, vs. redshift - requires “*standard candles*”
- **Angular diameter** as a proxy for the angular distance, vs. redshift - requires “*standard rulers*”
- **Source counts** as a function of redshift or flux (or magnitude), probing the evolution of a volume element - requires a population of sources with a constant comoving density - “*standard populations*”
- Indirect tests of age vs. redshift, usually highly model-dependent - “*standard clocks*”
- Local dynamical measurements of the mass density, Ω_{m0}
- If you measure H_0 and t_0 independently, you can constrain a combination of Ω_{m0} and Ω_Λ

The Hubble Diagram

magnitude



Model with a lower
density and/or $\Lambda > 0$

Model with a higher density
and/or $\Lambda \leq 0$

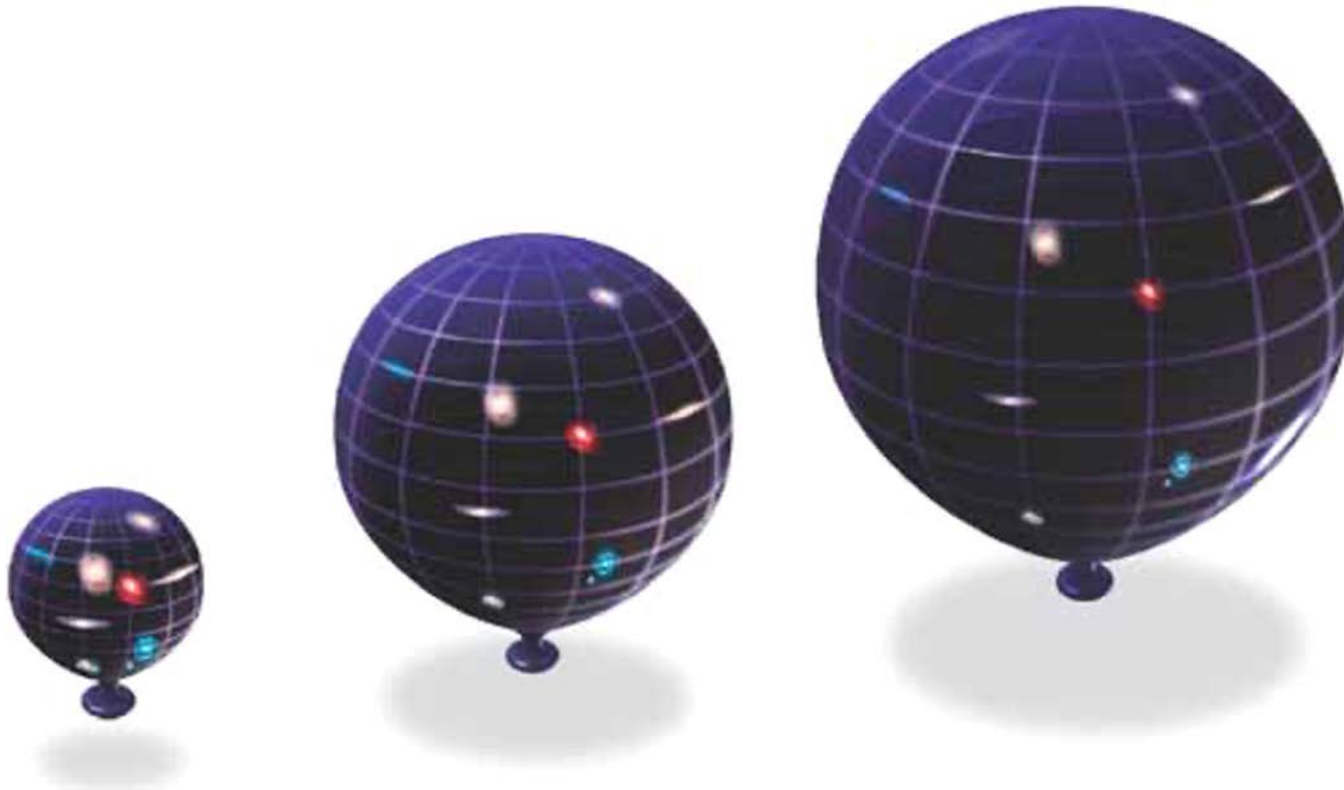
Requires a population on non-evolving
sources with a fixed luminosity -
“standard candles”. Some candidates:

- Brightest cluster ellipticals
- Supernovae of type Ia
- Luminosity functions in clusters
- GRB afterglows ??
- ...

redshift

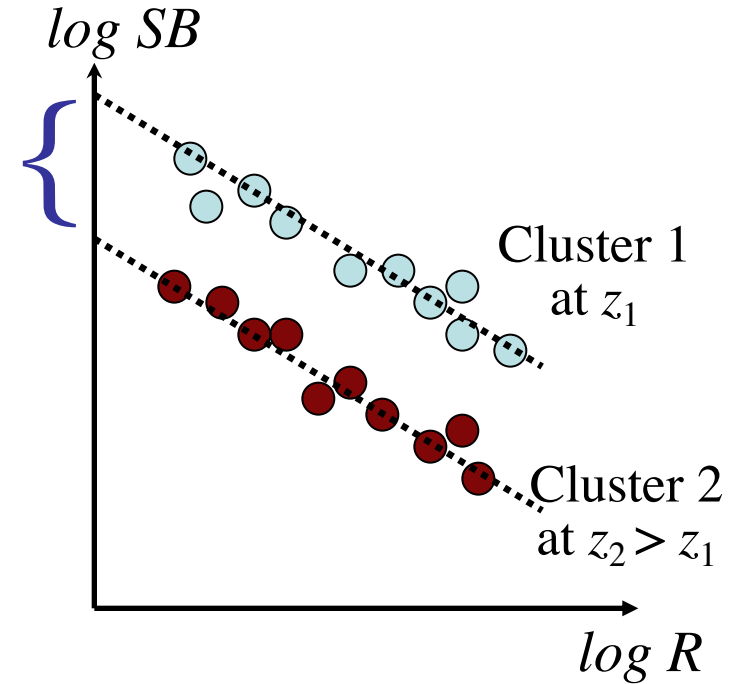
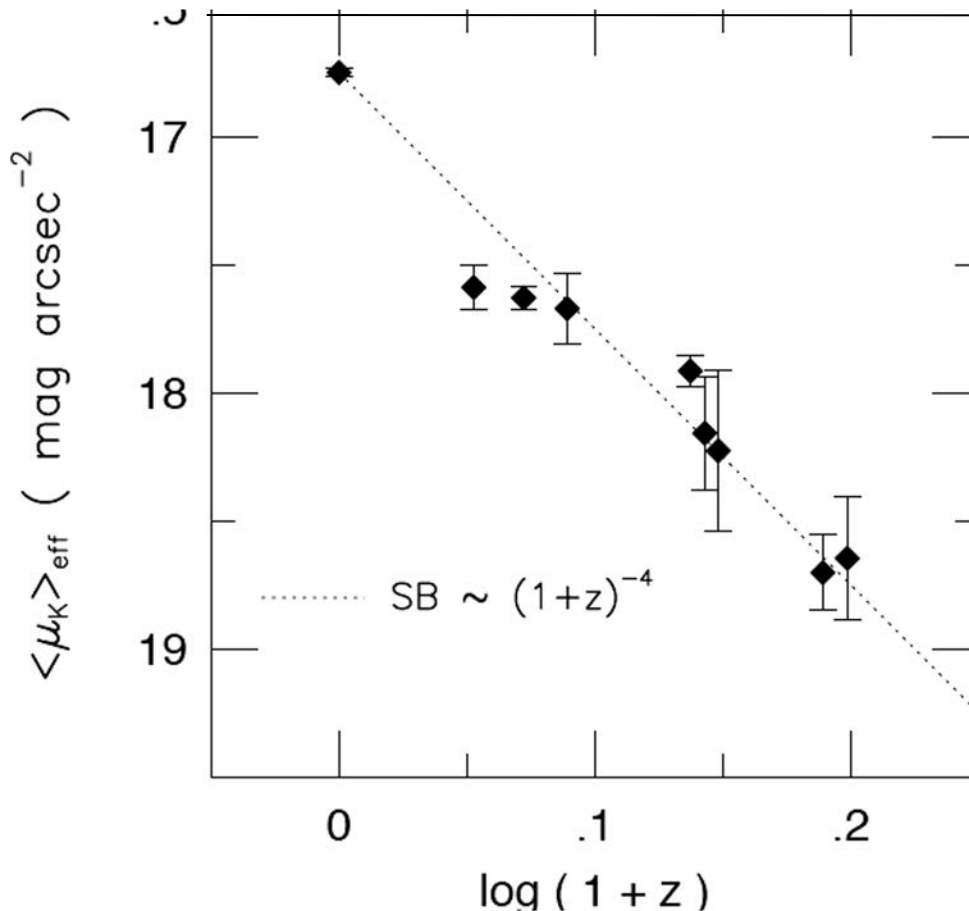
Tests for the Expansion of the Universe

- Tolman surface brightness (SB) test
 - In a stationary, Euclidean universe $SB = const.$, but in an expanding, relativistic universe it scales as $SB \sim (1+z)^{-4}$
- Time dilation of Supernova light curves
 - Time stretches by a factor of $(1+v/c) = (1+z)$



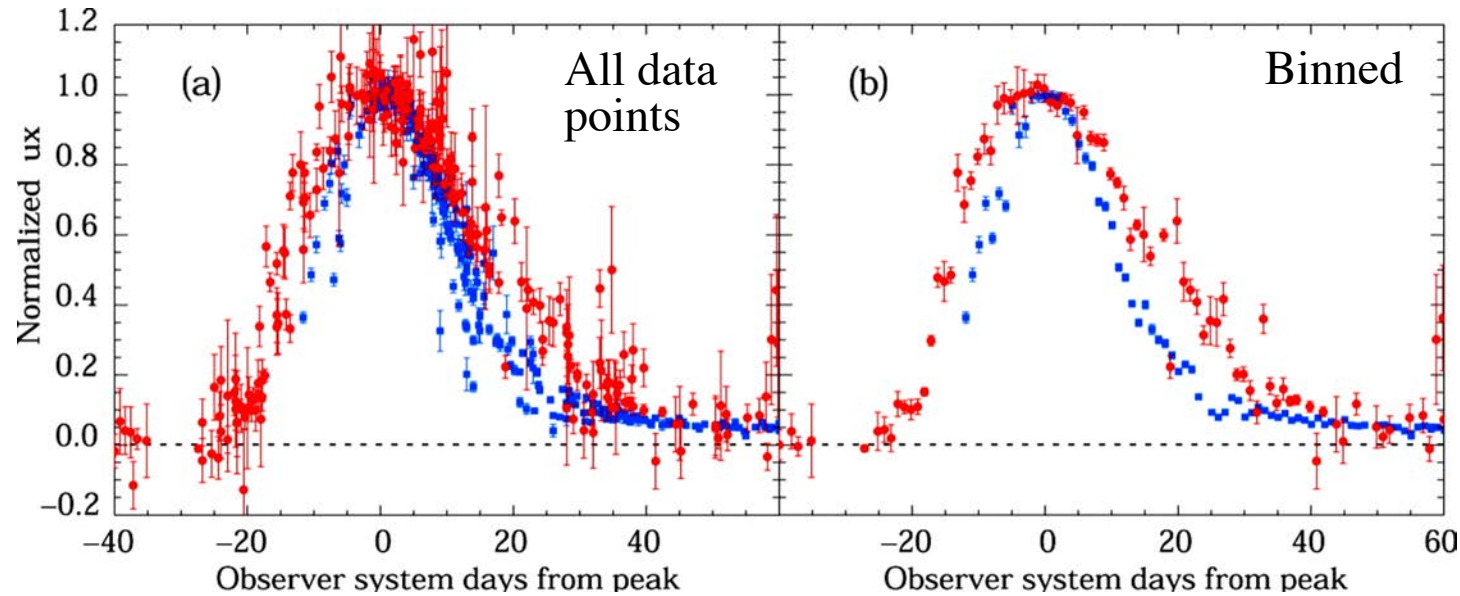
Performing the The Tolman Test

Use the SB-Radius and the Fundamental Plane correlations, with SB on the Y axis:



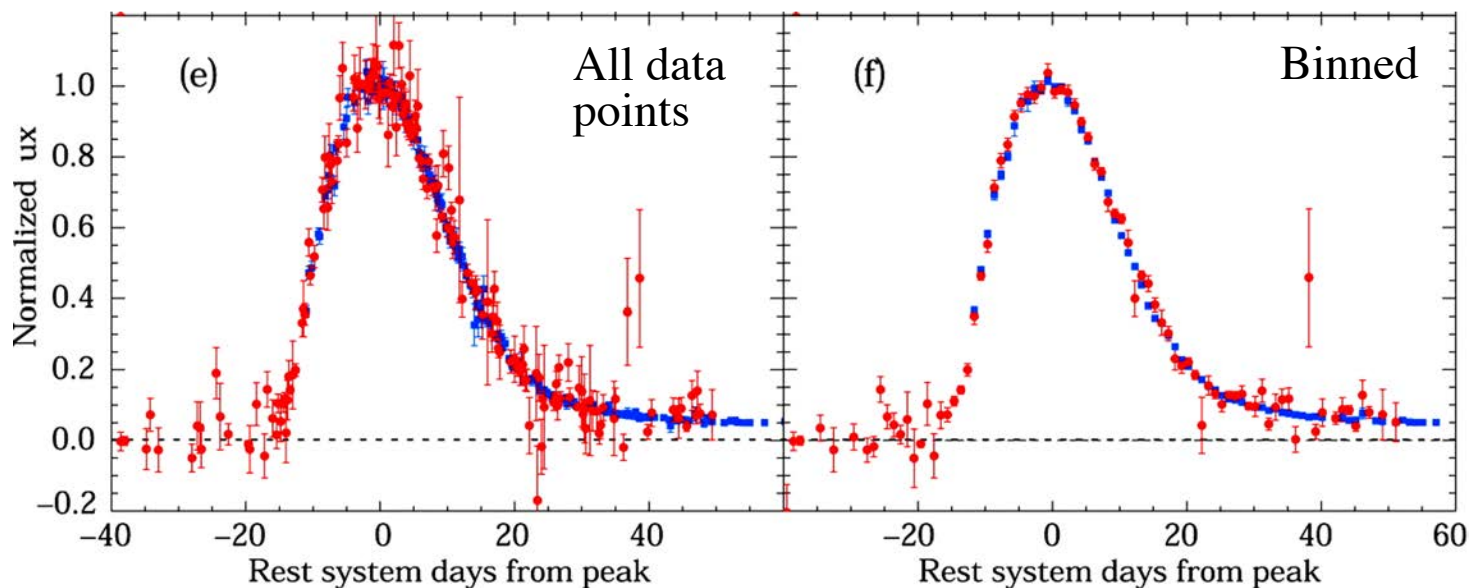
← After a mild evolution correction, the results confirm the prediction of the relativistic expansion

Time Dilation of Supernova Lightcurves



Blue dots: a low- z dataset

Red dots: a high- z dataset

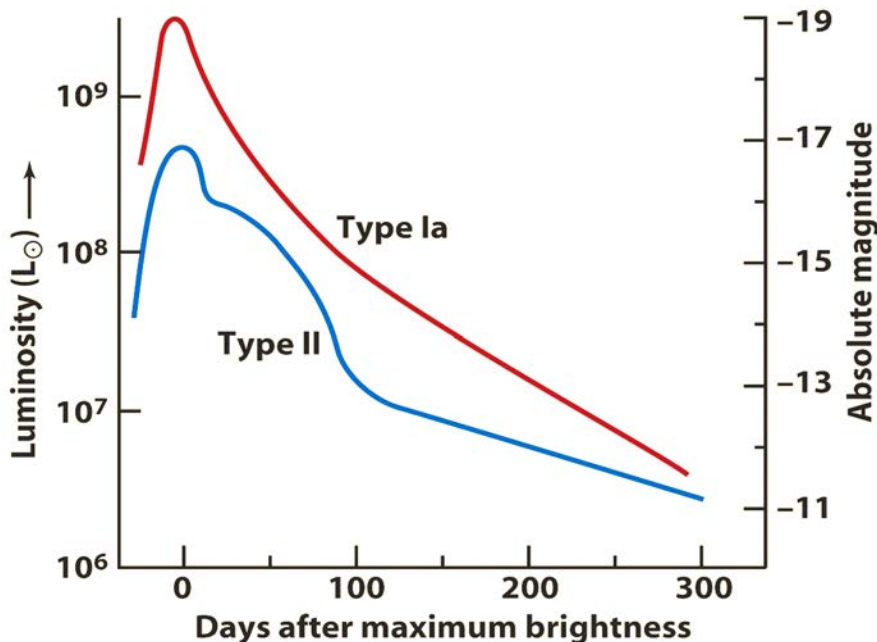


After applying the proper stretch factor

(Goldhaber *et al.*)

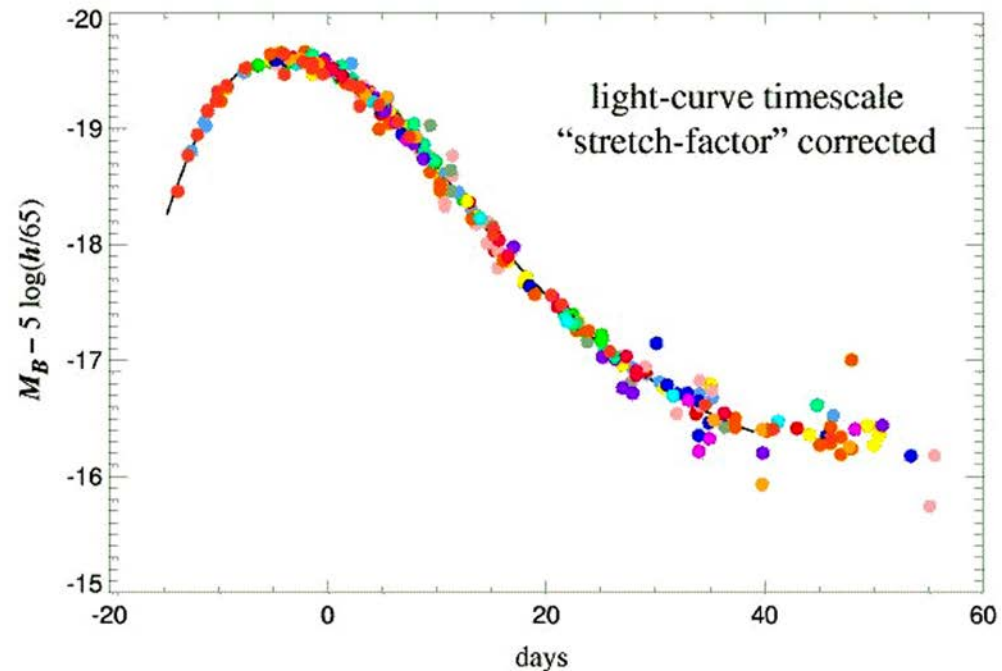
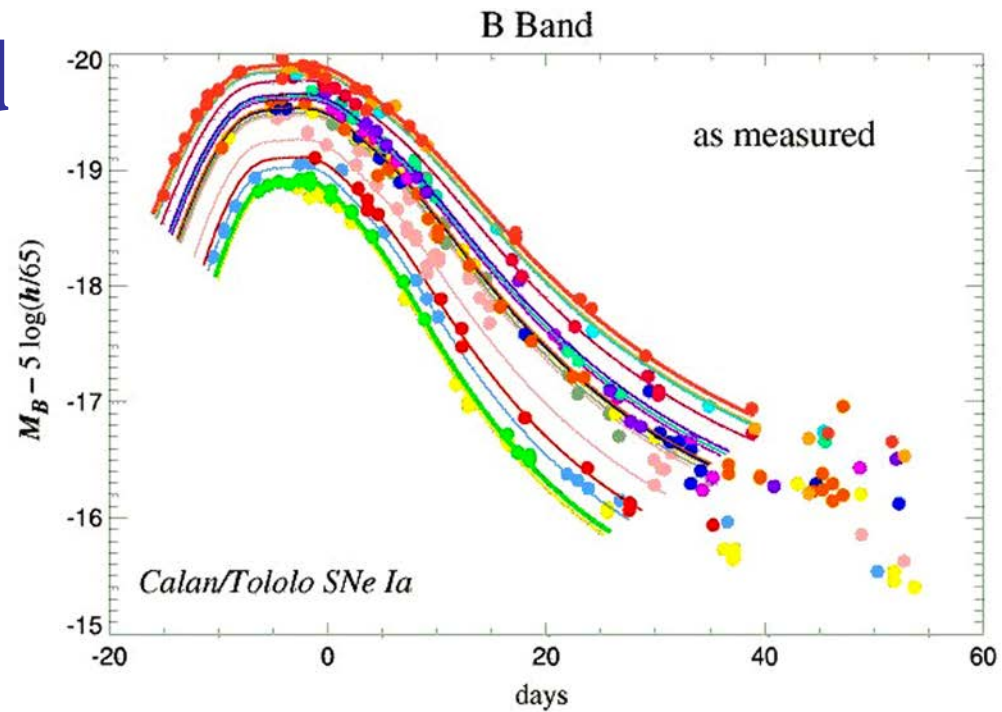
Supernovae (SNe) as Standard Candles

- Bright and thus visible far away
- **Type Ia** SNe are used as standard candles:
 - Binary white dwarfs accreting material and detonating
 - Pretty good standard candles, peak $M_V \sim -19.3$
 - Their scatter can be removed by using a light curve shape stretch factor to a peak magnitude scatter of $\sim 10\%$

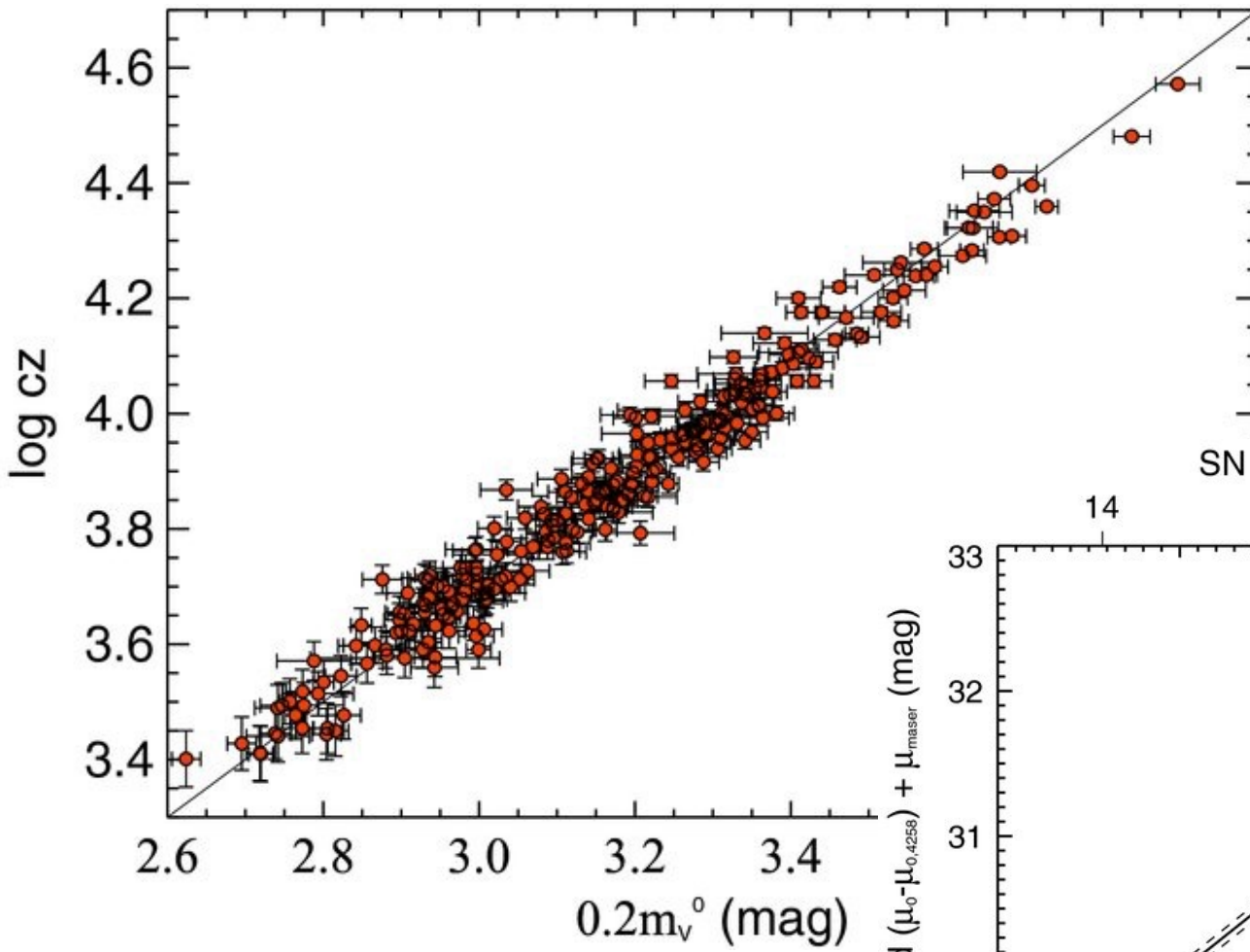


SNe Ia as Standard Candles

- The peak brightness of a SN Ia correlates with the shape of its light curve (steeper \rightarrow fainter)
- Correcting for this effect standardizes the peak luminosity to $\sim 10\%$ or better
- However, the absolute zero-point of the SN Ia distance scale has to be calibrated externally, e.g., with Cepheids



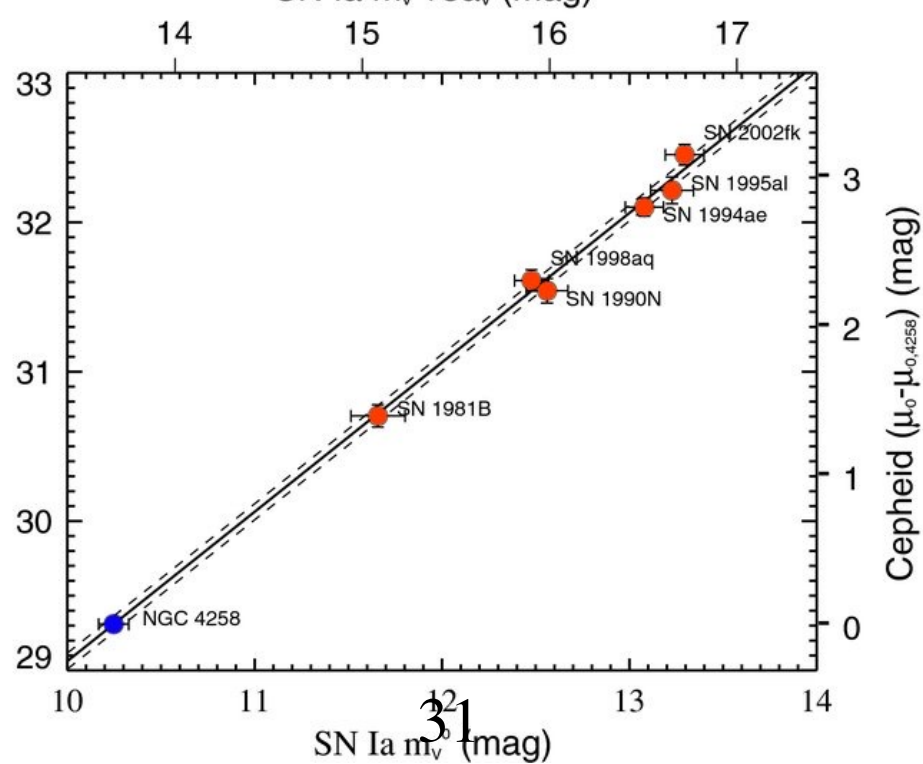
The Low-Redshift SN Ia Hubble Diagram



Riess et al. 2009
(also Riess et al. 2011
– slight changes)

Cepheid calibration

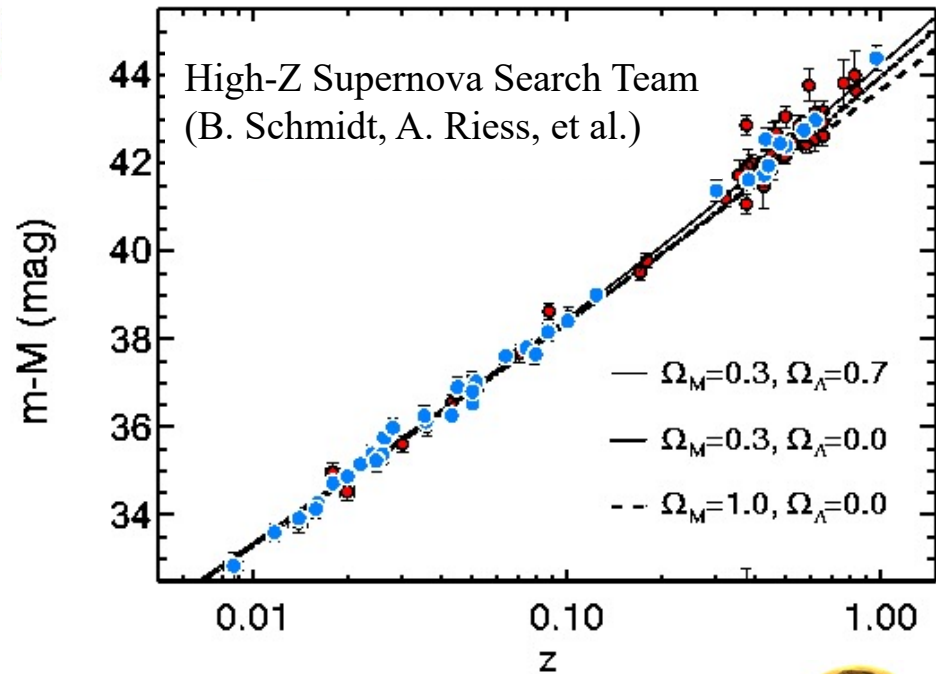
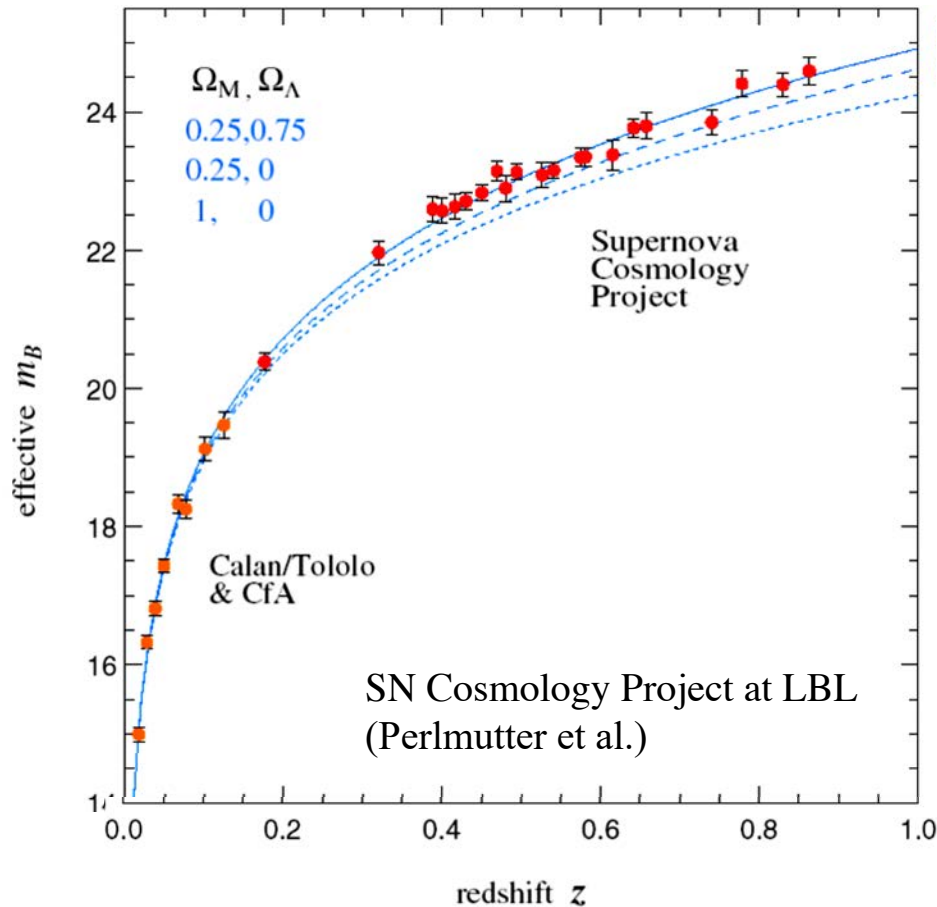
SN Ia $m_v^0 + 5a_v$ (mag)



$$H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

SN Hubble Diagram and the Dark Energy

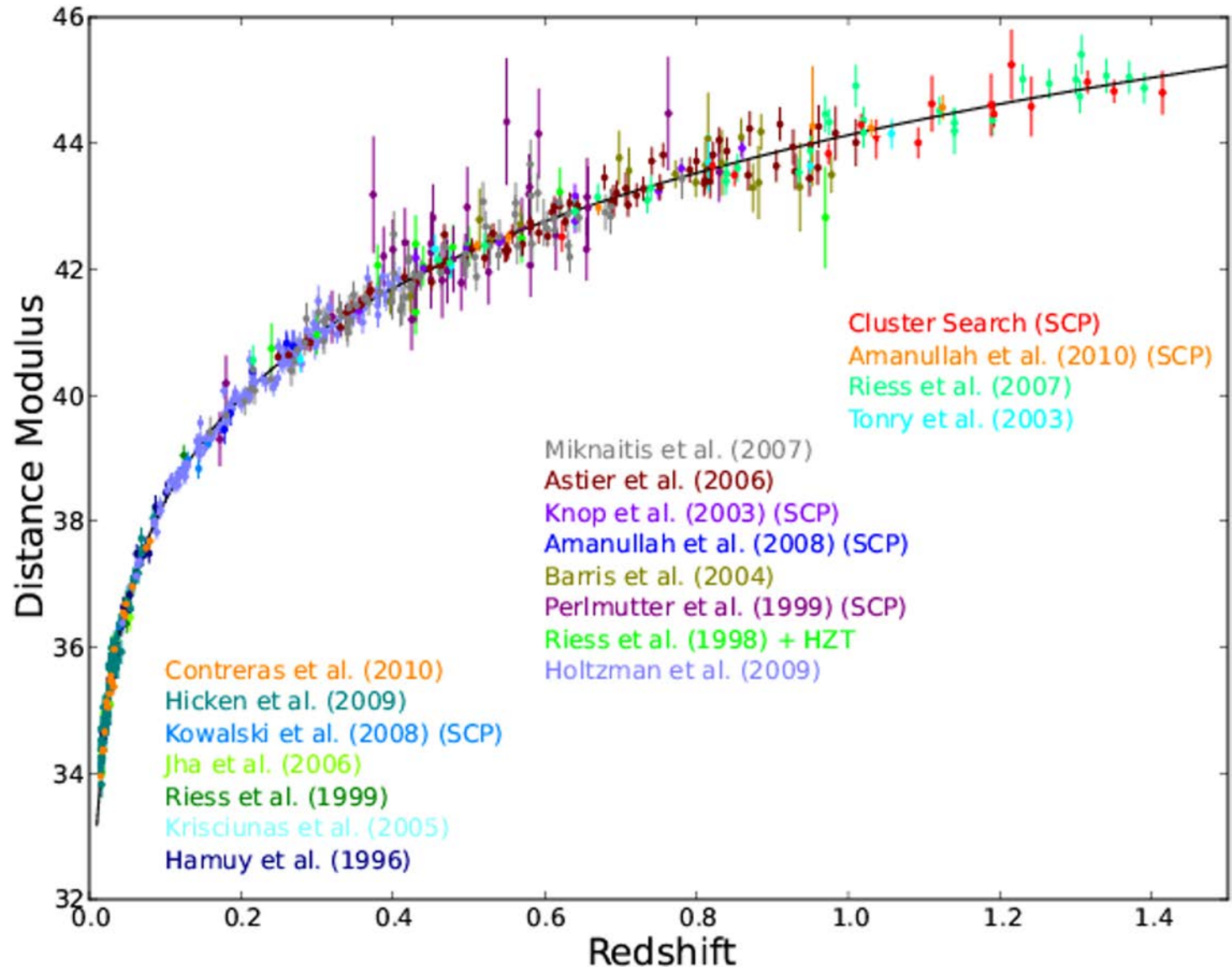
Pushing it to the higher redshifts it yielded a **convincing evidence for an accelerating universe and the positive cosmological constant**, independently and simultaneously by two groups:



For which Schmidt, Riess, and Perlmutter got a Nobel Prize in 2011



A Modern Version of the SN Hubble Diagram



The Angular Diameter Test

Angular
size

Requires a population on non-evolving sources with a fixed proper size - “standard rulers”.

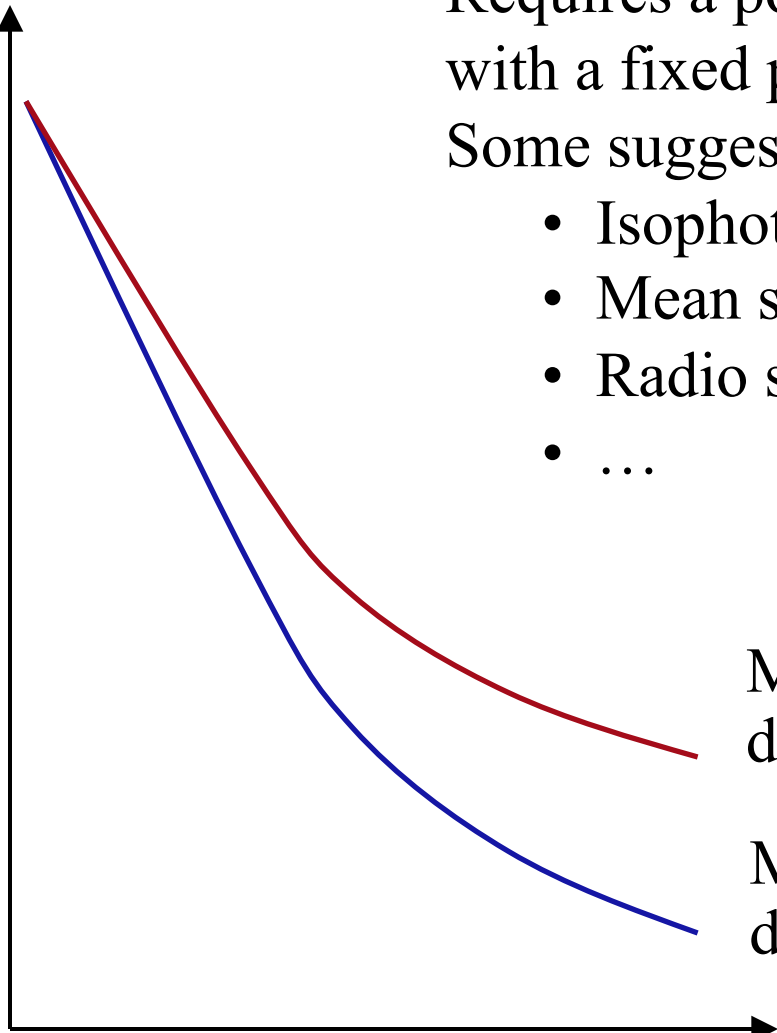
Some suggested candidates:

- Isophotal diameters of brightest cluster gal.
- Mean separation of galaxies in clusters
- Radio source lobe separations
- ...

Model with a higher
density and/or $\Lambda \leq 0$

Model with a lower
density and/or $\Lambda > 0$

redshift



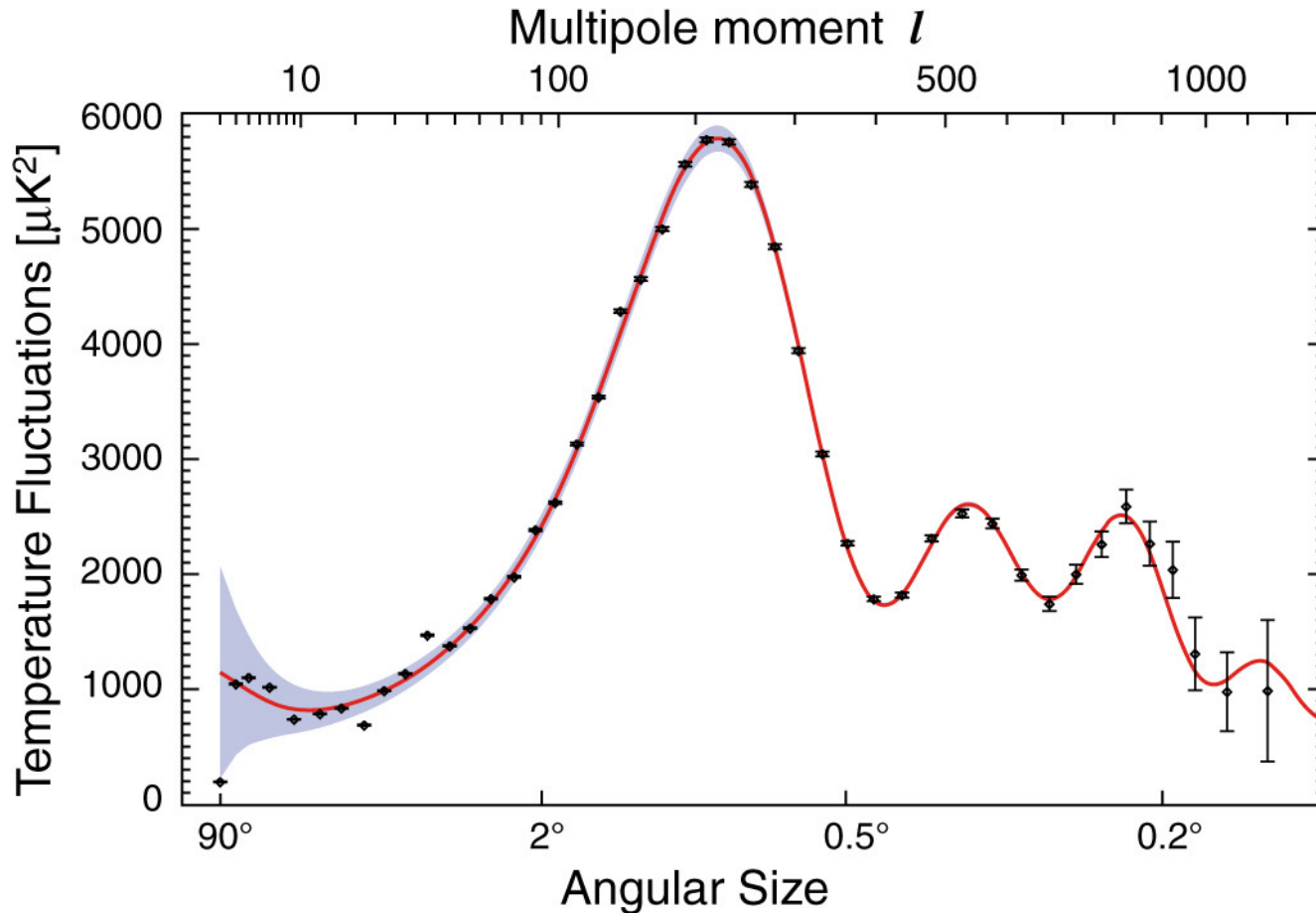
The Modern Angular Diameter Test: CMBR Fluctuations

- Uses the size of the particle horizon at the time of the recombination (the release of the CMBR) as a standard ruler
- This governs the largest wavelength of the sound waves produced in the universe then, due to the infall of baryons into the large-scale density fluctuations
- These sound waves cause small fluctuations in the temperature of the CMB ($\Delta T/T \sim 10^{-5} - 10^{-6}$) at the appropriate angular scales (\sim a degree and less)
- They are measured as the angular power spectra of temperature fluctuations of the CMBR

Is the Universe Flat, Open, or Closed?

Doppler peaks define a physical scale of the particle horizon at recombination. The corresponding angular size depends on the geometry of the universe

$$l = 220 \Rightarrow \Omega_{total} = 1.02 \pm 0.02$$

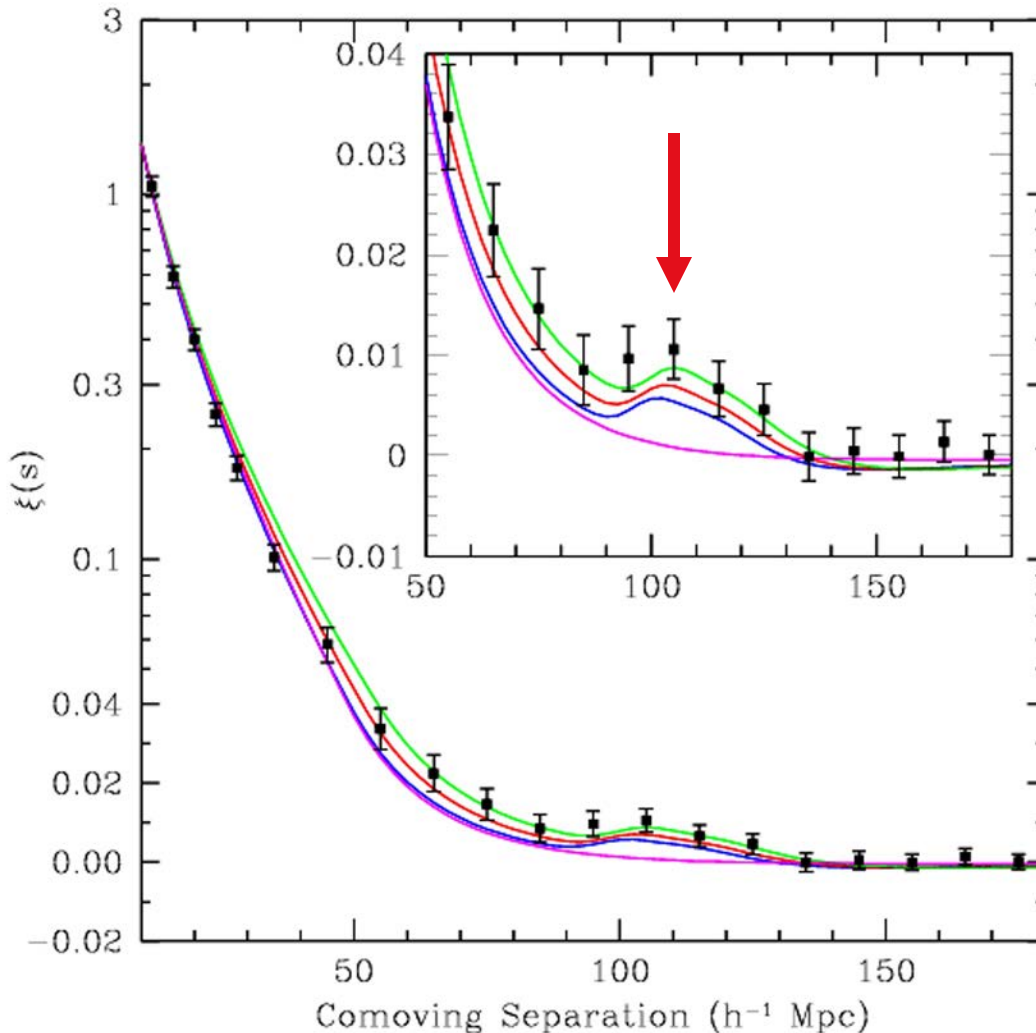


The universe is flat (or very close to flat)

WMAP 7 yr data

Baryon Acoustic Oscillations (BAO)

Eisenstein et al. 2005 (using SDSS red galaxies); also seen by the 2dF redshift survey



The 1st Doppler peak seen in the CMBR imprints a preferred scale for clustering of galaxies.

Detection of this feature in galaxy clustering at $z \sim 0.3$ gives us another instance of a “standard ruler” for an angular diameter test, at redshifts $z < 1100$

Future redshift surveys can do much better yet

All Matter and Energy in the Universe

There are several components:

- Luminous matter in galaxies: stars and gas (“luminous baryons”)
 - All normal matter not accounted by the luminous component (“dark baryons”)
 - Non-baryonic dark matter (DM)
 - “Dark energy” (recall that $\rho_{energy} = \rho_{matter} c^2$)
 - Radiation (all photons, mostly CMB)
 - Also neutrinos and gravitational waves
- Diagrammatic labels: A large right-facing curly bracket groups the first two items as “baryons”. A second, larger right-facing curly bracket groups the first two items and the DM item as “matter”.

Each has a mean density ρ_i and density parameter $\Omega_i = \rho_i / \rho_{crit}$

$$\text{where } \rho_{crit} = 3H^2 / (8\pi G) = 0.921 \times 10^{-29} h_{70}^2 \text{ g cm}^{-3}$$

The total density parameter is their sum: $\Omega_{total} = \sum \Omega_i$

Luminous Mass Density

Add up all of the starlight in galaxies to get the mean luminosity density:

$$\rho_{light} \approx (1.6 \pm 0.2) \times 10^8 h_{70} L_{\odot}/\text{Mpc}^3$$

Convert to mass density using a mean mass to light ratio of stellar populations, $\langle M/L \rangle \approx 5$, and correct for the fraction of the gas in the ISM, $f_{\text{gas}} \approx 10\%$

$$\rho_{lum} = \rho_{light} \times \langle M/L \rangle \times \langle 1 + f_{\text{gas}} \rangle \approx (7 \pm 2) \times 10^8 h_{70} M_{\odot}/\text{Mpc}^3$$

$$\rho_{lum} \approx (4.7 \pm 1.3) \times 10^{-32} h_{70} \text{ g cm}^{-3}$$

$$\text{Thus, } \Omega_{0,lum} \approx (0.0051 \pm 0.0015) h_{70}^{-1}$$

All of the visible matter amounts to only half a percent of the total mass/energy content of the universe!



The Total Baryon Density

It is measured in two independent ways:

1. The cosmic nucleosynthesis:

- ✧ Reaction rates are $\sim \rho_{\text{baryon}}^2$, so the abundances of D, He, and Li are very sensitive to ρ_{baryon} (especially for D)
- ✧ Measured in spectra of distant QSOs (actually Ly α forest clouds), star forming dwarf galaxies, halo stars, etc.

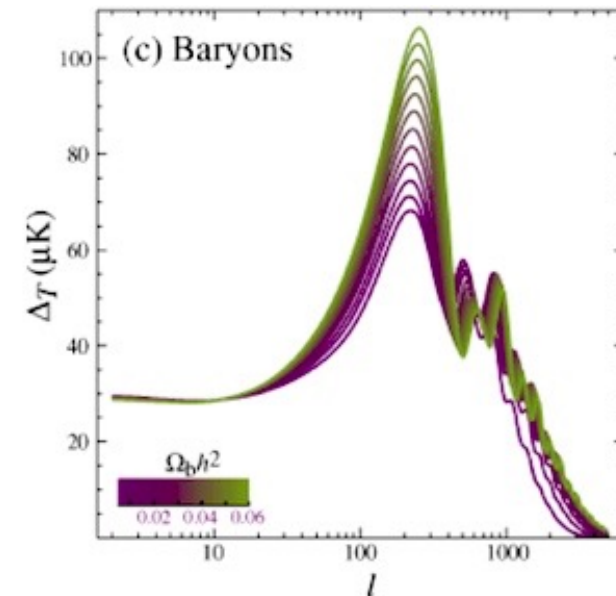
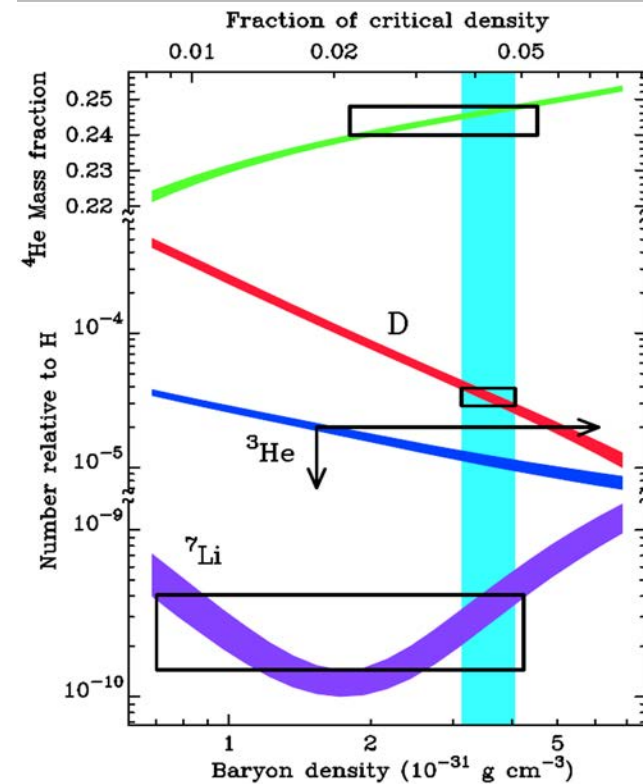
Result: $\Omega_{\text{baryons}} h^2 = 0.021 \rightarrow 0.025$

2. Acoustic peaks in the CMB

- ✧ Amplitude is sensitive to ρ_{baryon}

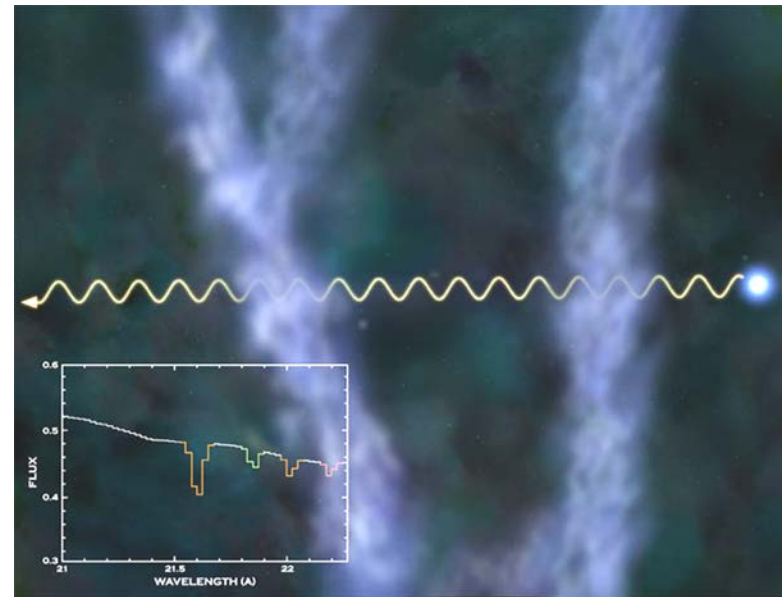
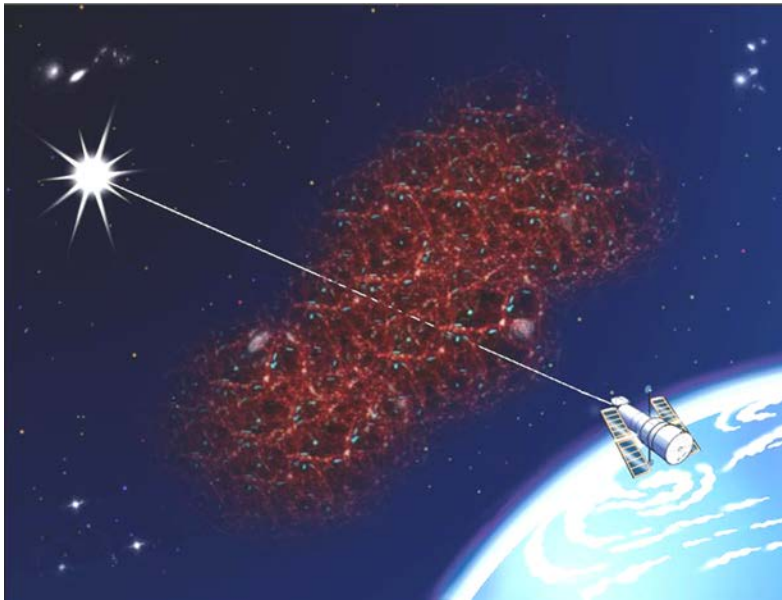
Result: $\Omega_{\text{baryons}} h^2 = 0.0221 \pm 0.0003$

Thus, $\Omega_{0,\text{baryons}} \approx (0.048 \pm 0.005) h_{70}^{-1}$



Missing Baryons in Warm/Hot IGM?

This hypothetical Baryon reservoir would have Virial temps. of $\sim 10^5 - 10^6$ K, where the peak emission is in FUV/soft-X, which is effectively absorbed by the ISM in our Galaxy, and is thus essentially impossible to detect in emission ...



However, it might have been *detected in absorption* in the UV (HST and FUSE) and X-Rays (Chandra), using O VI, O VII, and O VIII lines

The Total Matter Density

It is measured in several *independent* ways:

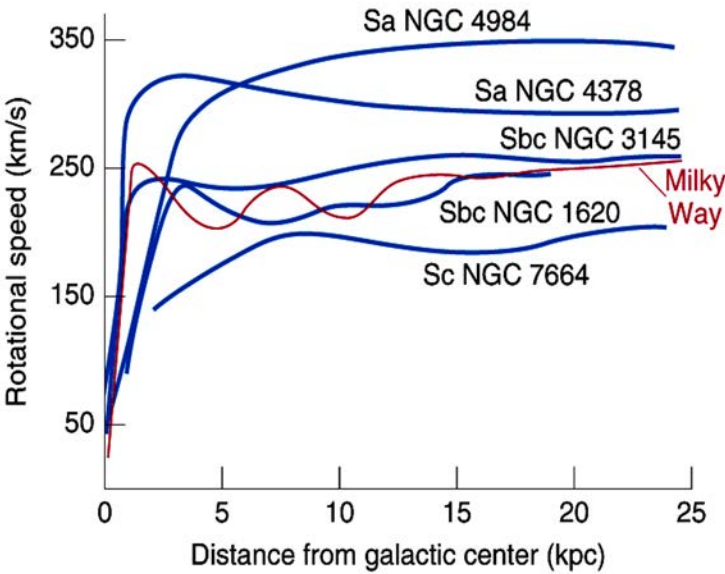
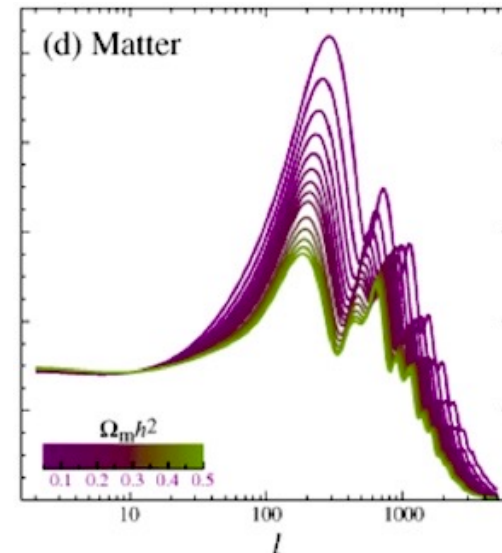
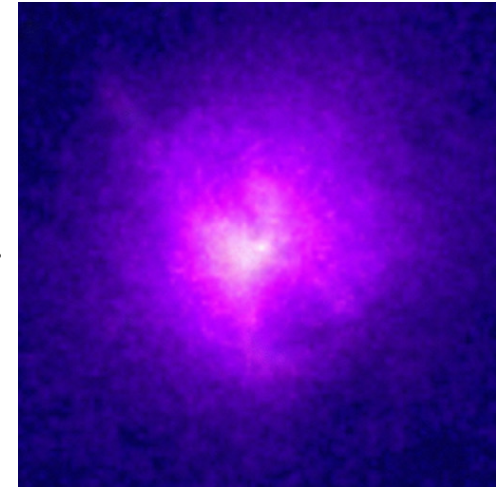
< Galaxy dynamics:
rotation curves,
velocity dispersions...

Cluster masses >
from the X-ray gas

< Cluster masses from
gravitational lensing

CMB fluctuations >

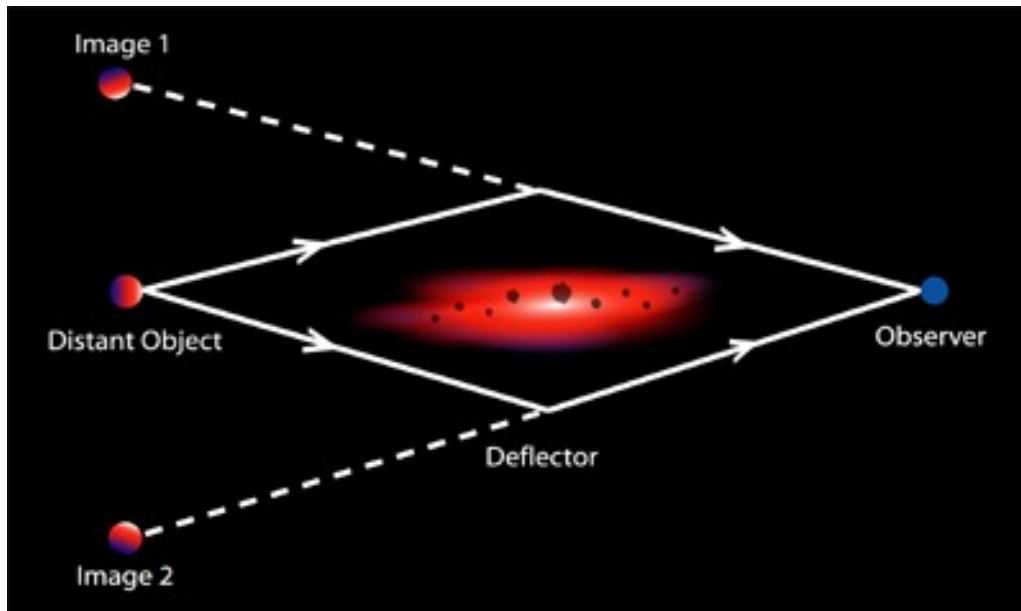
+ Large-scale structure...



Gravitational Lensing:

Mapping the Distribution of the Dark Matter

- We know from general relativity that mass - whether it is visible or not - bends light. This opens a possibility of “seeing” the distribution of dark matter
- Chowlson (1924) and Einstein (1936) predicted that if a background object is directly aligned with a point source mass, the light rays will be deflected into an “Einstein Ring”



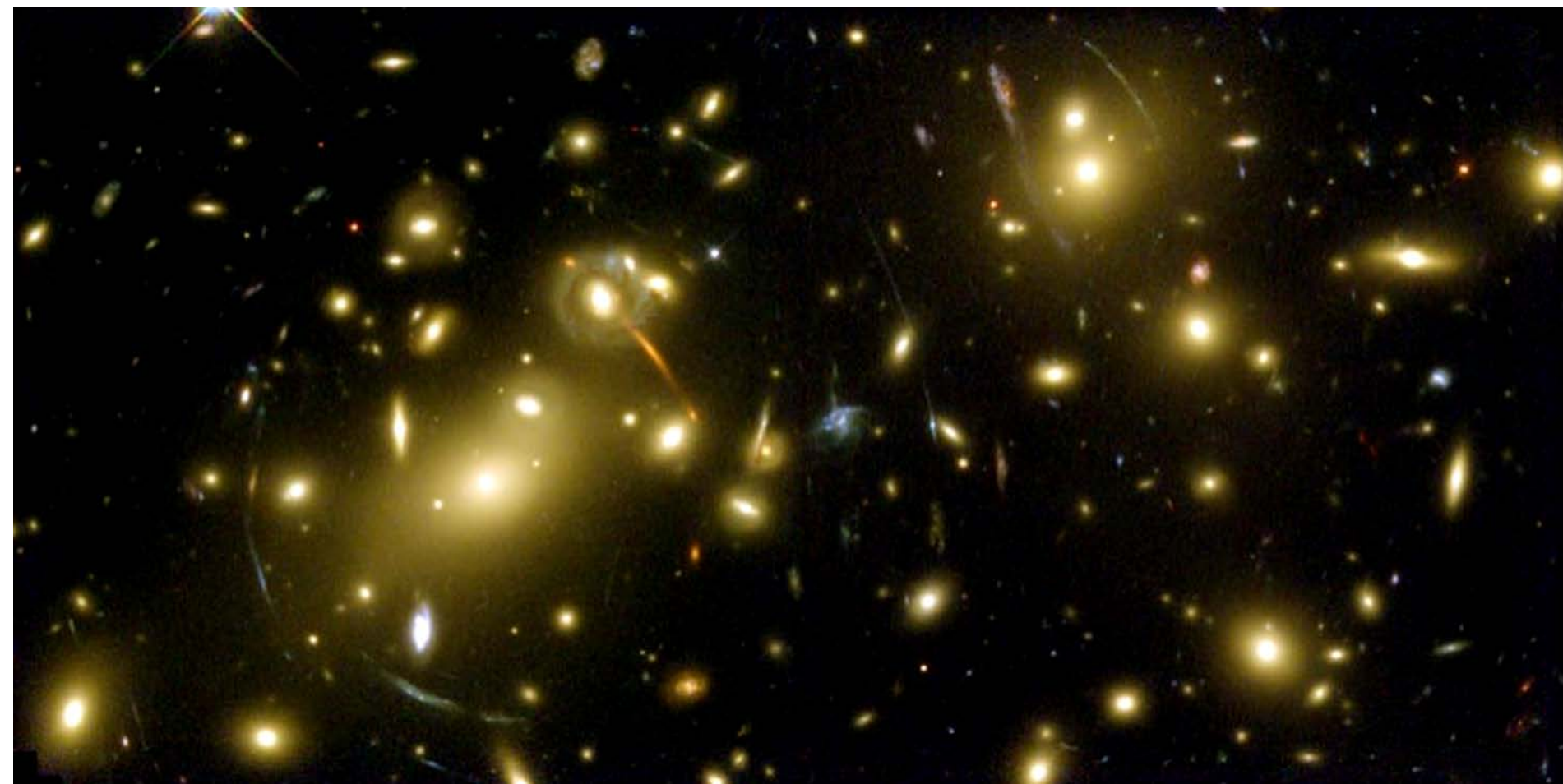
The first gravitational lens

QSO 0975+561
THE DOUBLE QUASAR

Walsh, Carswell & Weymann 1979

Gravitationally Lensed Galaxies - “Arcs”

In 1937, Zwicky predicted that one could study the mass distribution (dark matter) in clusters by studying background galaxies that are lensed by the dark matter in the cluster. This was not observationally feasible until the mid-1990' s



Galaxy Masses From Gravitational Lensing

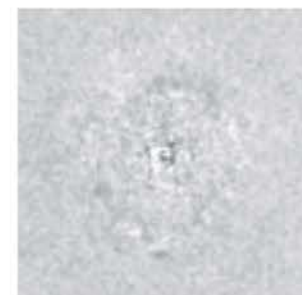
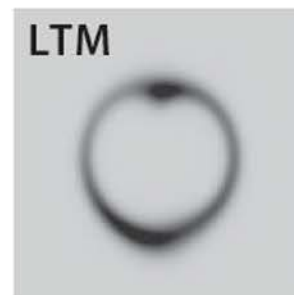
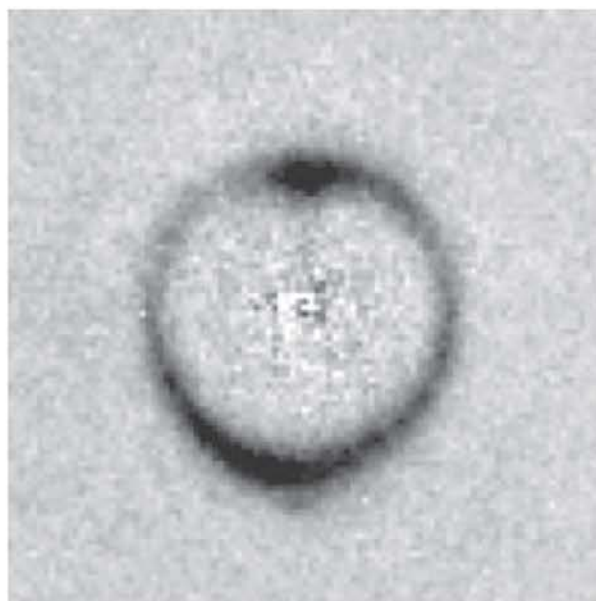
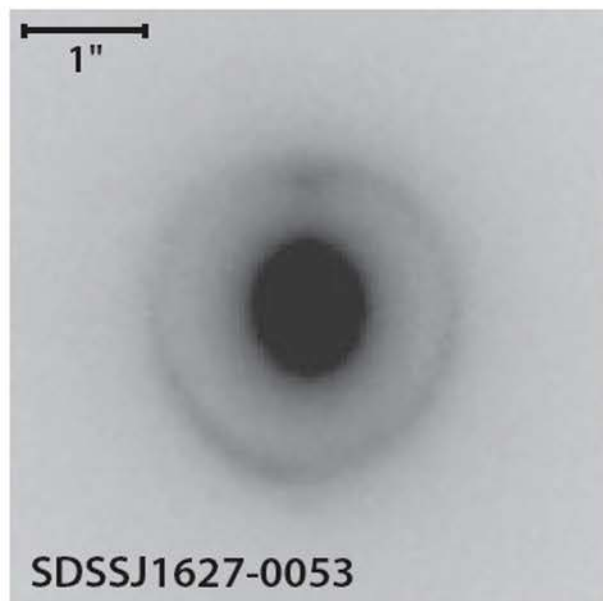
Treu et al. (the SLACS collaboration)

Original

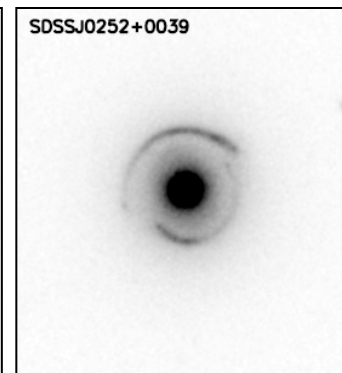
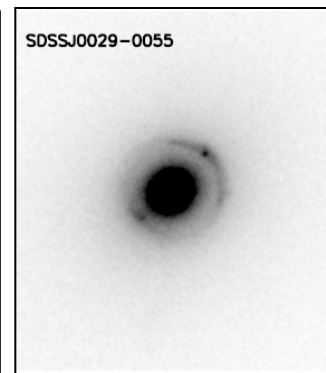
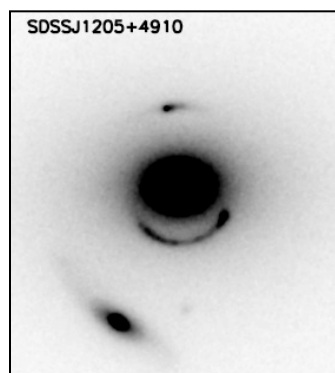
Lens galaxy subtracted

Lens models

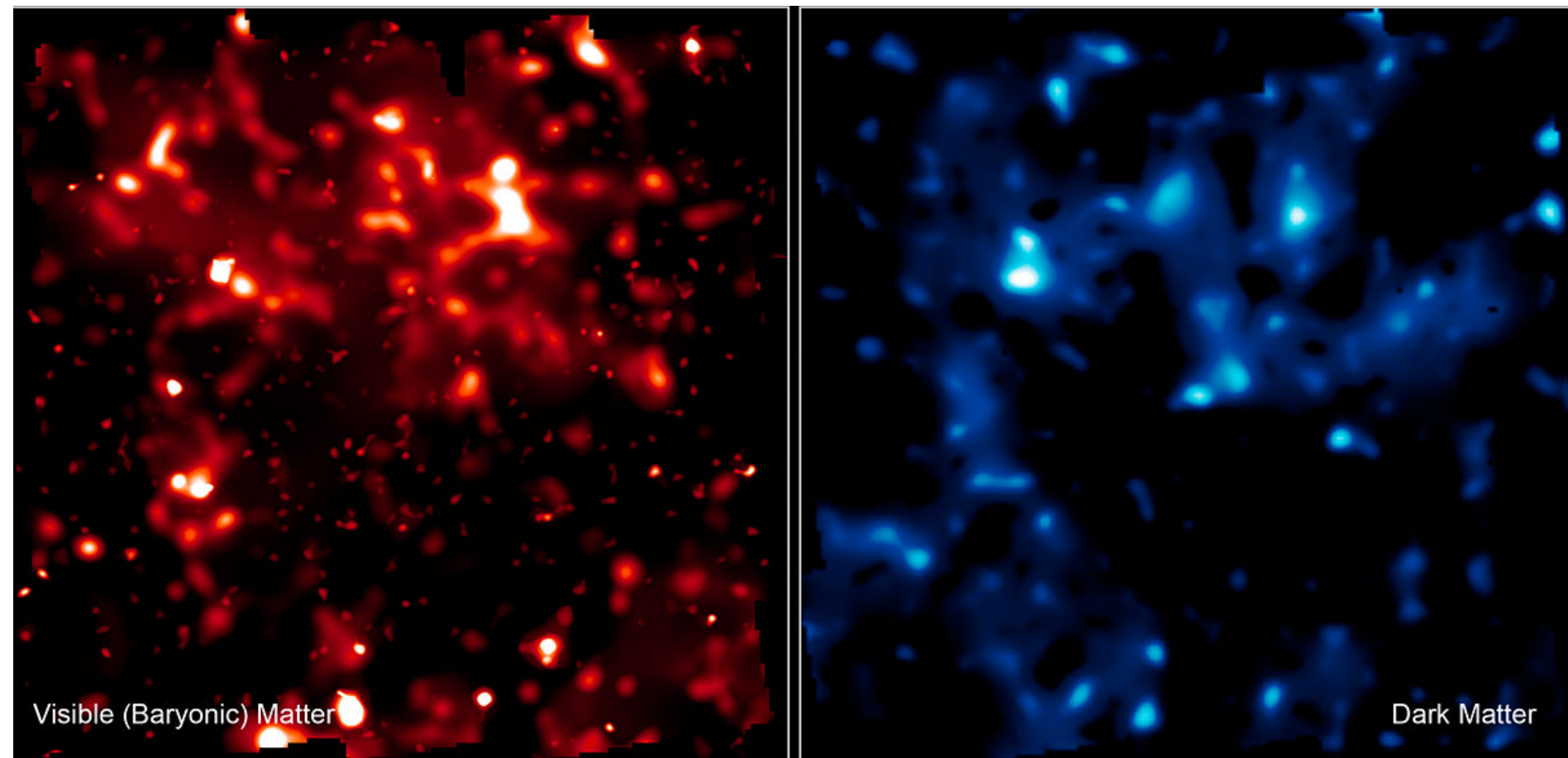
Residuals



Typically using a Singular Isothermal Ellipsoid (SIE) as a lens mass model

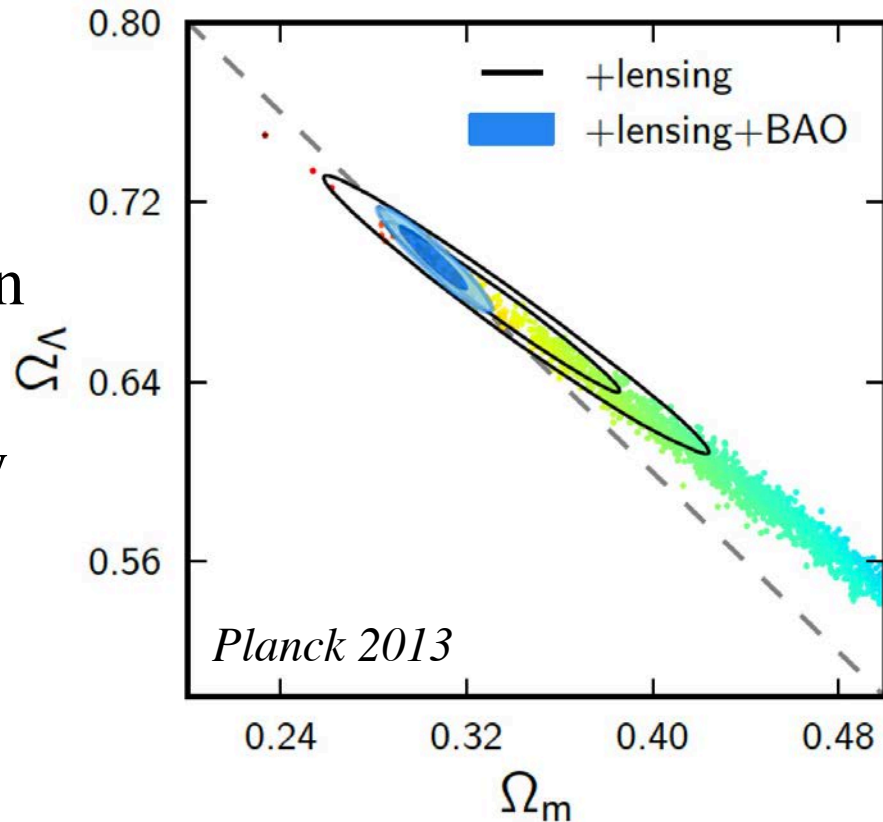


Visible and DM Distribution From the COSMOS Survey (Scoville, Massey et al. 2007)



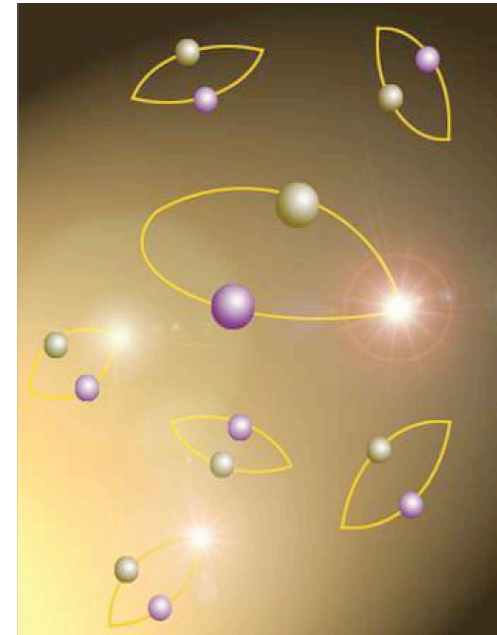
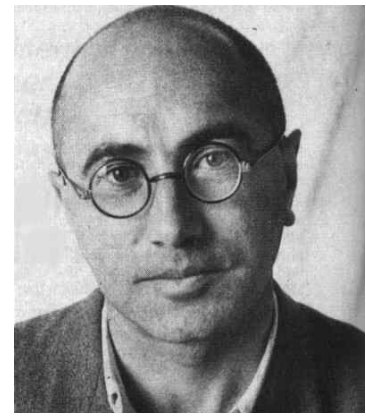
The Dark Energy

- The **dominant component** of the observed matter/energy density: $\Omega_{0,DE} \approx 0.7$
- Causes the accelerated expansion of the universe
- May affect the growth of density perturbations
- Effective only at cosmological distances
- Its physical nature is as yet ***unknown***; this may be the biggest outstanding problem in physics today
- ***Cosmological constant*** is just one special case; a more general possibility is called ***quintessence***



Cosmological Constant as a Quantum Field Phenomenon

- Proposed by Yakov Zeldovich (1967)
- A modern view of the physical vacuum is that it is not really empty - it is filled with virtual particle-antiparticle pairs, which annihilate within $\Delta t < \hbar/mc^2$, and their fluctuations give rise to a net energy density - a ground(?) state of the physical vacuum
- This is essentially the same mechanism proposed as the origin of the inflation
- But to really estimate the value of this vacuum energy density, we need a quantum theory of gravity, which we don't have yet
- Nevertheless, eager minds do try ...



The Worst Scientific Prediction Ever

- A “natural” Planck system of units expresses everything as combination of fundamental physical constants; the Planck density is:

$$\rho_{Planck} = c^5 / (\hbar G^2) = 5.15 \times 10^{+93} \text{ g cm}^{-3}$$

- The observed value is:

$$\rho_{vac} = \Omega_{vac} \rho_{crit} \approx 6.5 \times 10^{-30} \text{ g cm}^{-3}$$

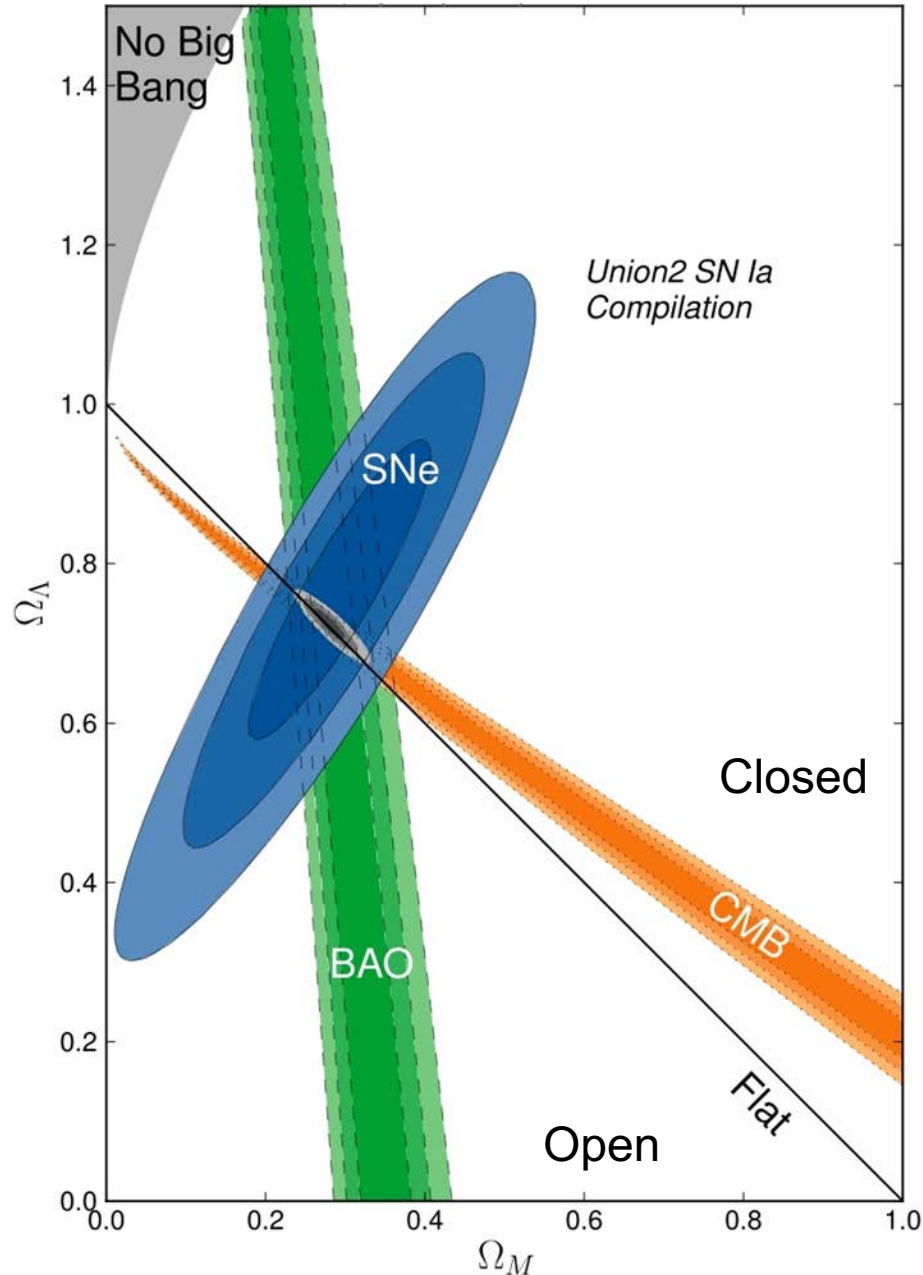
Ooops! Off by 123 orders of magnitude ...

- This is modestly called “the fine-tuning problem” (because it requires a cancellation to 1 part in 10^{123})
- The other “natural” value is zero
- So, lacking a proper theory, physicists just declared the cosmological constant to be zero, and went on...

Physical Origins of the Dark Energy

- ... are completely unknown at this time, and not for the lack of trying: there are literally thousands of papers about it, and more being published every day
- Many of the proposed models are based on one of the following:
 - Decay of some scalar field, similar to the inflation mechanism
 - Modified theories of gravity
 - Holographic models, connecting the vacuum energy density to the area of the event horizon and thermodynamics
 - Landscape or multiverse models that postulate the existence of $\sim 10^{500}$ separate universes, with different (random) values of the physical constants, Λ included
 - Models connecting DM and DE *... etc., etc.*
 - One measurement that might help eliminate some possibilities is a possible deviation (evolution) of the EOS parameter w

The Cosmic Concordance



Supernovae alone

⇒ Accelerating expansion

⇒ $\Lambda > 0$

CMB alone

⇒ Flat universe

⇒ $\Lambda > 0$

Any two of SN, CMB, LSS

⇒ Dark energy $\sim 70\%$

Also in agreement with the age estimates (globular clusters, nucleocosmochronology, white dwarfs)

The Component Densities

at $z \sim 0$, in critical density units, assuming $h \approx 0.7$

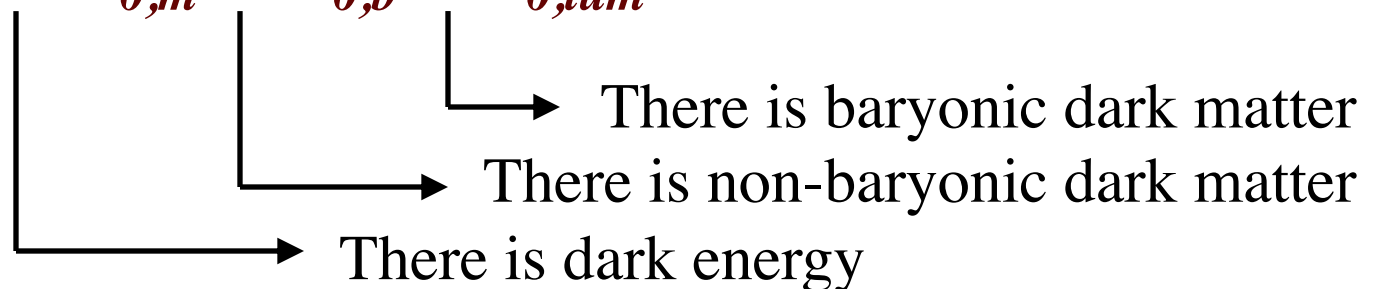
Total matter/energy density: $\Omega_{0,tot} \approx 1.00$ From CMB, and consistent with SNe, LSS

Matter density: $\Omega_{0,m} \approx 0.31$ From local dynamics and LSS, and consistent with SNe, CMB

Baryon density: $\Omega_{0,b} \approx 0.045$ From cosmic nucleosynthesis, and independently from CMB

Luminous baryon density: $\Omega_{0,lum} \approx 0.005$ From the census of luminous matter (stars, gas)

Since: $\Omega_{0,tot} > \Omega_{0,m} > \Omega_{0,b} > \Omega_{0,lum}$



Today's Best Estimates of the Cosmological Parameters

Age:

$$t_0 = 13.80 \pm 0.02 \text{ Gyr}$$

Best fit CMB model - consistent with ages of oldest stars

Hubble constant:

$$H_0 = 69 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

CMB + HST Key Project to measure Cepheid distances

Density of ordinary matter:

$$\Omega_{baryon} = 0.045$$

CMB + nucleosynthesis

Density of all forms of matter:

$$\Omega_{matter} = 0.31$$

Cluster dark matter estimate
CMB power spectrum

Cosmological constant:

$$\Omega_{\Lambda} = 0.69$$

Supernova data, CMB evidence for a flat universe plus a low matter density