DJORGOVSKI: Now let's talk about distances, because that's the fundamental physical quantity that you need in order to convert observed quantities-- like fluxes into luminosities, or angle or size into physical size-- and it turns out to be the basis of all cosmological tests. Hubble himself already devised a bunch of these, and then more have been added, at first. And you begin with this r of t curves as a function of cosmic time. Now, r of t is really 1 over 1 plus redshift, so that's just redshift in disguise.

Now, measuring time, looking back at some galaxy, at some redshift, is actually really difficult. So instead of measuring time, we use the distance. Because multiply the time with speed of light, and you get the distance. All right, speed of light being constant. So then you can transform this diagram, and now the x-axis is the redshift, because that's the real, earthly easy one to measure. So the scale factor is now the independent variable, because that's what we can measure easily. And convert time into distance by multiplying it by speed of light.

And so, if you can measure how distances to different things change as a function of time, then you can map out this curve. And that's the job of cosmology. Now this is curvature, the absolute scaling is given by the Hubble constant. But the shape of the curve is given by the other parameters, and by actually measuring what's going on you can see which curve fits the best.

So how do we get the distances? All right. First, let me give you exact values for what Hubble length, and Hubble time are. And here I introduced this scaling parameter, little h subscript 70 versus if Hubble constant is 70 kilometers per second per megaparsec, and it is within a couple kilometers per-- Then, Hubble length is 10 to the 28 centimeters. And Hubble time is 4.4 times 10 to the 17 seconds, turns out to be about 13 or 14 billion years.

And we can take 4 million that are obtained from solving Friedmann equation, and follow how these things change in time. Now, in general, those have to be done as numerical integration. The integrals of these, formally, do not have close analytical solution, but this is a straightforward matter to do. And so here is how distance, in units of the Hubble length changes out to a given redshift. The three curves are three different cosmological models.

The solid one is-- also called Einstein de Sitter model-- where density is exactly equal to critical, and it's just matter, and nothing else. And the dashed one is now with cosmological constant, together with the matter adding up a mega of 1, and that's actually not so different from what it really is. And the last one is an almost empty universe, no dark energy, and just 5% of the critical density in a regular matter.

Because more matter means more gravity, more deceleration, universes with higher density will be smaller at any given time than universes that are nearly empty. And cosmological constant can mess with this in either direction. So this is how we compute the distances, but that's not what we measure. What we measure use either inverse-square law for sources of light-- those measure relative distances. Or angular diameter as a functional distance, in relativistic form.

Now, you understand the inverse-square law, in plain Euclidean space. But now, because the universe is expanding, things get a little more complicated. First of all, because the source is moving. There is a relativistic time dilation. Time is stretched by factors of 1 plus redshift.

And, each photon gets stretched by that factor too. So, if you compute flux from some source of luminosity, L it will be L divided by 4 pi, times the distance squared-- for simple Euclidean case. And two extra powers of 1 plus redshift. One because of time dilation, one because of the stretching of the photons.

So people bundle those, and take real distance, times 1 plus redshift, and call it luminosity distance. And then that's the distance that you can use in relativistic equivalent of inverse square law. So if you had sources of light, that are intrinsically the same luminosity, so-called standard candles, then, by measuring how the flux changes as a function of redshift, you can map that back, how luminosity distance changes as a function of redshift. And that's called Hubble diagram.

Now here are all our three cosmological models. And now plotting the luminosity distance, because its real distance times 1 plus z, if you notice these numbers are higher. With regular distance, it was kind of 1 or 2 times the Hubble length. Now it's more like 10 times the Hubble length. And that's because of 1 plus z factor.

Another thing that you can measure-- and those are really the only two things we can measure-- would be angular diameters. So if you have a source of physical size x, and you put into the distance of d, the - it will subtend an angle of 10 angular diameter of x divided by D, in radians. Ah, but if it's fixed in proper coordinates and the universe has expanded since then, then it was bigger-- relative to the co-moving coordinates-- back then, in opposite sense of what happened with luminosity distance.

So you multiply-- I'm sorry, you divide the regular distance by 1 plus redshift, and we call that the angular diameter distance. So if you use that, in relativistic equivalent, or 1 over distance law, then you can compute how angular diameter will change. So if you had sources of standard size-- and it turns out the entire universe is a good one-- and look at how that length changes as a function of redshift, then you can map the relativistic version of angular diameter change, and map that back again into the distance, and then into R, T, and so on.

So these diagrams are what cosmologies are trying to map out, by looking at relative distances to some standardized kinds of things. Supernova turned out to be really good for this. We'll talk about that next time-- well, time after that. Now we cannot measure directly how clocks are ticking in other places, but it's useful to know what's the age at any given redshift if you're doing things like galaxy evolution. So you measure cosmological parameters somewhere else, and that tells you how to map redshift into the actual age.

And here are the friendly models, the sets of curves that is closer to the origin is the look-back time-- I'm sorry, it's the age of the universe. And you can see this today, they're around one Hubble time, so that's about right. And the other is the look-back time. The further you look in the past, the further you look in redshift, also the further look in past. And the big bang happens at the redshift of infinity. So these curves have to bend over, and flatten like that.

You can also compute things called the volume changes. So if you are looking at some populations, say galaxies, in comoving space, you can have how many are there per unit volume, and then look at it as well. So that's what forms the basis for cosmological tests, to which we'll come back later. But this is the takeaway point. That cosmological models that have higher density and/or negative cosmological constant, are those the show maximum deceleration.

Therefore, at any given time, the universe is smaller in those models. And vice versa for models with lower density, and their positive cosmological constant, because it accelerates expansion, they're always bigger at any given moment. So if the lengths are, say, smaller, that means the distances tend to be closer, regardless of whether cosmology luminosity or angular diameter distance.

And so, in the denser, and/or negative cosmological constant models, things will look bigger in the sky, and they'll be brighter in the sky. And because volume is smaller, there will be fewer of them. And exactly the opposite if we're looking at the low-density models, and/or with positive cosmological constant. Those will be larger, things would look smaller, they would look fainter, and volumes will be bigger, so there will be more things out of high redshifts. And so in this way, we can connect theoretical models with observational quantities. And that's how observational cosmology is done.