DJORGOVSKI: So first let's start with some basic ideas about how stars work. And there are four important ideas or issues to consider. First, since most stars seem to last a very long time, that means they're stable. And that, among other things, means they have to be in hydrostatic equilibrium. And that means that gas pressure inside stars has to balance the gravity that tries to keep the start together.

That is provenance of classical physics, a little bit of maybe modern atomic physics. But even back in the early 20th century people like Eddington and so on figured out how stars must work even though they didn't know exactly what the source of energy was.

The second thing is the stars have to be in thermal equilibrium, meaning the amount of energy they produce in the middle has to match the luminosity, otherwise either a star gets extinguished or it explodes. The origin of the energy, or the thermonuclear reactions, fusion of lighter elements inside the stellar core which is hot and dense enough to happen. And then finally, the energy has to get out somehow, and that's known as the energy transport.

So these things are encapsulated in what's called the equations of the stellar structure. And in a somewhat simplified version, it goes like this. Well, first of all, there is continuity of mass. As you look at the radial shell, somewhere at the radius R, the element of the mass is the area of the shelf times the element of the radius, or that gives you the gradient of mass as a function of radius, which is related to the local density.

The same thing with luminosity. In principle, some layer inside the sun, usually near the core, energy is being generated. The amount of energy that's been generated is similar to the amount of mass, but here we use a quantity called the energy generation rate per unit mass, which is something that's computed from nuclear physics. And once you know that from nuclear reactions, then you can establish the gradient of luminosity inside a star.

Now, hydrostatic equilibrium. If you look at a shell of material inside the sun or in a star. So there is a gravitational force pulling it down. There is pressure on the inside and the outside from thermal motions, and the difference in those two pressures has to match the gravitational force that pulls the shell inwards.

So since we know what element of mass versus radius is, you can then simply differentiate this, and you find that the gradient of pressure with radius is proportional to the enclosed mass, inversely proportional to the square of the radius, simply Newtonian gravity, and the density, at that point.

Now, pressure of what? Gas obviously, but also radiation. Radiation has its own pressure. As photons would hit some surface, there's momentum transfer. So there is a force acting on a surface. And that radiation pressure can be computed in a moment, but for the gas you probably know from your statistical mechanics, or thermodynamics. It is simple relation that pressure is proportional to the temperature, and proportionality is the density of particles in the volume and Boltzmann Constant.

Another way to do it in terms of the gas density is that you replace the number density of particles with the mass density. And then because it's the number of particles, you divide the mass density with mean mass of particles. And we like to express this in the units of masses of hydrogen.

So this quantity mu is called the mean molecular weight, and it is the average mass of particles. If you had pure neutral hydrogen, that would be 1. If you have fully ionized hydrogen, now you have two particles and the same amount of mass, so mu will be 1/2. If you start adding heavier elements, then that's going to go up.

The composition of solar plasma is roughly speaking 70 or so percent of hydrogen, a little shy of 30% of helium, and couple percent of heavier elements that were inherited from some previous stellar generations. So it works out on average to be roughly 0.8. This is a reasonable number to remember.

So I remember what the ideal gas constant is. It's Boltzmann's constant divided by the atomic mass unit, or mass of hydrogen. And then you can express this as what's really of somewhat familiar relationship from thermodynamics, that pressure is gas constant times density times the temperature.

OK. Second component of this is the radiation pressure. And to a very good approximation, radiation field inside stars is like a black-body radiation. And formula for this comes directly from Planck distribution, and it's proportional to the fourth power of temperature, and the constant of proportionality is the Stefan Boltzmann constant.

And the question is why is this important. You'd think that radiation pressure would be too slight to matter, but, in fact, in a really hot star it dominates over the mass density. The weight of the photons, if you will, is bigger. So there is so much radiation that, actually, radiation pressure dominates all the dynamics. And that can be computed nicely.

So all together those four equations are called the equations of stellar structure. And they are differential equations so they're usually solved numerically, or you can make a toy model of what you think interior distribution of density might be. People do that too. And then plug-in some constants from atomic physics and so on, and you can compute what a star might look like. And if you know what the composition is you can compute opacity-- will not go into that-- the energy generation rate, and, voila, you can figure out how a star is.

So this is more or less complete treatment. Even so, we have neglected several important things. We assumed the stars are spherical-- well, they pretty much are but not exactly. We didn't take into account rotation. That turns out to be important because, in some sense, rotation works as anti-gravity. Centrifugal force tends to pull stuff out, and it depends where in a star you are. And we neglected magnetic fields which also can play a significant role in moving the stuff around. So in more modern models, people take that into account as best as they can.

But we can do even just back-of-the-envelope, simple estimates. So let's see if we can figure out what is the pressure and temperature inside the sun. Well, you need force acting upon an area, and so simple force will be G times square of the solar mass divided by square of solar radius. You know the area divide it too, and you estimate pressure of 10 to the 15 dynes per square centimeter. And that's for sure going to be underestimated because we took outer surface area. And so in reality, when you do a proper computation, it's 100 times more. It's 2 times 10 to the 17 in CGS units.

We do better with temperature. We can say, let's equate thermal kinetic energy, 3/2kT because this is essentially protons, right? So that's why 3/2 not 5/2. That's equal to, say, gravitational binding energy of a proton. So the radius. And multiply those numbers, and we come up with something like 16 million degrees Kelvin. Turns out that's more or less exactly right.

So, the density is also inside sun. In the middle, it's of the order of 150 grams per cubic centimeter. On the surface, it's much less. So high densities plus high temperatures is what's needed for stars to do nuclear reactions.

So here is a simplified version of what's now called the standard solar model. When people do all the computation properly. And this is how mass and luminosity generation goes a radius.

So in the very center you don't have anything. Then as you go further out, you encompass more and more volume in which nuclear reactions happen. At some point, density and temperature drop sufficiently that you can no longer do nuclear fusion, and then it remains constant. So solar luminosity is established somewhere deep inside the sun.

The mass is a function of radius. It depends on the density distribution, it's denser in the middle than outside, and so here you see roughly how that works. So by about half the solar radius, you have 90% of solar mass.

So that's the basic ideas behind stellar structure. Another plot. This is density and temperature as a function of radius. And note, temperature goes down from 16 million Kelvin to, essentially, 0 at photosphere. And the density is 160 grams per cubic centimeter, again goes to something very light on the surface.