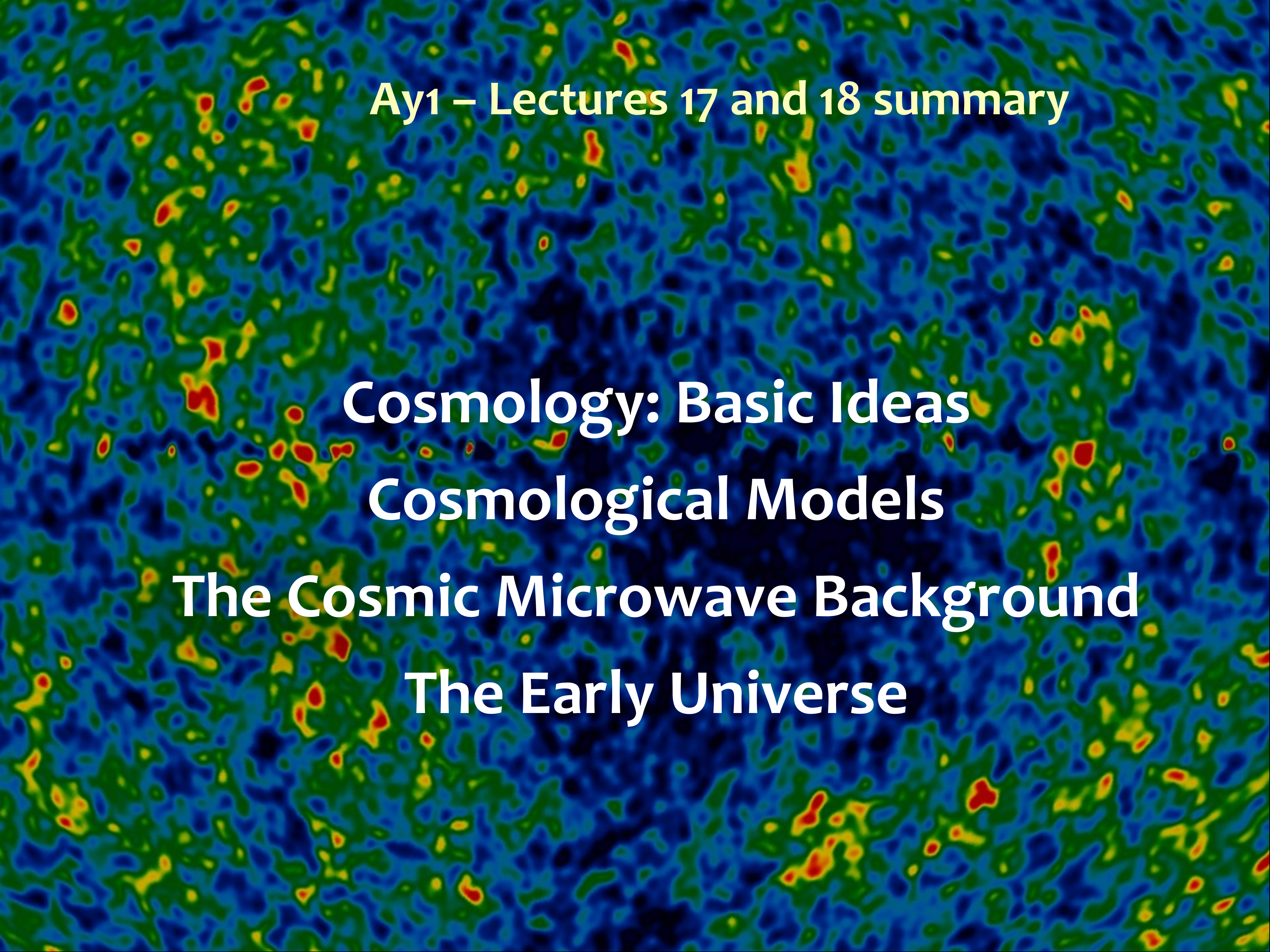


Ay1 – Lectures 17 and 18 summary

Cosmology: Basic Ideas
Cosmological Models
The Cosmic Microwave Background
The Early Universe

Discovery of the Expanding Universe

Based on an early work byVesto Melvin Slipher ->

< - Edwin Hubble discovered that galaxies recede from us with a velocity that is proportional to the distance

The ***Hubble’s law***:

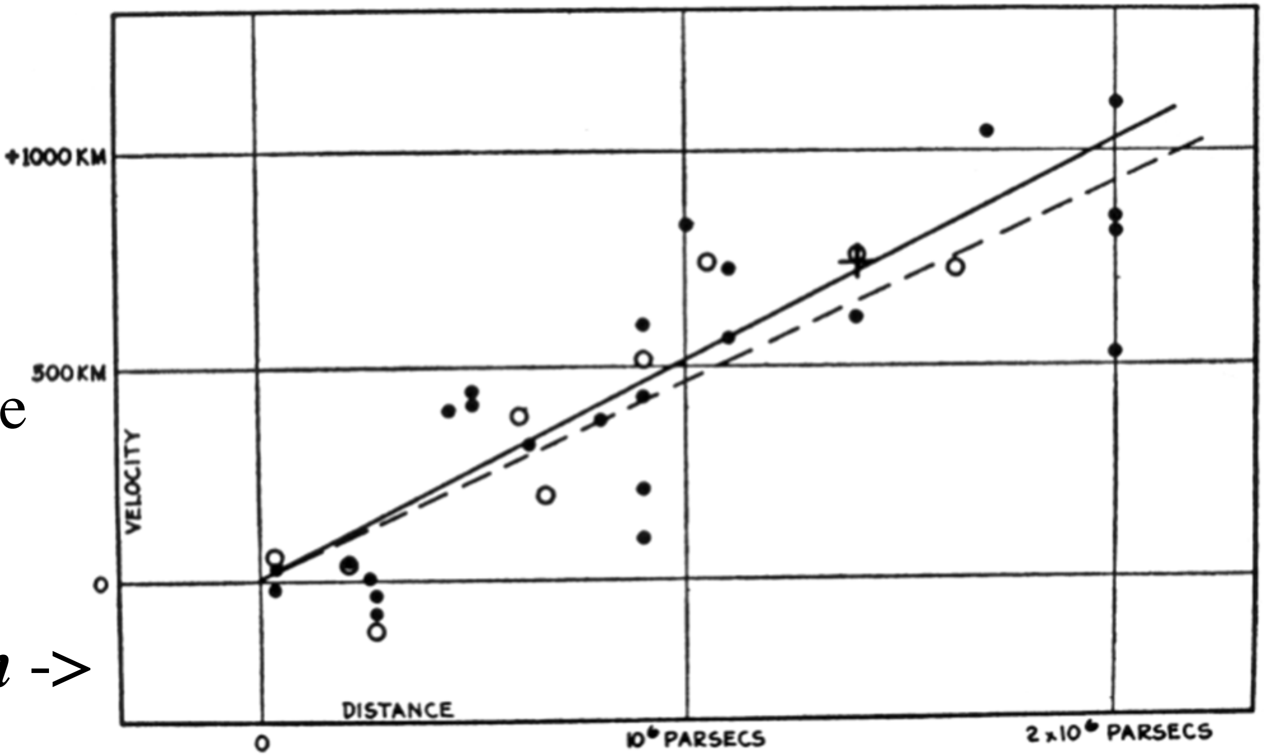
$$V = H D$$

Velocity

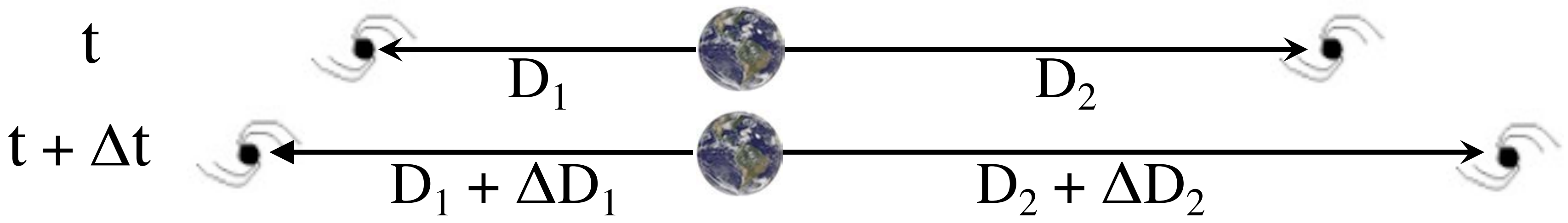
Distance

Hubble’s constant

The ***Hubble diagram*** ->

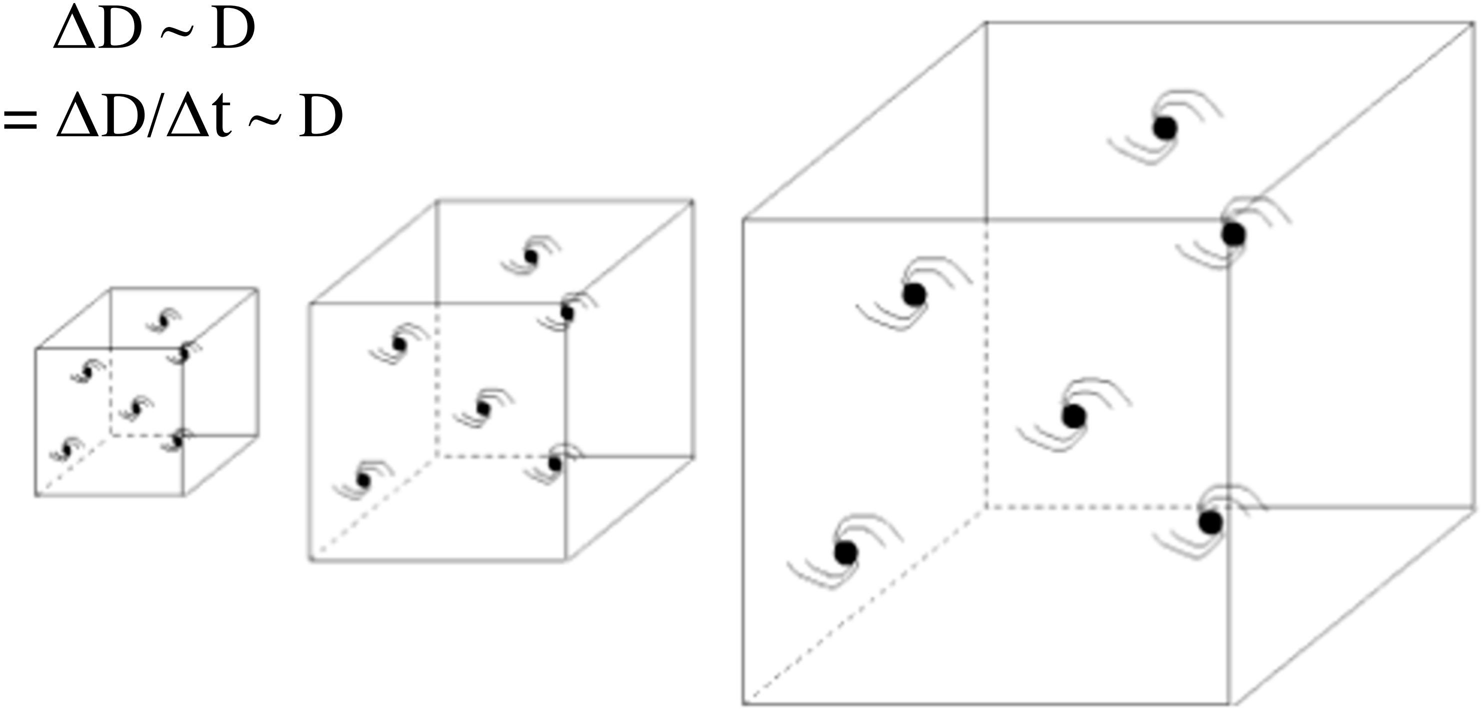


Expansion of the Universe



$$\Delta D \sim D$$

$$V = \Delta D / \Delta t \sim D$$



The ***space itself expands***, and carries galaxies apart

In a homogeneous, isotropic universe, there is no preferred center

The Cosmological Principle

Relativistic cosmology uses some symmetry assumptions or principles in order to make the problem of "solving the universe" viable. The **Cosmological Principle** states that

At each epoch, the universe is the same at all locations and in all directions, except for local irregularities

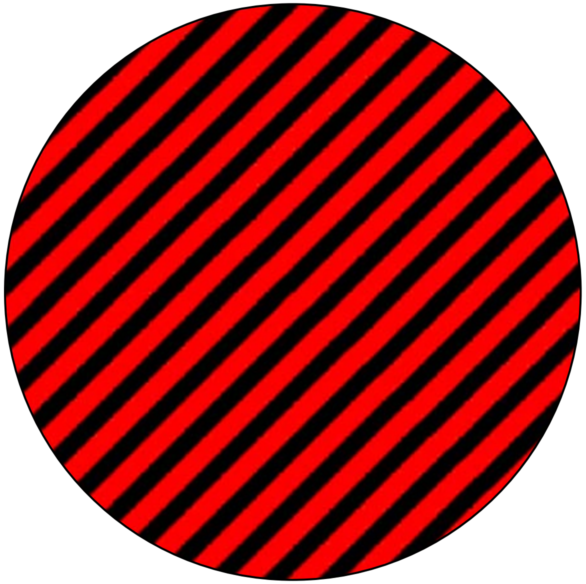
Therefore, globally the Universe is assumed to be **homogeneous** and **isotropic** at any given time; and its dynamics should be the same everywhere

Note: the **Perfect Cosmological Principle** states that the Universe appears the same at all times and it is unchanging - it is also homogeneous in time - this is the basis of the "Steady State" model

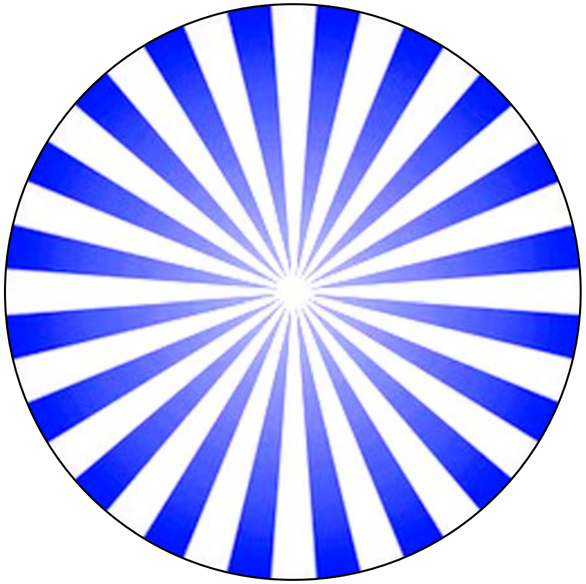
Homogeneity and Isotropy

(in *space*, but not in *time*)

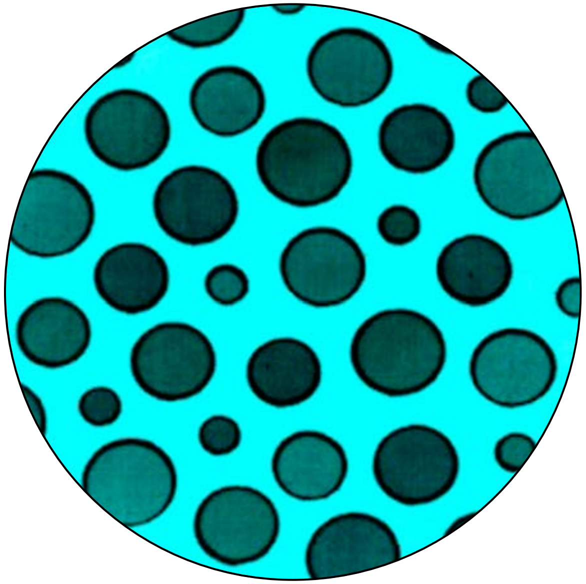
Homogeneous but not Isotropic



Isotropic but not Homogeneous



Isotropic and Homogeneous

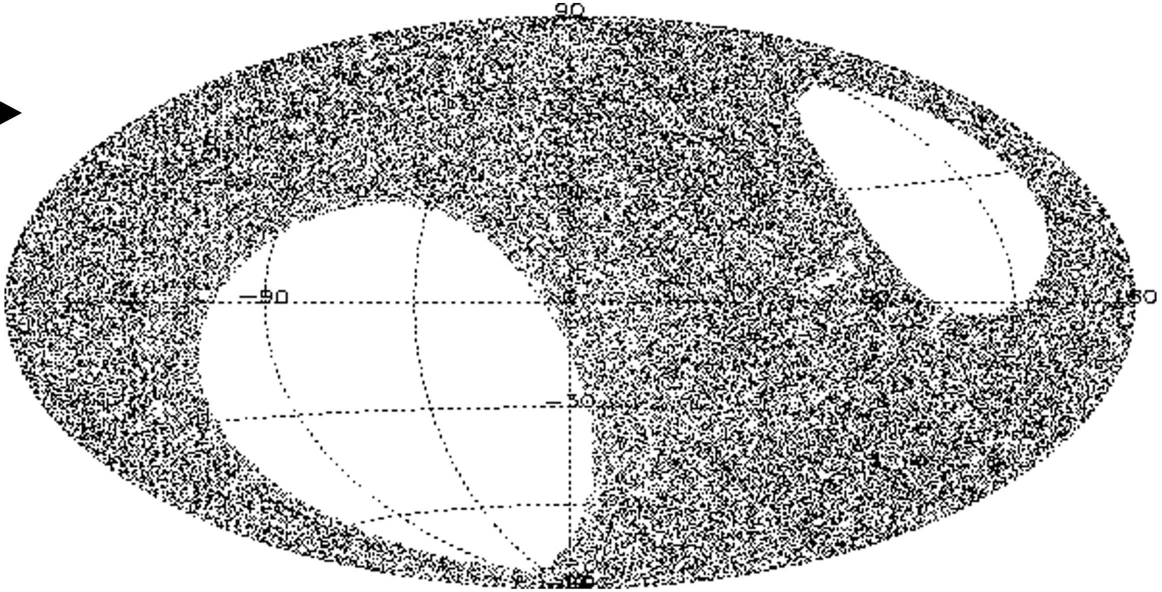


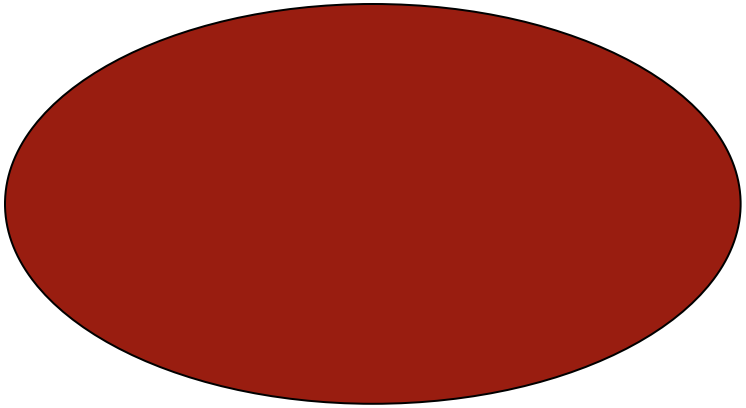
This simplifies the modeling, since only the radial coordinate matters, and the density of any mass/energy component is the same everywhere at a given time

So, is the Universe Really Homogeneous and Isotropic?

Globally, on scales larger than ~ 100 Mpc, say, it is - so the cosmological principle is valid

Distribution on the sky → of 65000 distant radio sources from the Texas survey, a cosmological population

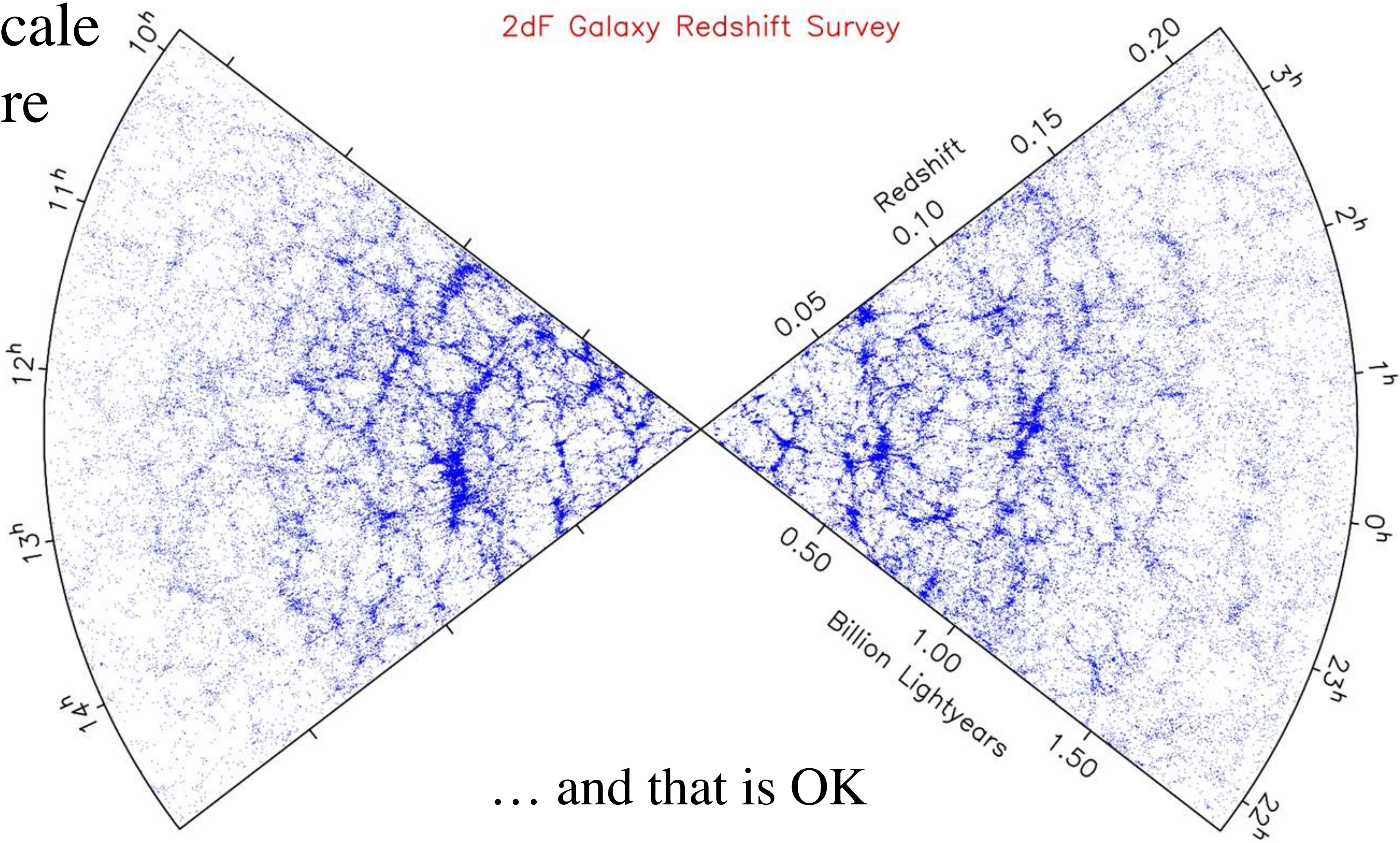




… and of course the CMBR, uniform to better than $\Delta T/T < 10^{-5}$, after taking the dipole out

So, is the Universe Really Homogeneous and Isotropic?

But not so on scales up to ~ 100 Mpc, as shown by the large-scale structure



… and that is OK

Expansion Relative to What? Comoving and Proper Coordinates

There are fundamentally two kinds of coordinates in a GR cosmology:

- ***Comoving coordinates*** = expand with the universe

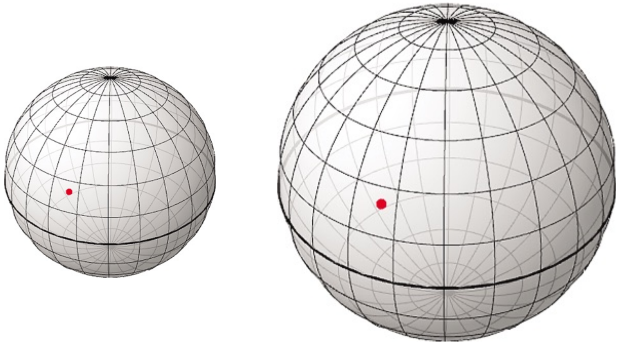
  Examples:
  - Unbound systems, e.g., any two distant galaxies
  - Wavelengths of massless quanta, e.g., photons
- ***Proper coordinates*** = stay fixed, space expands relative to them. Examples:
  - Sizes of atoms, molecules, solid bodies
  - Gravitationally bound systems, e.g., Solar system, stars, galaxies …

Expansion into What?

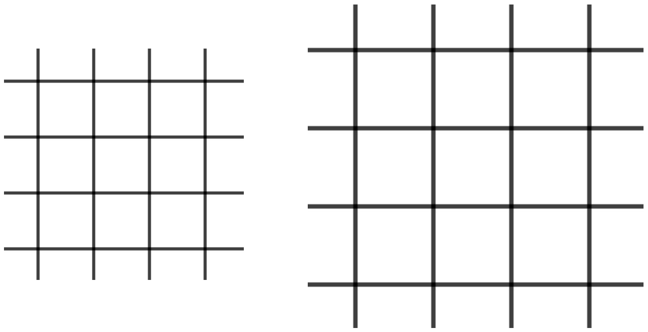
Into itself. There is nothing “outside” the universe

(Let’ s ignore the multiverse hypothesis for now)

A positive curvature universe is like the surface of a sphere, but in one extra dimension. Its volume is finite, but changes with the expansion of space



A flat or a negative curvature universe is infinite in all directions; the comoving coordinate grid stretches relative to the proper coordinates

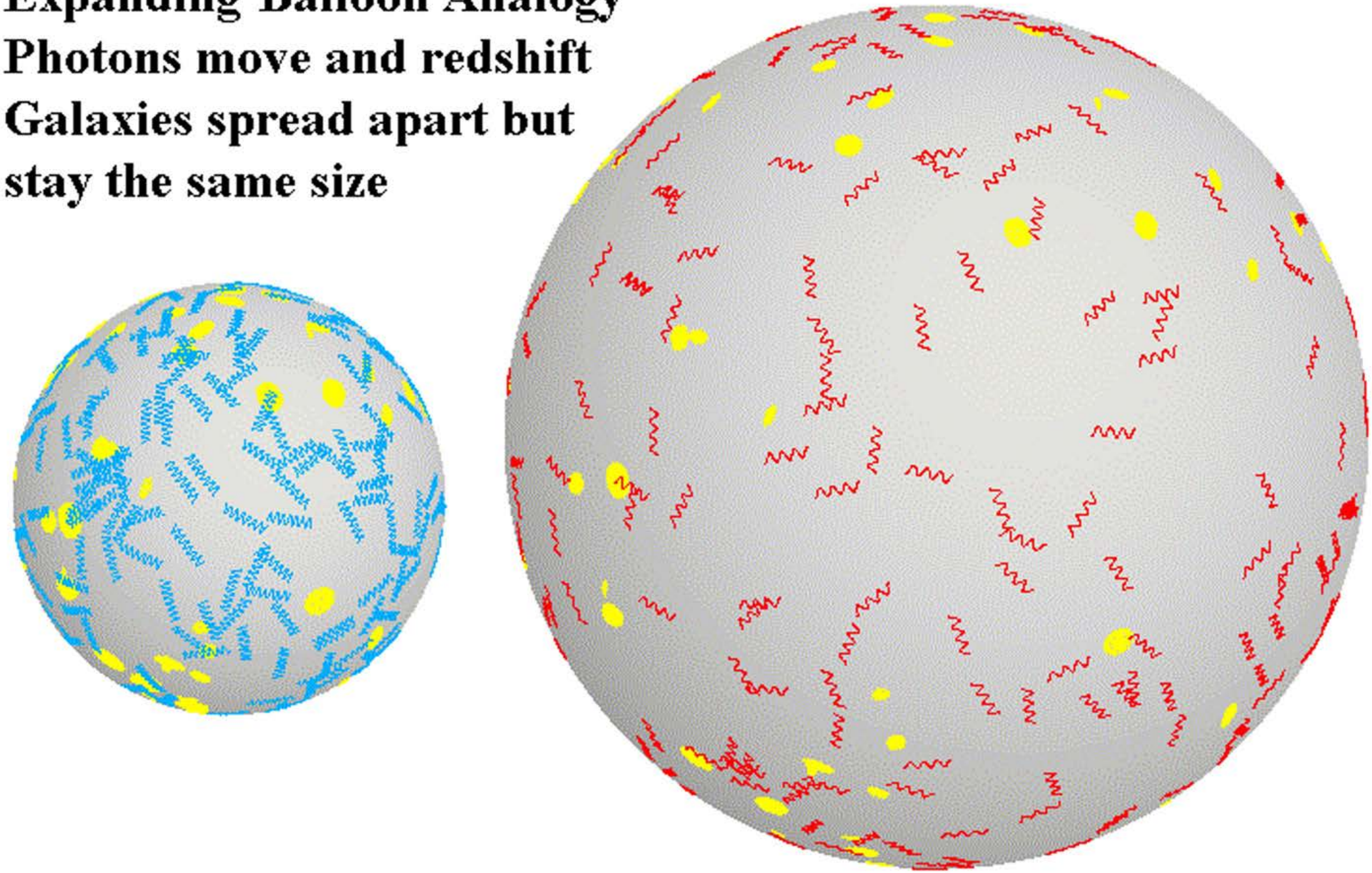


In either case, there is no “edge”, and there is no center

(homogeneity and isotropy)

The Cosmological Redshift

Expanding Balloon Analogy
Photons move and redshift
Galaxies spread apart but
stay the same size



Redshift as Doppler Shift

We define **doppler redshift** to be the shift in spectral lines due to motion:

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+v/c}{1-v/c}} - 1$$

which, in the case of **v<<c** reduces to the familiar

$$z = \frac{v}{c}$$

The **cosmological redshift** is something different, although we are often sloppy and refer to it in the same terms of the doppler redshift. The cosmological redshift is actually due to the **expansion of space** itself.

Cosmological Redshift

A more correct approach is to note that the wavelengths of photons expand with the universe:

$$\frac{R(t_0)}{R(t_e)} = \frac{\lambda_0}{\lambda_e}$$

Where *R(t)* is a separation between any two comoving points

Or, by our definition of redshift:

$$z = \frac{\Delta\lambda}{\lambda}$$

We get:

$$\boxed{\frac{R(t_0)}{R(t_e)} = (1+z)}$$

Thus, by measuring redshifts, we measure directly how much has the universe expanded since then

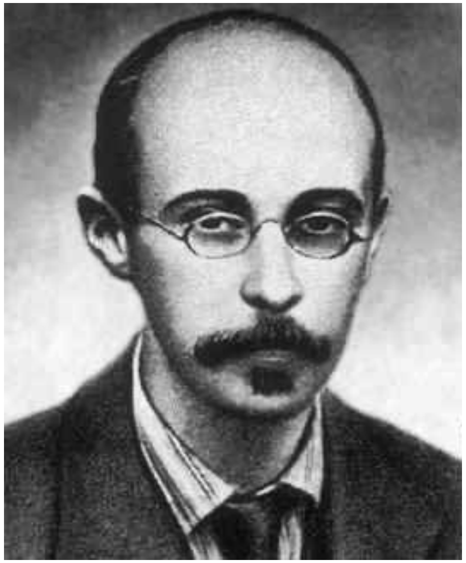
The two approaches are actually equivalent

Is Energy Conserved in an Expanding (or Contracting) Universe?

No!

- Consider energies of photons
- Consider potential energies of unbound systems

The Friedmann and Lemaitre Models



⇦ **Alexander Friedmann**

In 1922 developed the GR-based, expanding universe model. It was not taken very seriously at the time, since the expansion of the universe has not yet been established.

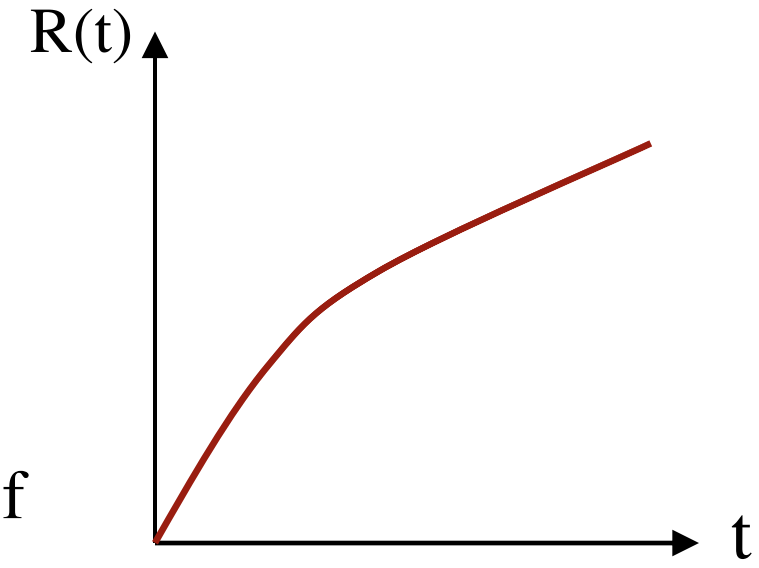
Georges Lemaitre ⇨

In 1927 independently developed cosmological models like Friedmann's. In 1933, he "ran the film backwards" to a hot, dense, early state of the universe he called "the cosmic egg". This early prediction of the Big Bang was largely ignored.

They used the homogeneity and isotropy to reduce the full set of 16 Einstein equations of GR to one: the Friedmann-Lemaitre eqn.

Kinematics of the Universe

We introduce a **scale factor**, commonly denoted as **R(t)** or **a(t):** a spatial distance between any two unaccelerated frames which move with their comoving coordinates



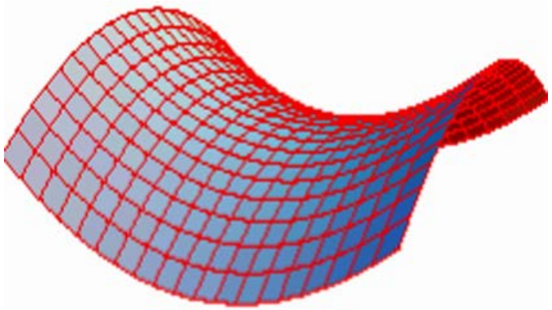
This fully describes the evolution of a homogeneous, isotropic universe

Computing **R(t)** and various derived quantities defines the **cosmological models**. This is accomplished by solving the **Friedmann (or Friedmann-Lemaitre) Equation**

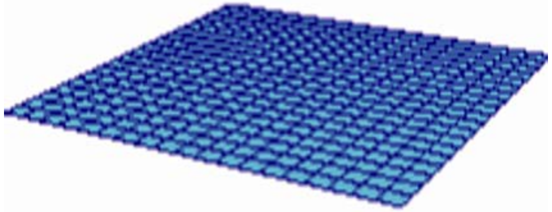
The equation is parametrized (and thus the models defined) by a set of **cosmological parameters**

Geometry and the Fate of the Universe

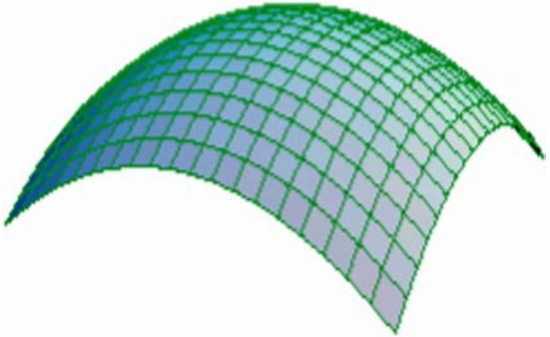
Matter and energy content of the universe determines its geometry (curvature of space), and the ultimate fate



$\rho < \rho_{crit}, k = -1$
negative curvature
expands for ever

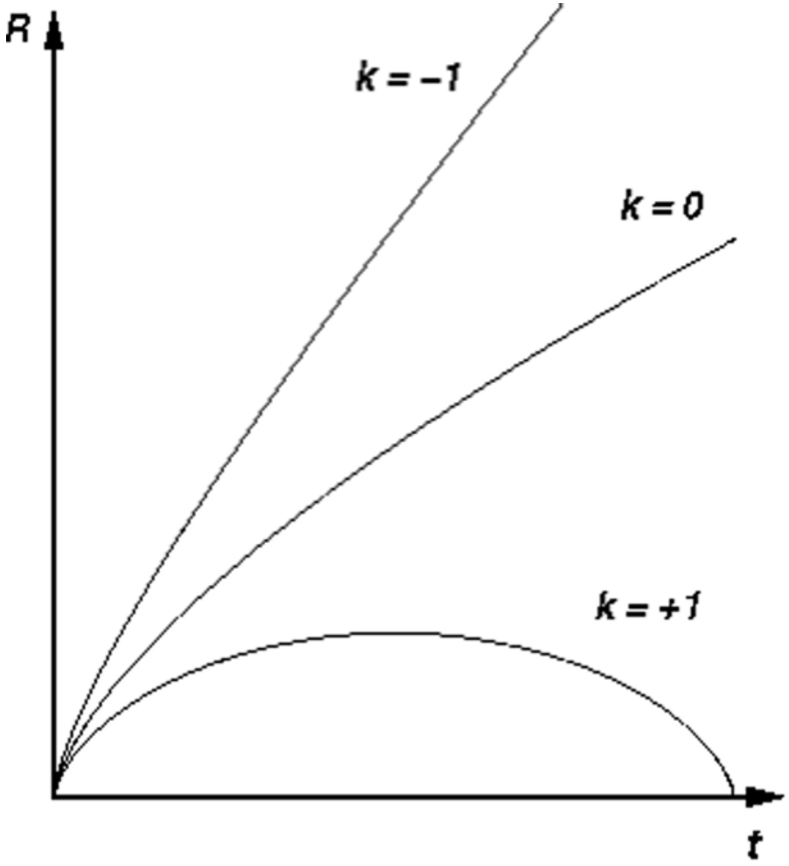


$\rho = \rho_{crit}, k = 0$
flat (Euclidean)
expands for ever



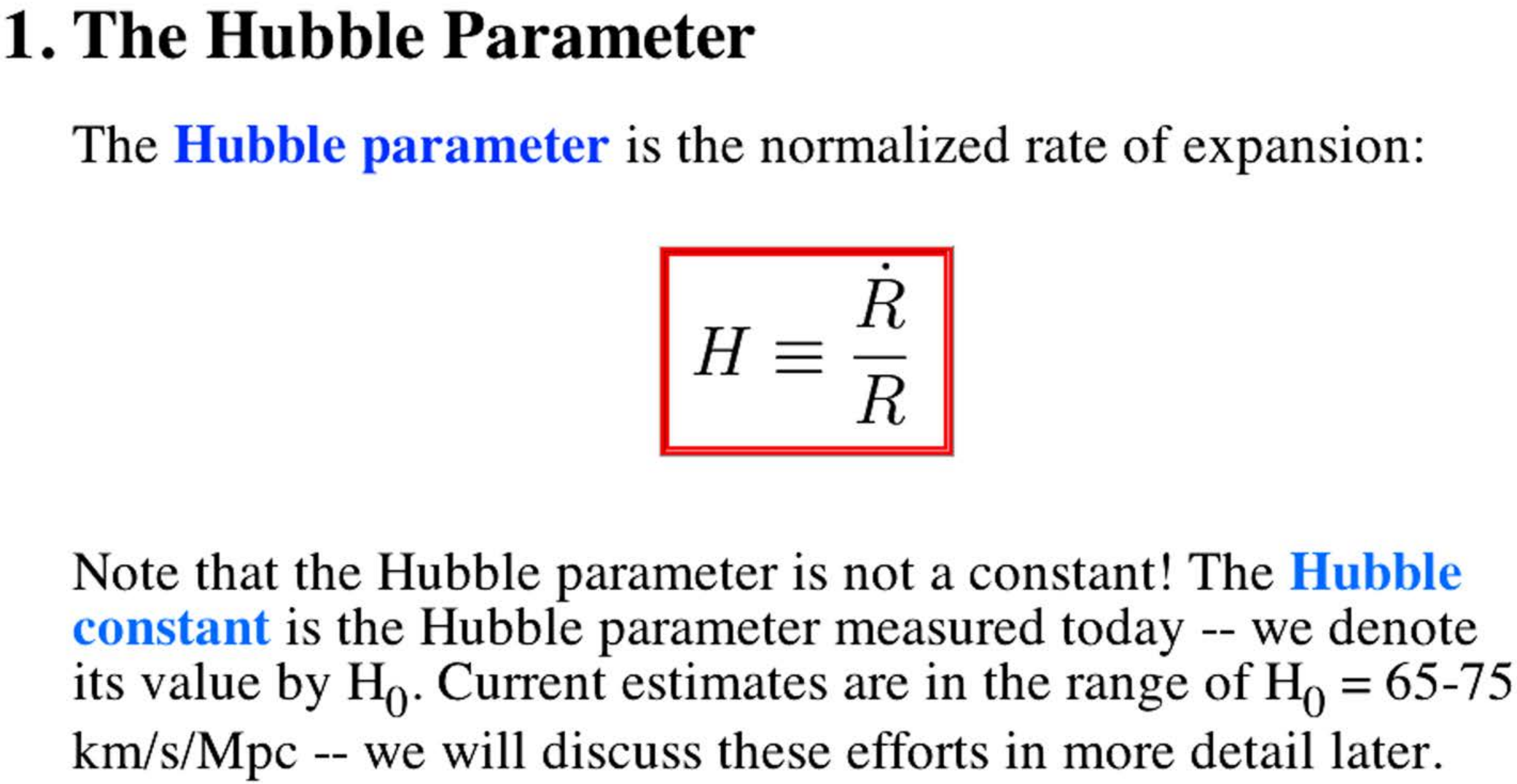
$\rho > \rho_{crit}, k = +1$
positive curvature
collapses

Possible expansion histories:



Cosmological Parameters

Cosmological models are typically defined through several handy key parameters:



1. The Hubble Parameter

The **Hubble parameter** is the normalized rate of expansion:

$$\boxed{H \equiv \frac{\dot{R}}{R}}$$

Note that the Hubble parameter is not a constant! The **Hubble constant** is the Hubble parameter measured today -- we denote its value by H_0. Current estimates are in the range of $H_0 = 65\text{-}75$ km/s/Mpc -- we will discuss these efforts in more detail later.

Cosmological Parameters

2. The Matter Density Parameter.

Rewriting the Friedmann Eqn. using the Hubble parameter, and for now set $\Lambda = 0$:

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R}$$

The Universe is flat if k=0, or if it has a critical density of

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the **matter density parameter** as

$$\boxed{\Omega_M = \frac{\rho}{\rho_{crit}}}$$

Cosmological Parameters

3. The "dark energy" density parameter

We can express a similar density parameter for lambda again by using the Friedmann equation and setting $\rho = 0$. We then get

$$\boxed{\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}}$$

The total density parameter is then

$$\boxed{\Omega = \Omega_M + \Omega_L}$$

4. The deceleration parameter

$$\boxed{q = -R\ddot{R}/\dot{R} = \frac{\Omega_M}{2} - \Omega_L}$$

Cosmological Parameters

A few notes:

The Hubble parameter is usually called the Hubble constant (even though it changes in time!) and it is often written as: **h** = H_0 / (100 km s^{-1} Mpc^{-1}), or $\mathbf{h_{70}}$ = H_0 / (70 km s^{-1} Mpc^{-1})

The current physical value of the critical density is

$$\rho_{0,crit} = 0.921 \times 10^{-29}\, h_{70}^2 \;\; \mathrm{g\ cm^{-3}}$$

The density parameter(s) can be written as:

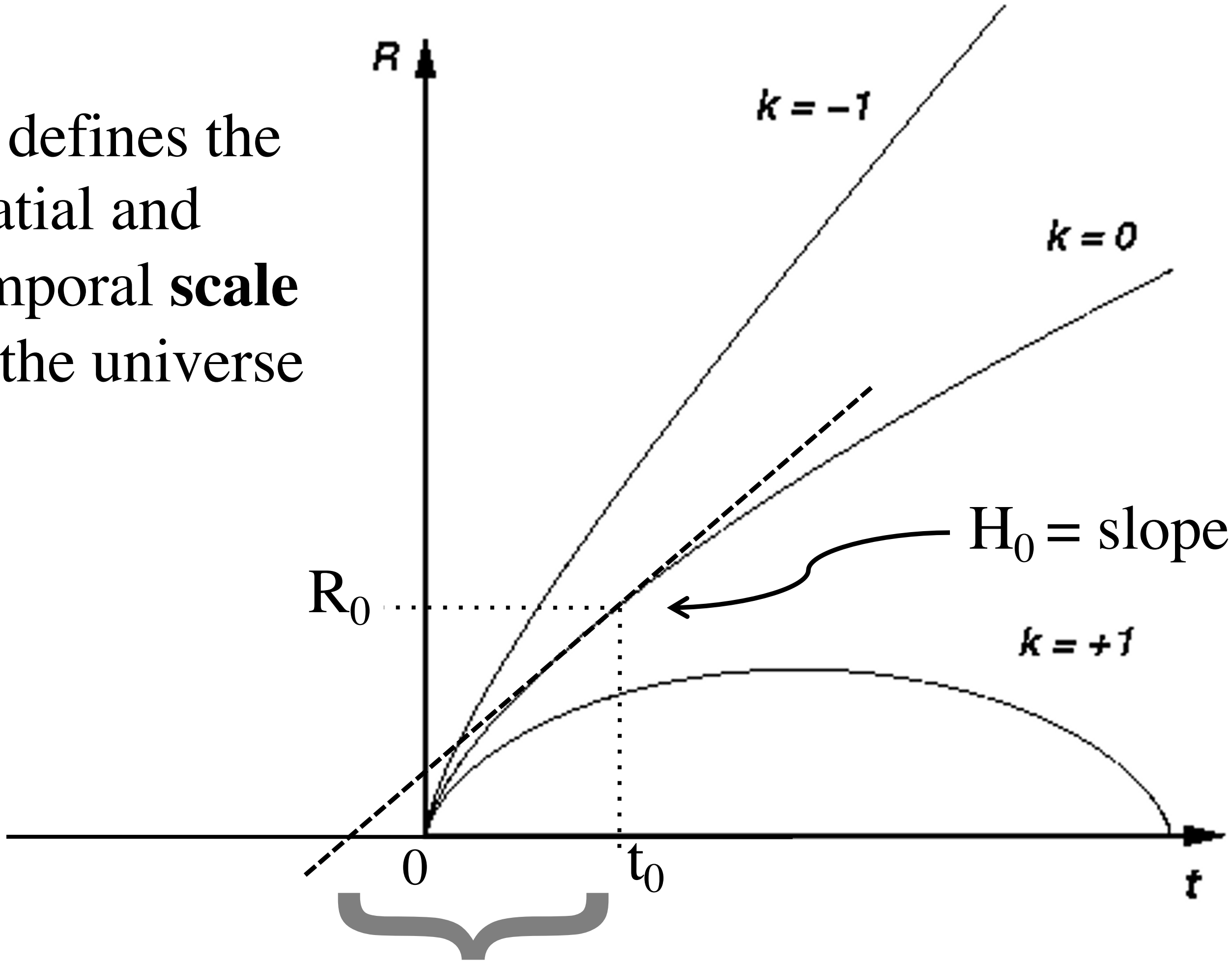
$$\Omega_m + \Omega_k + \Omega_\Lambda = 1$$

where Ω_k is a fictitious "curvature density"

Cosmological Parameters

H_0 defines the spatial and temporal **scale** of the universe

The other parameters (Ω_x) determine the **shape** of the R(t) curves



H_0 = slope at t_0

$1 / H_0$ = Hubble time

c / H_0 = Hubble length

Evolution of the Density

Densities of various matter/energy components change with the stretching of the volume (~ R^3) according to their equation of state (EOS):

$$\rho \sim R^{-3(w+1)}$$

where w is the EOS parameter (need not be constant):

- **Matter** dominated ($w = 0$): $\rho \sim R^{-3}$
- **Radiation** dominated ($w = 1/3$): $\rho \sim R^{-4}$
- **Cosmological constant** ($w = -1$): $\rho = constant$
- Dark energy with $w < -1$ e.g., $w = -2$: $\rho \sim R^{+3}$
  - Energy density *increases* as is stretched out!
  - Eventually would dominate over even the energies holding atoms together! (“Big Rip”)

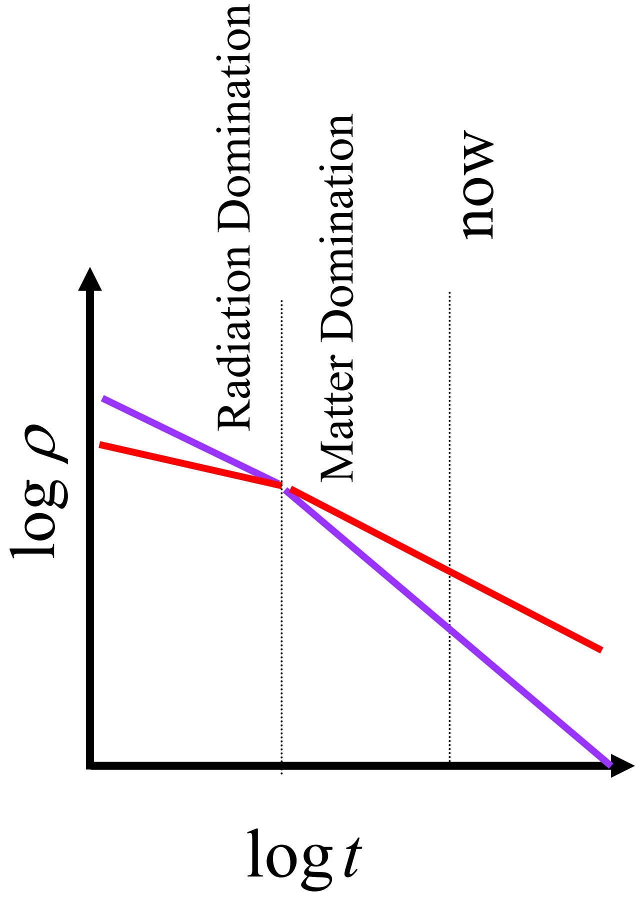
In a mixed universe, different components will dominate the global dynamics at different times

Models With Both Matter & Radiation

Harder to solve for $\rho\,(t)$

However, to a good approximation, we can assume that $k = 0$ and either radiation or matter dominate

	γ-dom	m-dom
$a(t)$	$\propto t^{1/2}$	$\propto t^{2/3}$
$\rho_{\rm m} \propto a^{-3}$	$\propto t^{-3/2}$	$\propto t^{-2}$
$\rho_\gamma \propto a^{-4}$	$\propto t^{-2}$	$\propto t^{-8/3}$



Generally, $$\frac{8\pi G\rho}{3} = H_0{}^2\left(\Omega_{\Lambda,0} + \Omega_{{\rm m},0}a^{-3} + \Omega_{\gamma,0}a^{-4}\right)$$

What is Dominant When?

Matter dominated ($w = 0$): $\rho \sim R^{-3}$

Radiation dominated ($w = 1/3$): $\rho \sim R^{-4}$

Dark energy ($w \sim -1$): $\rho \sim constant$

- Radiation density decreases the fastest with time
  - Must increase fastest on going back in time
  - Radiation must dominate early in the Universe
- Dark energy with $w \sim -1$ dominates last; it is the dominant component now, and in the (infinite?) future

Radiation domination → Matter domination → Dark energy domination

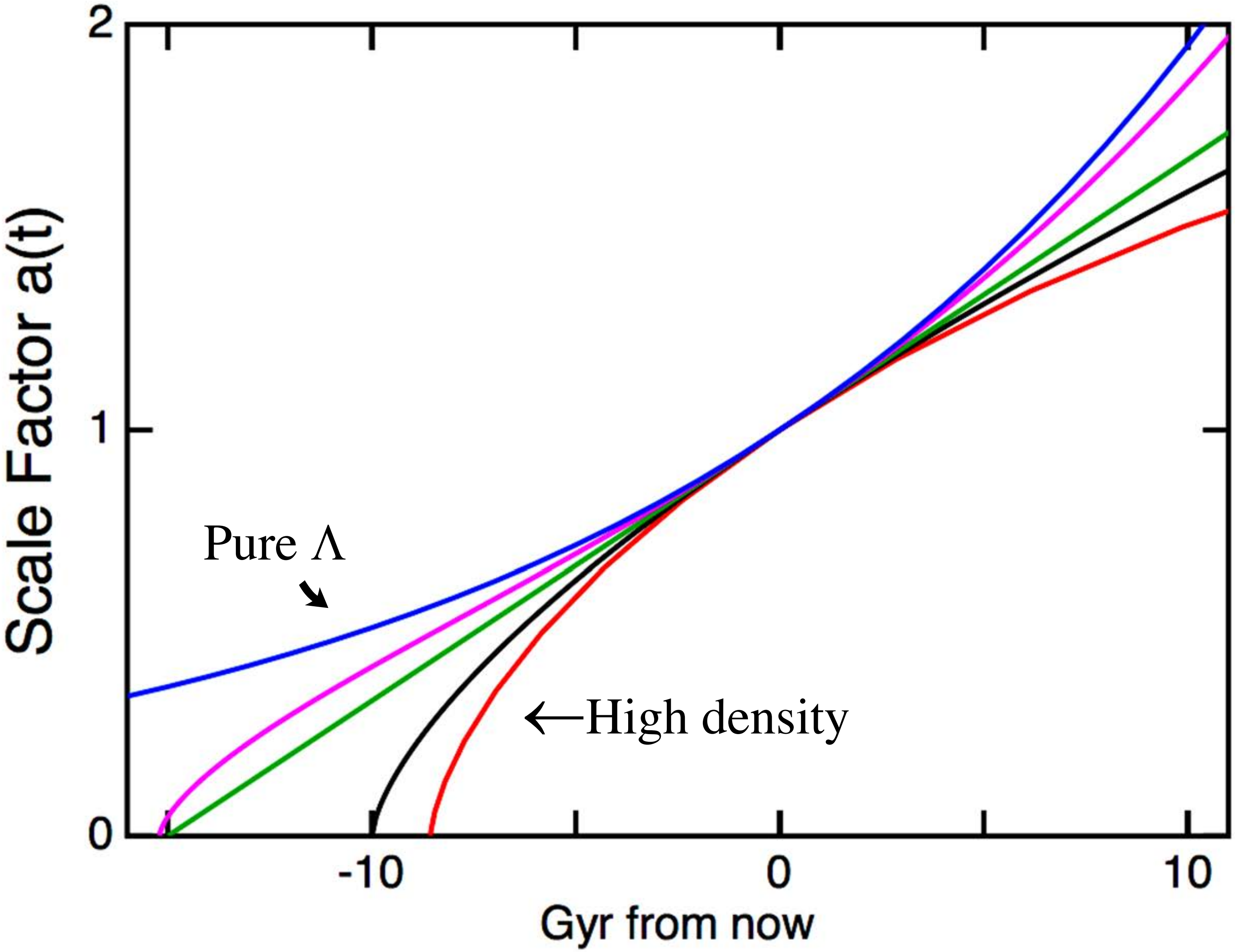


Fig. 10.— Scale factor *vs.* time for 5 different models: from top to bottom having $(\Omega_{m\circ}, \Omega_{v\circ}) = (0, 1)$ in blue, (0.25,0.75) in magenta, (0,0) in green, (1,0) in black and (2,0) in red. All have $H_\circ = 65$.

Distances in Cosmology

A convenient unit is the **Hubble distance**,

$$D_H = c / H_0 = 4.283\ h_{70}^{-1}\ \mathrm{Gpc} = 1.322 \times 10^{28}\ h_{70}^{-1}\ \mathrm{cm}$$

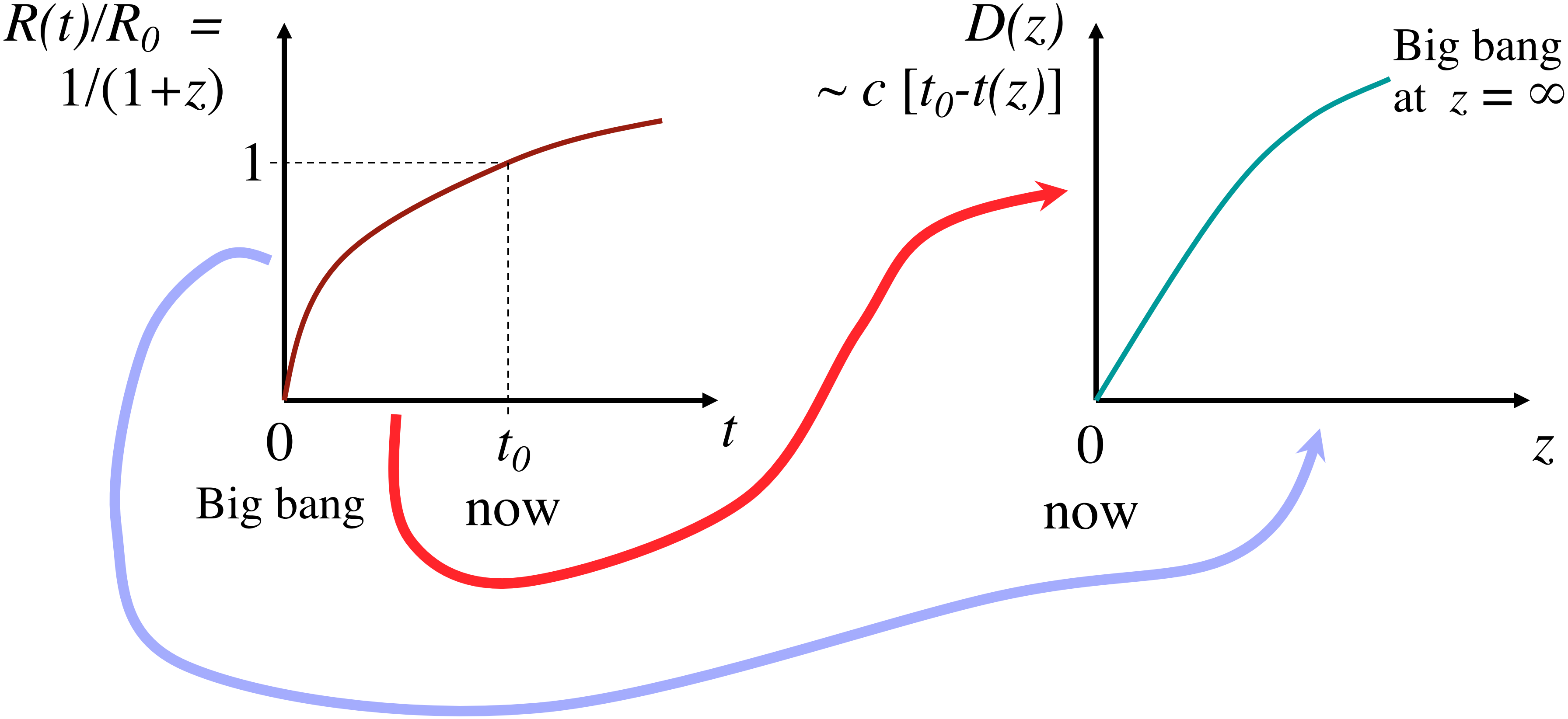
and the corresponding **Hubble time**,

$$t_H = 1 / H_0 = 13.98\ h_{70}^{-1}\ \mathrm{Gyr} = 4.409 \times 10^{17}\ h_{70}^{-1}\ \mathrm{s}$$

Note: these are only ***comparable*** to the actual size and age of the observable universe; the exact values depend on the ***cosmological models***.

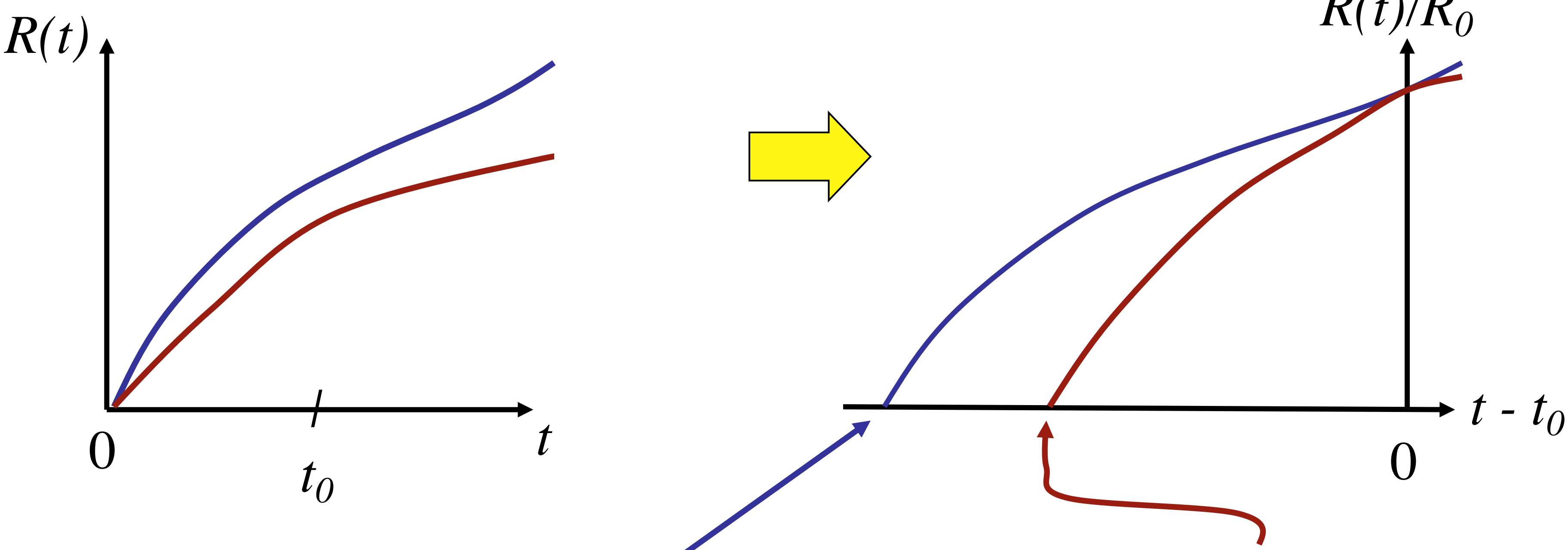
At low redshifts, distance to an object at the redshift z is: $D \approx z\, D_{H,}$ but ***the exact distance depends on the cosmological models***, and that forms the basis of the cosmological tests.

The Basis of Cosmological Tests



All cosmological tests essentially consist of comparing some measure of (relative) distance (or look-back time) to redshift. Absolute distance scaling is given by the H_0.

Cosmological Tests: Expected Generic Behavior of Various Models



Models with a lower density and/or positive Λ expand faster, are thus larger, older today, have more volume and thus higher source counts, at a given *z* sources are further away and thus appear fainter and smaller

Models with a higher density and lower Λ behave exactly the opposite

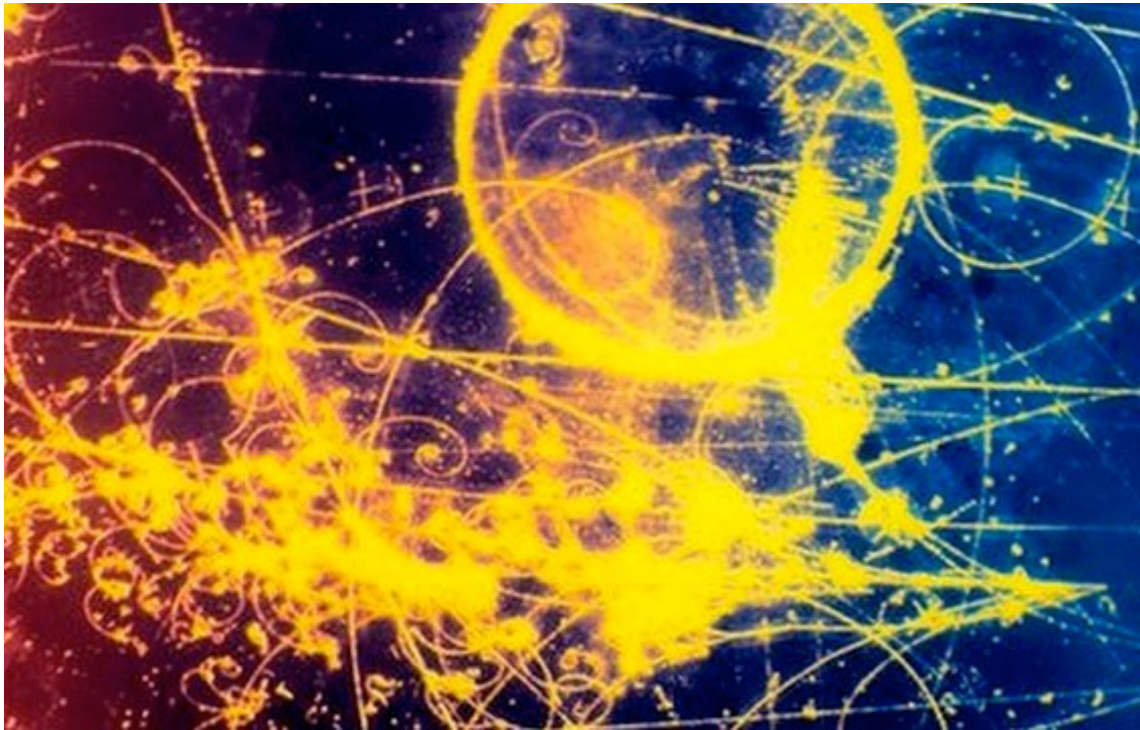
The Early Universe: Key Ideas

- Pushing backward in time towards the Big Bang, the universe was hotter and denser in a fairly predictable manner (aside from he surprising “glitches” such as the inflation…)
- At any given time, the radiation temperature translates into characteristic masses of particles, with the particle-antiparticle pairs created, which then annihilate and dominate that epoch: the universe as the ultimate accelerator?

$$mc^2 \sim kT$$

The Early Universe: Key Ideas

- As the energies increase, **different physical regimes and different fundamental interactions come into play**
- **The closer we get to the Big Bang** (i.e., further away from the experimentally probed regime), **the less certain the physics**: the early universe as the laboratory of physics beyond the standard model?
- **Our extrapolations must break down** by the epoch of ~10^{-43} sec ~ Planck time, where **quantum gravity** must be important



Some Key Moments in the Thermal History of the Universe:

- **Planck era, t ~ 10 $^{-43}$ sec**: quantum gravity, … ??? …
- **Inflation, t ~ 10 $^{-33}$ sec**: vacuum phase transition, exponential expansion
- **Grand Unification, t ~ 10 $^{-32}$ sec**: strong and electroweak interactions split
- **Baryogenesis , t ~ 10 $^{-6}$ sec**: quark-hadron transition
- **Nucleosynthesis , t ~ 1 ms to 3 min**: D, He, Li, Be form
- **Radiation to matter dominance transition, t ~ 10 ^{5} yr**: structure begins to form
- **Recombination, t ~ 380,000 yr**: hydrogen becomes neutral, CMBR released, dark ages begin
- **Reionization, t ~ 0.3 - 1 Gyr**: first galaxies and QSOs reionize the universe, the cosmic renaissance

Empirical Evidence

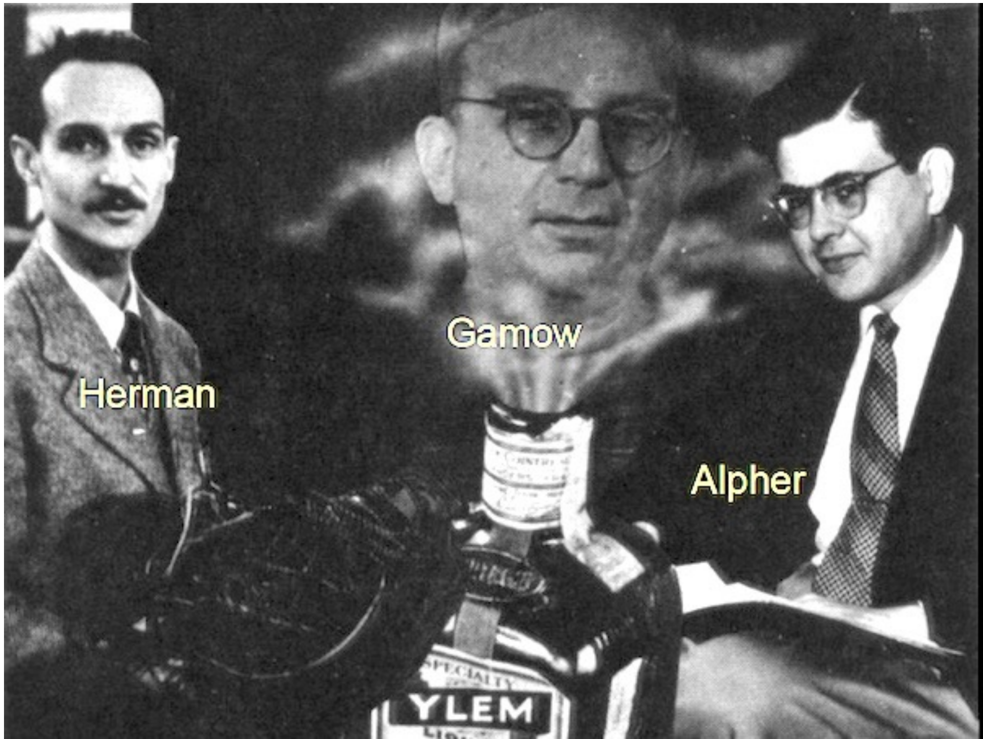
- **The CMBR:** probes the recombination era, $t \sim 10^5$ yr, $z \sim 1100$, based on a well understood atomic and macroscopic physics
- **Nucleosynthesis:** probes the $t \sim 10^{-3}$ - 10^2 sec era, $z \sim 10^9$, compare the model predictions with observed abundances of the lightest elements, based on a well understood nuclear physics
- **Matter-antimatter asymmetry:** probes the baryogenesis era, $t \sim 10^{-6}$ sec, $z \sim 10^{12}$, but only in suggesting that some symmetry breaking did occur
- **Predictions of the inflationary scenario:** flatness, uniformity of CMBR, absence of monopoles, the right type of density fluctuation spectrum - it all supports the idea that inflation did happen, but does not say a lot about its detailed physics
- Cosmological observations can indicate or constrain physics well outside the reach of laboratory experiments

CMB and the Recombination Era

Prediction of the Cosmic Microwave Background (CMB) is trivial in Hot Big Bang model:

- Hot, ionised initial state should produce thermal radiation
- Photons decouple when universe recombines (last scattering)
- Expansion by factor *R* cools a blackbody spectrum $T \Rightarrow T/R$

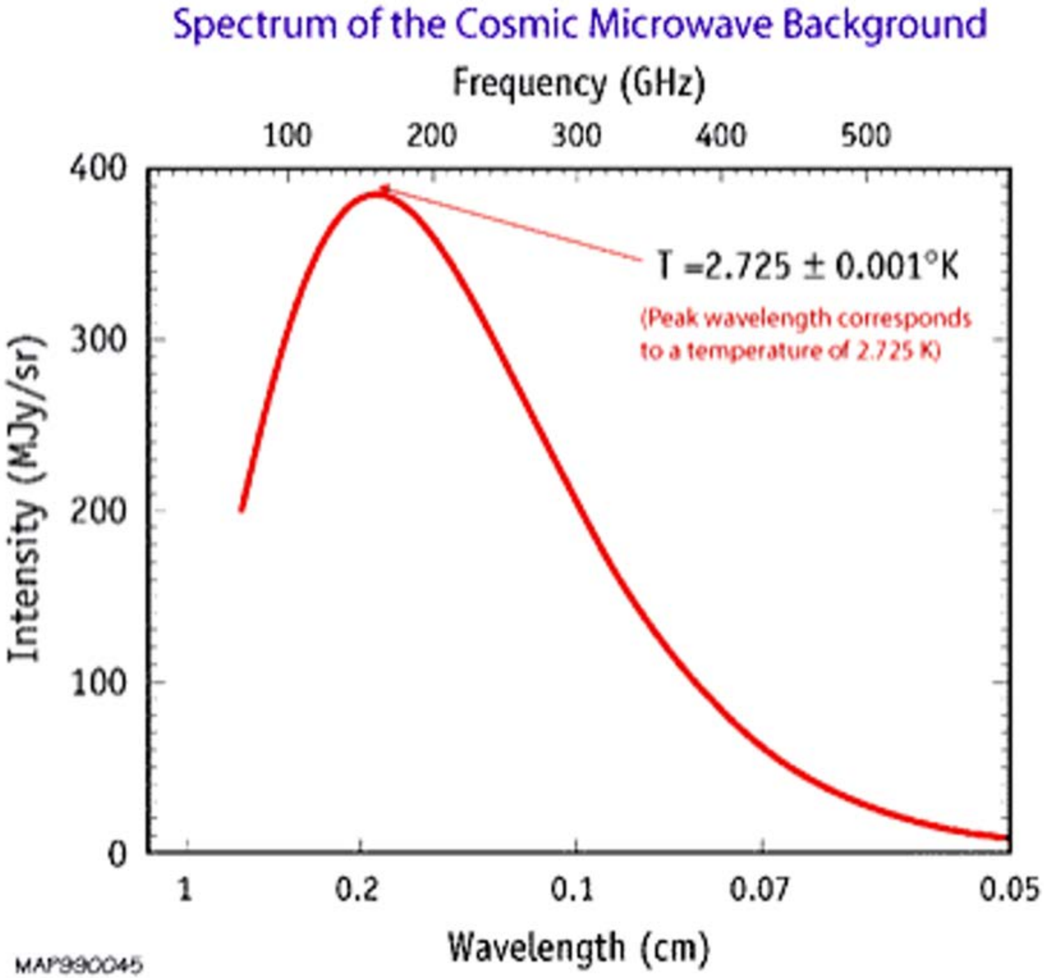
- Early prediction by Gamow, Alpher, and Herman in 1949, estimated T ~ 5 K
- It took the actual detection of the CMB to convince the cosmologists that the Big Bang really happened.



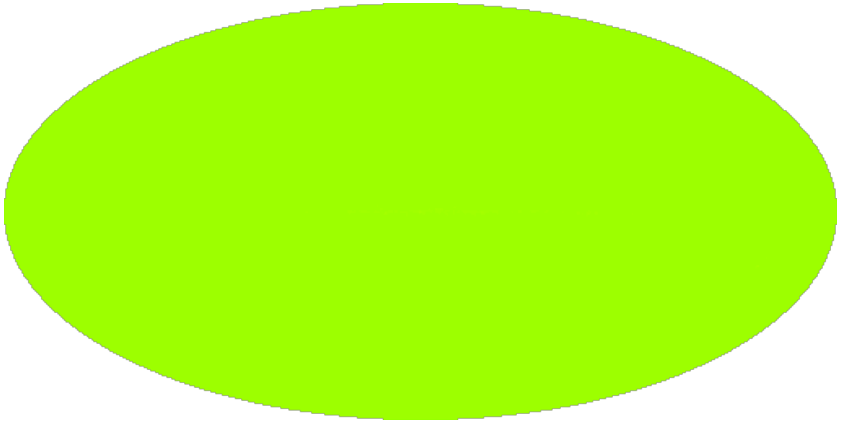
Discovery of the Cosmic Microwave Background (CMB):
A Direct Evidence for the hot Big Bang

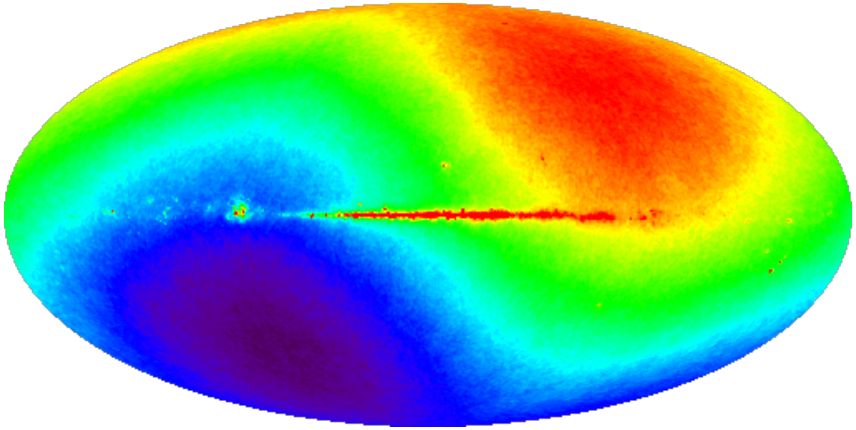
A. Penzias & R. Wilson (1965)

Nobel Prize, 1978



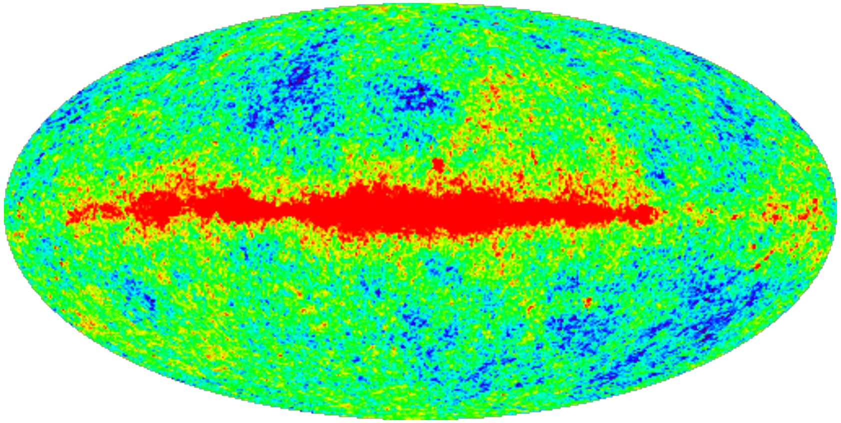
The CMBR sky from WMAP:
Remarkably uniform ➡

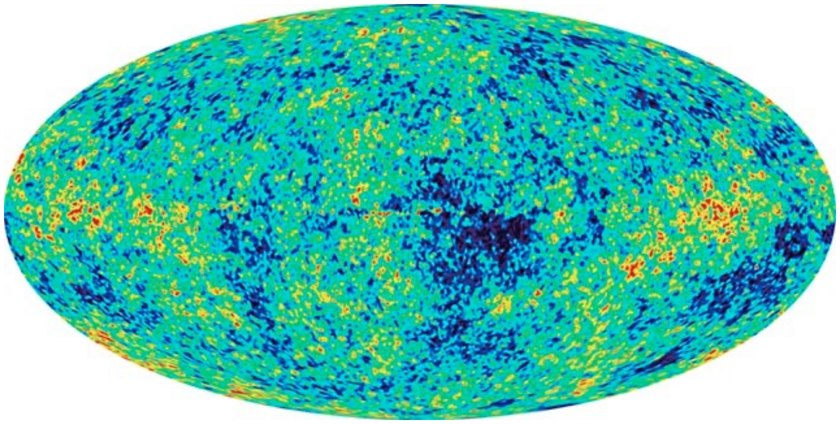




⬅ Enhance the contrast by 10^3
See the **dipole** due to our
peculiar velocity, ~ 600 km/s

Remove the dipole and
enhance the contrast to 10^5
See the Galactic foreground ➡

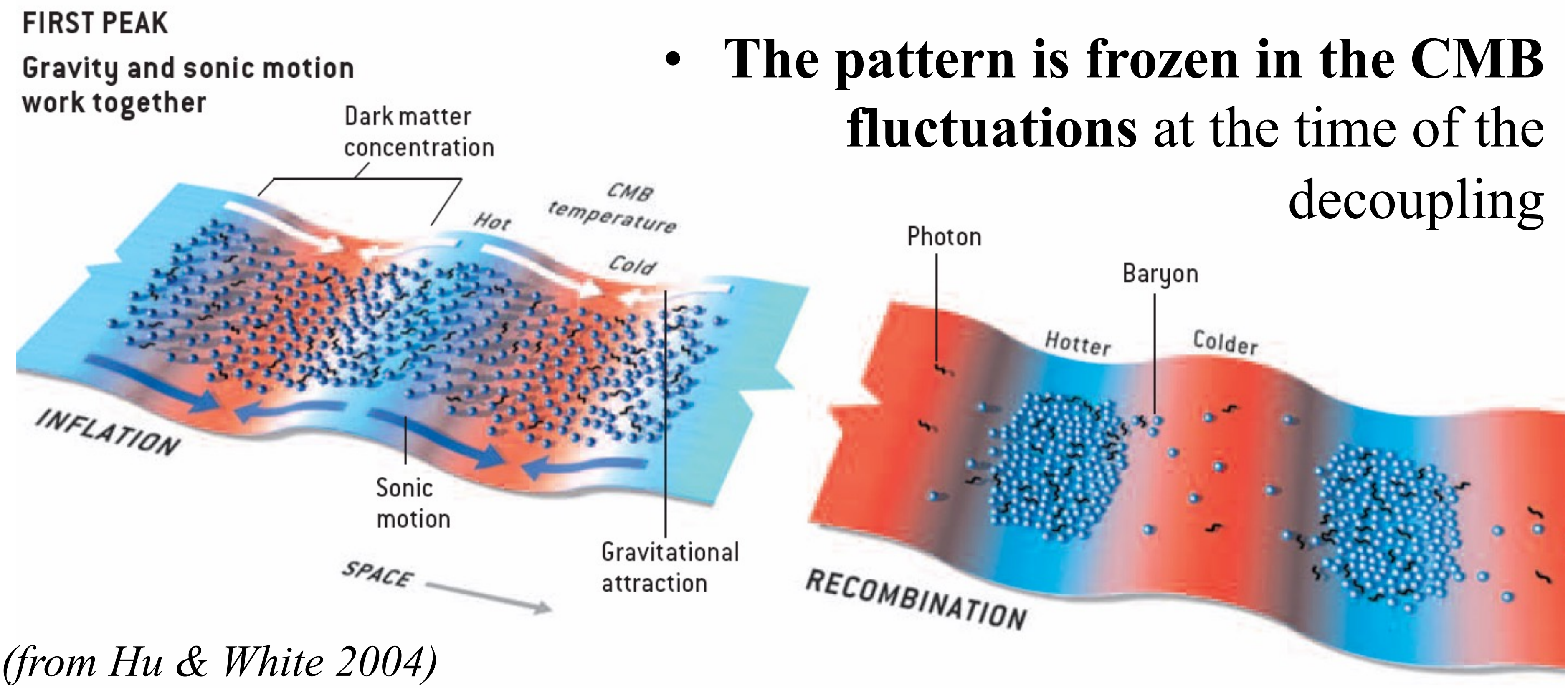




⬅ Remove the Galaxy, increase
the contrast to 10^6 and see the
primordial density fluctuations

The Cosmic Sound

- Large-scale density fluctuations in the early universe attract baryons and photons. Their streaming motion, compression, generate sound waves.
  - The largest wavelength corresponds to **the size of the particle horizon at the time**
  - **The pattern is frozen in the CMB fluctuations** at the time of the decoupling

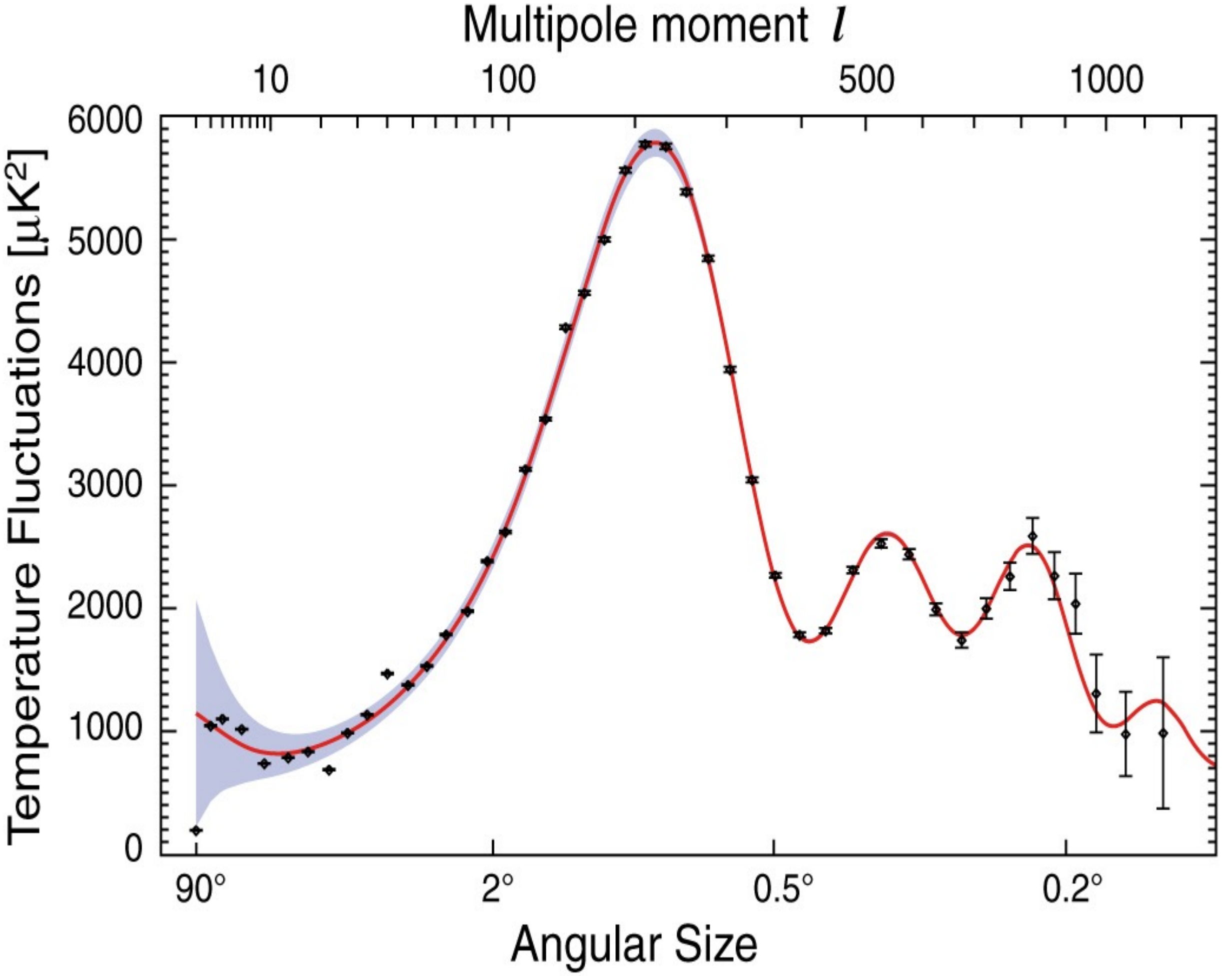


(from Hu & White 2004)

Observed CMB Angular Power Spectrum

We quantify the distribution of the fluctuations with the angular power spectrum. Doppler peaks define a **physical scale** of the particle horizon at recombination.

The corresponding **angular size** depends on the geometry of the universe



WMAP 7 yr data

Into the Nucleosynthesis Era

- In the pre-nucleosynthesis universe, the radiation produces pairs of electrons and positrons, as well as protons and antiprotons, neutrons and antineutrons, and they can annihilate; $e+$ $e-$ reactions produce electron neutrinos (ν_e) and antineutrinos:

$$e^{-} + e^{+} \longleftrightarrow \nu_e + \nu_e$$

$$e^{-} + p \longleftrightarrow n + \nu_e \,,\; \nu_e + p \longleftrightarrow n + e^{+}$$

$$n \longleftrightarrow p + e^{-} + \nu_e$$

$$e^{-} + e^{+} \longleftrightarrow \gamma + \gamma$$

- This occurs until the temperature drops to T~10^{10} K, t ~ 1 sec
- In equilibrium there will slightly more protons than neutrons since the neutron mass is slightly (1.293 MeV) larger
- This leads to an asymmetry between protons and neutrons …

Big Bang Nucleosynthesis (BBNS)

Free neutrons are unstable to beta decay, with *mean lifetime* = 886 sec, $n \rightarrow p + e^- + \nu_e$. This destroys ~ 25% of them, before they can combine with the protons

$$n + p \leftrightarrow {}^2H + \gamma$$

$${}^2H + {}^2H \leftrightarrow {}^3He + n$$

$${}^3He + n \leftrightarrow {}^3H + p$$

$${}^3H + {}^2H \leftrightarrow {}^4He + n$$

When the temperature drops to ~ 10^9 K (t=230s), neutrons and protons combine to form deuterium, and then helium:

Note that these are ***not*** the same reactions as in stars (the *pp* chain)!

Photons break the newly created nuclei, but as the temperature drops, the photodissociation stops

At $t \sim 10^3$ sec and T < 3×10^8 K, the density also becomes too low for fusion, and BBN ends. This is another "freeze-out", as no new nuclei are created and none are destroyed

BBNS Predictions

- The BBNS makes ***detailed predictions*** of the abundances of light elements: ^{2}D, ^{3}He, ^{4}He, ^{7}Li, ^{8}Be
- These are generally given as a function of the baryon to photon ratio $\eta = n_n/n_\gamma$, usually defined in units of 10^{10}, and directly related to the baryon density Ω_b
- As the universe evolves η is preserved, so that what we observe today should reflect the conditions in the early universe
- Comparison with observations (consistent among the different elements) gives:

$$\Omega_{baryons} h^2 = 0.021 \rightarrow 0.025$$

- This is in a spectacularly good agreement with the value from the CMB fluctuations:

$$\Omega_{baryons} h^2 = 0.024 \pm 0.001$$

BBNS Predictions

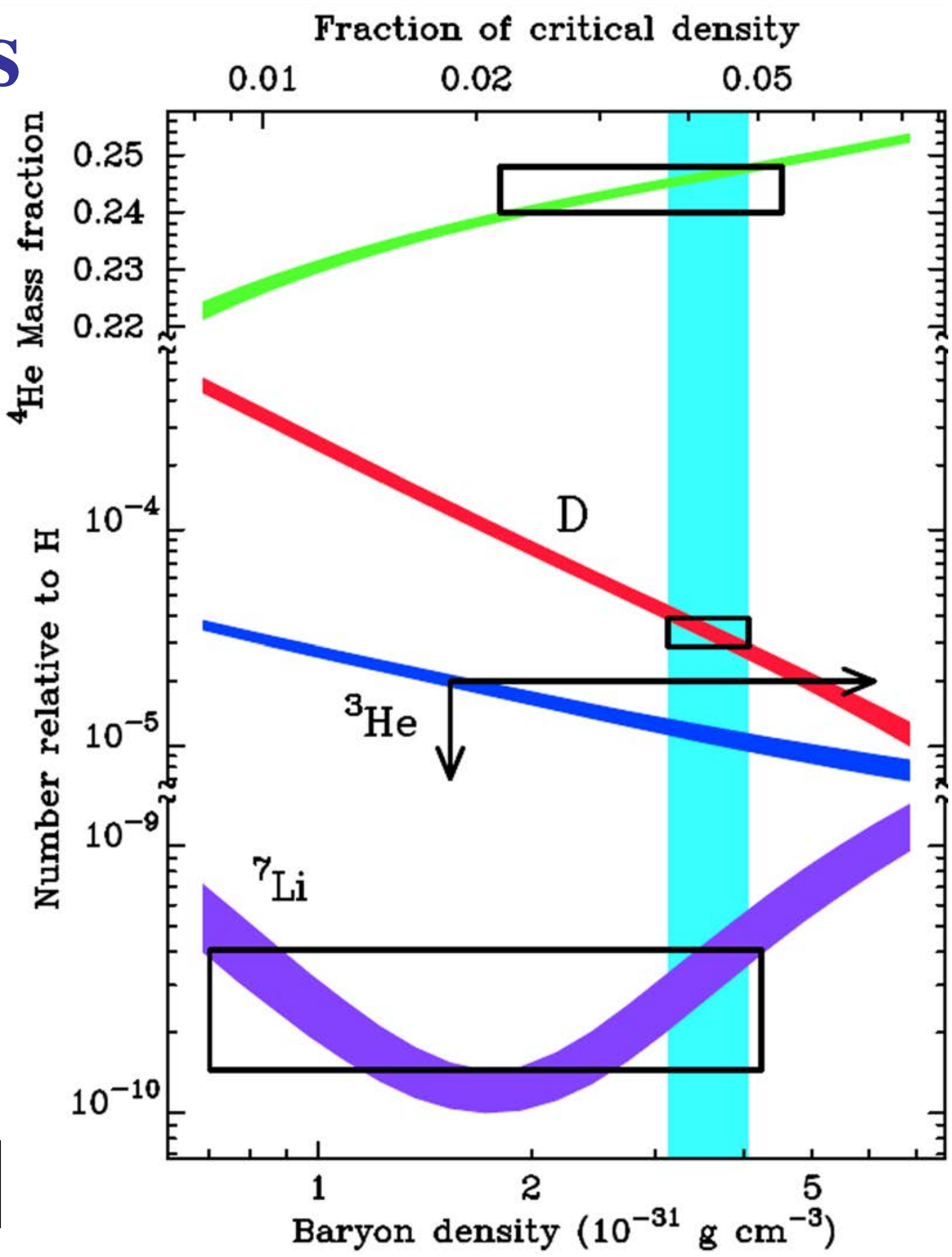
^{4}**He**: the higher the density, the more of it is made →

^{2}**D**, ^{3}**He**: easily burned into ^{4}He, so abundances are lower at higher densities →

Note: the ***Deuterium*** curve is the steepest, so it is ***the best indicator*** of the baryon density

^{7}**Li**: … complicated →

Boxes indicate observed values



The Inflationary Scenario

It solves ***3 key problems*** of the Big Bang cosmology:

1. **The flatness problem**: why is the universe so close to being flat today?
2. **The horizon problem**: how comes the CMBR is so uniform?
3. **The monopole problem**: where are the copious amounts of magnetic monopoles predicted to exist in the BB cosmology?

… It also *accounts naturally for the observed power spectrum of the initial density perturbations*

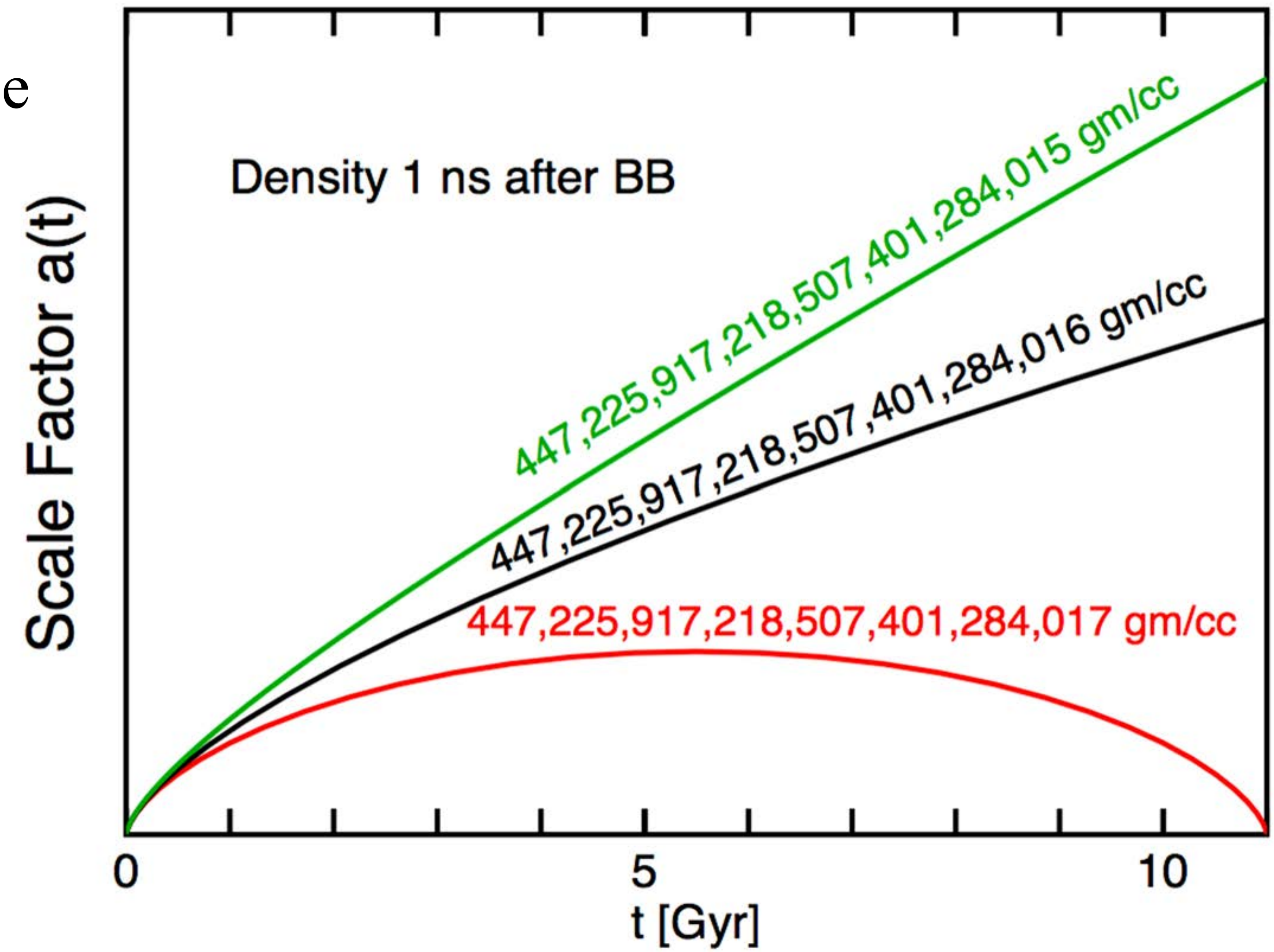
… *It predicts* a similar, scale-invariant spectrum for the cosmic gravitational wave background

… And it implies a much, much(!) bigger universe than the observable one

The Flatness Problem

The expanding universe always *evolves away* from $\Omega_{tot} = 1$

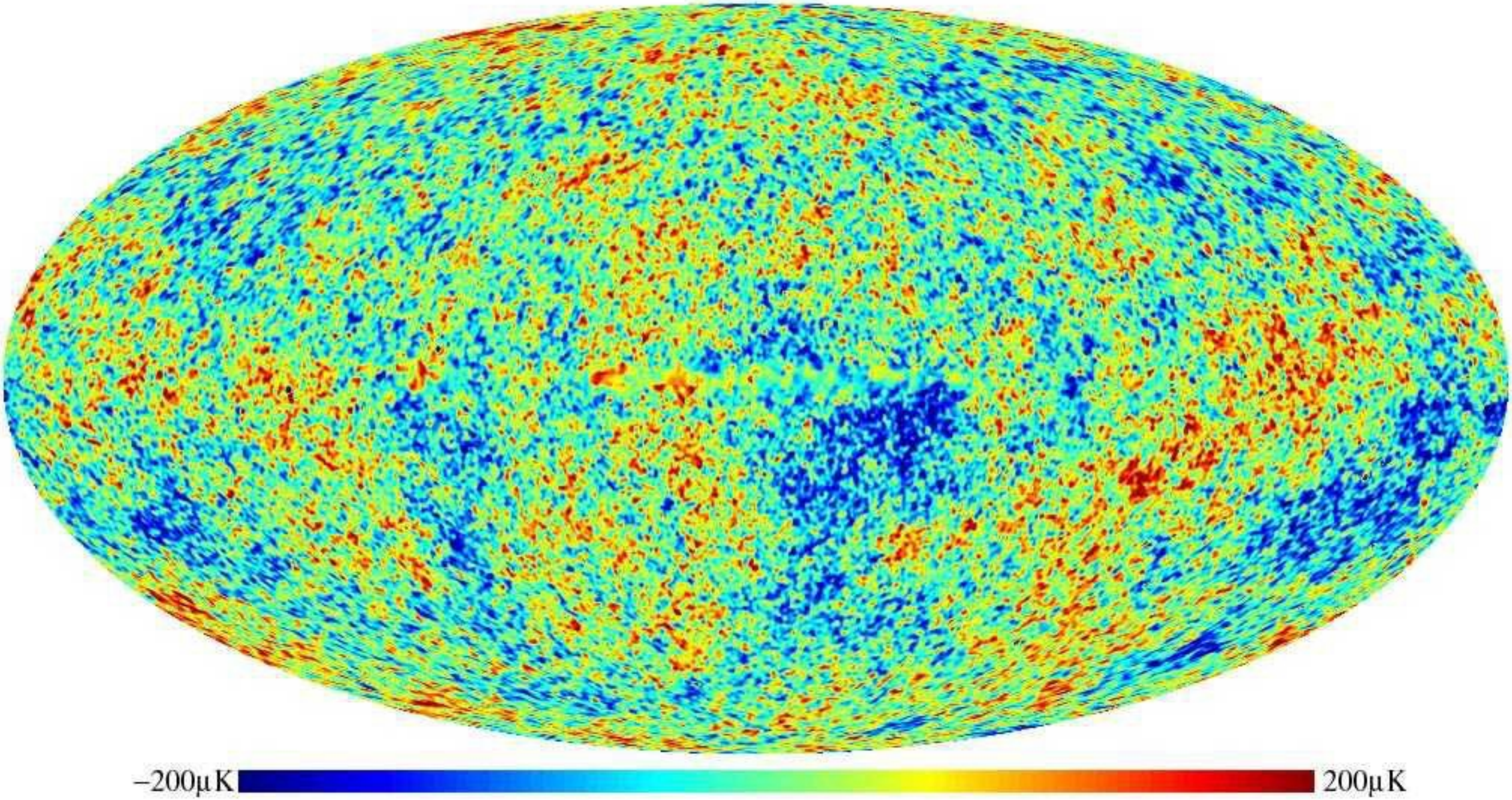
This creates an *enormous fine-tuning problem*: the early universe must have been remarkably close to $\Omega_{tot} = 1$ in order to have $\Omega_{tot} \sim 1$ today !



(from N. Wright)

CMBR is Uniform to $\Delta T/T \sim 10^{-6}$

Yet the projected size of the particle horizon at the decoupling was ~ 2° - these regions were causally disconnected - so how come?



Inflationary Universe Scenario

- At $t \sim 10^{-34}$ sec after the Big Bang, the universe was in a ***"false vacuum":*** the energy level higher than the lowest, ground state
- The false vacuum is a metastable state, and the universe undergoes a ***phase transition*** from a state of a false vacuum, to a ground state; this releases ***enormous amounts of energy*** ("latent heat") which drives an exponential expansion
- This also generates ***all of the energy content of the universe***
- There may be many such inflating "bubbles" of the physical vacuum, each corresponding to a separate universe in a larger multiverse (this is *highly speculative*!)
- This theory is called the ***chaotic inflation***

The Cosmic Inflation

Recall that the energy density of the physical vacuum is described as the ***cosmological constant***. If this is the dominant density term, the Friedmann Eqn. is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3}$$

(a = scaling factor, Λ = cosmological constant)

The solution is obviously: $$a(t) \propto e^{H_i t}$$

In the model where the GUT phase transition drives the inflation, the net expansion factor is:

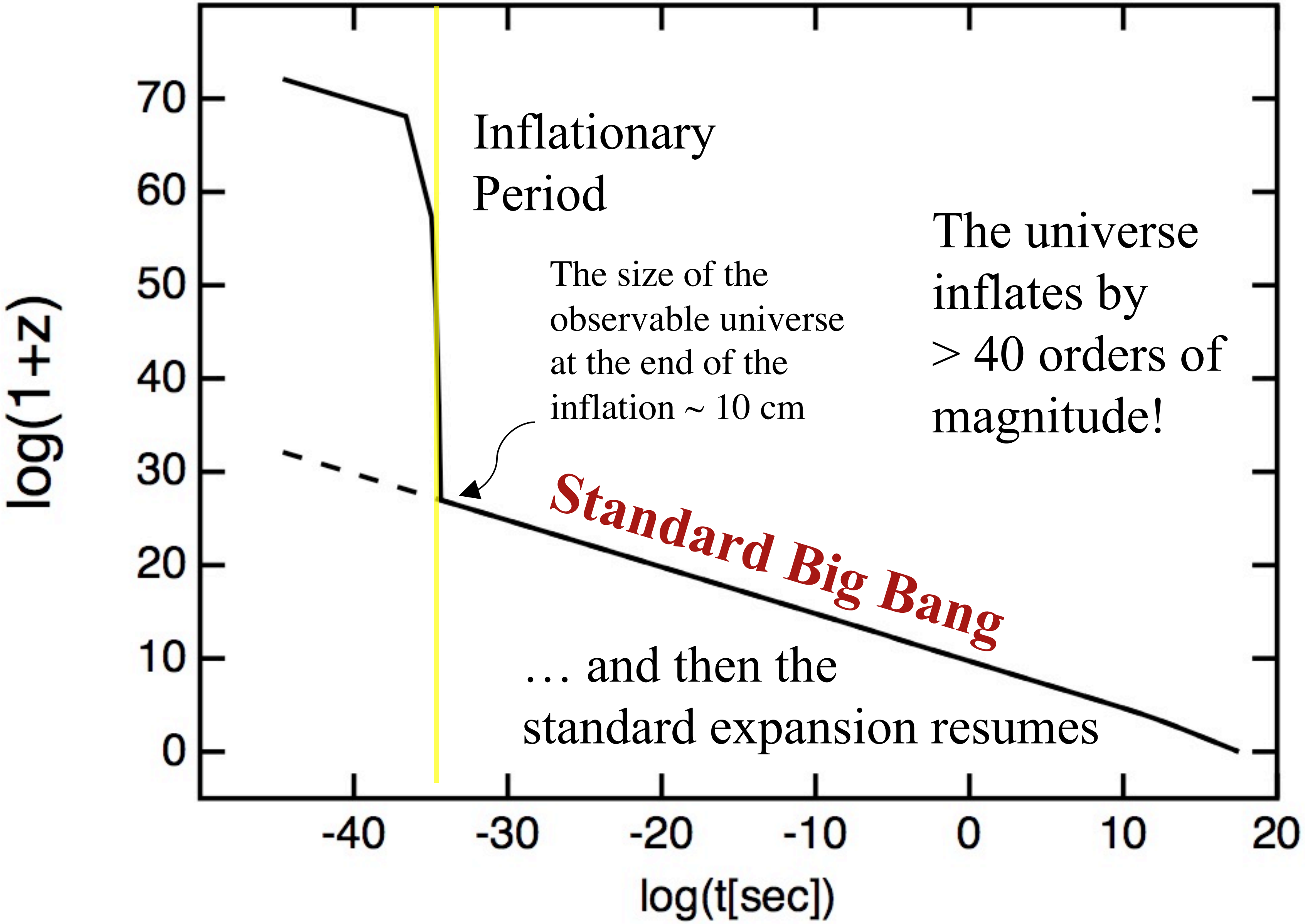
$$\frac{a(t_f)}{a(t_i)} \sim e^{100} \sim 10^{43}$$

The density parameter evolves as: $$|1 - \Omega(t)| \propto e^{-2H_i t}$$

Thus: $$|1 - \Omega(t_f)| \sim e^{-2N} \sim e^{-200} \sim 10^{-87}$$

The universe becomes ***asymptotically flat***

The Inflationary Scenario



Inflationary
Period

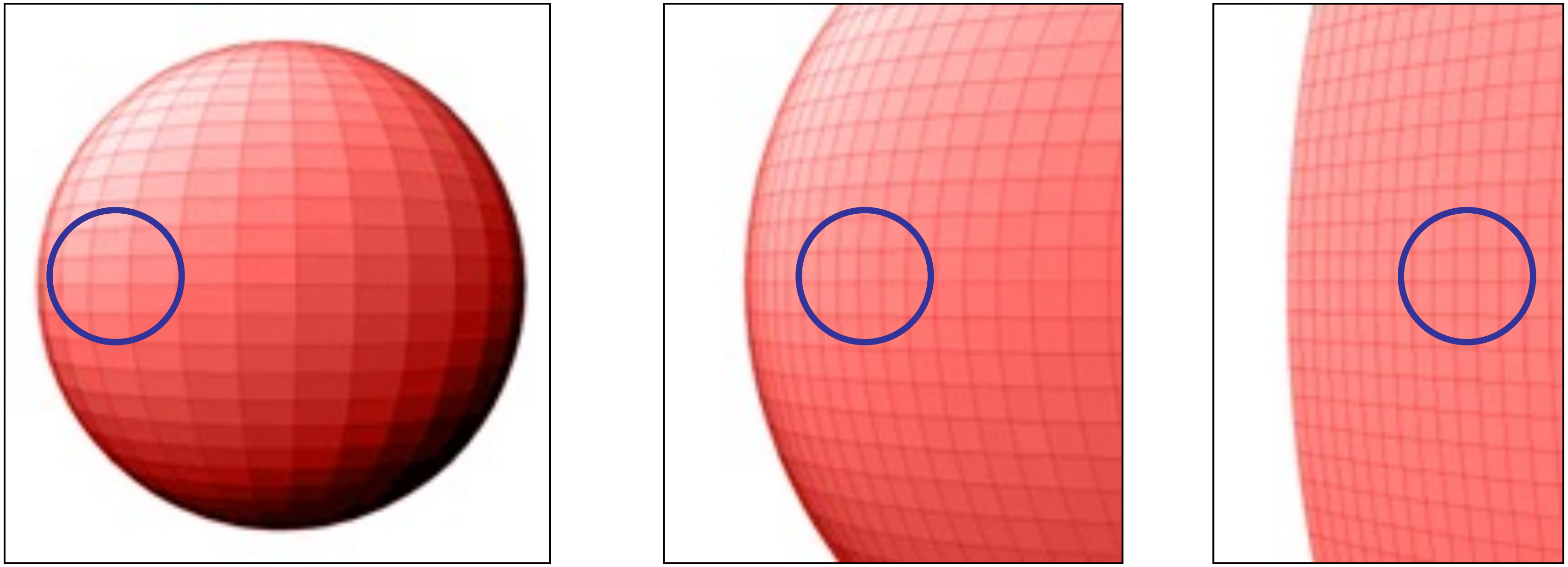
The size of the observable universe at the end of the inflation ~ 10 cm

The universe inflates by > 40 orders of magnitude!

Standard Big Bang

… and then the standard expansion resumes

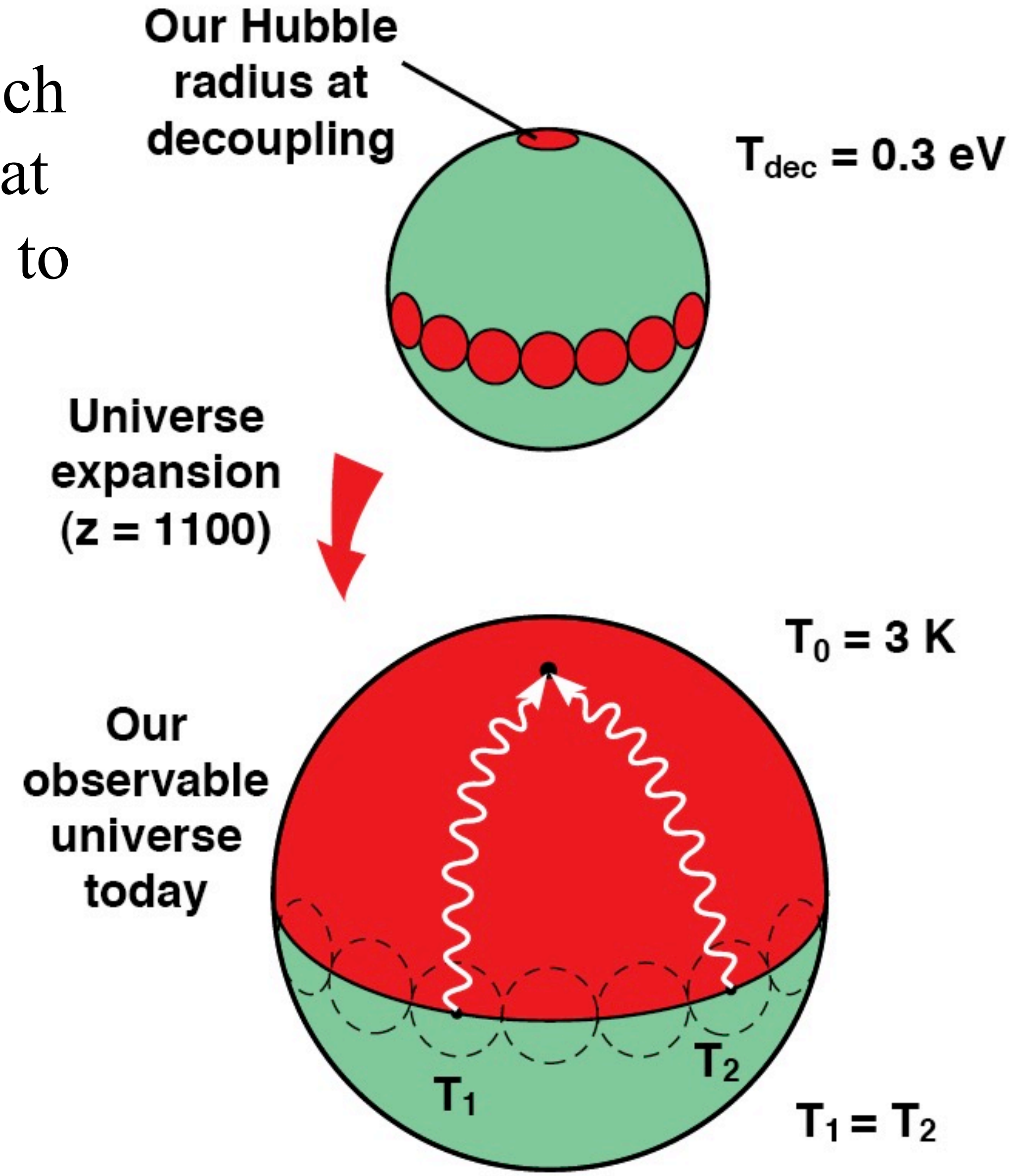
Inflation Solves the Flatness Problem



As the universe inflates, the local curvature effects become negligible in comparison to the vastly increased “global” radius of curvature: the universe becomes extremely close to flat locally (which is the observable region now). Thus, at the end of the inflation, $\Omega = 1 \pm \varepsilon$

Inflation Solves the Horizon Problem

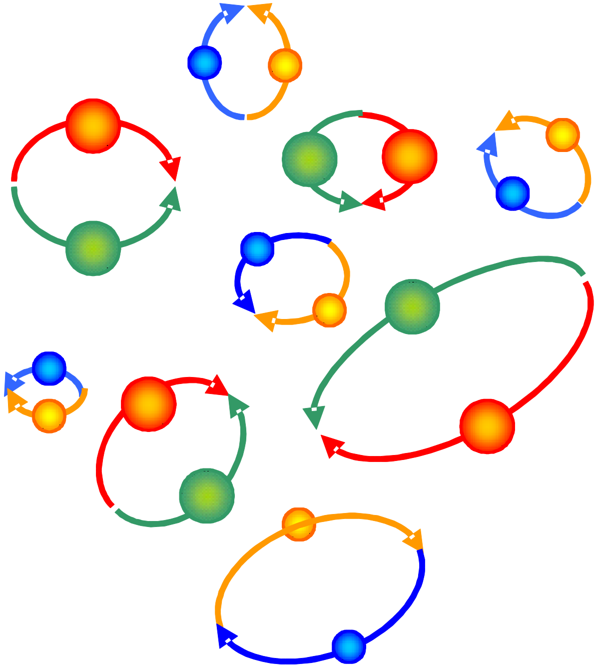
Regions of the universe which were causally disconnected at the end of the inflation used to be connected before the inflation - and thus in a thermal equilibrium



Note that the *inflationary expansion is superluminal*: the space can expand much faster than ***c***

Inflation and Structure Formation

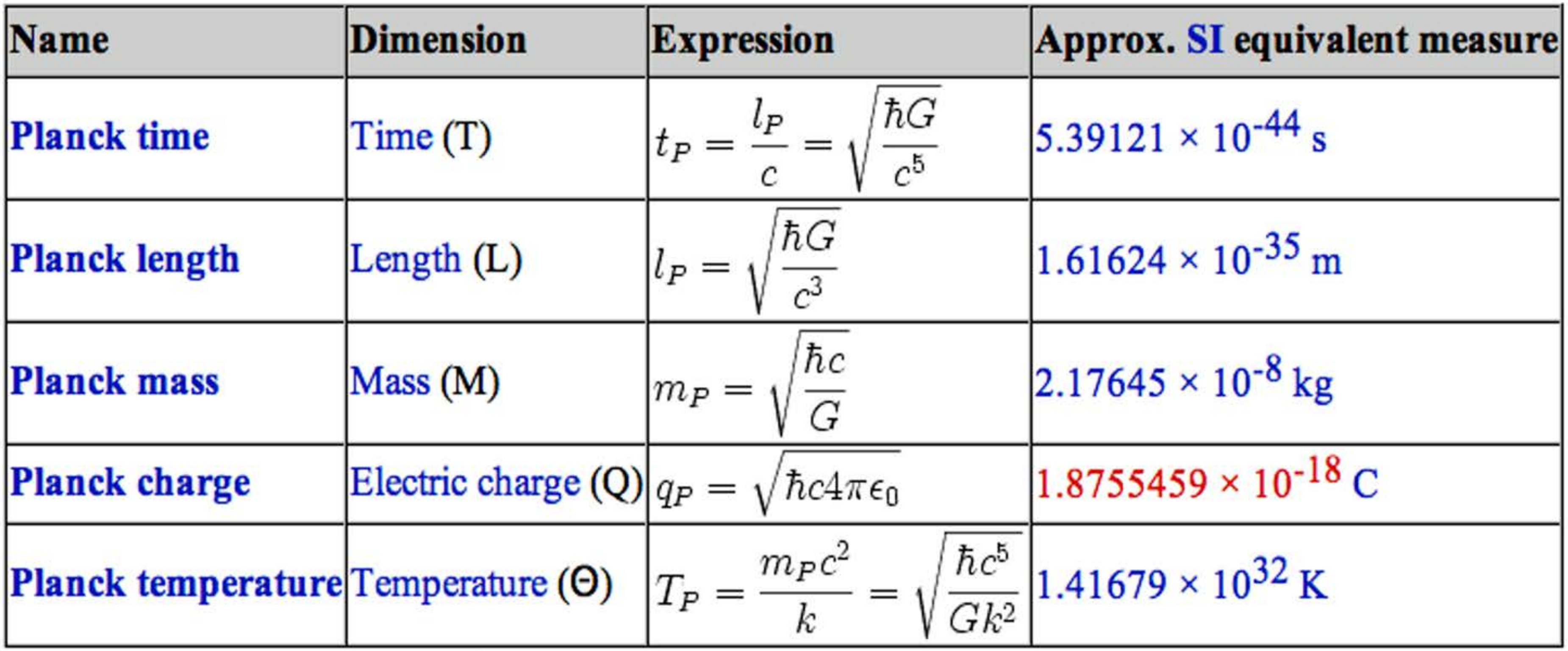
- Uncertainty Principle means that in quantum mechanics vacuum constantly produces temporary particle-antiparticle pairs
  - This creates minute density fluctuations
  - Inflation blows these up to macroscopic size
  - They become the seeds for structure formation



- Expect the mass power spectrum of these density fluctuations to be approximately scale invariant
  - This is indeed as observed!
  - Not a “proof” of inflation, but a welcome consistency test

Planck Units

Proposed in 1899 by M. Planck, as the “natural” system of units based on the physical constants:



Name	Dimension	Expression	Approx. SI equivalent measure
Planck time	Time (T)	$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}}$	5.39121×10^{-44} s
Planck length	Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	1.61624×10^{-35} m
Planck mass	Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	2.17645×10^{-8} kg
Planck charge	Electric charge (Q)	$q_P = \sqrt{\hbar c 4\pi\epsilon_0}$	$1.8755459 \times 10^{-18}$ C
Planck temperature	Temperature (Θ)	$T_P = \frac{m_P c^2}{k} = \sqrt{\frac{\hbar c^5}{G k^2}}$	1.41679×10^{32} K

They may be indicative of the physical parameters and conditions at the era when gravity is unified with other forces … assuming that ***G*, *c***, and ***h*** do not change … and that there are no other equally fundamental constants