DJORGOVSKI: Let's move on then and look in a little more detail how the universe expands or evolves in time. So as soon as there were first-- as general activity was recognized to be essentially theory of universe at large scales, Einstein himself came up with some cosmological models. He realized that left to itself, space filled with matter will either expand or contract. It has to do one or the other depending on how much mass energy there is.

So he introduced a quantity called cosmological constant, which you can think of it as an integration constant that can have any value whatsoever. And so he chose the value just so that it balances the gravity of regular stuff and the universe will static, because in 1917 people thought the universe was static. Turned out not to be the case, so Einstein missed predicting the expansion of the universe and he regretted it later. Now, this cosmological constant term in his equations turns out is actually there. But that required much, much more sophisticated measurements, which we now have. And it's probably one of the most outstanding mysteries of science.

At the same time, the Dutch physicist Willim de Sitter also computed similar things. And in fact, when Hubble discovered expansion of the universe, it was called it the de Sitter effect because it was recognized that space can expand itself. And then the two of them later have a joint model called Einstein-de Sitter model, which is neither one of these two. It's something else. Now, the real pioneers of this were two scientists, one in the Soviet Union-- Alexander Friedmann-- and one in Belgium-- Georges Lemaitre.

Friedmann was a meteorologist and he didn't live very long, maybe because of conditions in Leningrad or wherever it was, or maybe Soviet Union was the way it was. But in 1922 he looked at models of the universe based on general relativity. He used assumptions of symmetry that we've mentioned. If things are homogeneous and isotropic, then the only coordinate that matters is just the radio coordinate between any two points. That simplifies his life enormously.

The original Einstein equations are 16 equations-- standard equations, four by four. And so if there is only one coordinate that's important, and there is isotropy, that reduces to a single equation which people call the Friedmann equation.

Independent of him-- because people didn't realize Friedmann's models were at that front-- Georges Lemaitre, a Jesuit in Belgium, developed essentially the same model. And so sometimes we call them Friedman-Lemaitre model, Friedmann-Lemaitre equation, but sometimes just Friedmann to give him priority. And so these are the cosmological models we still use today because they seem to describe a universe of large scale perfectly fine. Essentially there are direct consequence of general activity and in the simplest possible case-- homogeneity and isotropy.

So if you want to quantify what's going to happen with the universe, again you use any two distant points that are not gravitationally bound-- they are kind of riding on the expanding space-- and measure how's the separation of them changing as a function of time. So we call it scale factors and usually it's called R of t and sometimes a of t. And so this will apply for any two unbound points, and so therefore you don't have to worry about units. Now you can just look at qualitative behavior. And if you can solve for that, then you can make predictions about the universe, what it's going to do.

And that's exactly what was done with Friedmann, or Friedmann-Lemaitre, equation. The solution of that equation gives you one of these curves. And parameters determine how the curve looks like, include matter and energy with content of the universe. Now remember the presence of matter and density of anything is related to geometry. And so it turns out that there are three distinct cases.

There is Euclidean case, which is called spatially flat. This is spaces as you used to think about and it's denoted with so-called curvature constant of 0. There is closed space-- three dimensional equivalent of two dimensional sphere, and that's positive curvature. And there is the opposite one, sort of a hyperbolic surface like a saddle, but again generalized in one extra dimension. It's called open negative or open curvature.

So that those with positive curvature will be closed, therefore it will have finite volume, just like a sphere. Those with the negative curvature go on to infinity. And whether the universe is infinity in extent depends on the value of the curvature.

Now what determines that is the matter and energy content of the universe. And let's ignore cosmological constant for the moment, and so think there is just regular matter in the universe. The only quantity that matters then is the mean density, because what else can you possibly have? And if there is too much gravity, if the density is more than certain value, the universe will expand for awhile, turn around, and collapse back into the reverse of the Big Bang-- so-called Big Crunch.

On the other hand, if density is less than the critical value, it's just going to go expanding forever. I mean, it gets slowed down. This is why these curves are all bent a little bit depending on how much mass there is in the universe. And the case between them, the dividing case, is the flat one.

Now if you remember orbits, this is the equivalent of elliptical orbits for a gravitationally bound pair of mass points. Critical case, the parabolic orbit, and then hyperbolic orbit when they're completely unbound. So this is four-dimensional space time equivalent of that three-dimensional consideration of Newtonian orbits.

So let's define some parameters that will help us to quantify these things. The first one is the Hubble Constant, which can be written as the ratio of the time derivative, a scale factor, because it is the rate of expansion divided by the scale factor itself because it doesn't matter on what scale you're measuring it. And if you think about units-- so the velocity, which is length over time, is Hubble Constant times length. And so therefore Hubble Constant has to have the measures of 1 over time. And as you can see, that is certainly the case. So r is dimensionless number but its time derivative is not.

And so essentially, Hubble Constant is the normalized slope of these curves at any given time. Because at any given time, we can declare r to be 1 in whatever units.

And today, its value is somewhere around 70 kilometers per second per megaparsecal distance. So the galaxy is exactly one megaparsecal-- well, that's not a good idea. Let's say it's 10 megaparsecs away. Then it's moving away with 700 kilometers per second, give or take a little peculiar velocity.

So this is what Hubble's constant does. It sets the scale of the universe because it's a particular value of the slope in 1 over time units. And so we draw a tangential line on the curve, and it intersects the time axis. That length is called Hubble time and multiplied by speed of light gives you so-called Hubble length. Those turn out to be useful units in which to measure a scale of the universe.

Now because these lines have some curvature, it's not exactly Hubble time. If it was perfectly flat, it might be. But it's still that same order. And so that's what sets the scale of the universe, both in space and in time. How fast is it expanding at any given moment?

Now let's look at matter-- the content of it. And this is what Friedmann equation with just plain matter looks like, and you don't have to know that. But if the universe is flat, the curvature constant little k is equal to 0. Then there is a critical density which is given by this formula-- 3 times Hubble Constant squared, divided by 8 pi G. G Is gravitational constant.

And then you divide the actual density of the universe, whatever it is, with this unit of density, the critical density. And if they're exactly equal, then this is a flat universe. If it's less than 1, then there isn't enough matter to close it. It's an open or hyperbolic universe. If It's more than 1, then there is more than enough matter to close it. It's a closed universe with positive curvature that eventually recollapses, unless you do something special to it.

So cosmologists use notation of omega, capital omega with subscript of-- they were talking, like, m for matter-- to denote ratio of the density, to the critical density. Because what matters for those curves, qualitatively, is the value of this dimensionless number. Units scale as you wish. Likewise, this cosmological constant actually corresponds to uniform energy density in space. And energy, being equivalent to matter, for productivity equals mc squared. It has gravity.

So whatever physical nature of this uniform energy is, it has some gravity. Now it can have either sign-- this is the interesting part. Unlike regular matter and gravity that is only attractive, this one could be attractive or repulsive. Just like in electromagnetism, there are two components. There is electrostatic and there is magnetism. And you can have two different signs for electrostatic. So this is sort of gravitational equivalent thereof.

So there is energy density that you can convert by dividing it by c squared. A kind of mass like density and you divide that with critical density and get this omega sub lambda. Lambda is traditional notation for cosmological constant. So then the total density parameter is the sum of these two.

And to answer an earlier question, we can define dimensionless number called deceleration parameter, which is sort of now secondary derivative of those curves. The Hubble Constant was giving you the first derivative, the slope, and this would be curvature. So if the universe is slowing down its expansion, then this number will be positive the way it's defined here. And if the universe is accelerating, it will be negative.

So this is what these cosmological parameters do. They define the shape of these curves. The Hubble Constant sets the scale, gives you the basic unit of the universe at any given time. But the values of these omegas define on which curve it will sit. And so the job of cosmology was to measure these parameters and it turns out this is a very difficult thing to do, because we're talking about scales that are much, much bigger than anything that we can probe in other ways.

So, a couple useful numbers to know. That sometimes you will see Hubble Constant expressed in dimensionless units, like a little h. And that means it's divided-- it's in units-- so 100 kilometers per second per megaparsec. Or sometimes h with subscript 70, because u is correct value Hubble Constant, of about 70 kilometers per second per megaparsec. The reason we do this is to take it out of the equations. It's just confusion factor. So you choose units you can set it up, but qualitative behavior does not depend on the actual value of Hubble Constant.

The critical density-- remember it's proportional to the square of Hubble Constant in a given time. And I should have pointed out the subscript of 0 means today. When there is no subscript, it could be at any time in history, or if its subscript is 0, like h0 or q0 or r0, it's today. And so today the value of critical density is, roughly speaking, 10 to the minus 29 grams per cubic centimeter.

And that sounds like a real small number, and it is, but there are many cubic centimeters in the universe. And so this gets to be very important when you start going on many hundreds of megaparsec scale. And so just to make it all look nice, you can say that some of the three density parameters-- the one of matter, the one of cosmological constant or dark energy-- need not be constant, actually-- plus some fictitious number called omega sub curvature add up to 1. The job of this omega k is just to make this equation equal to 1. And so when cosmologists look at deviations from flatness in the universe with precision measurements using my career background, they sometimes use this little omega sub k, which is not the physical quantity itself. It is just telling you how different is the global geometry from being flat.

OK. So those are the numbers that we'll see occasionally. And what we need to look at now is how is the density of matter and energy content of the universe changing in time? Because deceleration depends on the density and the nature of the gravitational force and that is related to kinematics. There is intimate relationship between the changing scale of the universe and curvature and all that with its energy that is the content. Now this may sound silly because of course, if you expand something by a factor in the density goes down by cube of that factor.

This is true in Euclidean case for regular matter. But not if it is radiation. So suppose you have a box full of photons. There is some amount of energy there. You can divide by c squared to get mass equivalent. You expand the box-- sure, you'll dilute the number density of photons according to the cube, but they're also stretched by that factor. They lose energy.

So the absolute value of density does not as third but as fourth power of the scale factor. And in different times in history of universe, dominant component was radiation and matter then comes to be cosmological constant. Cosmological constant, as the name implies, is density stays constant. The whole space is filled with constant energy. Constant energy then is the field. And even though space expands, it always stays the same.

Now there is another aspect how energy is not conserved in expanding space. And other different ways you can parametrize this, but what's usually done is to express density behaves a scale factor to some power-- which is minus 3 for organic matter-- and then this little w plus 1-- called equation of state parameter-- and turns out, if it's less than minus 1, then universe goes into super exponential expansion, so-called Big Rip. That's unlikely to happen.

Now we know that in real universe, we have mixture of radiation and regular matter. And now we know there is also cosmological constant or dark energy. And so how's this going to play out? Well, let's first look just at the matter and radiation. In the early universe, the density of matter, and for that matter radiation, were so high that value of cosmological constant was just trivial. Completely negligible. And so let's ignore it for now.

Now energy density of radiation declines as the fourth power of expansion. Matter is the third power, so those two curves are about to cross. And that turns out to be at the right redshift of several thousand. Before then, universe is dominated by the radiation density. After that, it's dominated by the matter density.

Just as we call it in stellar structure and very massive stars, pressure has two components. There is gas pressure, there is radiation pressure. And for really hot stellar interiors, radiation pressure is larger than gas pressure.

So depending then on how this goes-- recall here how density scales is my third or fourth power-- the expansion will actually have different behavior. And in case of just pure radiation dominated universe-- just photons, nothing else-- it goes as the square root of time. And if it's matter dominated, it goes as 2/3 power of time. OK.

So there is a more general formula I'm showing you to just for completeness, but you don't have to know that.

And so the history of the universe is then dominated by three different components. It's going to be first dominated by the one that's declining fastest as the space expands and gets diluted, and that will be radiation. Then you'll be dominated by the one that's next fastest, which will be matter-- goes as a cube of scale factor. And eventually they'll drop down so low that this constant level of cosmological constant, dark energy, whatever it is, is bigger than either one of the two.

So today, cosmic microwave background-- which is a remnant of the photons from the early universe-- is so diluted that its density parameter is something 10 to the minus 4, 10 to the minus 5. Whereas that one for the regular matter is more like a little shy of 0.3. And that one of dark energies is about 0.7.

And so we now live in a universe which is dominated-- the economics is dominated by the dark energy. And it works in a sense of accelerated expansion. So there is transition from radiation dominated to matter dominated to cosmological constant, or dark energy, dominated and the expansion law changes depending on how you do it. So these are examples of some of the models.

They're normalized. The scale factor is 1 today, and plotted in billions of years and so highest density models bend most. And lower maybe get flatter. But if you have pure cosmological constant-- which is positive in sign-- corresponds to positive energy density, it works as a repulsive force, sort of like elastic force-- proportional to the separation. And in principle, drives exponential expansion.

So it turns out there is a real mixture. And in the early days, the universe was dominated by the matter and radiation. It was decelerating expansion. At some point, those became equal or comparable to the energy density of the dark matter-- sorry, dark energy-- and at that point, the universe started accelerating again. And now it's in an accelerated expansion phase. It's been so for several billion years.