DJORGOVSKI: Let's now talk about what was essentially the first scientific thing in astronomy and where does it come from, which is Kepler's laws. The strange contraption on the left here is Kepler's model of nested Platonic solids. Who knows what are Platonic solids? These are volumes that can be composed out of equilateral polygons, and tetrahedron, and cube, and so on.

And so when Kepler figured out, using Tycho Brahe's data, how far the planets are relative to the Sun, he also sorted out that if you can take combinations of these Platonic solids sitting inside of spheres and put spheres inside of them, you can find a combination that kind of approximates the actual relative sizes of orbits of planets in the solar system. Which is totally mystical and complete nonsense, actually, but Kepler was a mystic, and an alchemist, and also a court astrologer. And this was a perfectly normal thing to be then.

Isaac Newton spent most of his life doing alchemy. He did this whole stuff that he's famous for a few years while he was a young guy, and then later on he was just messing with alchemy, and then he was Master of the Mint, and stuff like that.

Anyway, back to Kepler. So Tycho Brahe's data were best at the time, but even so, we're talking about the positions of planets seen on the sky as they move around, and so things are all rotating. Earth's rotating, going around the Sun. Planets are going at the different speeds and so on. So it gets to be pretty messy. And yet from that set of data, Kepler managed to deduce these three laws.

First, that the orbits are elliptical, and the Sun is at the center. So this was like first real proof of the Copernican system. You can say Galileo pretty much demonstrated it, but this was a good thing to add. And it's the ellipsis and not collections of epicycles, circles rolling on other circles, and the Sun sits in a focus of those.

Then something that's truly amazing that he can figure out is that radius vector, from the Sun to the planet, rotates around, and it doesn't rotate with the same angular speed. It rotates faster when planets closer and slower when planets further out, but in such a way that the distance and speed together match, and the area of the little triangle swept by this radius per unit time is the same. This is a completely unnatural thing if you start from nothing, and it's correct. It's just amazing that he figured this one out.

He also found this proportion, that the squares of periods are proportional to the cubes of orbits. So this was empirical data. There was no theoretical understanding. He had the Copernican system as a geometrical framework, if you will, but nobody knew why this is. Kepler did this just like a few years after Galileo's initial discoveries. They're pretty much contemporaries. So it's the early 17th century when science really started.

Now, all these are explained by Newton, who is justly famous. And there he is. Then in 1676, something like that-- let's see, that Greek number is 1687, sorry-- a Roman number, he published the famous book, the *Mathematical Principles of Natural Philosophy,* where he put forward his four laws, the three famous ones, and the fourth one is the law of gravity, which for some reason never gets the number, as well as calculus.

He was pushed into this by the news that Leibniz, who apparently, independently, discovered calculus, was about to publish his ideas. And some of the ideas of Newton's may have been actually put forward by others, by Hooke, for example. Anyway, you know the three Newton's laws, the law of inertia, and then force is defined as the product of mass and acceleration, and that action equals reaction.

Now, the first two can be actually related to what then became really understood in the 19th century, the conservation laws of energy, linear momentum, and angular momentum. And we'll make heavy use of those.

Now, as far as the third one, that you have match of the forces, if you have, say, a mass moving in a gravitational field of another mass, then its centrifugal force is mass times velocity squared divided by the radius. And that has to be matched by the gravitational force, which was the unnamed fourth Newton's law of gravity, an amazing achievement in and of itself. And that is something that we'll be using very heavily as well.

The law of gravity, there is this legend about an apple falling from a tree on Newton's head, and then figuring out it's the same thing as the Moon going around the Earth, that's probably just urban legend from the 17th century. But one way or the other, he was able to connect simple observations right here on planet Earth and motion of the Moon and so on, and to deduce what looks like a pretty natural formula.

If you think there is some kind of attraction force and acts between different masses, well, it's kind of reasonable it will be proportional to each of the masses. And in some sense, that its effect will be diluted as a square of the radius because of the surface of the sphere in which it is. OK?

So it's a reasonable guess for a formula, and it turns out to be correct. There is a constant that you can measure in front, and this worked fine, well, it was superseded by the theory of relativity, but for the most part, we can use Newton's gravity for just about anything.

And the conservation of energy is that the total energies, combination of two parts, kinetic and potential, for-- well, minus sign for the potential energy. And for gravitational field, it's proportional to the masses and inverse proportional to the radius.

And the last quantity is angular momentum, which, for a point mass going in rotation around some center, for whatever reason, say, gravity, is proportional to mass, velocity, and radius. If you have a solid body integrated over all mass points, you get that. All right. Keep this in mind.

So Newton, as a part of *Principia*, figured out that if you have two mass points moving under the influence of each other's gravity, their orbits-- let's call them orbits whether or not they actually stay together-- are always conic sections.

Conic sections are curves you get when you take a plane and slice a cone. If you do it obliquely, you'll get an ellipse. Circle is a special case of an ellipse. If you do it parallel to the angle of the cone, you'll get the parabola. And if you do it vertical along the cone's axis, you get the hyperbola. There is some tedious proof of this using polar coordinates.

But the important point was that this major shape of the orbit is directly related to the total energy of the system. If the two mass points are such that potential energy overcomes the kinetic energy, they're bound. They just have to go around each other, so the orbits are ellipses.

If the net energy is positive, there is more kinetic than potential, they'll fly apart according to a hyperbola. And the boundary condition between the two when it's exactly kinetic equal potential, it's a parabola. So you can think of a parabola as an ellipse with an infinitely long semi-major axis.

OK. So he immediately explained Kepler's first law, which up until then was very mysterious. You know, why ellipses and not something else? Well, you can find in books the tedious derivation in polar coordinates, and there is this all manner of angles that you'll never need in your life. But here is a very simple, intuitive way to think about it.

So as, say, a planet goes around the Sun in an ellipse, you can decompose its velocity into a radial component and a tangential component. And so the motion will be the sum of the two. Now, think of a simple harmonic oscillator, like a pendulum. You probably know that the period of the pendulum would not depend on the amplitude. It only depends on the length. So if you give it initial momentum in two different directions, the pendulum is not going to go according to just the slice of a circle, it's going to make a little ellipse.

This exactly same thing happens here. In some sense, you can think of it as a planet going around the Sun, and also going in and out, in and out. And it has to be the exact same period because we're talking about the same system, and so that kind of gives you [INAUDIBLE]. Now, this is not exactly like analogy, but it kind of gives you a good intuitive idea of where it comes from. OK?

Well, all right. So we have orbits, and we'll just talk about closed orbits, those that keep things together. And their size will depend on the energy. Let's say the planets, some can go through each other, for simplicity's sake. All right? Physicists love to make these kind of assumptions, like the famous case of spherically symmetric homogeneous chicken, right? And say there is a center of attraction, mass 1. There is mass 2. Mass 2 has no kinetic energy whatsoever. It just sits there in the middle because that's the center of attraction.

Now, if you give it some energy, it's going to move away. How far it would go, it depends on the amount of kinetic energy you have. If you throw a rock up, the height is going to depend on how much energy you give it. Well, same thing applies here. So the higher the net energy of a planet or a system of sun and a planet, the larger orbit you're going to get. And when the kinetic energy is equal to the potential one, they'll fly apart. Bye-bye. The planet goes in parabolic orbit or hyperbolic, either it acts as kinetic energy. OK.

So the size of the orbit, in some sense, depends on the energy. The shape of a closed orbit, turns out, depends on angular momentum. So there is a fixed amount of energy, whatever it is, and that defines the semi-major axis or radius of the Earth.

Now, the minimum angular momentum you can have is 0. So the planet can just run out and come back on a linear trajectory, which is maximum, so that's an infinitely thin ellipse. So that's the 0 angular momentum, pure radial orbit.

The other extreme is a pure circular orbit. And for a circular orbit, there is a maximum value that angular momentum can have because, you remember, kinetic energy is mv squared, or a 2, and angular momentum is mv divided by r. r was given because of total energy. So because the energy is conserved-- sorry, energy is fixed, angular momentum has to be fixed too.

And so there is a certain critical amount of angular momentum. A planet cannot go on an orbit that's more circular than circular. It can only have less angular momentum than that. If there's none, it's in linear orbit. If it has the maximum, it's in circular orbit. And orbits of planets in the solar system, our solar system at least, are pretty close to circular.

This ellipse business is very subtle actually. And the reason for this, the reason why there are circular orbits and discs in astrophysics and all different kinds of phenomena, planetary systems, accretion discs for quasars, and so on and so forth, is that many of these systems are made by dissipating energy. They can radiate energy away and so on, but you cannot radiate angular momentum. There are no photons of angular momentum.

And so things like planets or comets or what have you settle on the orbit that has the lowest possible energy for the amount of angular momentum they've got, which is a circle. And so this is why there are disc galaxies and planetary systems and accretion discs. OK. So that's as far as Kepler's first law.

What about the second one? Well, this turns out to be a beautiful, little derivation, right? So there is angular momentum, and here assuming the Sun is much heavier than planets so the Sun doesn't essentially move, and so the angular momentum is mass of the planet times the velocity times the radius at any given time. So in the case of elliptical orbit, the radius changes. So because mass is fixed, product of velocity and radius is fixed. That's also known as adiabatic invariant.

Now, if you look at the tiny little angle, element of angle, that's swept per unit time, the length of the arch is proportional to the velocity in a given element of time. And so we multiply that by the radius and divide it by 2, you have the area. And therefore, the product is independent of mass.

And so therefore, Kepler's second law that applies in more in elliptical orbits, inscribed elements of equal area per unit time moving slower and then further away-- obviously don't move slower and then further away because distance is larger and gravitational pull is smaller. Therefore centrifugal force has to be smaller. Velocity has to the smaller. This is a simple and direct consequence of the conservation of angular momentum. Kepler had no idea about a thing called angular momentum. He just empirically found this is the case.

What about the third law? Well, we can do that too. So the centripetal force is given by the law of gravity, adding up the semi-major axis of the planet and the Sun. They're actually both moving around a common center of the mass, which is so close to the center of the Sun that it doesn't really matter. So we can just use the semi-major axis of the planetary orbit. OK?

Now, centrifugal force is given by the product of mass, square of velocity, and divided by the semi-major axis. And velocity is circumference of the orbit divided by the period. So this is how I transform. Centrifugal force is proportional then to mass, semi-major axis, and inversely proportional to the square root of the period, right?

Make them equal, and what you get is that 4 pi square, cube of the semi-major axis is equal to G, mass of the Sun, because mass of the planet cancels, times period square. And so that's exactly Kepler's third law. So it's a simple consequence of conservation of energy. We can go about this in different ways, like you can do energy way as opposed to the centrifugal, centripetal force, but you still get the same result. Now, this actually tends to carry on in many other situations, including even pulsating stars.

So that's how Newtonian physics, simple mechanics and Newtonian gravity, explained motion inside the solar system. Well, all right. Actually things are little more complicated because it's not a whole bunch of two bodies, it's many planets and other stuff. And it turns out that there are no analytical solutions for the gravitational motion when there is more than two bodies.

But 2-body problems fill up half a page. Add a third mass point, there is no analytical solution in closed form. Poincare has proven this in the 19th century, and it still holds. But since masses of planets are pretty small relative to the mass of the Sun, you can use the gravitational pull of the other planets just as a small perturbation on the orbit that's governed by the big mass, the Sun.

And there is a whole theory called perturbation theory that you can integrate as far as you want in different number of terms and get approximations, which can be very close. Turns out, actually, this is not so difficult to do, and this is how Neptune was discovered.

The two astronomers, Adams and Le Verrier, looked at the orbit of Uranus, which was discovered empirically by Herschel, and found out that Uranus is not moving exactly as you'd expect, according to Kepler's laws. But if there was another massive planet behind, then its gravitational pull could explain variations in the orbit of Uranus. And then computing this backward, they figured out where this new planet should be.

And an astronomer named Galle has found it. And so this was a fantastic prediction of Newtonian mechanics. The new planet was discovered as a direct prediction of simple Newtonian gravity.

More recently, also, we see things like impacts of comets on Jupiter or Earth for that matter. Jupiter is the only other mass in the solar system that really matters when it comes to these things and can deflect orbits of comets and asteroids. We've now seen at least three different cases of a comet being redirected from its unperturbed orbit and making an impact into Jupiter. And so when that happens on planet Earth, bad things happen, depending on the size of it. So this is how astronomy can actually kill you. That's one of the few ways.

Now, we can also measure things really precisely and can find out that there are effects that cannot be explained through Newtonian gravity, no matter how good perturbation theory is. And those are due to the theory of relativity, which is a better approximation of physical reality than the Newton's physics was.

And in fact, this was the first significant evidence for the theory of relativity is correct. That, for many years, until early 20th century, astronomers found out the orbit of Mercury doesn't stay the same. That the ellipse itself rotates. That's called the advance of the perihelion. And there was no way to explain why this was the case. But Einstein explained it and predicted it, and it was the first reason why people started paying attention.

Things actually can get interesting in just good, old Newtonian mechanics. Now, there are different systems, like Saturn's rings, that consist of gazillions of particles orbiting the planet, and they form the rings. And the rings are not solid. They're are a whole bunch of little, thin ringlets.

And the gaps between them are all governed by the gravitational pull of Saturn's satellites. So you have very complex, multi-body dynamics that produces this sort of like LP record look of Saturn's rings, and astrophysicists, including Peter Goldreich here, have explained why this happens.

An even more interesting question is so-called dynamical chaos. Now, we tend to kind of indoctrinate you. How once you know these laws and initial conditions, you can compute everything to infinite future. And Laplace, in the 19th century, even invented a thing called the Laplace demon, which is a hypothetical theoretical physicist who knows positions and masses and velocities of every single mass in the universe, and therefore can compute exactly what's going to happen to infinite future.

Turns out that's not the case. That in mechanical systems, you can have genuine chaotic motions past a certain threshold of complexity. And the solar system, which looked like a nice, little clockwork mechanism, according to Kepler's laws, Newton's laws, only looks that way now. But if you integrate through billions of years, some of the members of the solar system may not actually stay with us. These perturbations keep jiggering them, and sooner or later, you can kick a planet out. This probably happens to comets and asteroids quite a bit.

So solar systems need not be stable in the long term. And that actually is not too much of a problem for our solar system. We're pretty stable. But now that we have planetary systems around other stars, some of those are definitely not stable. And so there is some interesting stellar dynamics to be worked out there.