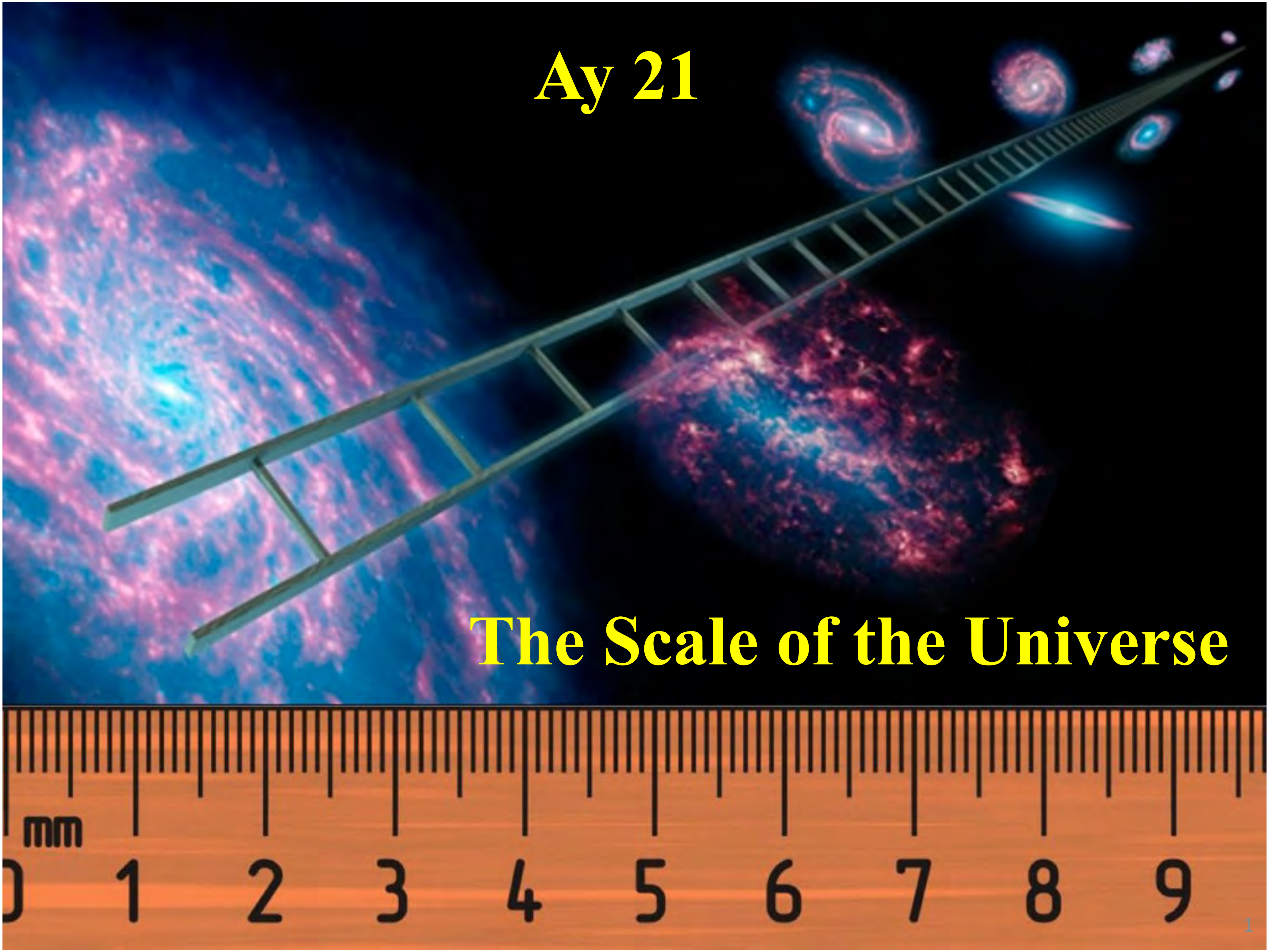

Ay 21
The Scale of the Universe
mm
0 1 2 3 4 5 6 7 8 9

The Scale of the Universe

- The **Hubble length**, $D_H = c/H_0$, and the **Hubble time**, $t_H = 1/H_0$ give the approximate spatial and temporal scales of the universe
- H_0 is a "scale parameter", and is *independent* of the "shape parameters" (expressed as density parameters) Ω_m, Ω_Λ, Ω_k, w, etc., which govern the global geometry and dynamics of the universe
- Distances to galaxies, quasars, etc., scale linearly with H_0, $D \approx cz / H_0$. They are necessary in order to convert observable quantities (e.g., fluxes, angular sizes) into physical ones (luminosities, linear sizes, energies, masses, etc.)

Measuring the Scale of the Universe

- The ***only*** clean-cut distance measurements in astronomy are from trigonometric parallaxes. Everything else requires physical modeling and/or a set of calibration steps (the ***"distance ladder"***), and always some statistics:

Use parallaxes to calibrate some set of distance indicators

→ Use them to calibrate another distance indicator further away

→ And then another, reaching even further

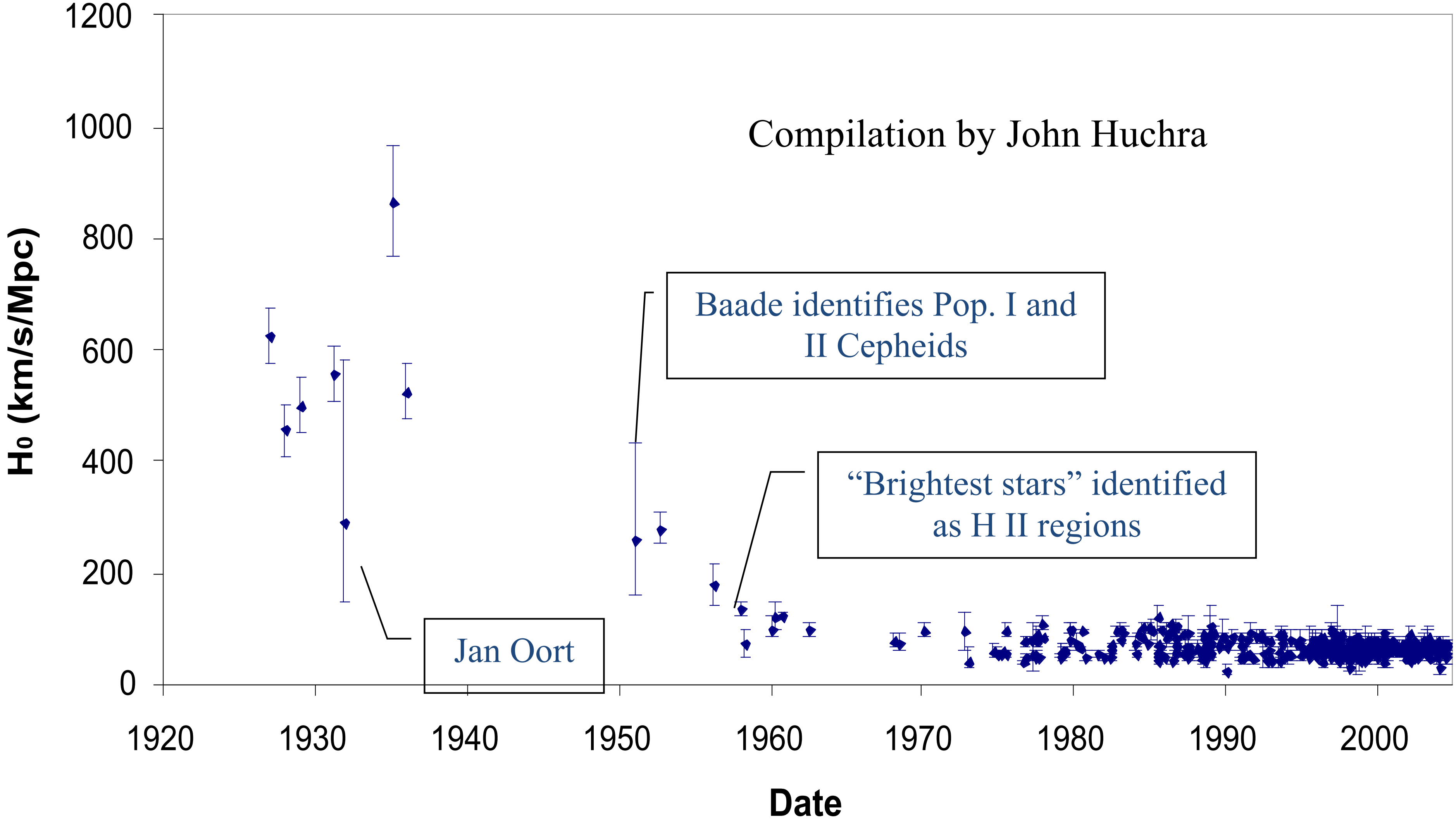
→ etc. etc.

→ Until you reach a "pure Hubble flow"

- The age of the universe can be constrained independently from the H_0, by estimating ages of the oldest things one can find around (e.g., globular clusters, heavy elements, white dwarfs)

The History of the H_0 Measurements

Major revisions downwards happened as a result of recognizing some major systematic errors



Modern value: $H_0 \sim 70$ km/s/Mpc

Distance Ladder: Methods

Methods yielding absolute distances:

- Parallax (trigonometric, secular, and statistical)
- The moving cluster method - has some assumptions
- Baade-Wesselink method for pulsating stars
- Expanding photosphere method for Type II SNe
- Sunyaev-Zeldovich effect
- Gravitational lens time delays

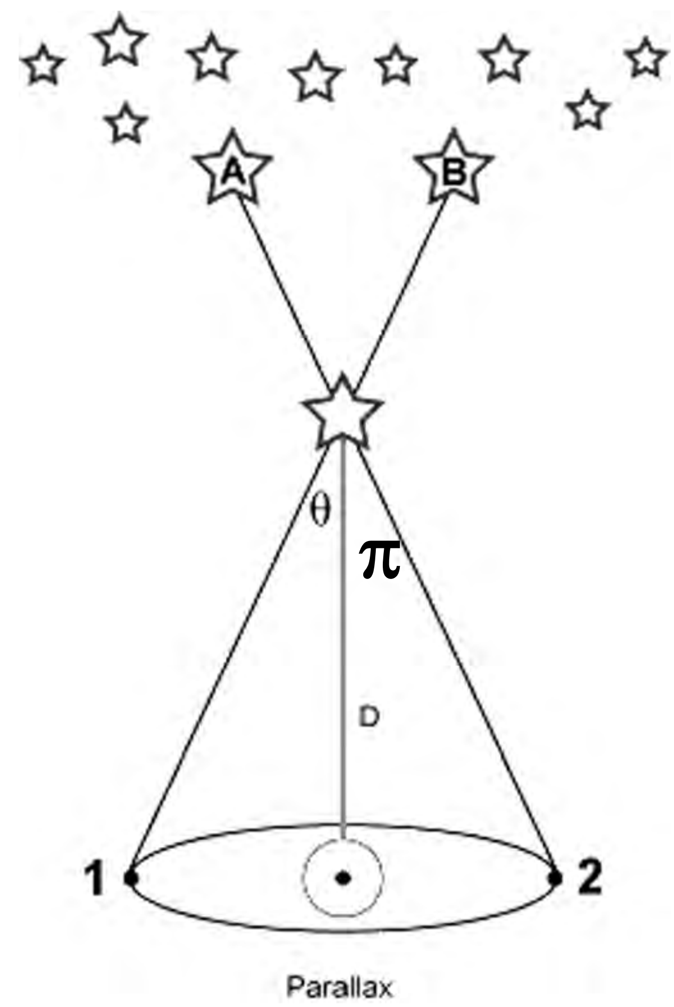
} ***Model dependent!***

Secondary distance indicators: *"standard candles"*, requiring a calibration from an absolute method applied to local objects - ***the distance ladder***:

- Pulsating variables: Cepheids, RR Lyrae, Miras
- Main sequence fitting to star clusters
- Tip of the red giant branch
- Planetary nebula luminosity function
- Globular cluster luminosity function
- Surface brightness fluctuations
- Tully-Fisher, D_n-σ, FP scaling relations for galaxies
- Type Ia Supernovae
- … etc.

Trigonometric Parallax

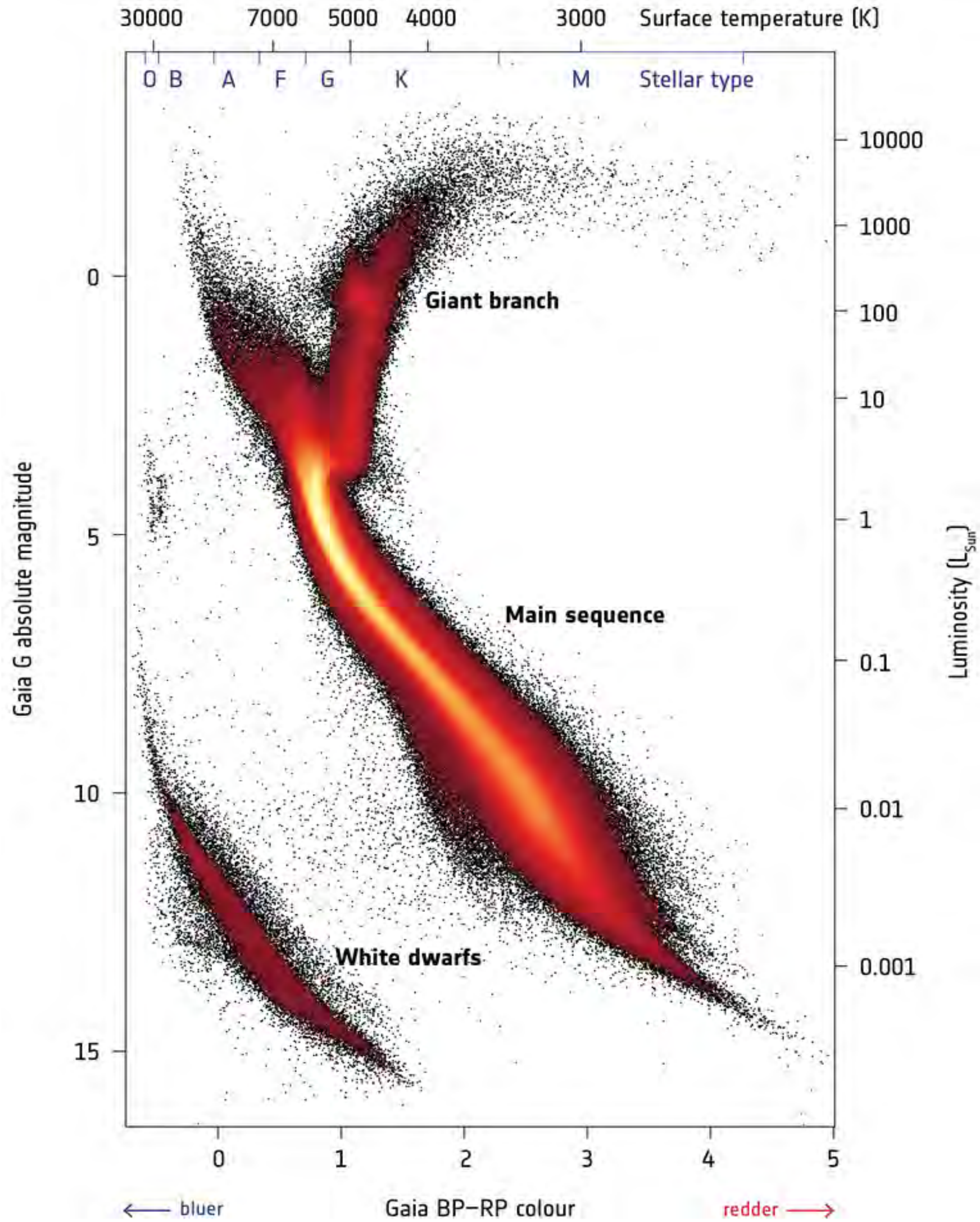
- Straightforward, geometric, and the only “true” method - the fundament of the distance scale
- Measure the shift in observed position of nearby stars relative to background stars as earth moves in orbit around the Sun
- Can get distances of out to ~ 10 kpc from the ***Gaia*** satellite, for ~ 10^9 stars
- Parallaxes provide absolute calibrations to the next rung of the distance ladder – subdwarfs, Cepheids, nearby star clusters



D [pc] = **1** / π [arcsec]

The *Gaia* Revolution

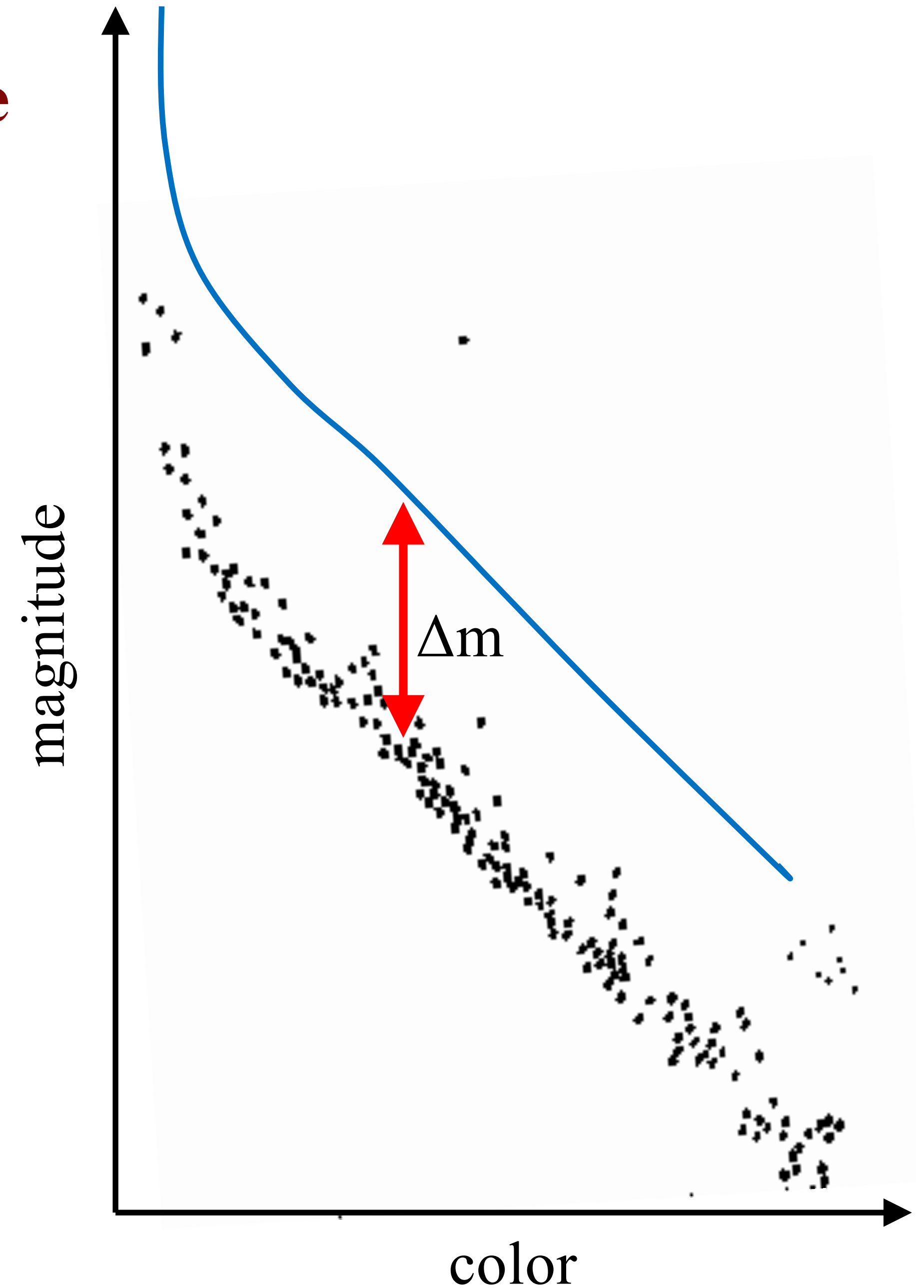
Gaia mission measured parallaxes on nearly 2 billion stars, thus fully calibrating the Galactic distance scale, the absolute calibration of the H-R diagram (giving the distances to star clusters), and including Cepheids and other variables useful for the extragalactic distance scale measurements



Main Sequence Fitting for Star Clusters

Luminosity (distance dependent) vs. temperature or color (distance independent)

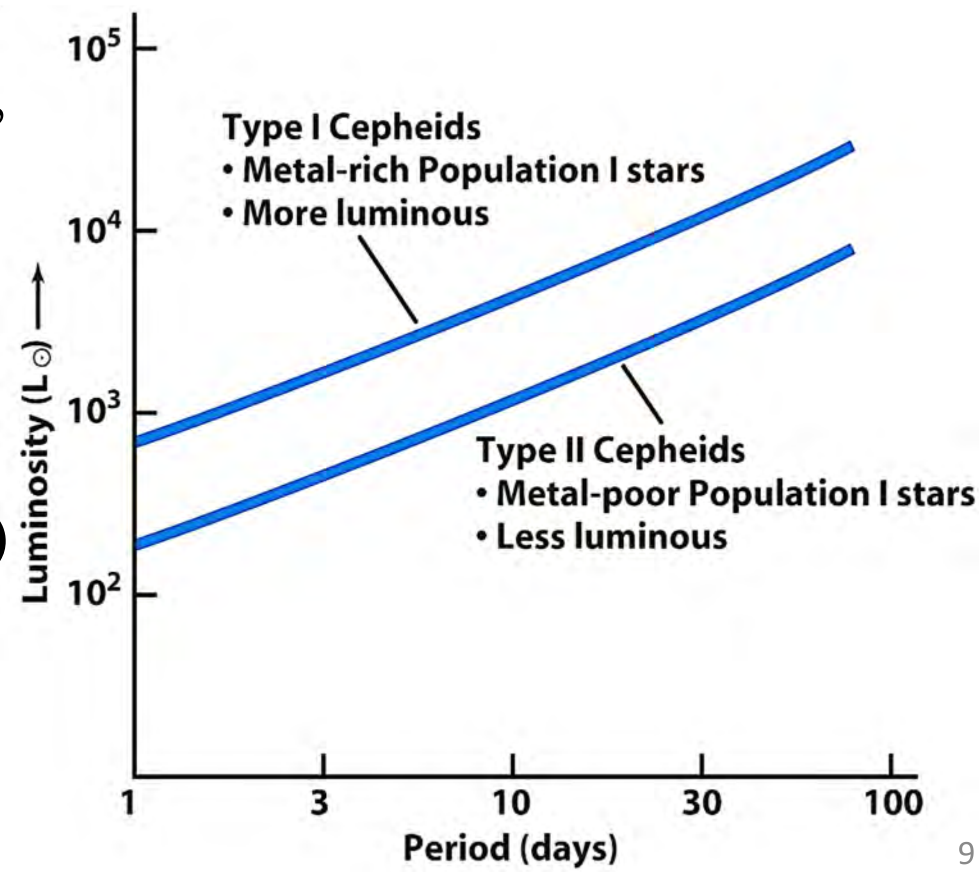
- Can measure distance to star clusters (open or globular) by fitting their main sequence with clusters with known distances from Gaia
- The apparent magnitude difference gives the ratio of distances, as long as we know the reddening (extinction)!
- For globular clusters we use parallaxes to nearby subdwarfs (metal-poor main sequence stars)



Pulsating Variables

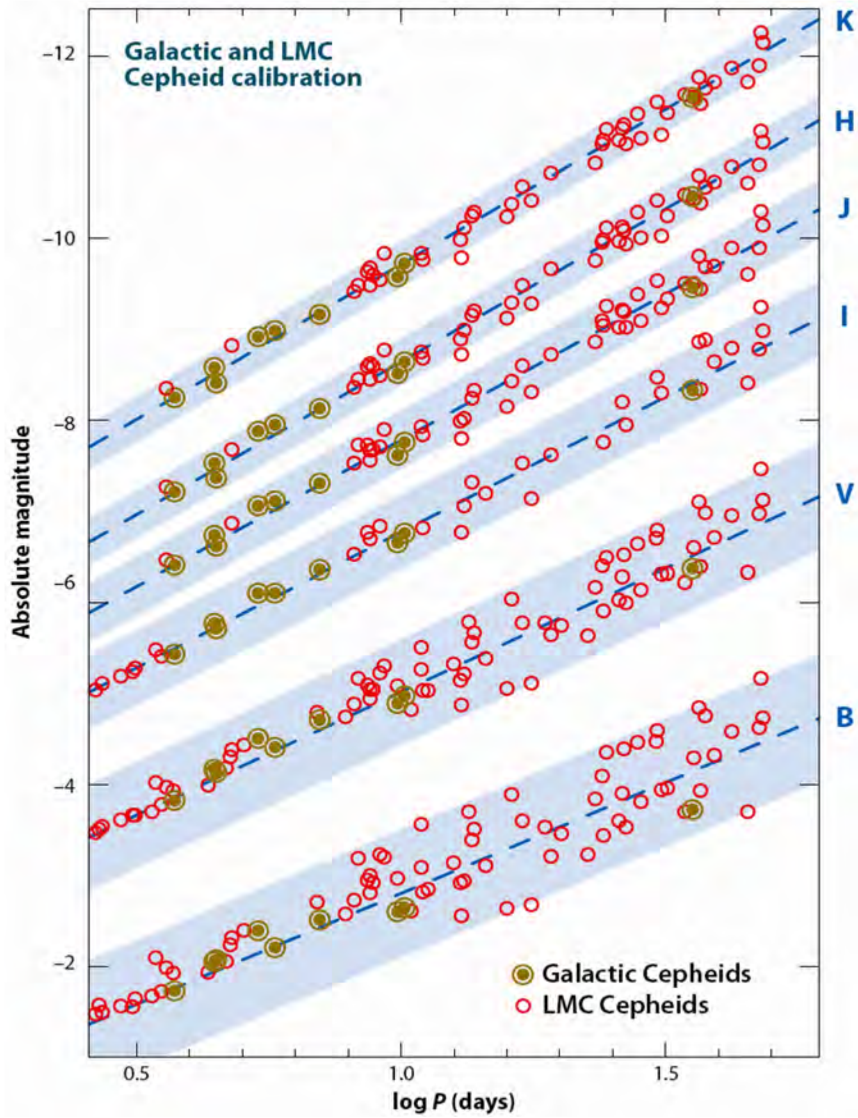
- Stars in the instability strip in the HR diagram
- All obey empirical period - luminosity (distance independent vs. dependent) relations which can be calibrated to yield distances
- Different types (in different branches of the HRD, different stellar populations) have different relations

- Cepheids are high-mass, luminous, upper MS, Pop. I stars
- RR Lyrae are low-mass, metal-poor (Pop. II), HB stars, often found in globulars
- Long-period variables (e.g., Miras) pulsate in a fashion that is less well understood



Cepheids

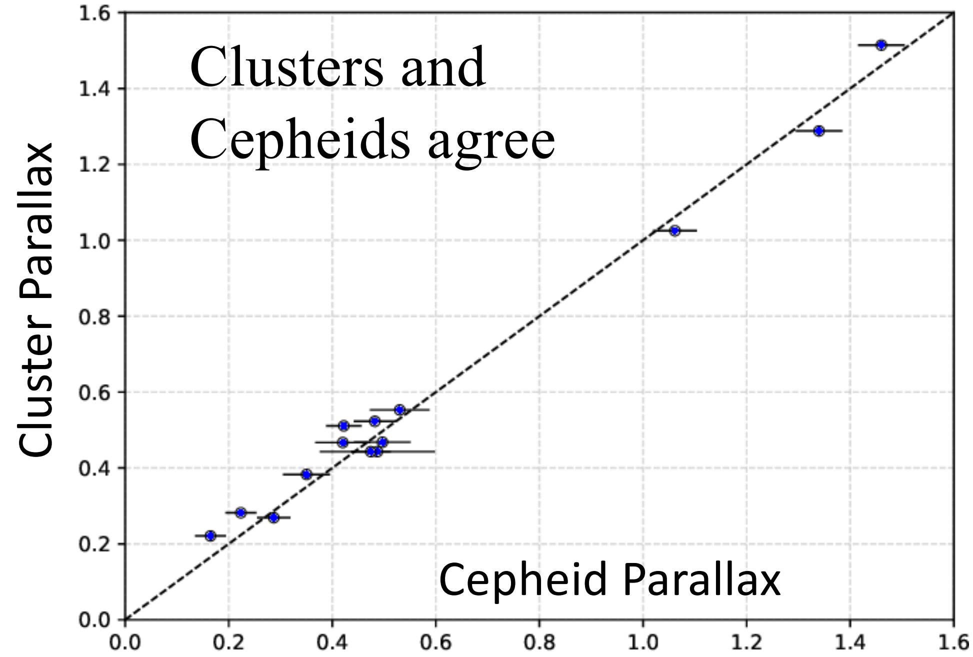
- Luminous ($M \sim -4$ to -7 mag), pulsating variables, evolved high-mass stars on the instability strip in the H-R diagram
- *Henrietta Leavitt* (1912) found a period-luminosity relation for Cepheids in the SMC: brighter ones have longer periods
- **Advantages:** Bright and easily seen in galaxies(out to ~25 Mpc with the HST, stellar pulsation is well understood
- **Disadvantages:** Relatively rare, period may depend on metallicity or color, need multiple epoch observations, found near star forming regions, so extinction corrections are necessary
- Redder bands have smaller scatter, but also shallower slope



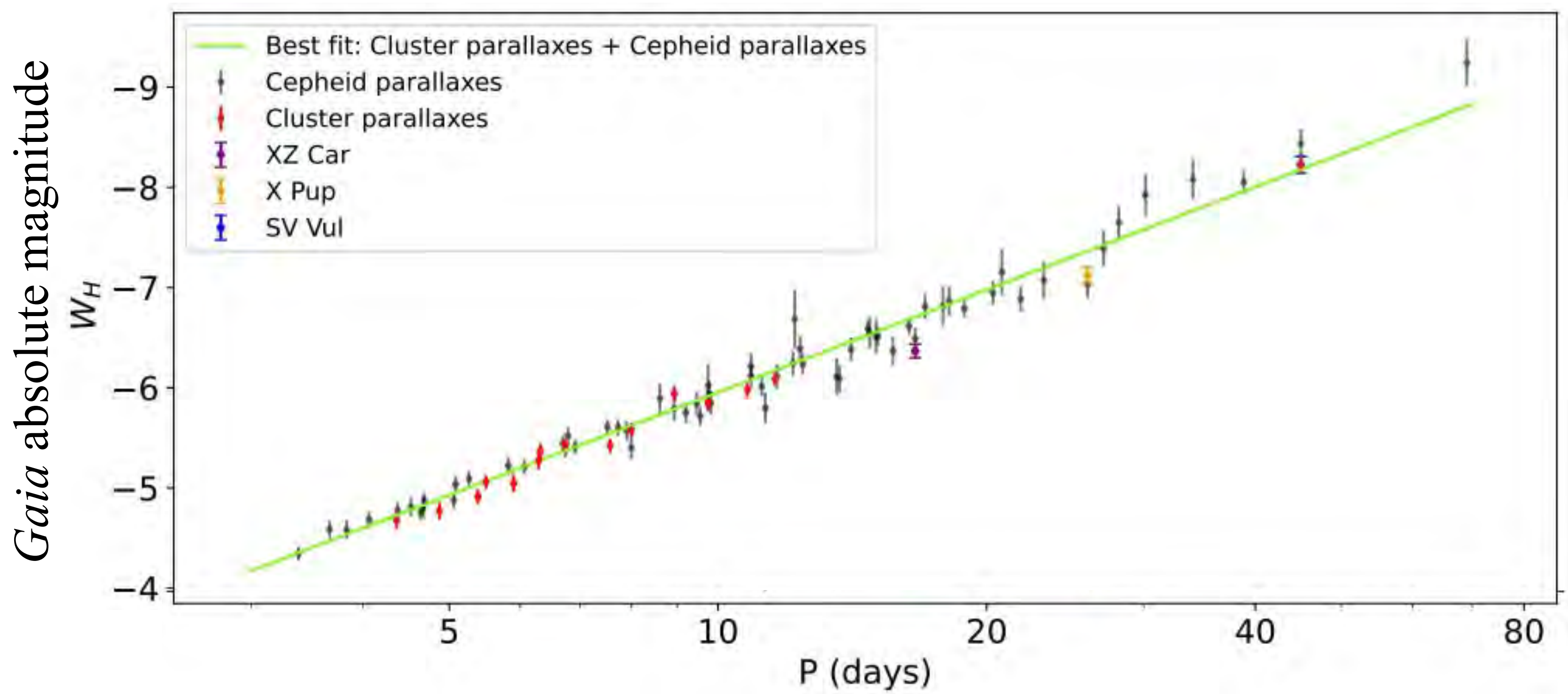
- Calibrated using parallaxes or the H-R diagram

Gaia Calibration of the Cepheid Period-Luminosity Relation

The basic calibration of the extragalactic distance scale

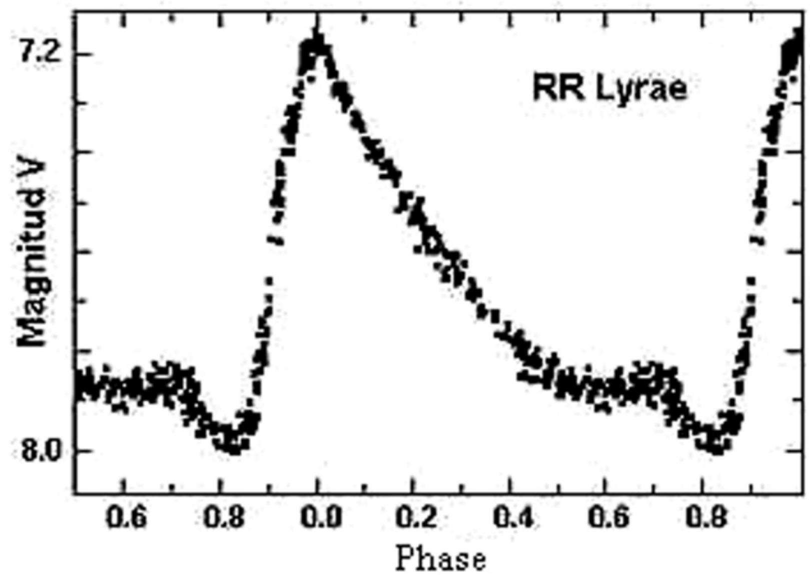


Cepheid period vs. luminosity:



RR Lyrae Stars

- Pulsating variables, evolved old, low mass, low metallicity stars
  - Pop II indicator, found in globular clusters, galactic halos
- Lower luminosity than Cepheids, $M_V \sim 0.75 +/- 0.1$
  - There may be a metallicity dependence
- Have periods of 0.4 – 0.6 days, so don't require as much observing to find or monitor
- **Advantages:** less dust, easy to find
- **Disadvantages:** fainter (2 mag fainter than Cepheids). Used for Local Group galaxies only. The calibration is still uncertain (uses globular cluster distances from their main sequence fitting; or from Magellanic Clouds clusters, assuming that we know their distances)

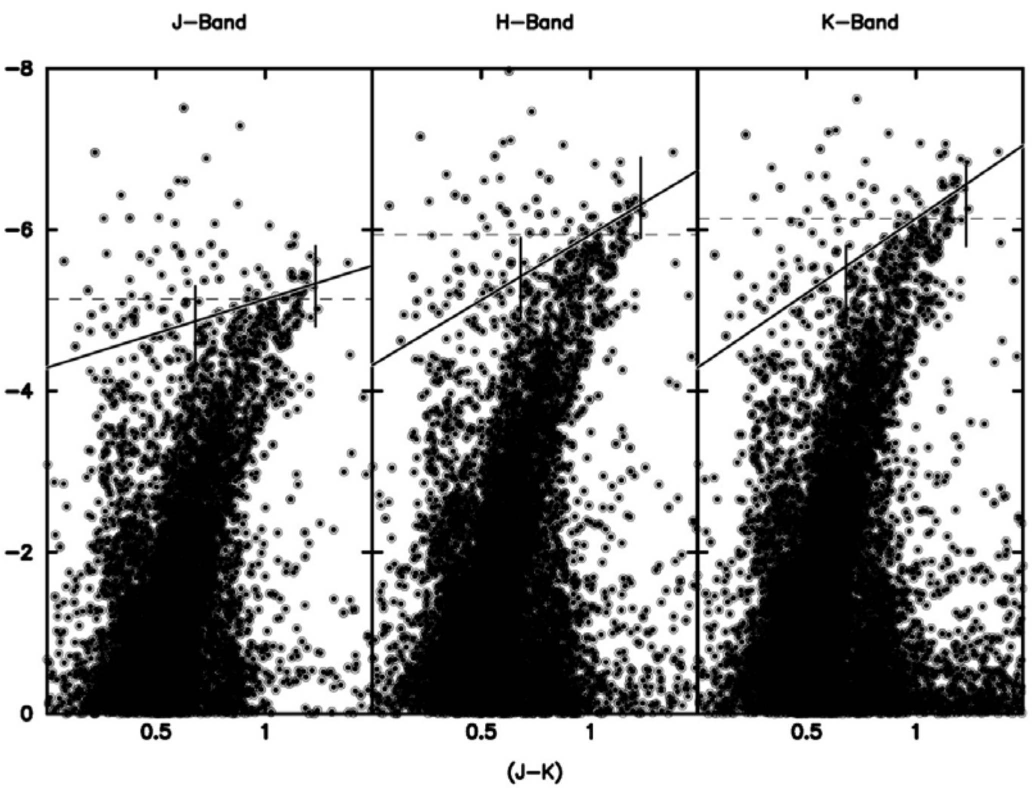


Tip of the Red Giant Branch

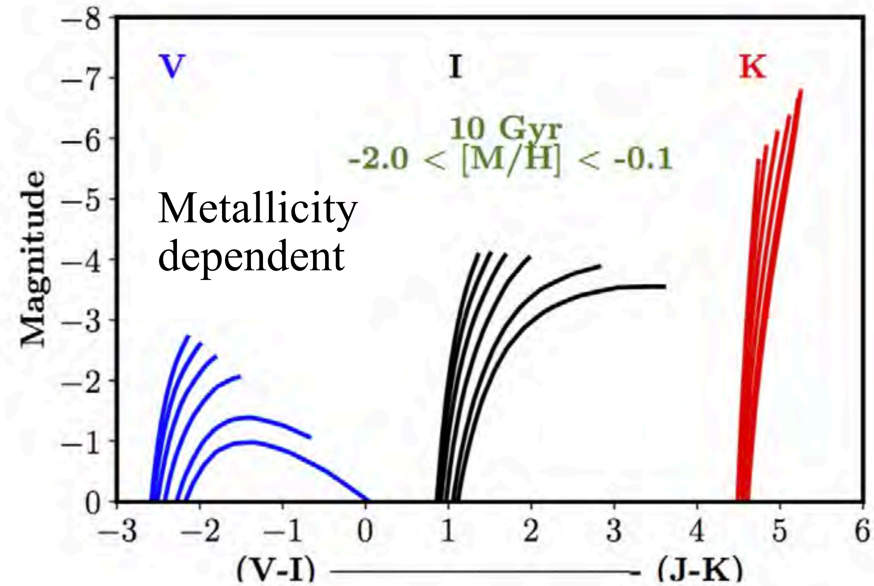
Well understood from stellar evolution (transition from H burning in a shell to He burning in the core)

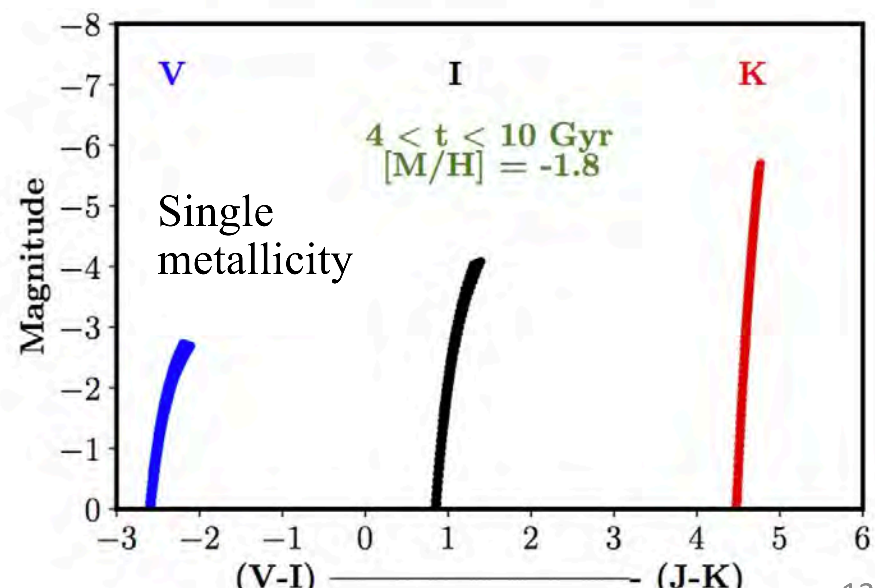
Bright and competitive with the Cepheids to comparable distances

Examples from Galactic star clusters:



Freedman et al. (2020)





The Baade-Wesselink Method

Luminosity from the Stefan-Boltzmann formula: $L = \sigma R^2 T^4$

Consider a ***pulsating star*** at a *minimum*, with a measured temperature T_1 and observed flux f_1 with radius R_1, then:

$$f_1 = \frac{4\pi R_1^2 \sigma T_1^4}{4\pi D^2}$$

At a *maximum*, with a measured temperature T_2 and observed flux f_2 with radius R_2:

$$f_2 = \frac{4\pi R_2^2 \sigma T_2^4}{4\pi D^2}$$

Note: T_1 , T_2 , f_1 , f_2 are directly observable! Just need the radius… So, from spectroscopic observations we can get the photospheric velocity $v(t)$, from this we can determine the change in radius, ΔR:

$$R_2 = R_1 + \Delta R = R_1 + \int_{t_1}^{t_2} v(t)\, dt$$

➞ **3 equations, 3 unknowns, solve for R_1, R_2, and D**

Difficulties: the effects of the stellar atmospheres (not a perfect black body), and deriving the true radial velocity from what we observe

The Expanding Photosphere Method (EPM)

Similar to the Baade-Wesselink method, and used for ***Type II SNe***, but *not as accurate* as the ***Type Ia SN*** standard candles

Assume that SN photospheres radiate as dilute blackbodies:

$$\theta_{ph} = \frac{R_{ph}}{D} = \sqrt{\frac{F_\lambda}{\zeta^2 \pi B_\lambda(T)}}$$

Apparent Diameter

Fudge factor to account for the deviations from blackbody, from spectra models

Determine the radius by monitoring the expansion velocity

$$R_{ph} = v_{ph}(t - t_0) + R_0$$

And solve for the distance:

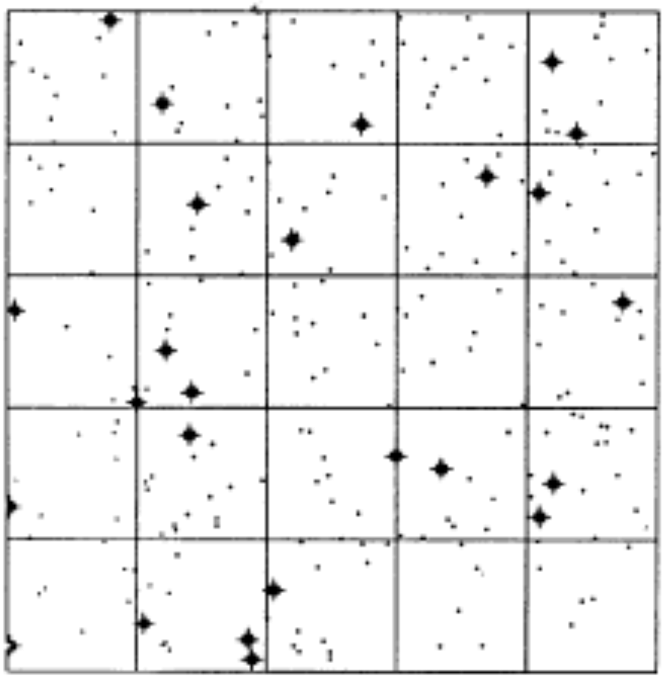
$$t = D\left(\frac{\theta_{ph}}{v_{ph}}\right) + t_0$$

Surface Brightness Fluctuations

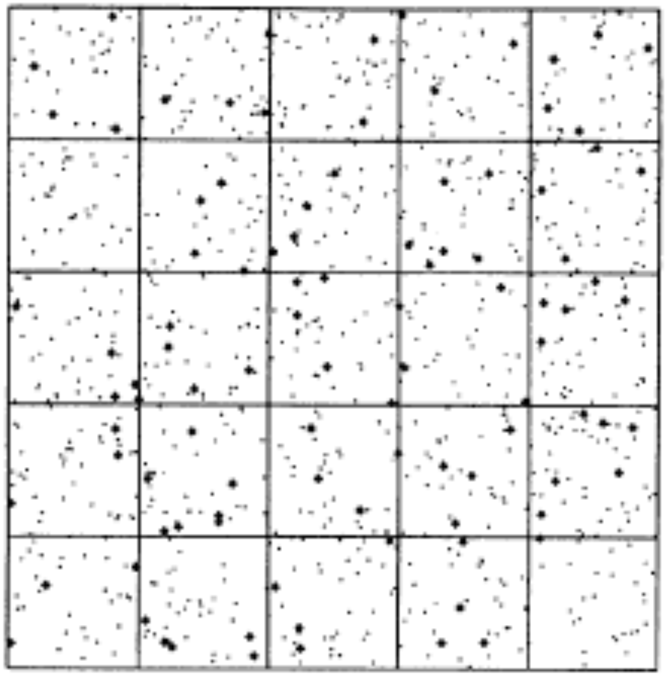
Useful for the distances to old stellar populations, e.g., bulges, ellipticals

Consider stars projected onto a pixel grid of your detector:

A nearby galaxy



A galaxy twice farther away



The more distant galaxy appears "smoother" at the same surface brightness: more stars per pixel have smaller √N fluctuations

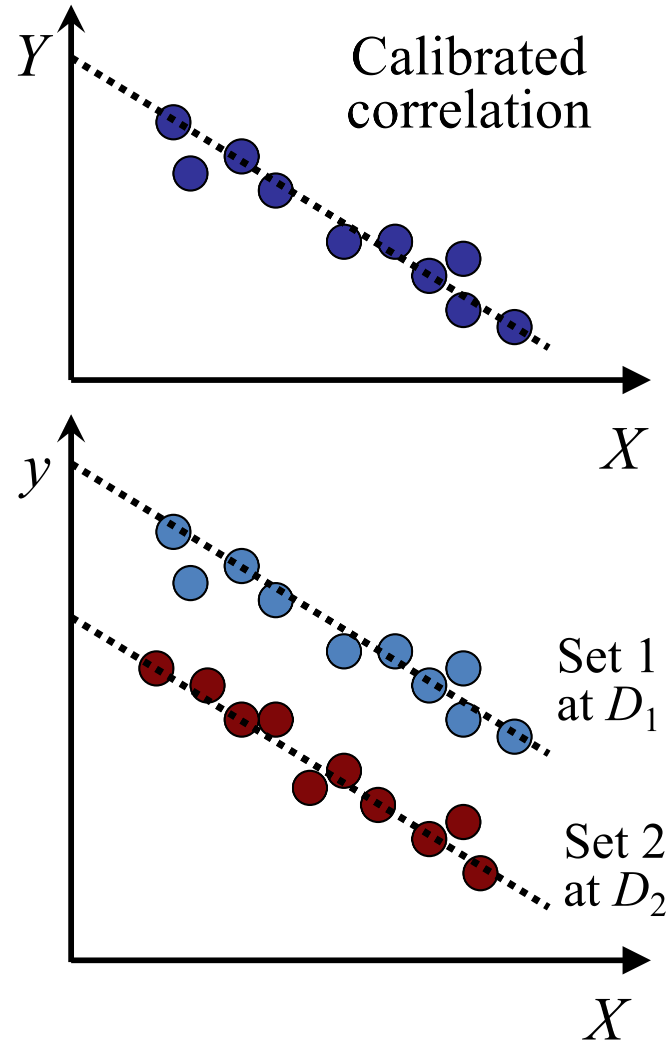
This gives us the *relative* distances, to get the *absolute* distances, we need a set of galaxies with the known distances to calibrate the relation

Galaxy Scaling Relations

- Once a set of distances to galaxies of some type is obtained, one finds *correlations between distance-dependent quantities* (e.g., luminosity, radius) *and distance-independent ones* (e.g., rotational speeds for disks, or velocity dispersions for ellipticals and bulges, surface brightness, etc.). These are called ***distance indicator relations.*** Examples:
  - **Tully-Fisher relation** for spirals (luminosity vs. rotation speed)
  - **Fundamental Plane** relations for ellipticals (radius vs. a combination of velocity dispersion and surface brightness)
- These relations *must be calibrated locally* using other distance indicators, e.g., Cepheids or surface brightness fluctuations; then they can be extended into the general Hubble flow regime
- Their origins - and thus their universality - are not yet well understood. There may be some systematic variations

The Basic Idea:

- Need a correlation between a distance-independent quantity, "X", (e.g., temperature or color for stars in the H-R diagram, or the period for Cepheids), and a distance-dependent one, "Y", (e.g., stellar absolute magnitude, *M*)



- Two sets of objects at different distances will have a systematic shift in the *apparent* versions of "y" (e.g., stellar apparent magnitude, *m*), from which we can deduce their ***relative distance***

- This works for stars (main sequence fitting, period-luminosity relations), but can we find such relations for galaxies?

The Tully-Fisher Relation

- A well-defined luminosity vs. rotational speed (often measured as a H I 21 cm line width) relation for spirals:

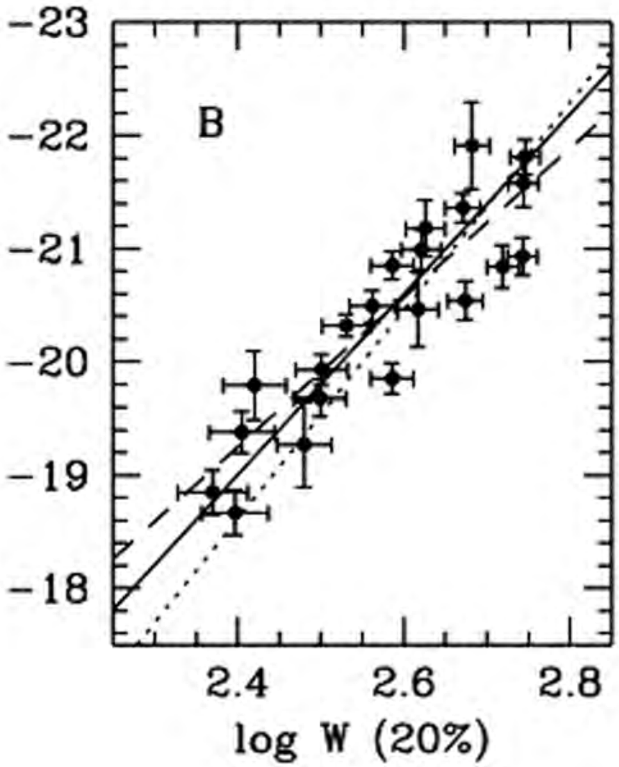
$$\boldsymbol{L \sim v_{rot}^{\gamma}, \gamma \approx 4}, \text{ varies with wavelength}$$

Or: $M = b \log(W) + c$, where:

  - M is the absolute magnitude
  - W is the Doppler broadened line width, typically measured using the HI 21cm line, corrected for inclination $W_{true} = W_{obs} / \sin(i)$
  - Both the slope b and the zero-point c can be measured from a set of nearby spiral galaxies with well-known distances
  - The slope b can be also measured from any set of galaxies with roughly the same distance - e.g., galaxies in a cluster - even if that distance is not known
- *Scatter* is ~ 10-20% at best, which limits the accuracy
- Problems include dust extinction, so working in the redder bands is better

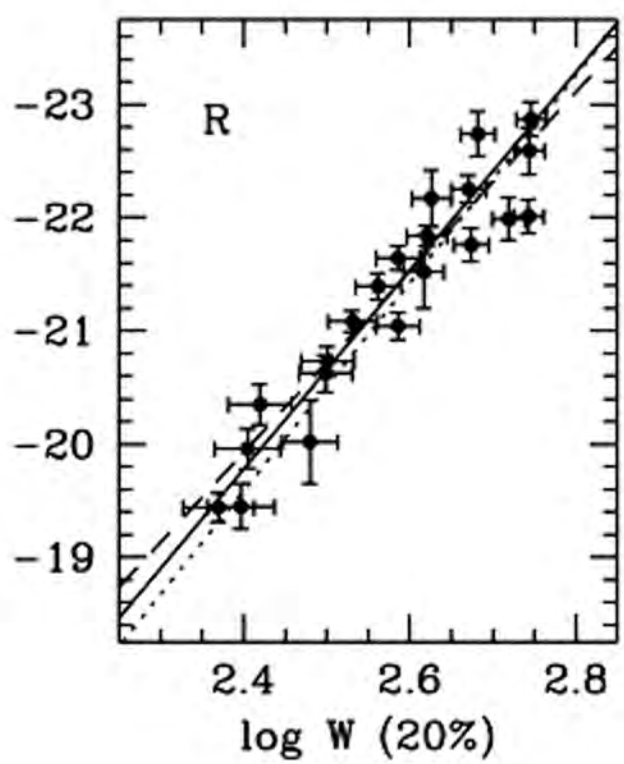
Tully-Fisher Relation at Various Wavelengths

Blue



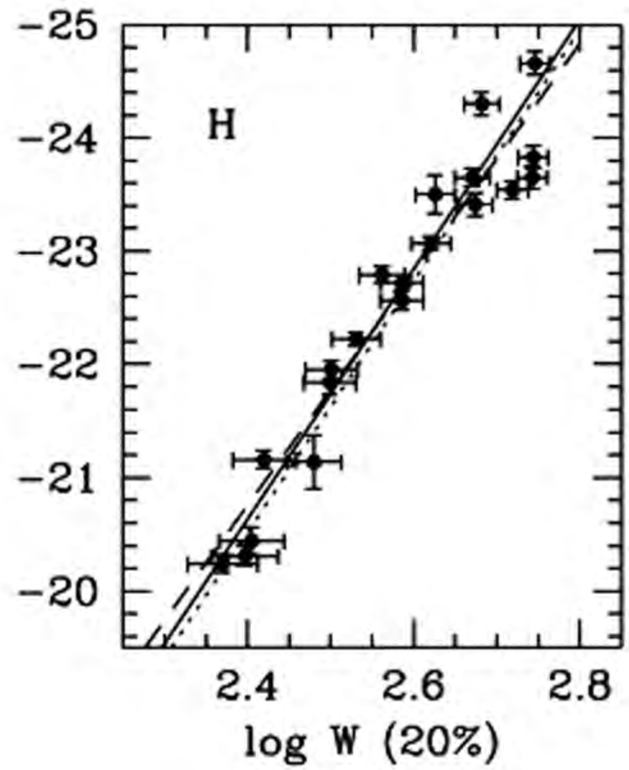
Slope= 3.2
Scatter=0.25 mag

Red



Slope= 3.5
Scatter=0.25 mag

Infrared

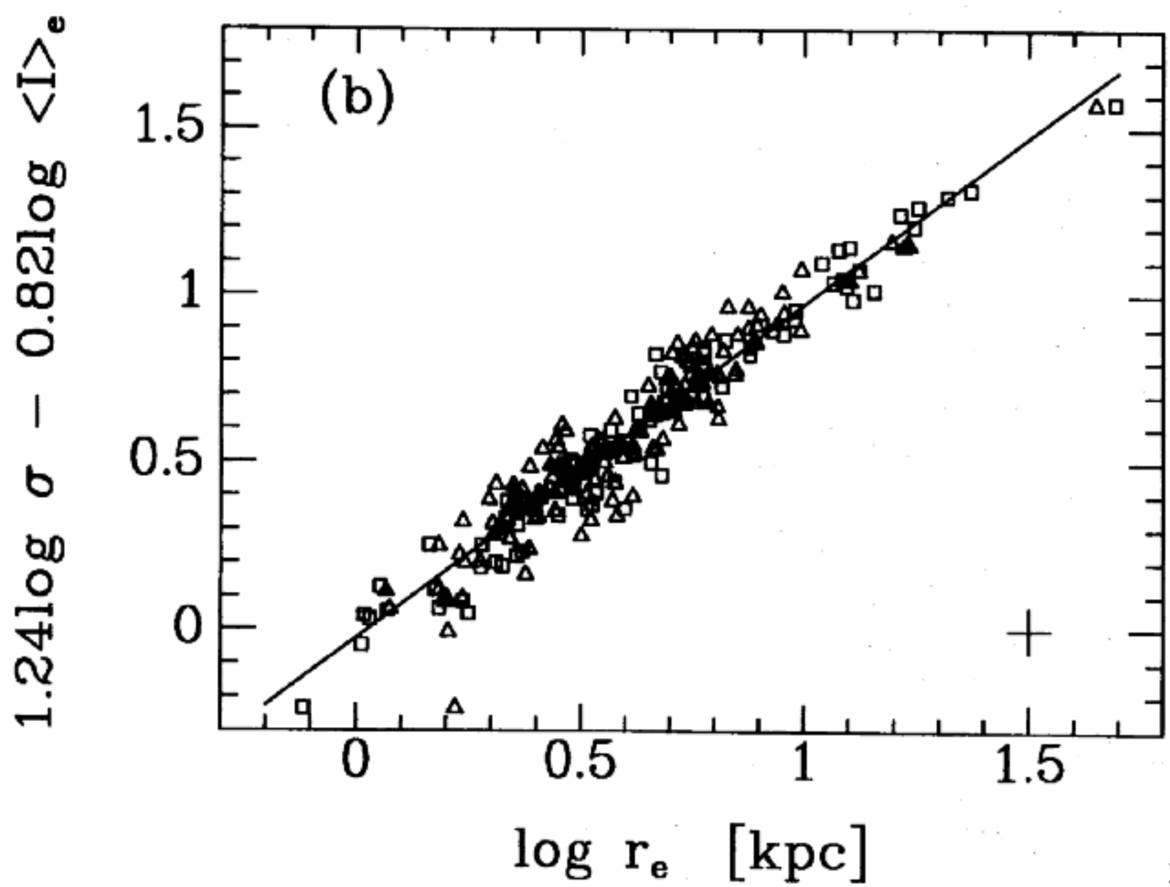


Slope= 4.4
Scatter=0.19 mag

Fundamental Plane Relations

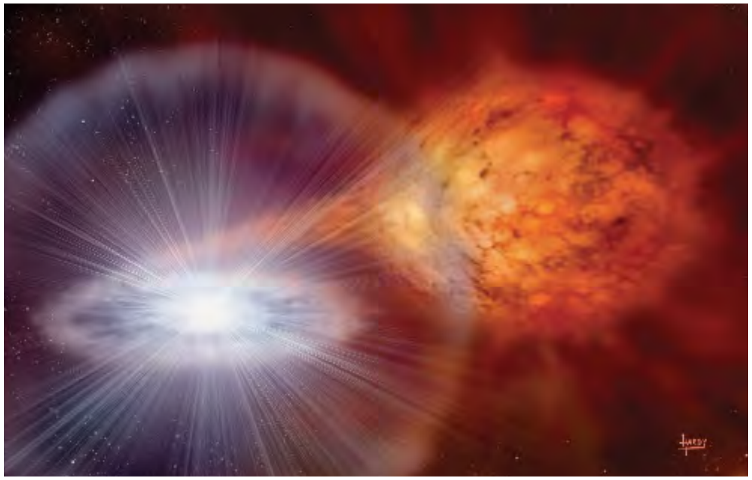
- A set of bivariate scaling relations for elliptical galaxies, including relations between distance dependent quantities such as radius or luminosity, and a combination of two distance-independent ones, such as velocity dispersion or surface brightness

- Scatter ~ 10%, but it could be lower?
- Usually calibrated using surface brightness fluctuations



Supernovae (SNe) as Standard Candles

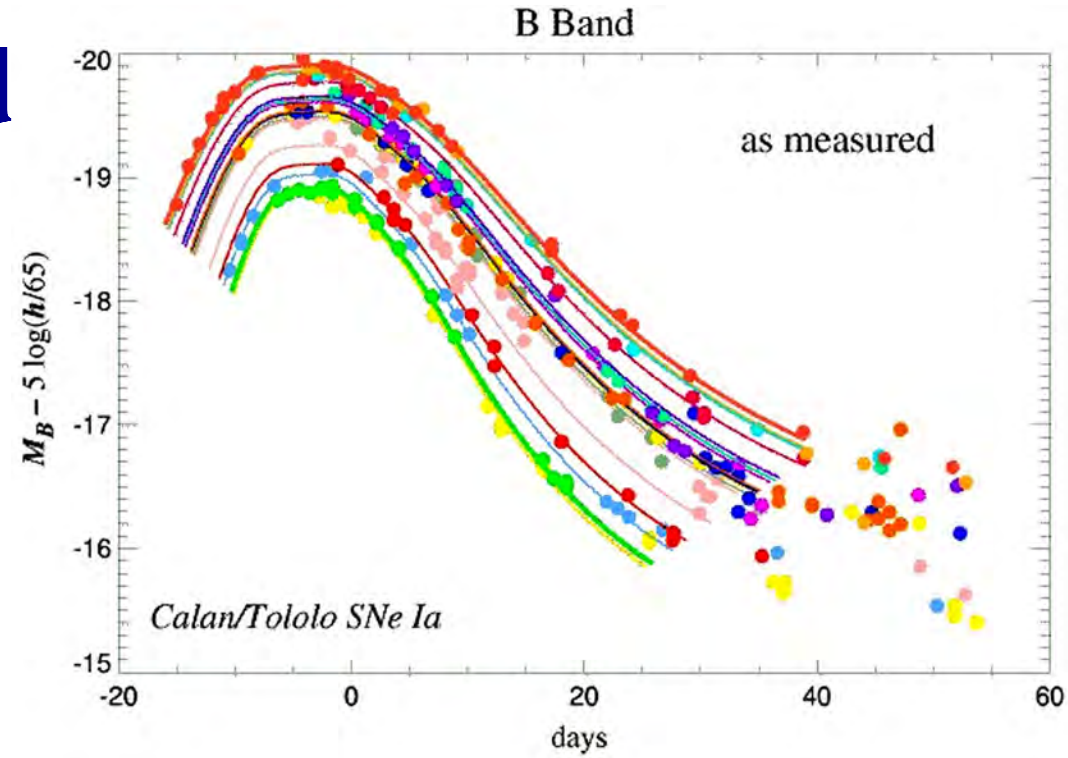
- *Bright and thus visible far away*
- *Two* different types of SNe are used as standard candles:
- **Type Ia** from a binary white dwarfs merging or accreting material, going over the Chandrasekhar limit, and detonating
  - Remarkably homogenous properties: Same type of an object exploding in each case; explains the uniformity of their light curves
  - Pretty good standard candles, peak $M_V \sim -19.3$
  - Light curves can be standardized to a peak magnitude scatter of ~ 10%

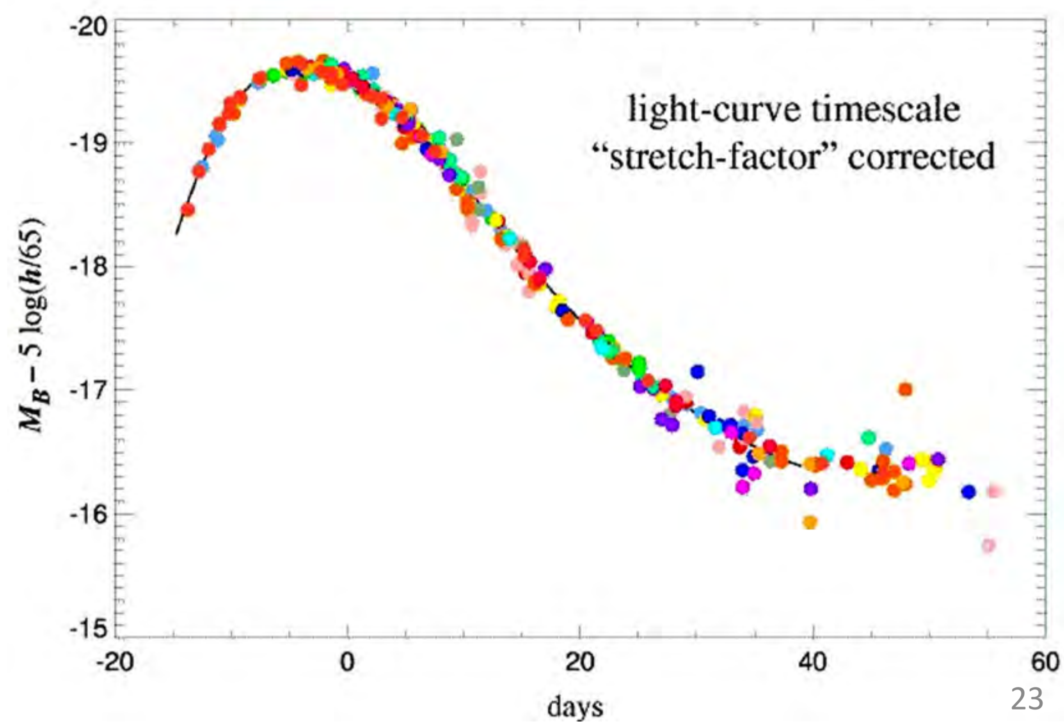


  - Old stellar population: observed in elliptical galaxies as well as spirals
  - Light curve fit by radioactive decay of about a Solar mass of ^{56}Ni
- **Type II** from collapse of massive stars (also Type Ib)
  - Not as bright as Type Ia's
  - Not good standard candles, but we can measure their distances using the "Expanding Photosphere Method"

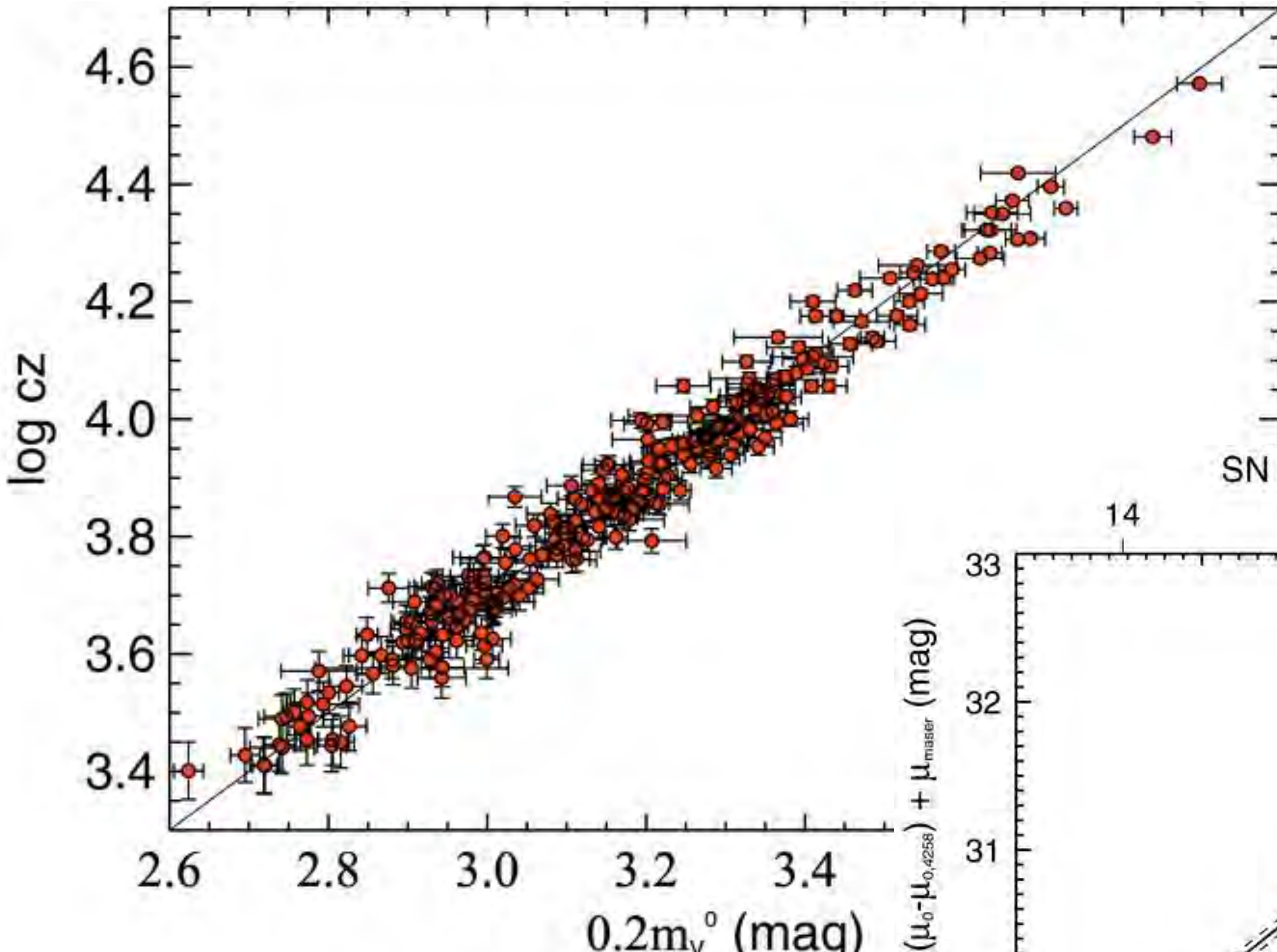
SNe Ia as Standard Candles

- The peak brightness of a SN Ia correlates with the shape of its light curve (steeper → fainter)
- Correcting for this effect standardizes the peak luminosity to ~10% or better
- However, the absolute zero-point of the SN Ia distance scale has to be calibrated externally, e.g., with Cepheids



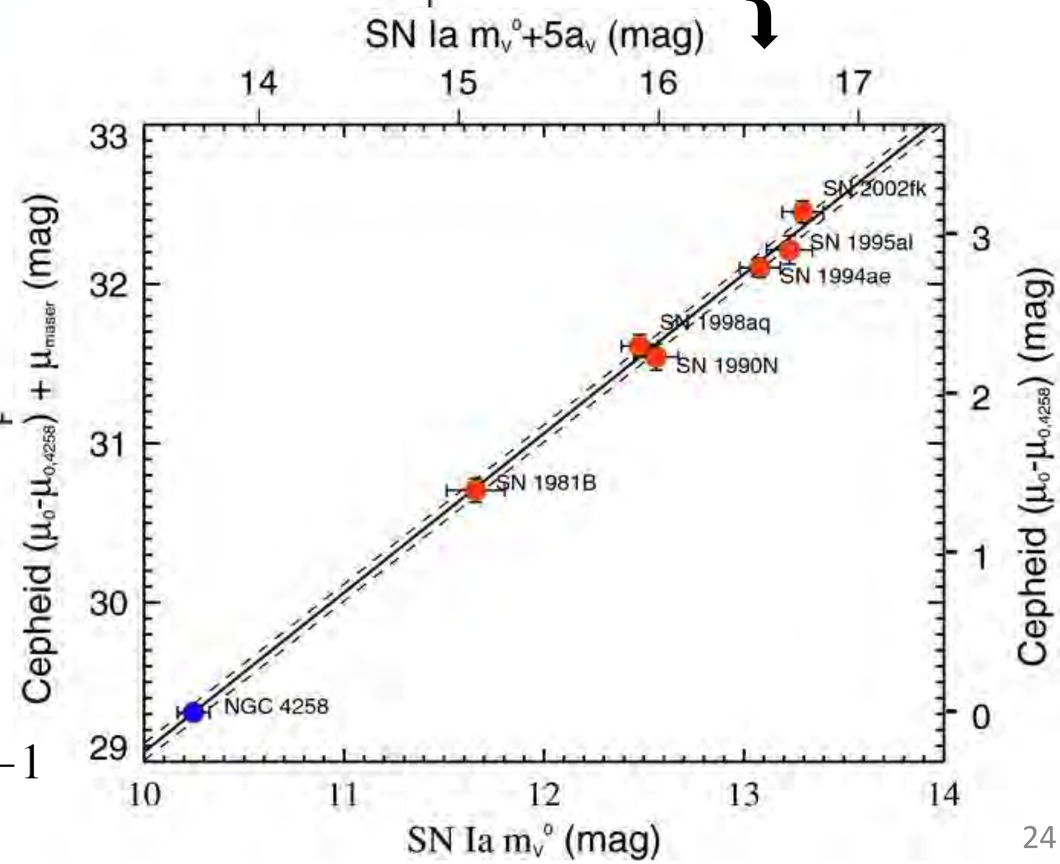


The Low-Redshift SN Ia Hubble Diagram



Riess et al. 2009
(also Riess et al. 2011
– slight changes)

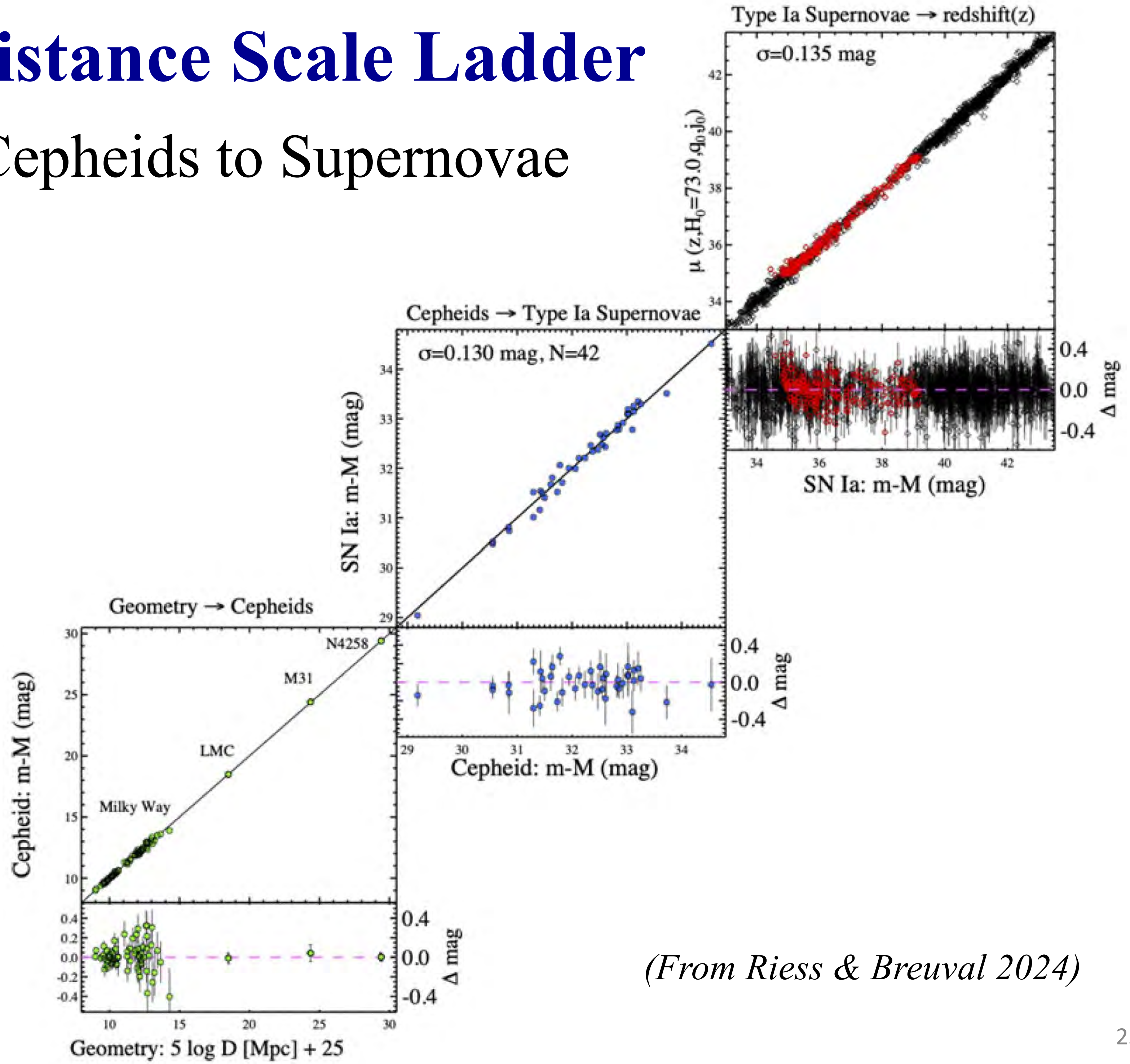
Cepheid calibration



$$H_0 = 74.2 \pm 3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Local Distance Scale Ladder

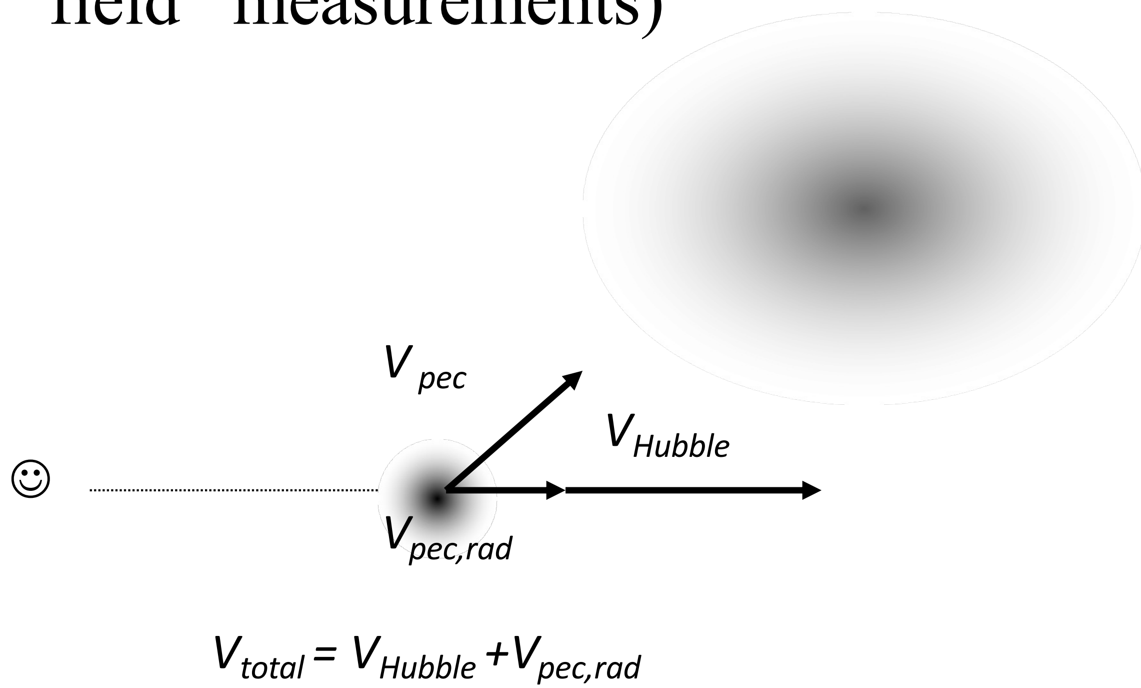
From Cepheids to Supernovae

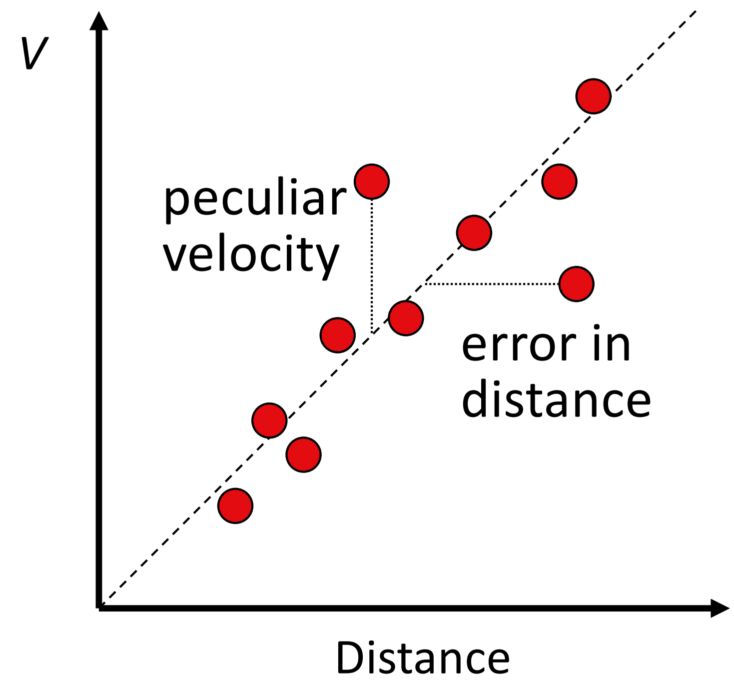


(From Riess & Breuval 2024)

Another Problem: Peculiar Velocities

- Large-scale density field inevitably generates a peculiar velocity field, due to the acceleration over the Hubble time
- Note that we can in practice only observe the radial component
- Peculiar velocities act as a noise (on the $V = cz$ axis, orthogonal to errors in distances) in the Hubble diagram - and could thus bias the measurements of the H_0 (which is why we want "far field" measurements)



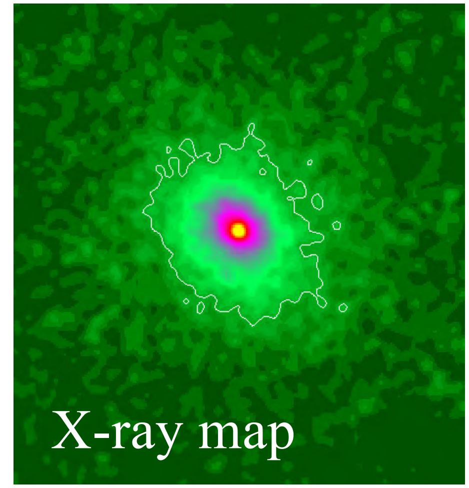


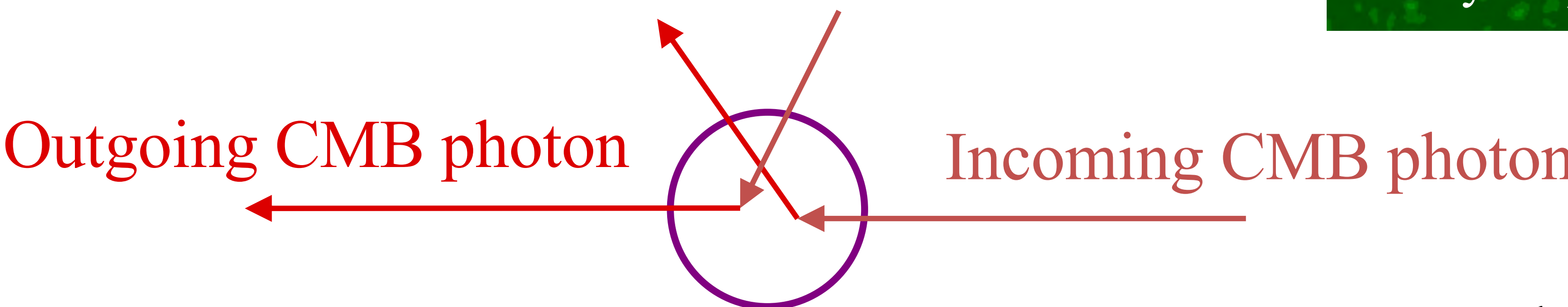
Pushing Into the Hubble Flow

- Hubble's law: $\boldsymbol{v = H_0 D}$
- But the total observed velocity v is a combination of the cosmological expansion, and the *peculiar velocity* of any given galaxy, $\boldsymbol{v = v_{cosmo} + v_{pec}}$
- Typically $v_{pec} \sim$ a few hundred km/s, produced bythe gravitational infall into the local large scale structures (e.g., the local supercluster), with the characteristic scales of tens of Mpc
- Thus, one wants to measure H_0 on scales greater than tens of Mpc, and where $\boldsymbol{v_{cosmo} >> v_{pec}}$. This is the "pure" Hubble flow regime
- This requires *luminous standard candles* - galaxies or Supernovae

Synyaev-Zeldovich Effect

- Clusters of galaxies are filled with hot X-ray gas
- The electrons in the intracluster gas will scatter the background photons from the CMBR to higher energies (frequencies) and distort the blackbody spectrum





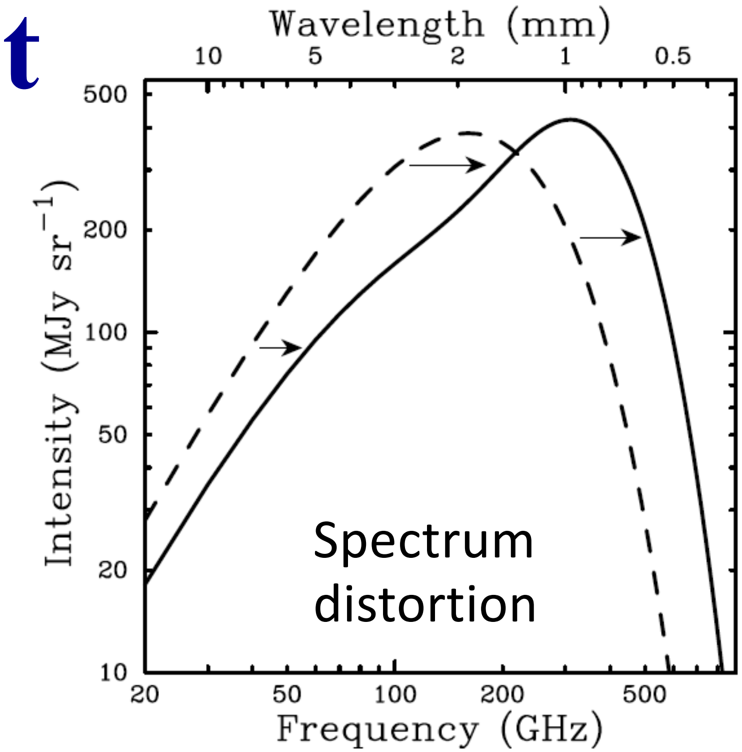
Galaxy Cluster with hot gas

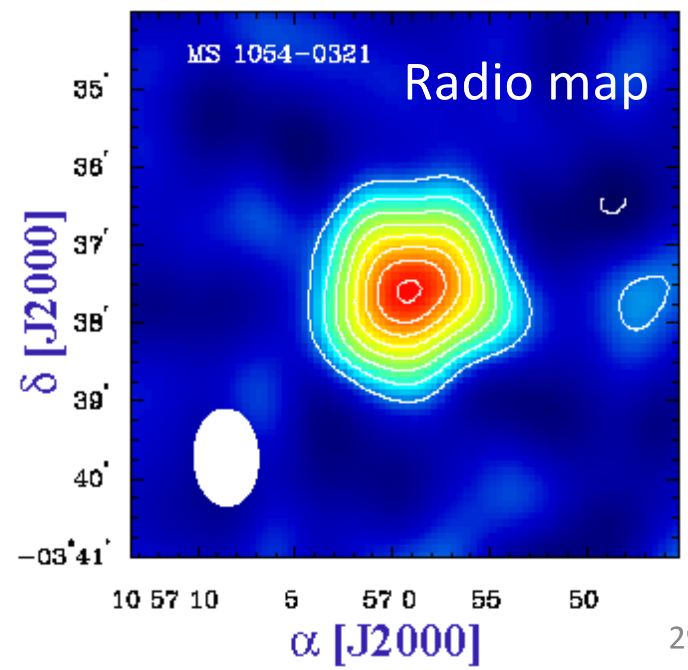
This is detectable as a slight temperature dip or bump (depending on the frequency) in the radio map of the cluster, against the uniform CMBR background

For every photon scattered away from the observer, there is another scattered towards.

Synyaev-Zeldovich Effect

- If we can measure the electron density and temperature of the X-ray emitting gas along the line of sight from X-ray measurements, we can estimate the path length (~ cluster diameter) along the line of sight
- If we assume the cluster is spherical (??), from its angular diameter (projected on the sky) we can determine the distance to the cluster
- Potential uncertainties include cluster substructure or shape (e.g., non-spherical). It is also non-trivial to measure the X-ray temperature to derive the density at high redshifts.

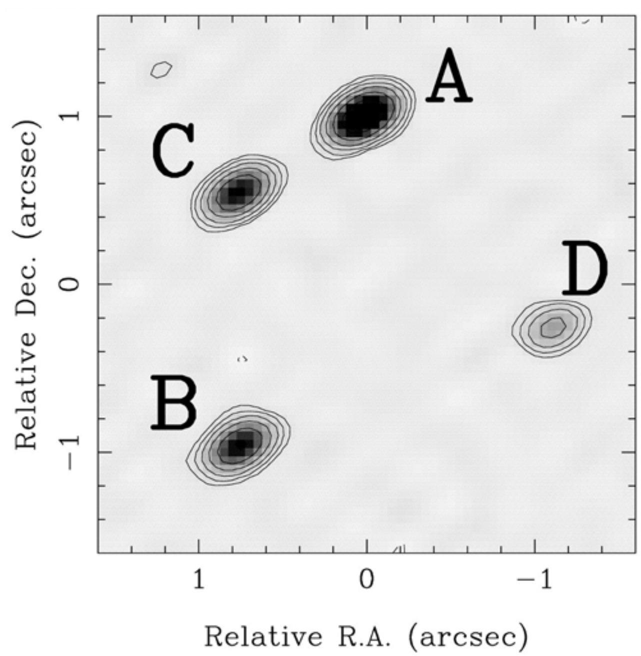


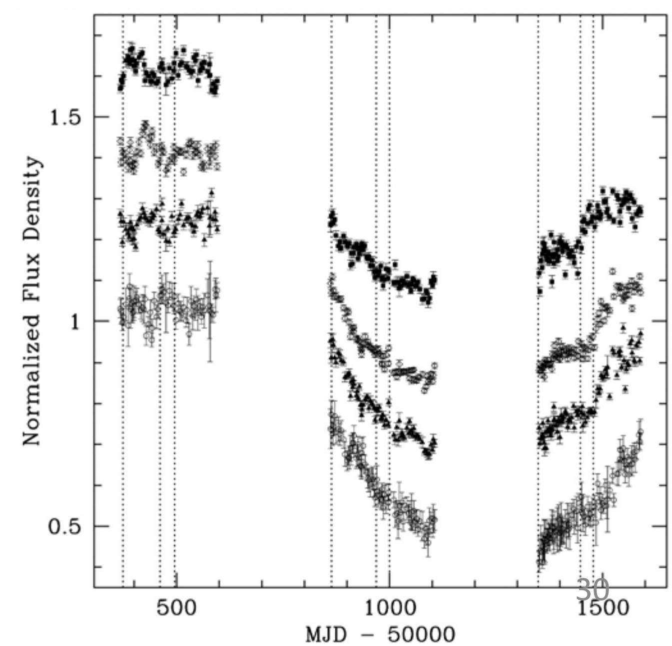


Gravitational Lens Time Delays

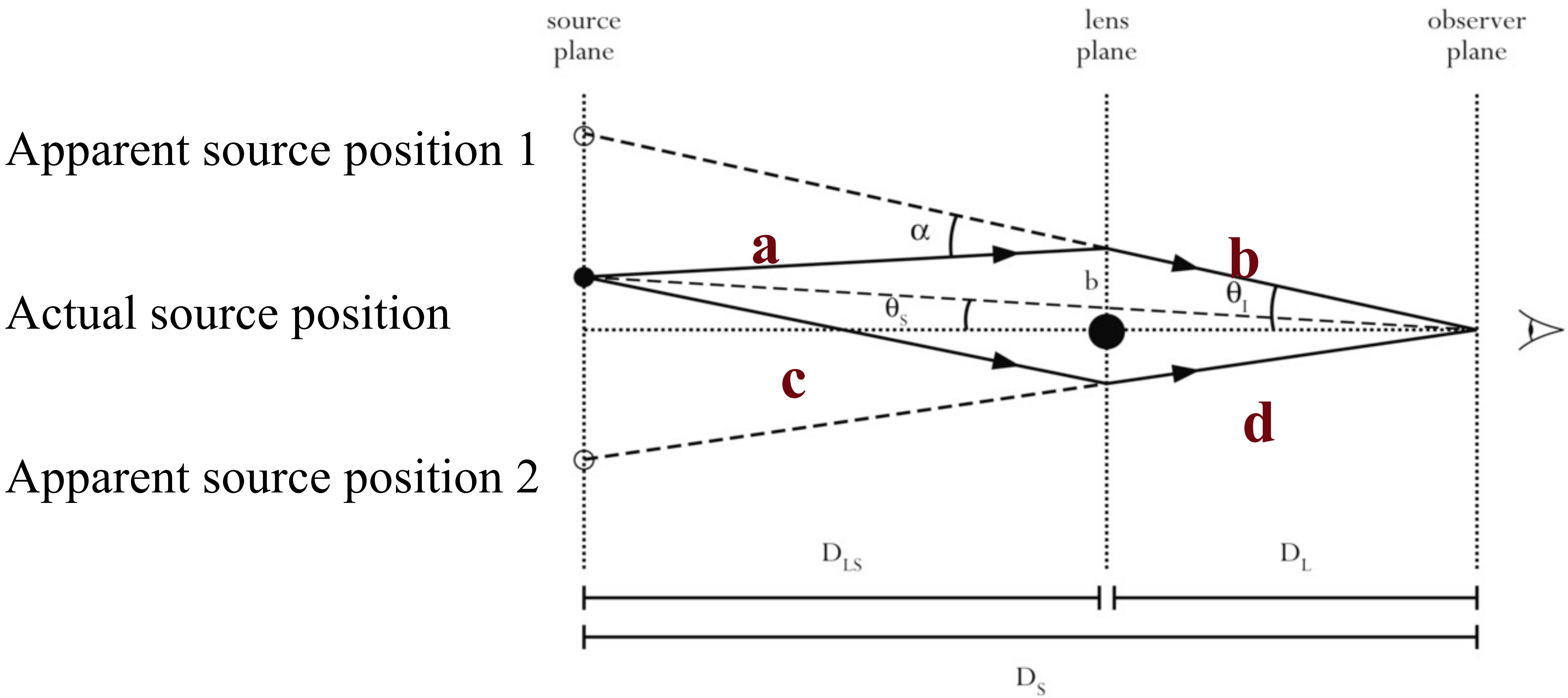
- Assuming the mass model for the lensing galaxy of a gravitationally lensed quasar is well-known (!?!), the different light paths taken by various images of the quasar will lead to time delays in the arrival time of the light to us. This be can be traced by the quasar variability
- If the lensing galaxy is in a cluster, we also need to know the mass distribution of the cluster and any other mass distribution along the line of sight. The modeling is complex!

Images and lightcurves for the lens B1608+656 (from Fassnacht *et al.* 2000)





How Does It Work?



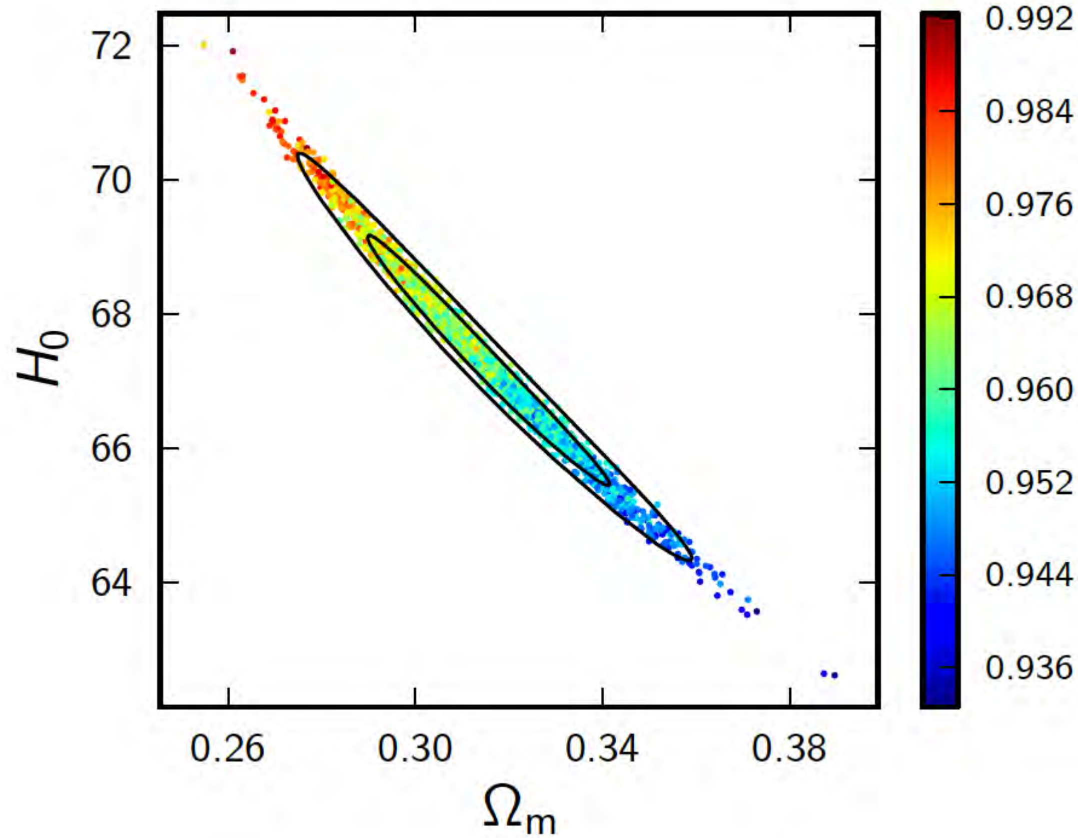
The difference in the light paths is (a+b) – (c+d) = ΔS = c Δt where Δt is the measured time delay

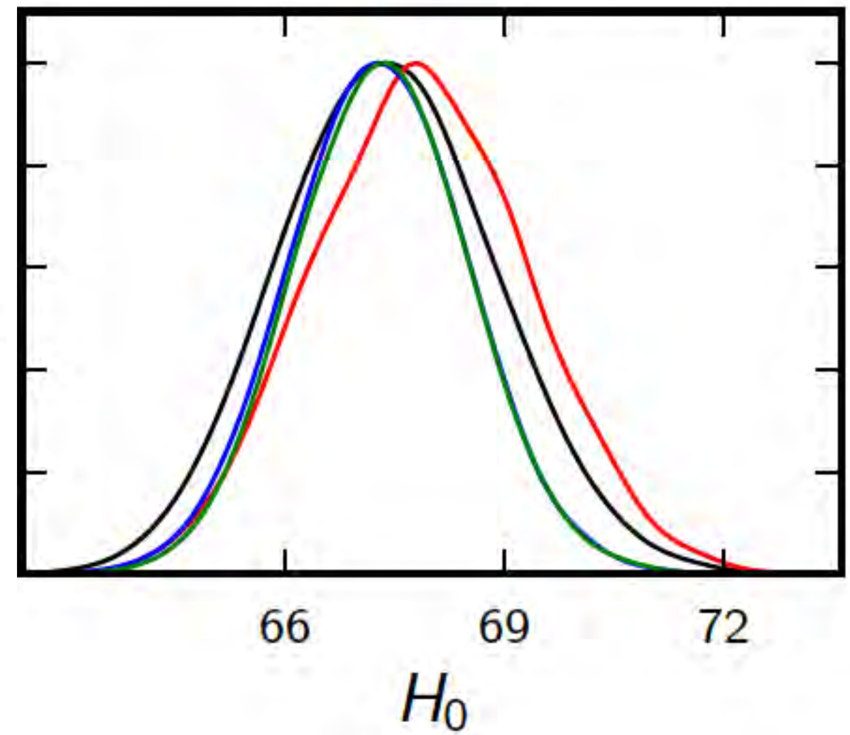
For a fixed lensing geometry, $\Delta S \sim D_L$ or D_S and the ratio $\Delta S/D_L$ or $\Delta S/D_S$ is also given by the geometry

Assuming that, measuring Δt gives ΔS, and thus D_L or D_S

H_0 From the CMB

- Bayesian solutions from model fits to CMB fluctuations – cosmological parameters are coupled





- Planck (2018) results:

$$H_0 = 67.37 \pm 0.54 \text{ km/s/Mpc}$$

$$\text{Age} = 13.801 \pm 0.024 \text{ Gyr}$$

The Hubble Tension

Between the **Cepheid-based** and **CMB-based** measurements of the Hubble Constant

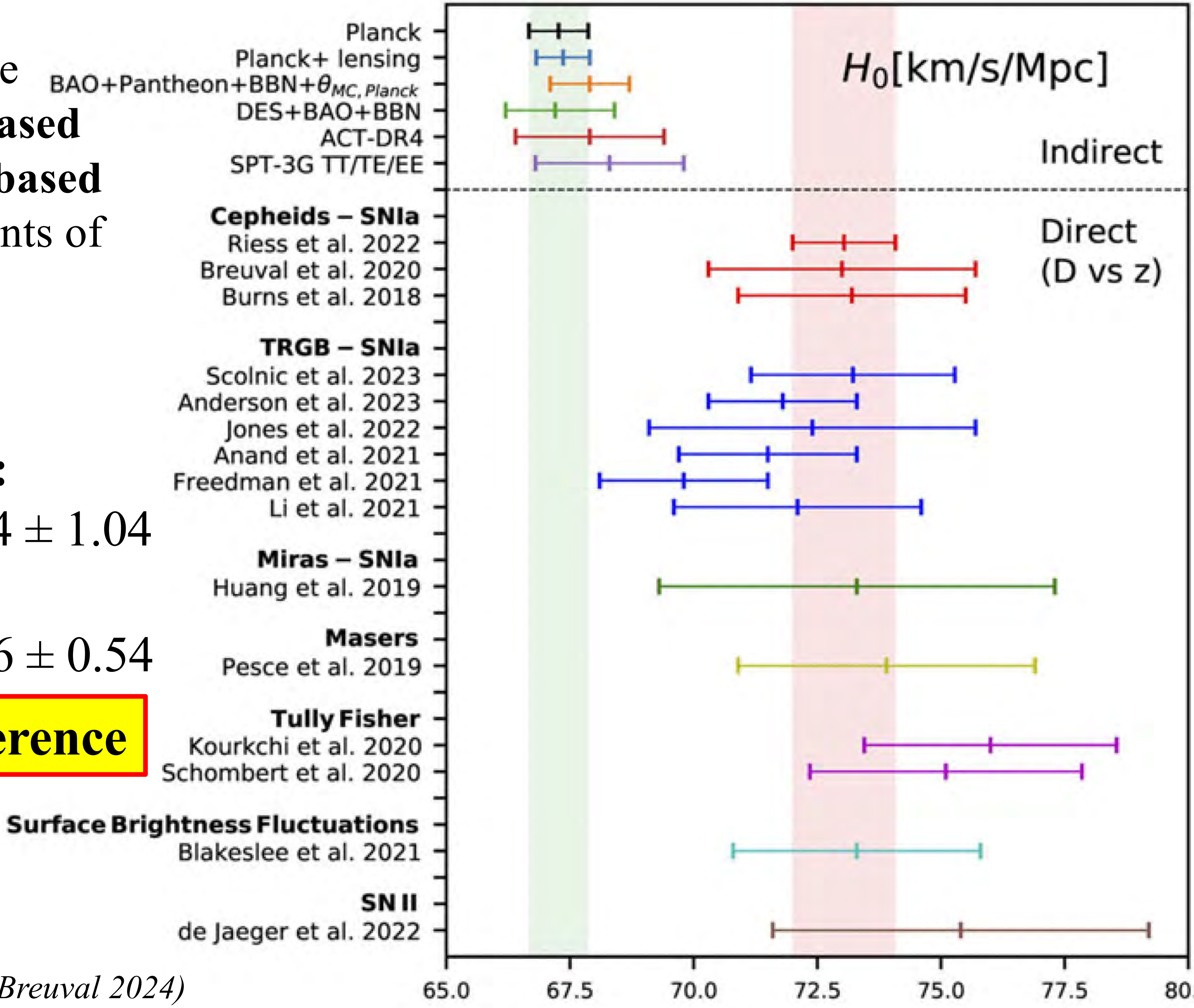
Cepheids:

$H_0 = 73.04 \pm 1.04$

CMB:

$H_0 = 67.36 \pm 0.54$

3.6 σ diference



(From Riess & Breuval 2024)

The Age of the Universe

- Several different methods (different physics, different measurements) agree that the *lower limit* to the age of the universe is ~ 12 – 13 Gyr
- This is in an excellent agreement with the age determined form the cosmological tests (~ 13.8 Gyr)

Measuring the Age of the Universe

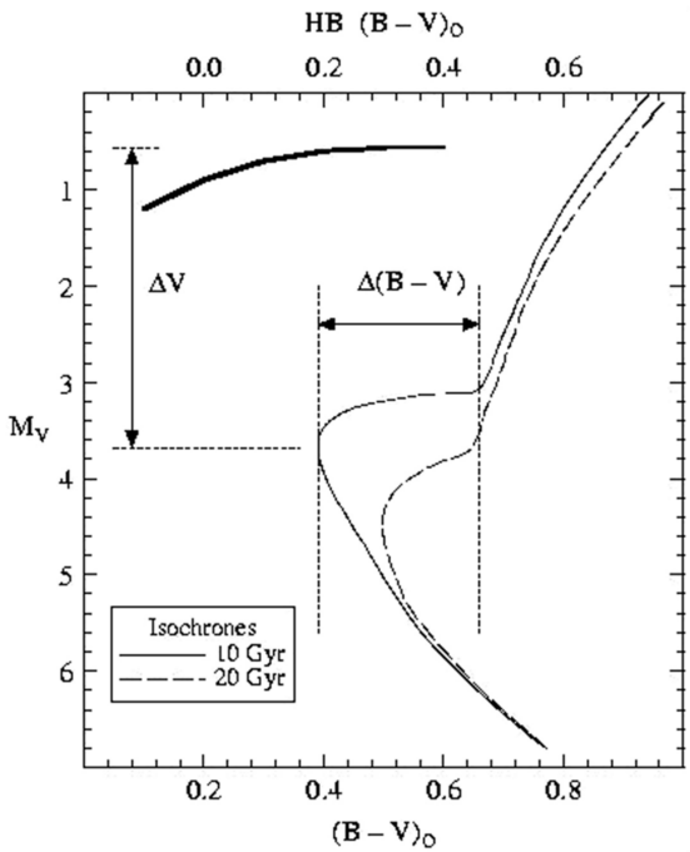
- Related to the Hubble time $t_H = 1/H_0$, but the exact value depends on the cosmological parameters
- Could place a ***lower limit*** from the ages of astrophysical objects (pref. the oldest you can find), e.g.,
  - **Globular clusters** in our Galaxy; known to be very old. Need stellar evolution isochrones to fit to color-magnitude diagrams
  - **White dwarfs**, from their observed luminosity function, cooling theory, and assumed star formation rate
  - **Heavy elements**, produced in the first Supernovae; somewhat model-dependent
  - Age-dating **stellar populations** in distant galaxies; this is very tricky and requires modeling of stellar population evolution, with many uncertain parameters

Ages of Globular Clusters

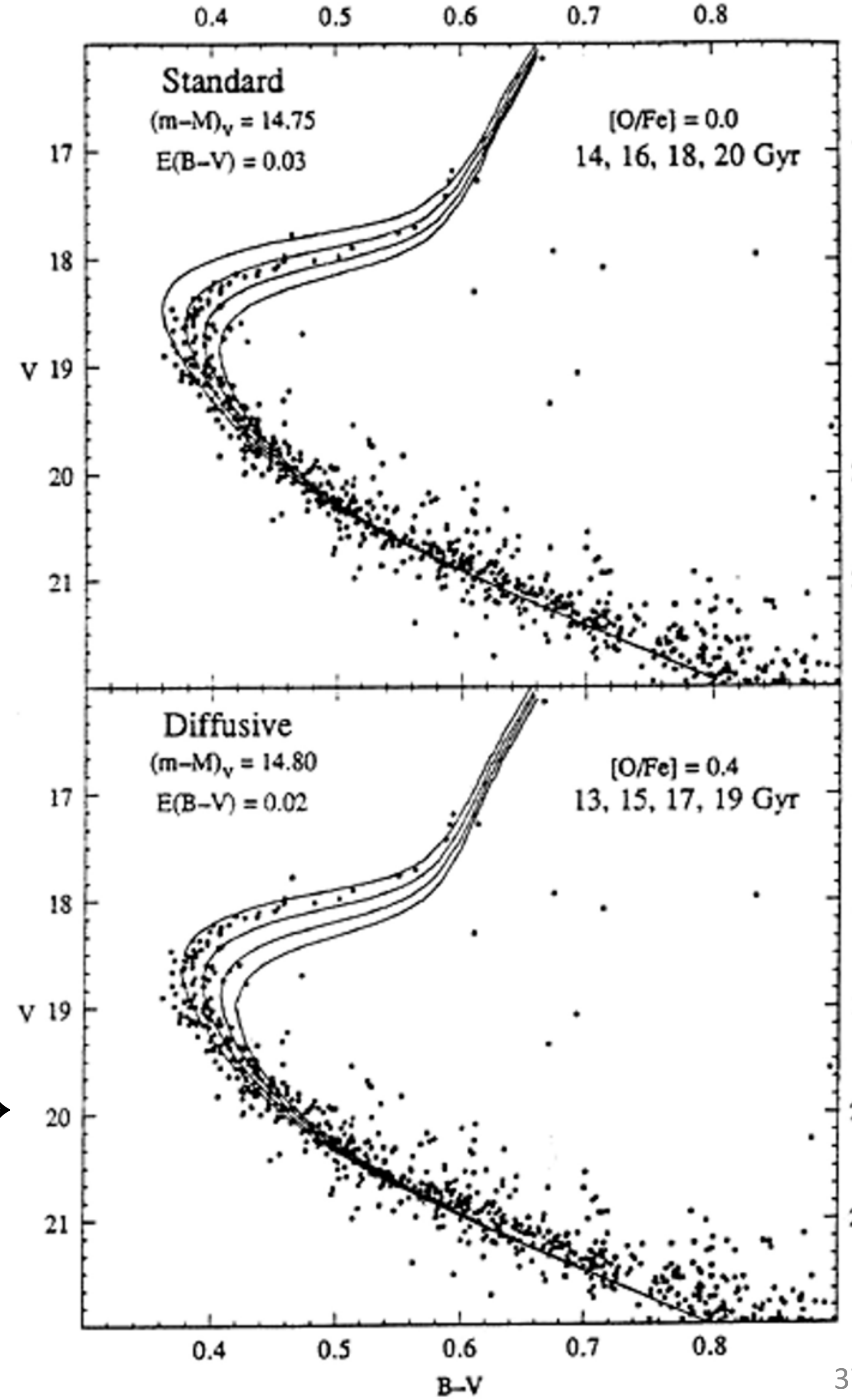
- We measure the age of a globular cluster from the magnitude of the main sequence turnoff or the difference between that magnitude and the level of the horizontal branch, and comparing this to stellar evolutionary models
- There are a fair number of uncertainties in these estimates, including errors in measuring the distances to the GCs and uncertainties in the isochrones used to derive ages (i.e., stellar evolution models)
- Inputs to stellar evolution models include: oxygen abundance [O/Fe], treatment of convection, He abundance, reaction rates of $^{14}N + p \rightarrow {}^{15}O + \gamma$, He diffusion, conversions from theoretical temperatures and luminosities to observed colors and magnitudes, and opacities; and especially ***distances***

Globular Cluster Ages

Schematic CMD and isochrones



Examples of actual isochrone fits →

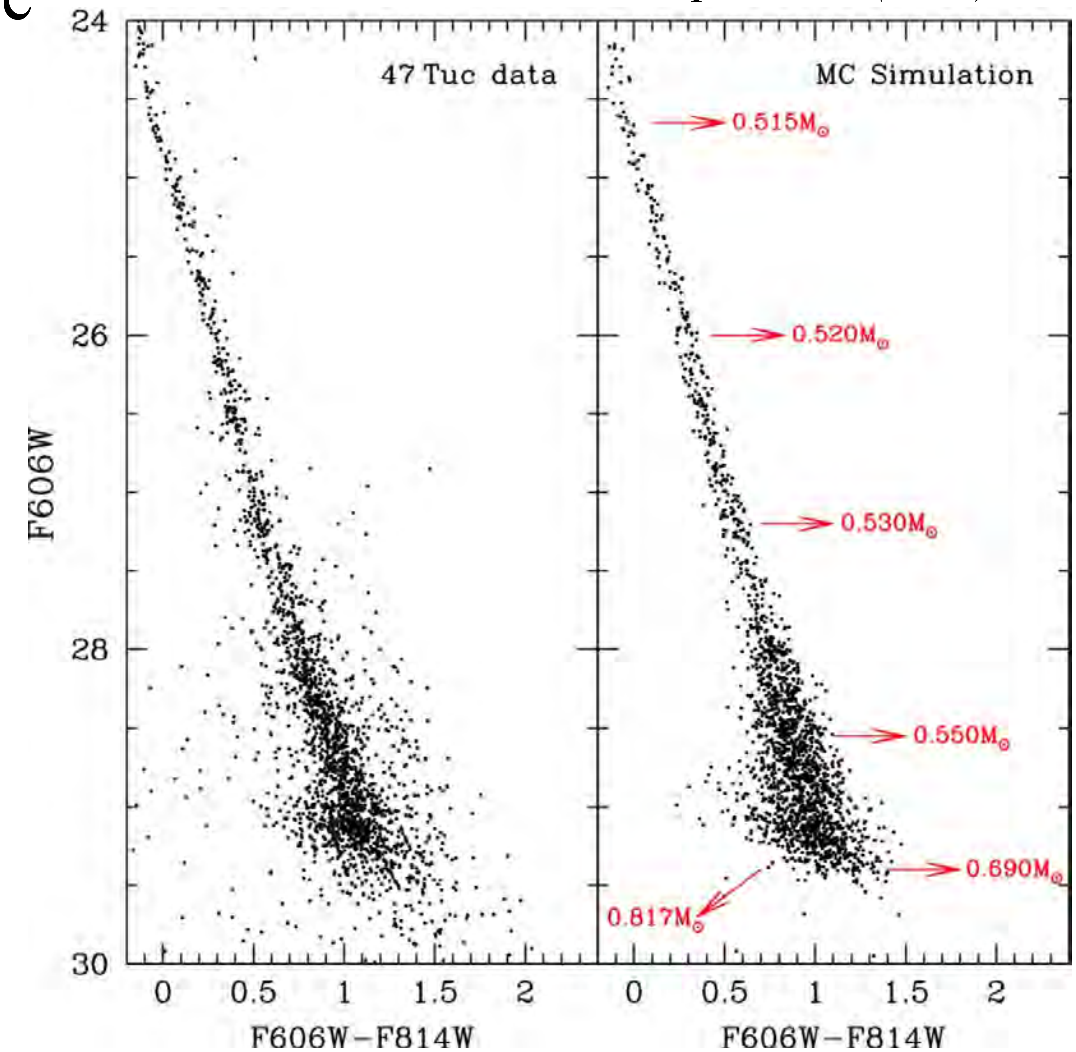


Results give ages ~ 12 ± 2 Gyr, and there is a real spread in ages

White Dwarf Cooling Curves

- White dwarfs are the end stage of stellar evolution for stars with initial masses < 8 $M_\odot$
- They are slowly cooling and fading as they radiate
- Use the *luminosity of the faintest WDs* in a cluster to estimate the cluster age *by comparing to theoretical cooling curves*
- WDs are faint, typically need deep HST observations

Campos et al. (2016)



The results for globular clusters *are fully consistent* with the results from isochrone fits to their H-R diagrams

Nucleocosmochronology

- Can use the radioactive decay of elements to age date the oldest stars in the galaxy. Analogous to age dating of rocks or carbon age dating
- Has been done with ^{232}Th (half-life = 14 Gyr) and ^{238}U (half-life = 4.5 Gyr) and other elements
- Abundances determined from spectral lines in the oldest stars, requires theoretical predictions of the initial abundance ratios
- The results are *consistent with the ages of star clusters*

An example for a single halo star:

Chronometer Pair	Predicted	Observed	Age (Gyr)	Solar[a]	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	$\geq$13.4	2.32	11.0
U/Ir	0.05036	0.0045	$\geq$15.5	0.0369	13.5
U/Pt	0.013	0.0019	$\geq$12.4	0.01846	14.6

Cosmic Distance Scale Summary

- Local measurements of the H_0 are now good to ~2%
- The concept of the ***distance ladder***; many uncertainties and calibration problems, model-dependence, etc.
- ***Cepheids*** as the key local distance indicator
- ***SNe*** as a bridge to the far-field measurements
- ***Far-field measurements*** (SZ effect, lensing, CMB)
- ***Ages*** of the oldest stars (globular clusters), white dwarfs, and heavy elements are consistent with the age inferred from CMB
- ***CMB*** provides more precise determinations of the H_0 (to ~ 0.5%) and other cosmological parameters. However, there is a ***persistent discrepancy*** ("Hubble tension") between the CMB based and Cepheid based measurements. This may be a sign of a new physics (or not), or some calibrations or data analysis issues