4.1 The Scale of the Universe: Basis of the Cosmological Distance Scale
The Scale of the Universe

• The Hubble length, $D_H = c/H_0$, and the Hubble time, $t_H = 1/H_0$ give the approximate spatial and temporal scales of the universe.

• $H_0$ is independent of the “shape parameters” (expressed as density parameters) $\Omega_m$, $\Omega_\Lambda$, $\Omega_k$, $w$, etc., which govern the global geometry and dynamics of the universe.

• Distances to galaxies, quasars, etc., scale linearly with $H_0$, $D \approx cz / H_0$. They are necessary in order to convert observable quantities (e.g., fluxes, angular sizes) into physical ones (luminosities, linear sizes, energies, masses, etc.)
Measuring the Scale of the Universe

- The *only* clean-cut distance measurements in astronomy are from trigonometric parallaxes. Everything else requires physical modeling and/or a set of calibration steps (the “distance ladder”), and always some statistics:

  Use parallaxes to calibrate some set of distance indicators
  → Use them to calibrate another distance indicator further away
  → And then another, reaching even further
  → etc. etc.
  → Until you reach a “pure Hubble flow”

- The age of the universe can be constrained independently from the $H_0$, by estimating ages of the oldest things one can find around (e.g., globular clusters, heavy elements, white dwarfs)
The Hubble’s Constant Has a Long and Disreputable History …

THE VELOCITY-DISTANCE RELATION AMONG EXTRA-GALACTIC NEBULAE


The new data extend out to about eighteen times the distance available in the first formulation of the velocity-distance relation, but the form of the relation remains unchanged except for the revision of the unit of distance. The relation is

\[
\text{Vel.} = \frac{\text{Dist. (parsecs)}}{1790}, \quad H_0 = 560 \text{ km/s/Mpc}
\]

and the uncertainty is estimated to be of the order of 10 per cent.

Since then, the value of the \( H_0 \) has shrunk by an order of magnitude, but the errors were always quoted to be about 10% (or less) …

Generally, Hubble was estimating \( H_0 \sim 600 \text{ km/s/Mpc} \). This implies for the age of the universe \( \sim 1/ H_0 < 2 \text{ Gyr} \) - which was a problem!
The History of $H_0$

Major revisions downwards happened as a result of recognizing some major systematic errors.

Compilation by John Huchra

Baade identifies Pop. I and II Cepheids

“Brightest stars” identified as H II regions

Jan Oort
The History of $H_0$, Continued …

But even in the modern era, measured values differed covering a factor-of-2 spread (larger than the quoted errors from every group)!

(from R. Kennicutt)
Distance Ladder: Methods

**Methods yielding absolute distances:**
- Parallax (trigonometric, secular, and statistical)
- The moving cluster method - has some assumptions
- Baade-Wesselink method for pulsating stars
- Expanding photosphere method for Type II SNe
- Sunyaev-Zeldovich effect
- Gravitational lens time delays

**Secondary distance indicators:** “standard candles”, requiring a calibration from an absolute method applied to local objects - the distance ladder:
- Pulsating variables: Cepheids, RR Lyrae, Miras
- Main sequence fitting to star clusters
- Brightest red giants
- Planetary nebula luminosity function
- Globular cluster luminosity function
- Surface brightness fluctuations
- Tully-Fisher, $D_n - \sigma$, FP scaling relations for galaxies
- Type Ia Supernovae
  … etc.
Trigonometric Parallax

- Straightforward, geometric, and the only “true” method - the fundament of the distance scale
- Measure the shift in observed position of nearby stars relative to background stars as earth moves in orbit around the Sun
- Can get distances of out to \( \sim 10 \text{ kpc} \) from the \textit{Gaia} satellite, for \( \sim 10^9 \) stars
- Parallaxes provide absolute calibrations to the next rung of the distance ladder – subdwarfs, Cepheids, nearby star clusters

\[
D \ [\text{pc}] = \frac{1}{\pi \ [\text{arcsec}]}
\]
Main Sequence Fitting for Star Clusters

Luminosity (distance dependent) vs. temperature or color (distance independent)

• Can measure distance to star clusters (open or globular) by fitting their main sequence of a cluster with a known distance (e.g., Hyades)

• The apparent magnitude difference gives the ratio of distances, as long as we know the reddening (extinction)!

• For globular clusters we use parallaxes to nearby subdwarfs (metal-poor main sequence stars)
Pulsating Variables

- Stars in the instability strip in the HR diagram
- All obey empirical period - luminosity (distance independent vs. dependent) relations which can be calibrated to yield distances
- Different types (in different branches of the HRD, different stellar populations) have different relations

- Cepheids are high-mass, luminous, upper MS, Pop. I stars
- RR Lyrae are low-mass, metal-poor (Pop. II), HB stars, often found in globulars
- Long-period variables (e.g., Miras) pulsate in a fashion that is less well understood
Cepheids

- Luminous ($M \sim -4$ to $-7$ mag), pulsating variables, evolved high-mass stars on the instability strip in the H-R diagram
- Shown by Henrietta Leavitt in 1912 to obey a period-luminosity relation (P-L) from her sample of Cepheids in the SMC: brighter Cepheids have longer periods than fainter ones
- **Advantages:** Cepheids are bright, so are easily seen in other galaxies, the physics of stellar pulsation is well understood
- **Disadvantages:** They are relatively rare, their period depends (how much is still controversial) on their metallicity or color (P-L-Z or P-L-C) relation; multiple epoch observations are required; found in spirals (Pop I), so extinction corrections are necessary
- P-L relation usually calibrated using the distance to the LMC and now using Hipparcos parallaxes. *This is the biggest uncertainty now remaining in deriving the $H_0$!*
- With HST we can observe to distances out to $\sim 25$ Mpc
Cepheid P-L Rel’ n in different photometric bandpasses

Amplitudes are larger in bluer bands, but extinction and metallicity corrections are also larger; redder bands may be better overall

**TABLE 3.** Galactic Leavitt Laws from fundamental distances. Table adapted from Fouqué et al. 2007.

<table>
<thead>
<tr>
<th>Band</th>
<th>Slope</th>
<th>Intercept</th>
<th>σ</th>
<th>N</th>
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<td>$B$</td>
<td>$-2.289 \pm 0.091$</td>
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<tr>
<td>$V$</td>
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<tr>
<td>$R_c$</td>
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<td>0.180</td>
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<tr>
<td>$I_c$</td>
<td>$-2.980 \pm 0.074$</td>
<td>$-1.726 \pm 0.022$</td>
<td>0.168</td>
<td>59</td>
</tr>
<tr>
<td>$J$</td>
<td>$-3.194 \pm 0.068$</td>
<td>$-2.064 \pm 0.020$</td>
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<td>59</td>
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<tr>
<td>$H$</td>
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<td>$-2.215 \pm 0.019$</td>
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<td>56</td>
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<tr>
<td>$K_s$</td>
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<td>58</td>
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<tr>
<td>$W_{vi}$</td>
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<td>58</td>
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<tr>
<td>$W_{bi}$</td>
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<td>$-2.401 \pm 0.023$</td>
<td>0.178</td>
<td>58</td>
</tr>
</tbody>
</table>
P-L relations for Cepheids with measured Hipparcos parallaxes, in different photometric bands

(from Freedman & Madore)

Typical fits give:

\[
\langle M_V \rangle = -2.76 \log P - 1.45
\]

\[
\langle M_I \rangle = -2.96 \log P - 1.88
\]

… with the estimated errors in the range of \(~ 5\% - 20\%\)

We expect major improvements from Gaia
RR Lyrae Stars

- Pulsating variables, evolved old, low mass, low metallicity stars
  - Pop II indicator, found in globular clusters, galactic halos
- Lower luminosity than Cepheids, $M_V \sim 0.75 +/- 0.1$
  - There may be a metallicity dependence
- Have periods of 0.4 – 0.6 days, so don’t require as much observing to find or monitor

**Advantages:** less dust, easy to find

**Disadvantages:** fainter (2 mag fainter than Cepheids). Used for Local Group galaxies only. The calibration is still uncertain (uses globular cluster distances from their main sequence fitting; or from Magellanic Clouds clusters, assuming that we know their distances)
Physical Parameters of Pulsating Variables

Star’s diameter, temperature (and thus luminosity) pulsate, and obviously the velocity of the photosphere must also change.
Baade-Wesselink Method

Consider a pulsating star at minimum, with a measured temperature $T_1$ and observed flux $f_1$ with radius $R_1$, then:

Similarly at maximum, with a measured temperature $T_2$ and observed flux $f_2$ with radius $R_2$:

Note: $T_1, T_2, f_1, f_2$ are directly observable! Just need the radius…

So, from spectroscopic observations we can get the photospheric velocity $v(t)$, from this we can determine the change in radius, $\Delta R$:

$$ R_2 = R_1 + \Delta R = R_1 + \int_{t_1}^{t_2} v(t) \, dt $$

$\rightarrow$ 3 equations, 3 unknowns, solve for $R_1, R_2, \text{ and } D$!

Difficulties lie in modeling the effects of the stellar atmosphere, and deriving the true radial velocity from what we observe.
Tip of the Red Giant Branch

- Brightest stars in old stellar populations are red giants
- In I-band, $M_I = -4.1 \pm 0.1 \approx$ constant for the tip of the red giant branch (TRGB) if stars are old and metal-poor ($[\text{Fe/H}] < -0.7$)
- These conditions are met for dwarf galaxies and galactic halos
- **Advantages:** Relatively bright, reasonably precise, RGB stars are plentiful. Extinction problems are reduced
- **Disadvantages:** Only good out to $\sim 20$ Mpc (Virgo), only works for old, metal poor populations
- Calibration from subdwarf parallaxes from Hipparcos and distances to galactic GCs

---

![Graph showing the relation between $M_I$ and $(V-I)$](VandenBerg et al. (2000) models)
Cepheids:

A. Are more luminous than RR Lyrae
B. Are found in the old stellar populations
C. Can be seen out to 200 Mpc
D. Have a period that depends on the star’s metallicity
E. Have larger amplitudes in the redder bands
F. None of the above
4.2 Distance Indicator Relations
Pushing Into the Hubble Flow

• Hubble’s law: \( D = H_0 \, v \)

• But the total observed velocity \( v \) is a combination of the cosmological expansion, and the *peculiar velocity* of any given galaxy, \( v = v_{\text{cosmo}} + v_{\text{pec}} \)

• Typically \( v_{\text{pec}} \sim \) a few hundred km/s, and it is produced by gravitational infall into the local large scale structures (e.g., the local supercluster), with characteristic scales of tens of Mpc

• Thus, one wants to measure \( H_0 \) on scales greater than tens of Mpc, and where \( v_{\text{cosmo}} \gg v_{\text{pec}} \). This is the Hubble flow regime

• This requires *luminous standard candles* - galaxies or Supernovae
Surface Brightness Fluctuations

Useful for the distances to old stellar populations, e.g., bulges, ellipticals

Consider stars projected onto a pixel grid of your detector:

Nearby Galaxy

A galaxy twice farther away is “smoother”
Surface Brightness Fluctuations

- Surface brightness fluctuations for old stellar populations (E’s, SO’s and bulges) are based primarily on their giant stars.
- Assume typical average flux per star $<f>$, the average flux per pixel is then $N<f>$, and the variance per pixel is $N<f^2>$. But the number of stars per pixel $N$ scales as $D^{-2}$ and the flux per star decreases as $D^{-2}$. Thus the variance scales as $D^{-2}$ and the RMS scales as $D^{-1}$. Thus a galaxy twice as far away appears twice as smooth. The average flux $<f>$ can be measured as the ratio of the variance and the mean flux per pixel. If we know the average $L$ (or $M$) we can measure $D$.
- $<M>$ is roughly the absolute magnitude of a giant star and can be calibrated empirically using the bulge of M31, although there is a color-luminosity relation, so $<M_I> = -1.74 + 4.5 [(V-I)_0 -1.15]$.
- Have to model and remove contamination from foreground stars, background galaxies, and globular clusters.
- Can be used out to $\sim 100$ Mpc in the IR, using the HST.
Galaxy Scaling Relations

• Once a set of distances to galaxies of some type is obtained, one finds correlations between distance-dependent quantities (e.g., luminosity, radius) and distance-independent ones (e.g., rotational speeds for disks, or velocity dispersions for ellipticals and bulges, surface brightness, etc.). These are called *distance indicator relations*.

• Examples:
  – Tully-Fisher relation for spirals (luminosity vs. rotation speed)
  – Fundamental Plane relations for ellipticals

• These relations must be calibrated locally using other distance indicators, e.g. Cepheids or surface brightness fluctuations; then they can be extended into the general Hubble flow regime.

• Their origins - and thus their universality - are not yet well understood. Caveat emptor!
The Basic Idea:

- Need a correlation between a distance-independent quantity, “X”, (e.g., temperature or color for stars in the H-R diagram, or the period for Cepheids), and a distance-dependent one, “Y”, (e.g., stellar absolute magnitude, $M$)

- Two sets of objects at different distances will have a systematic shift in the apparent versions of “Y” (e.g., stellar apparent magnitude, $m$), from which we can deduce their relative distance

- This obviously works for stars (main sequence fitting, period-luminosity relations), but can we find such relations for galaxies?
The Tully-Fisher Relation

• A well-defined luminosity vs. rotational speed (often measured as a H I 21 cm line width) relation for spirals:

\[ L \sim v_{\text{rot}} \gamma, \gamma \approx 4, \text{ varies with wavelength} \]

Or: \[ M = b \log(W) + c \], where:

– \( M \) is the absolute magnitude
– \( W \) is the Doppler broadened line width, typically measured using the HI 21cm line, corrected for inclination \( W_{\text{true}} = \frac{W_{\text{obs}}}{\sin(i)} \)
– Both the slope \( b \) and the zero-point \( c \) can be measured from a set of nearby spiral galaxies with well-known distances
– The slope \( b \) can be also measured from any set of galaxies with roughly the same distance - e.g., galaxies in a cluster - even if that distance is not known

• Scatter is \(~10-20\%\) at best, which limits the accuracy
• Problems include dust extinction, so working in the redded bands is better
Tully-Fisher Relation at Various Wavelengths

Blue

Slope = 3.2
Scatter = 0.25 mag

Red

Slope = 3.5
Scatter = 0.25 mag

Infrared

Slope = 4.4
Scatter = 0.19 mag
Fundamental Plane Relations

- A set of bivariate scaling relations for elliptical galaxies, including relations between distance dependent quantities such as radius or luminosity, and a combination of two distance-independent ones, such as velocity dispersion or surface brightness
- Scatter \( \sim 10\% \), but it could be lower?
- Usually calibrated using surface brightness fluctuations distances

![Graph showing relation between log \( r_e \) and \( 1.24 \log \sigma - 0.82 \log <I>_e \).]
The $D_n$-$\sigma$ Relation

- A projection of the Fundamental Plane, where a combination of radius and surface brightness is combined into a modified isophotal diameter called $D_n$ which is the angular diameter that encloses a mean surface brightness in the $B$ band of $\langle \mu_B \rangle = 20.75$ mag/arcsec$^2$
- $D_n$ is a standard yardstick, and can be used to measure relative distances to ellipticals
- Also calibrated using SBF
Tully-Fisher Relation:

A. Can be used to measure distances to elliptical galaxies only
B. Can be used to measure distances to both spiral and elliptical galaxies
C. Needs to be calibrated using Cepheids
D. Depends on the photometric bandpass
E. Has an intrinsic scatter of 10% or less
4.3 Supernova Standard Candles
The Basic Concept

- If two sources have the same intrinsic luminosity ("standard candles"), from the ratio of their apparent brightness we can derive the ratio of their luminosity distances.
- If two sources have the same physical size ("standard rulers"), from the ratio of their apparent angular sizes we can derive the ratio of their angular diameter distances.
Supernovae (SNe) as Standard Candles

• Bright and thus visible far away

• Two different types of SNe are used as standard candles:
  – **Type Ia** from a binary white dwarfs accreting material and detonating
    ✴ Pretty good standard candles, peak $M_V \sim -19.3$
    ✴ There is a diversity of light curves, but they can be standardized to a peak magnitude scatter of $\sim 10\%$
  – **Type II** from collapse of massive stars (also Type Ib)
    ✴ Not good standard candles, but we can measure their distances using the “Expanding Photosphere Method” (EPM), essentially the Baade-Wesselink method of measuring the expansion of the outer envelope
    ✴ Not as bright as Type Ia’s
Believed to be caused by an accretion of material from a binary companion star to a white dwarf (WD), pushing it over its Chandrasekhar limit, causing its collapse.

No H lines, Si lines in absorption: At most \( \sim 0.1 \, M_\odot \) of H in vicinity. Nuclear burning all the way to Si must occur.

- Observed in elliptical galaxies as well as spirals: Old stellar population – not from young massive stars.
- Remarkably homogenous properties: Same type of an object exploding in each case; explains the uniformity of their light curves.
- Lightcurve fit by radioactive decay of about a Solar mass of \( ^{56}\text{Ni} \).
The peak brightness of a SN Ia correlates with the shape of its light curve (steeper → fainter)
Correcting for this effect standardizes the peak luminosity to ~10% or better
However, the absolute zero-point of the SN Ia distance scale has to be calibrated externally, e.g., with Cepheids
SNe Ia as Standard Candles

- A comparable or better correction also uses the color information (the Multicolor Light Curve method)
- This makes SNe Ia a superb cosmological tool (note: you only need relative distances to test cosmological models; absolute distances are only needed for the $H_0$)
The Low-Redshift SN Ia Hubble Diagram
The Expanding Photosphere Method (EPM)

EPM assumes that SN photospheres radiate as dilute blackbodies:

\[ \theta_{ph} = \frac{R_{ph}}{D} = \sqrt{\frac{F_{\lambda}}{\zeta^2 \pi B_{\lambda}(T)}} \]

- **Apparent Diameter**
- **Fudge factor to account for the deviations from blackbody, from spectra models**

Determine the radius by monitoring the expansion velocity:

\[ R_{ph} = v_{ph}(t - t_0) + R_0 \]

And solve for the distance:

\[ t = D \left( \frac{\theta_{ph}}{v_{ph}} \right) + t_0 \]
4.4 The Hubble Space Telescope Distance Scale Key Project

... and Beyond
The HST $H_0$ Key Project

- Observe Cepheids in ~18 spirals to test the universality of the Cepheid P-L relation and greatly improve calibration of other distance indicators
- Their Cepheid P-L relation zero point is tied directly to the distance to the LMC (largest source of error for the $H_0$!)
- Combining different estimators, they find:
  $$H_0 = 72 \pm 3 \text{ (random)} \pm 7 \text{ (systematic)} \text{ km/s/Mpc}$$
- Since then, the Cepheid calibration has improved, and other methods yield results in an excellent agreement
The HST $H_0$ Key Project Results

Overall Hubble diagram, from all types of distance indicators →

$H_o = 72 \pm (3)_r \pm [7]_s$

From Cepheid distances alone ↓
The Low-Redshift SN Ia Hubble Diagram

Riess et al. 2009
(also Riess et al. 2011 – slight changes)

Cepheid calibration

\[ H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1} \]
The Carnegie Hubble Program

Friedman et al. 2012

Using Mid-IR Cepheid calibration from Spitzer

<table>
<thead>
<tr>
<th>HST Key Project</th>
<th>Carnegie Hubble Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>H = 72 +/- 7 [10%]</td>
<td>H = 74.3 +/- 2.1 [2.8%]</td>
</tr>
<tr>
<td>(Freedman et al. 2001)</td>
<td>(Freedman et al. 2012)</td>
</tr>
</tbody>
</table>

Optical: HST

Mid-Infrared: SPITZER
How Well Do the Different Distance Indicators Agree?

Consider the distance measurements to the Large Magellanic Cloud (LMC), one of the first stepping stones in the distance scale.

Different techniques give distance moduli \( (m-M) = 5 \log \left[ \frac{D}{\text{pc}} \right] - 5 \), in the range \( \sim 18.07 \) to \( 18.70 \) mag, (distance range \( \sim 41 \) to \( 55 \) kpc) with typical errors of \( \sim 5 - 10\% \)

Cepheids give:
\( (m-M) = 18.39 \pm 0.03 \) mag
Herrnstein et al. (1999) have analyzed the proper motions and radial velocities of NGC 4258’s nuclear masers. The orbits are Keplerian and yield a distance of $7.2 \pm 0.3$ Mpc, or $(m-M)_0 = 29.29 \pm 0.09$. This is inconsistent with the Cepheid distance modulus of $29.44 \pm 0.12$ at the $\sim1.2\sigma$ level.
Another Problem: Peculiar Velocities

- Large-scale density field inevitably generates a peculiar velocity field, due to the acceleration over the Hubble time.
- Note that we can in practice only observe the radial component.
- Peculiar velocities act as a noise (on the $V = cz$ axis, orthogonal to errors in distances) in the Hubble diagram - and could thus bias the measurements of the $H_0$ (which is why we want “far field” measurements).

$$V_{total} = V_{Hubble} + V_{pec, rad}$$
Bypassing the Distance Ladder

There are two methods which can be used to large distances, which don’t depend on local calibrations:

1. Gravitational lens time delays
2. Synyaev-Zeldovich (SZ) effect for clusters of galaxies

Both are very model-dependent!

Both tend to produce values of $H_0$ somewhat lower than the HST Key Project

... And finally, the CMB fluctuations (more about that later)
Synyaev-Zeldovich Effect

- Clusters of galaxies are filled with hot X-ray gas
- The electrons in the intracluster gas will scatter the background photons from the CMBR to higher energies (frequencies) and distort the blackbody spectrum.

This is detectable as a slight temperature dip or bump (depending on the frequency) in the radio map of the cluster, against the uniform CMBR background.
Synyaev-Zeldovich Effect

- If we can measure the electron density and temperature of the X-ray emitting gas along the line of sight from X-ray measurements, we can estimate the path length (~ cluster diameter) along the line of sight.
- If we assume the cluster is spherical (?), from its angular diameter (projected on the sky) we can determine the distance to the cluster.
- Potential uncertainties include cluster substructure or shape (e.g., non-spherical). It is also non-trivial to measure the X-ray temperature to derive the density at high redshifts.
Gravitational Lens Time Delays

- Assuming the mass model for the lensing galaxy of a gravitationally lensed quasar is well-known (!?!), the different light paths taken by various images of the quasar will lead to time delays in the arrival time of the light to us. This be can be traced by the quasar variability.

- If the lensing galaxy is in a cluster, we also need to know the mass distribution of the cluster and any other mass distribution along the line of sight. The modeling is complex!

Images and lightcurves for the lens B1608+656 (from Fassnacht et al. 2000)
How Does It Work?

The difference in the light paths is \((a+b) - (c+d) = \Delta S = c \Delta t\) where \(\Delta t\) is the measured time delay.

For a fixed lensing geometry, \(\Delta S \sim D_L\) or \(D_S\) and the ratio \(\Delta S/D_L\) or \(\Delta S/D_S\) is also given by the geometry.

Assuming that, measuring \(\Delta t\) gives \(\Delta S\), and thus \(D_L\) or \(D_S\).
**$H_0$ From the CMB**

- Bayesian solutions from model fits to CMB fluctuations – cosmological parameters are coupled

![Graph showing the relationship between $H_0$ and $\Omega_m$ with contours for different $n_s$ values](image)

- **Planck (2013) results:**

  $$H_0 = (67.3 \pm 1.2) \text{ km s}^{-1} \text{ Mpc}^{-1}$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Planck+WP</th>
<th>Planck+WP+highL</th>
<th>Planck+lensing+WP+highL</th>
<th>Planck+WP+highL+BAO</th>
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<td>$H_0$</td>
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<td>67.3 ± 1.2</td>
<td>67.3 ± 1.2</td>
<td>67.9 ± 1.0</td>
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<td>13.817 ± 0.048</td>
<td>13.813 ± 0.047</td>
<td>13.794 ± 0.044</td>
<td>13.798 ± 0.037</td>
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4.5 Estimating the Age of the Universe
Measuring the Age of the Universe

- Related to the Hubble time $t_H = 1/H_0$, but the exact value depends on the cosmological parameters
- Could place a *lower limit* from the ages of astrophysical objects (pref. the oldest you can find), e.g.,
  - **Globular clusters** in our Galaxy; known to be very old. Need stellar evolution isochrones to fit to color-magnitude diagrams
  - **White dwarfs**, from their observed luminosity function, cooling theory, and assumed star formation rate
  - **Heavy elements**, produced in the first Supernovae; somewhat model-dependent
  - Age-dating **stellar populations** in distant galaxies; this is very tricky and requires modeling of stellar population evolution, with many uncertain parameters
Ages of Globular Clusters

• We measure the age of a globular cluster by measuring the magnitude of the main sequence turnoff or the difference between that magnitude and the level of the horizontal branch, and comparing this to stellar evolutionary models of which estimate the surface temperature and luminosity of a stars as a function of time.

• There are a fair number of uncertainties in these estimates, including errors in measuring the distances to the GCs and uncertainties in the isochrones used to derive ages (i.e., stellar evolution models).

• Inputs to stellar evolution models include: oxygen abundance [O/Fe], treatment of convection, He abundance, reaction rates of $^{14}\text{N} + p \rightarrow ^{15}\text{O} + \gamma$, He diffusion, conversions from theoretical temperatures and luminosities to observed colors and magnitudes, and opacities; and especially distances.
Globular Cluster Ages

Schematic CMD and isochrones

Examples of actual model isochrones fits
Globular Cluster Ages From Hipparcos Calibrations of Their Main Sequences

Examples of g.c. main sequence isochrone fits, for clusters of a different metallicity (Graton et al.)

The same group has published two slightly different estimates of the mean age of the oldest clusters:

\[ \text{Age} = 11.8^{+2.1}_{-2.5} \text{ Gyr} \]

\[ \text{Age} = 12.3^{+2.1}_{-2.5} \text{ Gyr} \]
White Dwarf Cooling Curves

• White dwarfs are the end stage of stellar evolution for stars with initial masses < 8 M☉
• They are supported by electron degeneracy pressure (not fusion) and are slowly cooling and fading as they radiate
• We can use the luminosity of the faintest WDs in a cluster to estimate the cluster age by comparing the observed luminosities to theoretical cooling curves
• Theoretical curves are subject to uncertainties related to the core composition of white dwarfs, detailed radiative transfer calculations which are difficult at cool temperatures
• White dwarfs are faint so this is hard to do. Need deep HST observations
• It can be also done for the local Galactic disk, but that is an intrinsically younger population
An Example: White Dwarf Sequence of M4

Hansen et al. (2002) find an age of 12.7 ± 0.7 Gyr for the globular cluster M4

Blue = hydrogen atmosphere models
Red = helium atmosphere models for a 0.6 M☉ WD
Nucleocosmochronology

• Can use the radioactive decay of elements to age date the oldest stars in the galaxy
• Has been done with $^{232}$Th (half-life = 14 Gyr) and $^{238}$U (half-life = 4.5 Gyr) and other elements
• Measuring the ratio of various elements provides an estimate of the age of the universe given theoretical predictions of the initial abundance ratio
• This is difficult because Th and U have weak spectral lines so this can only be done with stars with enhanced Th and U (requires large surveys for metal-poor stars) and unknown theoretical predictions for the production of r-process (rapid neutron capture) elements
## Nucleocosmochronology:
An Example Isotope Ratios and Ages for a Single Star

<table>
<thead>
<tr>
<th>Chronometer Pair</th>
<th>Predicted</th>
<th>Observed</th>
<th>Age (Gyr)</th>
<th>Solar$^a$</th>
<th>Lower Limit (Gyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Th/Eu</td>
<td>0.507</td>
<td>0.309</td>
<td>10.0</td>
<td>0.4615</td>
<td>8.2</td>
</tr>
<tr>
<td>Th/Ir</td>
<td>0.0909</td>
<td>0.03113</td>
<td>21.7</td>
<td>0.0646</td>
<td>14.8</td>
</tr>
<tr>
<td>Th/Pt</td>
<td>0.0234</td>
<td>0.0141</td>
<td>10.3</td>
<td>0.0323</td>
<td>16.8</td>
</tr>
<tr>
<td>Th/U</td>
<td>1.805</td>
<td>7.413</td>
<td>$\geq13.4$</td>
<td>2.32</td>
<td>11.0</td>
</tr>
<tr>
<td>U/Ir</td>
<td>0.05036</td>
<td>0.0045</td>
<td>$\geq15.5$</td>
<td>0.0369</td>
<td>13.5</td>
</tr>
<tr>
<td>U/Pt</td>
<td>0.013</td>
<td>0.0019</td>
<td>$\geq12.4$</td>
<td>0.01846</td>
<td>14.6</td>
</tr>
</tbody>
</table>

$^a$ From Burris et al. 2001.

(from Cowan et al. 2002)

Mean = 13.8 +/- 4, but note the spread!
The Age of the Universe

• Several different methods (different physics, different measurements) agree that the lower limit to the age of the universe is $\sim 12 - 13$ Gyr

• This is in an excellent agreement with the age determined from the cosmological tests ($\sim 13.8$ Gyr)
The Age of the Galaxy is Approximately:

A. 7 Gyr
B. 10 Gyr
C. 14 Gyr
D. 17 Gyr
E. None of the above