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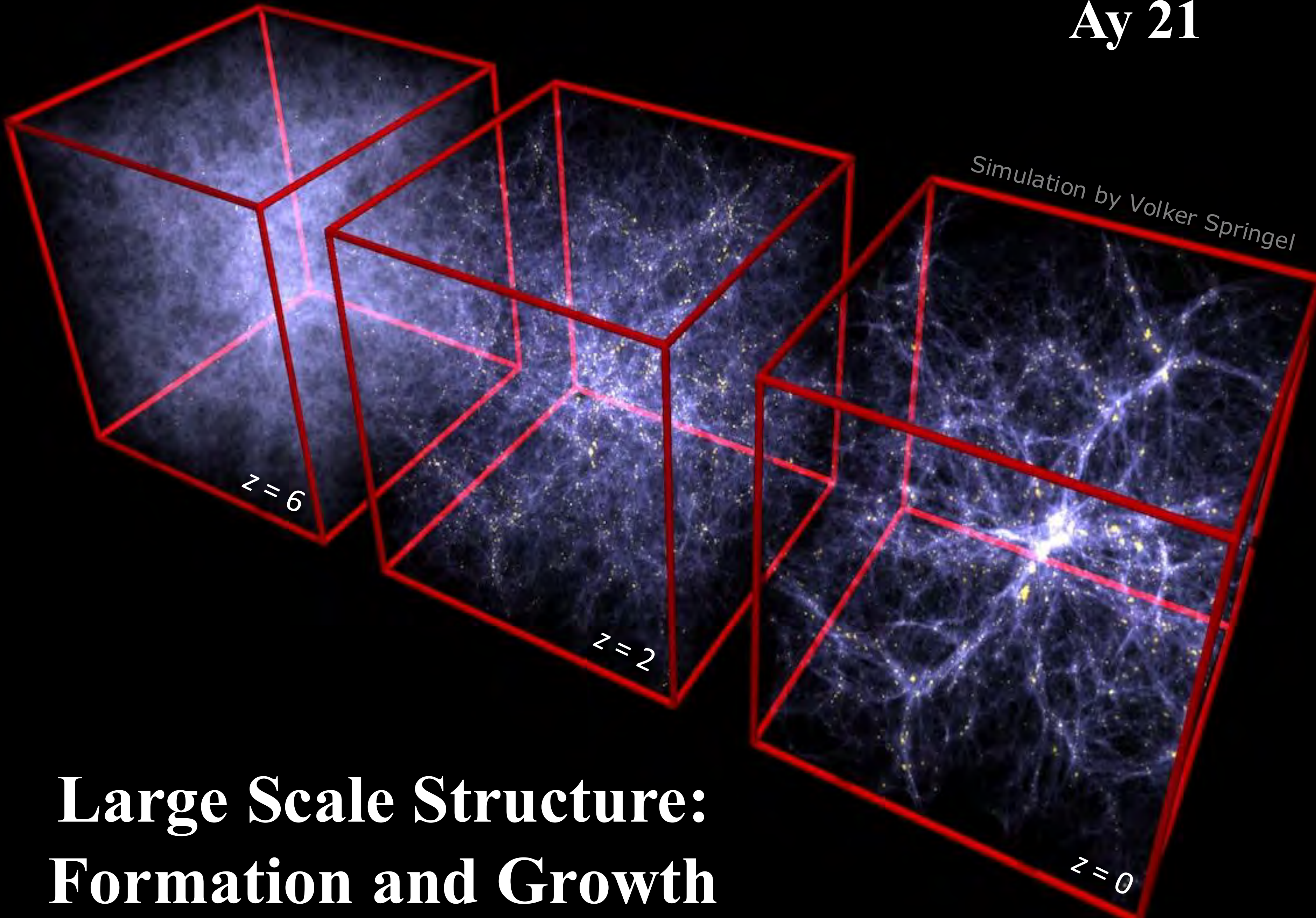
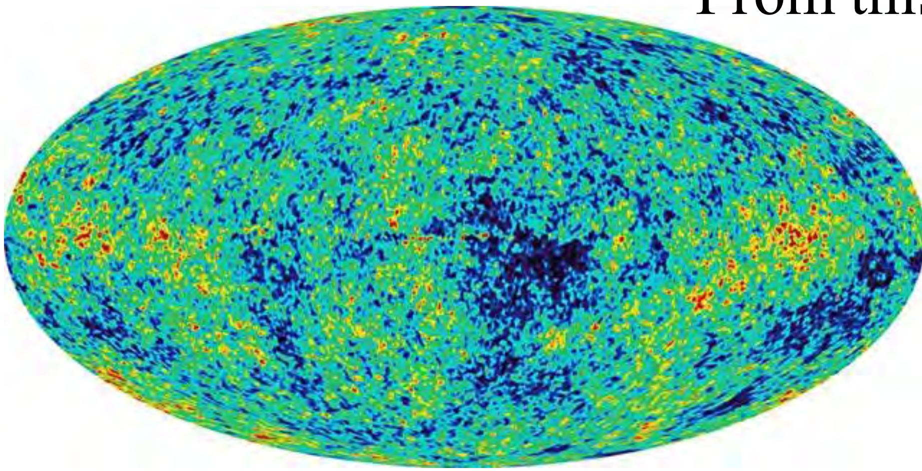


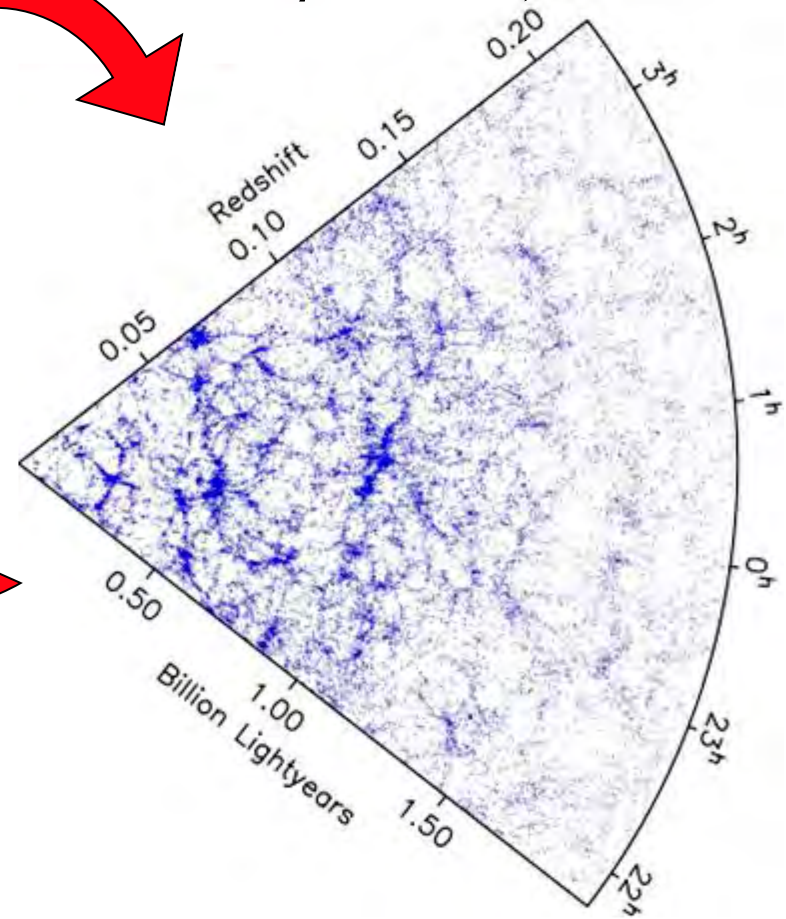

# Large Scale Structure: Formation and Growth

# Structure Formation and Evolution

From this $(\Delta\rho/\rho \sim 10^{-6})$



to this
$(\Delta\rho/\rho \sim 10^{+2})$

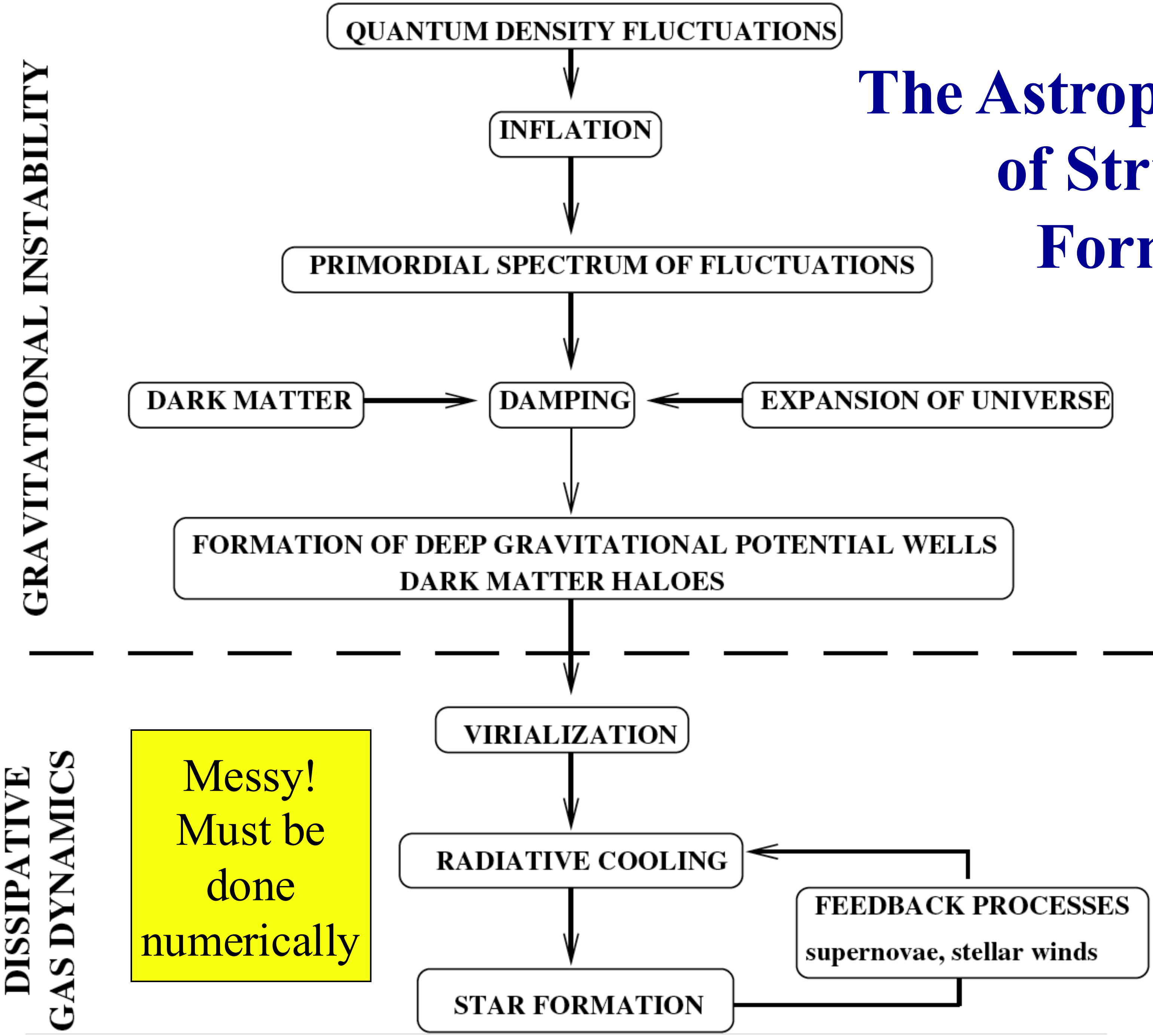



to this
$(\Delta\rho/\rho \sim 10^{+6})$

# Origin of Structure in the Universe

- Origin and evolution of the structure in the universe (galaxies, large-scale structures) is a central problem in cosmology
- Structure is generally thought to arise through ***a growth of density perturbations*** which originate in the early universe
  - We think they came from quantum fluctuations in the scalar field that caused inflation and were then amplified by the exponential inflation of the universe
- What do we know about the early structure formation?
  - We see CMB fluctuations with $\delta T/T \sim 10^{-6} \sim \Delta\rho/\rho$, since radiation and baryons are coupled before recombination
  - High-$z$ objects: We observe galaxies and quasars at $z > 10$. A galaxy requires an overdensity of $\sim 10^6$ relative to the mean
- Can we get to such large density enhancements required for galaxies, clusters, etc., by evolving the small fluctuations we see in the CMB?

# The Astrophysics of Structure Formation



# Linear Growth of Density Fluctuations

How do fluctuations in density evolve with time? For simplicity, consider a flat, matter-dominated universe with $\Omega_m = 1$ (a good approx. for the early times, and we can also ignore $\Lambda$).

The Friedmann equation is:
(Note: using $R$ instead of $a$)

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2} = 0$$

Now consider a small area with a slight overdensity of matter, it will evolve slightly differently:

$$H^2 - \frac{8}{3}\pi G\rho' = -\frac{k}{R^2}$$

Subtract the two equations:

$$-\frac{8}{3}\pi G(\rho' - \rho) = -\frac{k}{R^2}$$

Rewrite this as:

$$\rho' - \rho = -\frac{3k}{8\pi G R^2}$$

# Linear Growth of Density Fluctuations

We can define δ as the ***fractional overdensity:***

$$\delta \equiv \left(\frac{\rho' - \rho}{\rho}\right) = -\frac{3k}{8\pi G R^2 \rho}$$

But note that:

$$\delta \sim \frac{1}{R^2\rho} \sim \frac{1}{R^2 R^{-3}} \sim R$$

Since $R \sim (1+z)^{-1}$,

$$\boxed{\delta \sim (1+z)^{-1}}$$

And this is the ***linear growth of density fluctuations***

Evolution of the density contrast between two redshifts is then:

$$\frac{\delta_f}{\delta_i} = \frac{(1+z)_i}{(1+z)_f}$$

The overdense region behaves as a slightly denser Friedman model

# Linear Growth of Density Fluctuations

Let' s try it from recombination to $z = 10$: $\delta_f = 10^{-5}\,(1100+1)\,/\,(10+1) \approx 0.001$

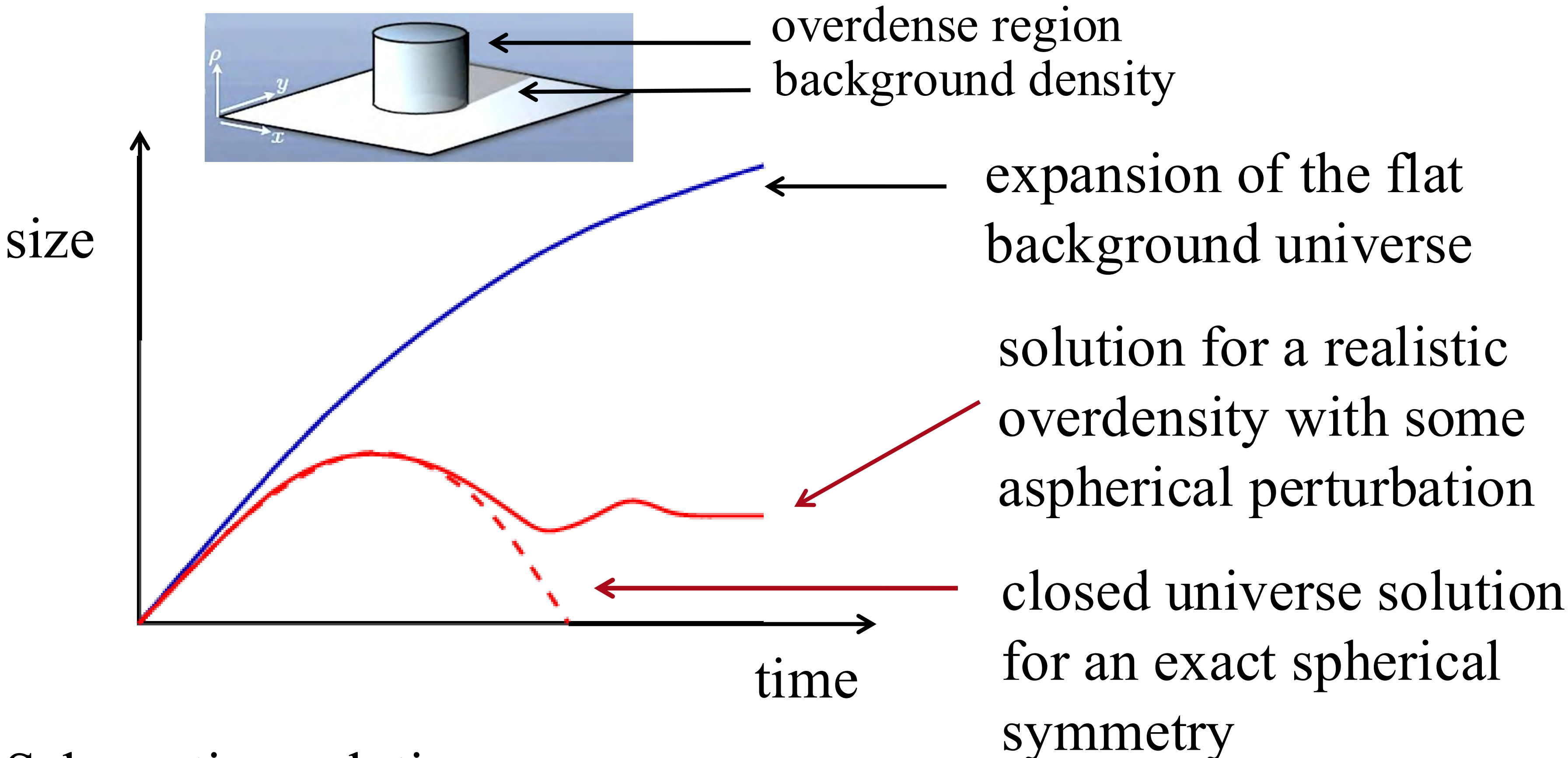
Oops! It should be ~ $10^6$ for galaxies - now what?

We need to start with larger fluctuations to get galaxies, etc. We neglected the effects of ***non-baryonic dark matter***. Before recombination, the radiation prevented the baryonic fluctuations of collapsing, but fluctuations in non-baryonic dark matter can start growing much earlier!

The fluctuations we see in the CMB (in the baryonic matter) are sitting on top of much stronger fluctuations in the non-baryonic matter. Once recombination occurs, the baryons can fall into the dark matter concentrations to form galaxies …

We also neglected the effects of energy dissipation that can occur for the baryonic matter, and that would lead to higher densities

# Evolution of the "Spherical Top-Hat Model"




Schematic evolution:

- Density contrast grows as universe expands
- Perturbation "turns around" at $R = R_{turn}$, $t = t_{turn}$
- If exactly spherical, collapses to a point at $t = 2\ t_{turn}$
- Realistically, bounces and **virializes** at radius $R = R_{virial}$

# Evolution of the Spherical Top-Hat Model

We can use the virial theorem to derive the final radius of the collapsed perturbation. Let perturbation have mass $M$, kinetic energy $<KE>$, and gravitational potential energy $<PE>$ :

**Virial theorem:** $<PE> + 2<KE> = 0$ (in equilibrium)

**Energy conservation:** $<PE> + <KE> = const.$

$$-\frac{GM^2}{R_{turn}} = -\frac{GM^2}{R_{virial}} + \langle KE \rangle = -\frac{GM^2}{2R_{virial}}$$

Potential energy at turnaround

Potential energy when virialized

Conclude: $R_{virial} = \frac{1}{2} R_{turn}$

Note: this assumes *no* energy dissipation!

# Evolution of the Spherical Top-Hat Model

We can apply the solution for a closed universe to calculate the final overdensity:

Friedmann equation: $$\dot{a}^2 = \frac{A^2}{a} - kc^2$$

where $$A^2 = \left(\frac{8pG}{3}\right) r_0 a_0^3$$

Solutions:

$$a = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}$$

$k = 0$ solution derived previously

$$a = \frac{1}{2}\frac{A^2}{c^2}\left(1 - \cos Y\right)$$

$$t = \frac{1}{2}\frac{A^2}{c^3}\left(Y - \sin Y\right)$$

Parametric solution for a closed, $k$ = 1 universe (see the derivation elsewhere, e.g., in Ryden's book), but you don't need to know this

# Evolution of the Spherical Top-Hat Model

Now we can calculate the turnaround time for the collapsing sphere by finding when the size of the small closed universe has a maximum:

$$\frac{da}{d\Psi} = \frac{1}{2}\frac{A^2}{c^2}\sin\Psi = 0 \quad \rightarrow \quad \Psi = 0, \pi, \ldots$$

At turnaround, $\Psi = \pi$, which corresponds to time

$$t_{turn} = \frac{1}{2}\frac{A^2}{c^3}\pi$$

The scale factors of the perturbation and of the background universe are:

$$a_{sphere} = \frac{1}{2}\frac{A^2}{c^2}\left(1 - \cos\pi\right) = \frac{A^2}{c^2}$$

$$a_{background} = \left(\frac{3A}{2}\right)^{2/3} t_{turn}^{2/3} = \left(\frac{3\pi}{4}\right)^{2/3}\frac{A^2}{c^2}$$

# Evolution of the Spherical Top-Hat Model

The density contrast at turnaround is therefore:

$$\frac{r_{sphere}}{r_{background}} = \left( \frac{a_{sphere}}{a_{background}} \right)^{-3} = \frac{9p^2}{16}$$

At the time when the collapsing sphere virialized, at $t = 2\ t_{turn}$:

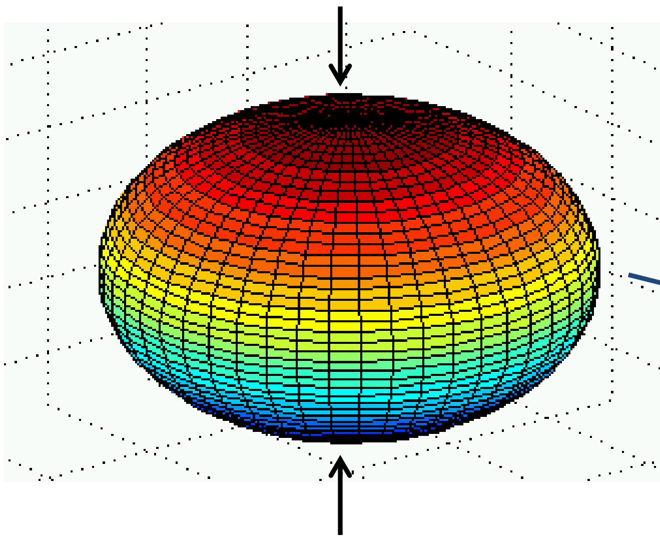
- Its density has *increased* by a factor of 8 (since $R_{turn} = 2\ R_{vir}$)
- Background density has *decreased* by a factor of $(2^{2/3})^3 = 4$

***A collapsing object virializes when its density is greater than the mean density of the universe by a factor of* 18 $\pi^2$ ~ 180**
(and this is about right for the large-scale structure today)

First objects to form are small and dense (since they form when the universe is denser). These later merge to form larger structures: **"bottom-up" structure formation**

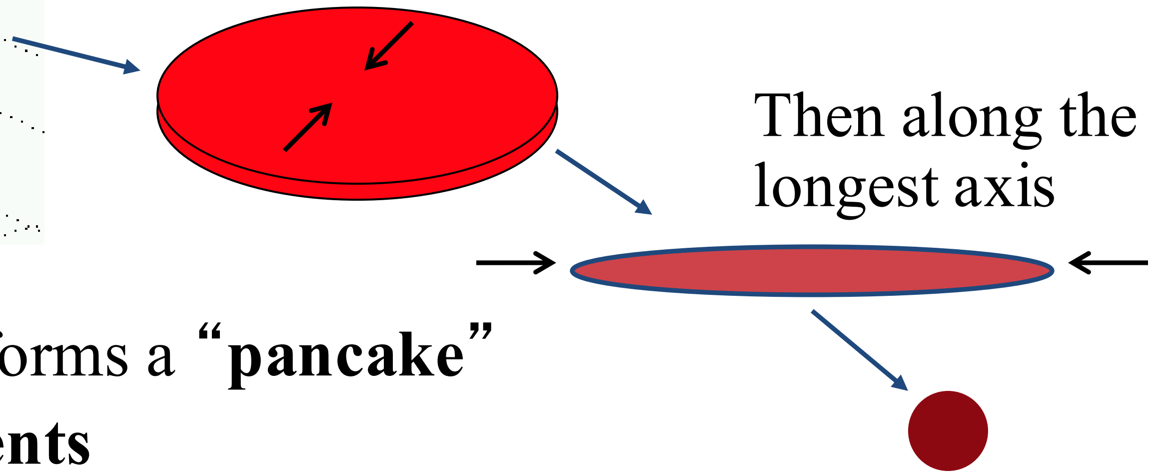
# Non-Spherical Collapse

Real perturbations will not be spherical. Consider a collapse of an ellipsoidal overdensity:



The expansion turns into collapse along the shortest axis first …

Then along the intermediate axis



Then along the longest axis

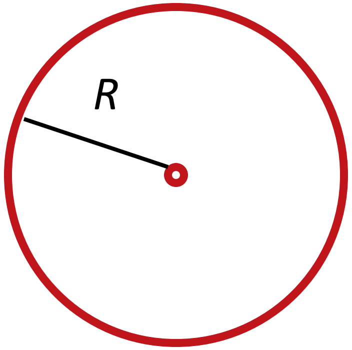
- Perturbation first forms a "**pancake**"
- Then forms **filaments**
- Then forms **clusters**

This kinds of structures are seen both in numerical simulations of structure formation and in galaxy redshift surveys

# How Long Does It Take?

Dissipationless gravitational collapse timescale is on the order of the **free-fall time, $t_{ff}$:**

The outermost shell has acceleration $g = GM/R^2$




It falls to the center in:

$$t_{ff} = (2R/g)^{1/2} = (2R^3/GM)^{1/2} \approx (2/G\rho)^{1/2}$$

Thus, low density lumps collapse more slowly than high density ones. More massive structures are generally less dense, take longer to collapse. For example:

For a galaxy: $t_{ff} \sim 240 \text{ Myr } (R/50\text{kpc})^{3/2} (M/10^{12}M_\odot)^{-1/2}$

For a cluster: $t_{ff} \sim 7.5 \text{ Gyr } (R/5\text{Mpc})^{3/2} (M/10^{15}M_\odot)^{-1/2}$

So, we expect that galaxies collapsed early (at high redshifts), and that clusters are still forming now. This is as observed!

# Quantifying the Density Field

- Consider the overall fluctuating density field as a superposition of waves with different wavelengths, phases, and amplitudes
- Then we can take a Fourier transform and measure the **Power** on different scales, expressed either as wavelengths ***l*** or frequencies or wave numbers $\boldsymbol{k = 1 / l}$
- ***Density fluctuations field :*** $d = \dfrac{r - \bar{r}}{\bar{r}}$
- ***Fourier Transform of density field :*** $d_k = \text{å}\ de^{-i\mathbf{k}\cdot\mathbf{r}}$
- ***Its Power Spectrum :*** $P(k) = \left\langle \left| d_k \right|^2 \right\rangle$

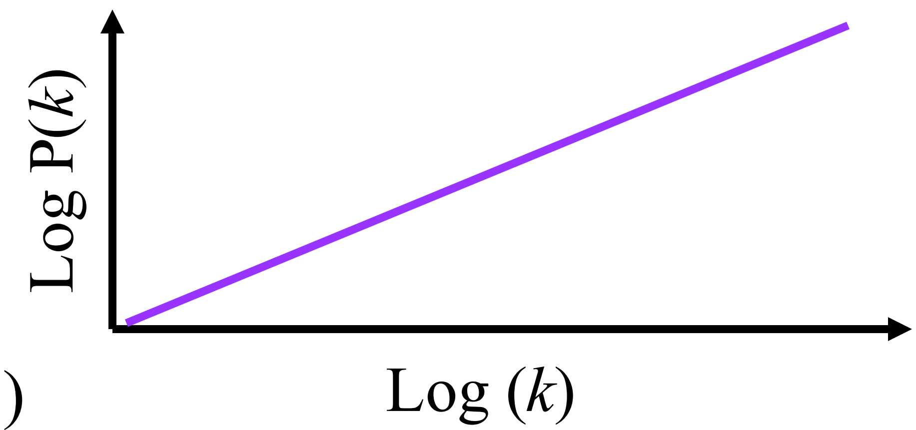
It measures the power of fluctuations on a given scale ***k***
Sometimes one uses the "mass scale"; $\boldsymbol{M \sim k^{-3\,(3+n)/4}}$

# Density Fluctuation Spectrum

- A common assumption is that the fluctuations have the same amplitude $\delta \sim 10^{-4}$ when they enter the horizon
- This gives a ***scale-free*** or ***Harrison-Zeldovich*** spectrum
- Such spectrum is also predicted in the inflationary scenario
- More generally, it can be a power-law:

$$\boxed{P(k)=\left\langle\left|\delta_k\right|^2\right\rangle \propto k^n}$$

(for Harrison-Zeldovich, $n = 1$)



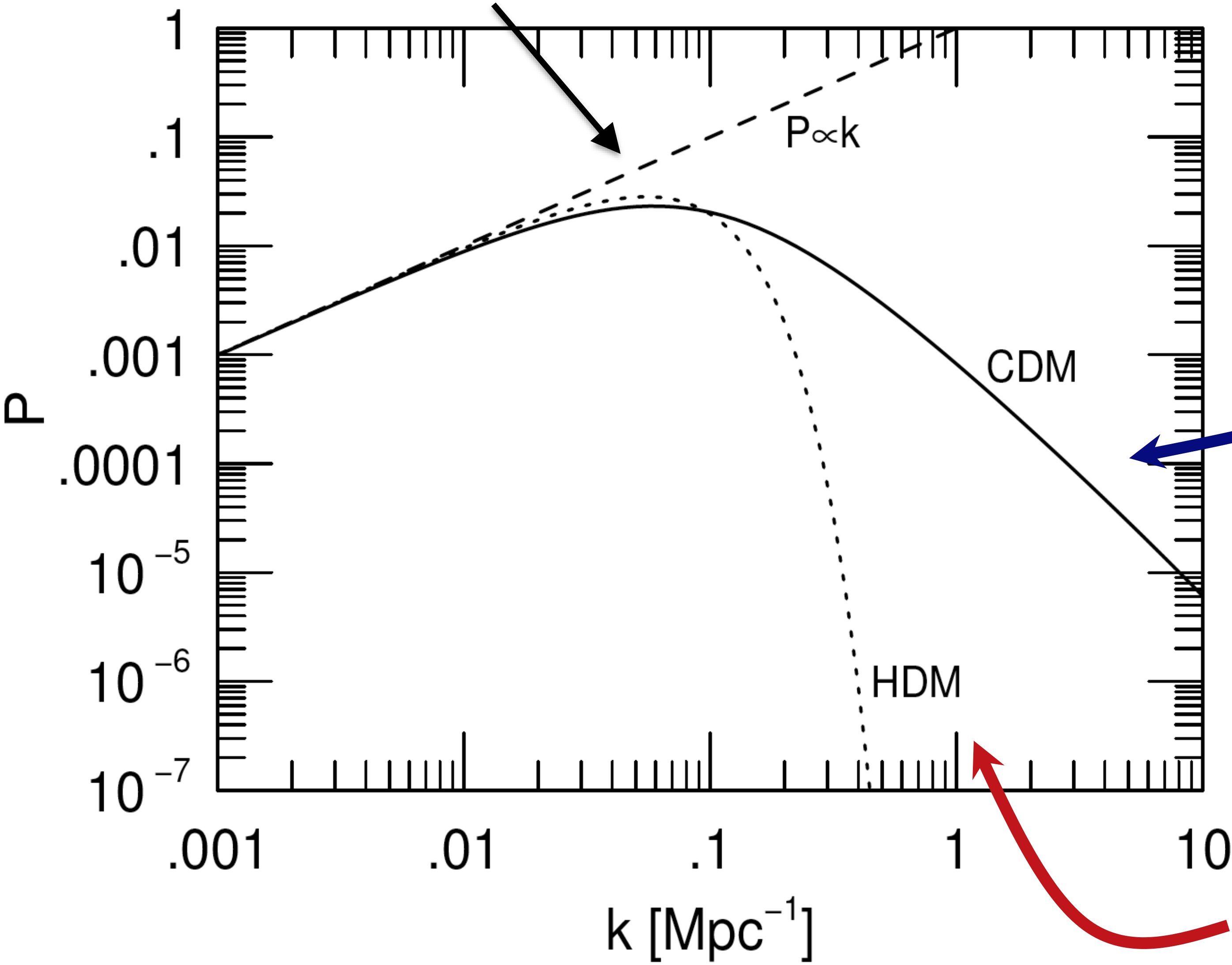

- The ***fluctuations grow under self-gravity***, so the power increases (if all scales grow equally, the spectrum just shifts up in *log*), but ***they can be also erased or damped*** through various processes - the shape of the spectrum changes

# Dark Matter and Damping of Fluctuations

- ***Different types of dark matter form structure differently***
- Baryonic dark matter is coupled to radiation, so it does not help in forming structure prior to the recombination
- Fluctuations can be erased or damped by ***sound waves*** (this is also called the Meszaros effect). This is important for slowly moving DM particles, i.e., cold dark matter (CDM)
- They can be erased by ***free streaming*** of relativistic particles, i.e., hot dark matter (HDM); diffusion of photons, which then "drag along" the baryons in the radiation-dominated era, does the same thing (this is also called the Silk damping)
- Thus ***HDM vs. CDM make very different predictions*** for the evolution of structure in the universe!
- In any case, ***the smaller fluctuations are always erased first***

# Damping of Fluctuations

The primordial (H-Z) spectrum




Sound waves diminish the strength of small scale fluc’s for the CDM case

Relativistic streaming and photon diffusion erase them completely for the HDM case

# Structure Formation in the CDM Scenario

- CDM density fluctuations exist on all scales, large and small
- Small lumps collapse first, big things collapse later.  Larger overdensities will incorporate smaller ones as they collapse, via **merging**
- Structure forms early with CDM, and it forms "**bottom-up**" Galaxies form early, before clusters, and clusters are still forming now
- This picture is known as ***hierarchical structure formation***
- This closely matches what we observe
- It also produces the right kind of CMB fluctuations
- Thus, while we know that massive neutrinos (which would constitute HDM) do exist, ***most of the dark matter must be cold***!

# The Growth of Fluctuations

- Prior to matter-radiation equality perturbations are prevented from growing due to ***radiation pressure***
- ***Pressure opposes gravity*** effectively for all wavelengths below the Jeans Length:

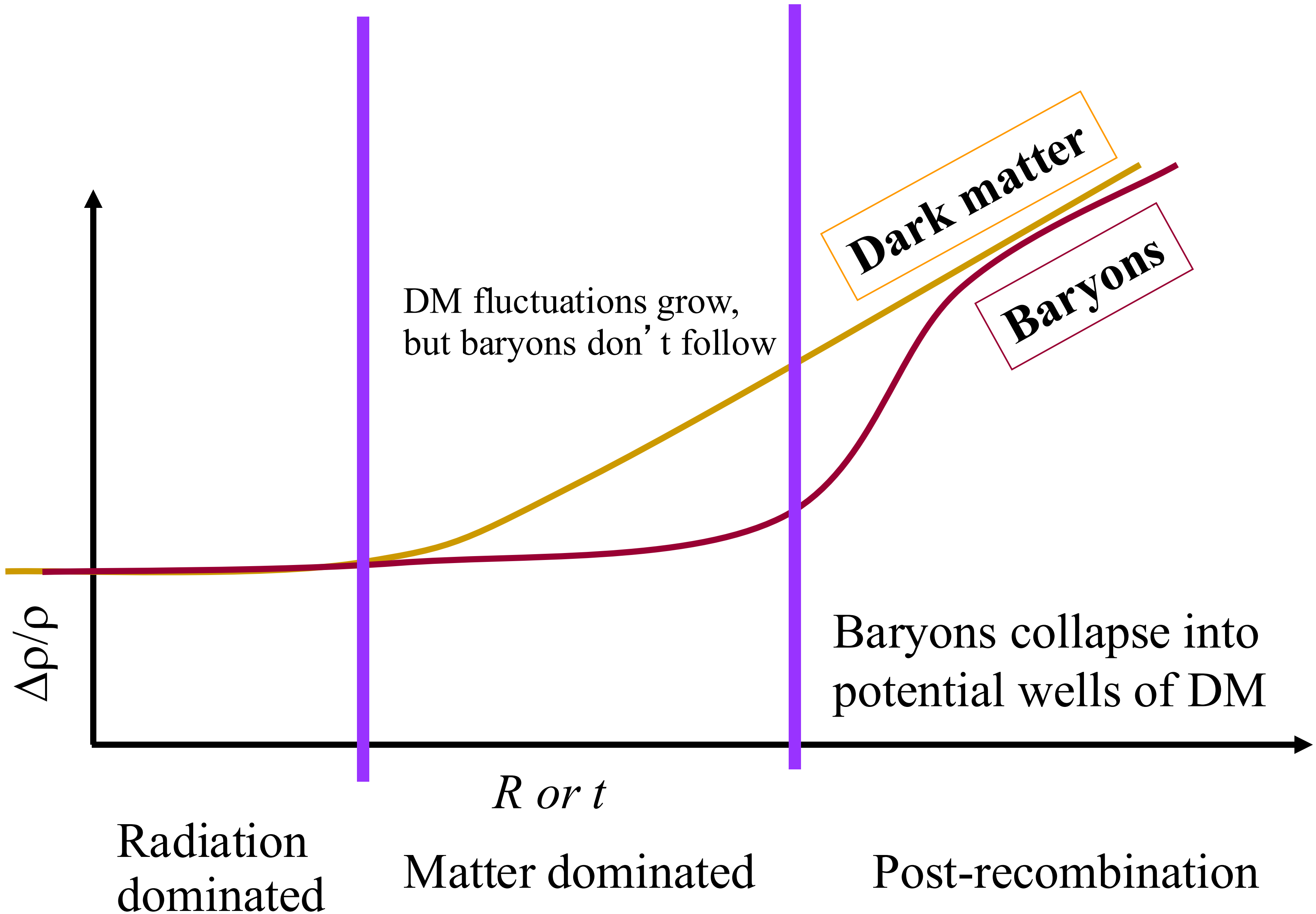
$$\boxed{l_J = c_s \sqrt{\frac{p}{Gr}}}$$

where $\boldsymbol{c_s}$ is the speed of sound, and the equation of state is:

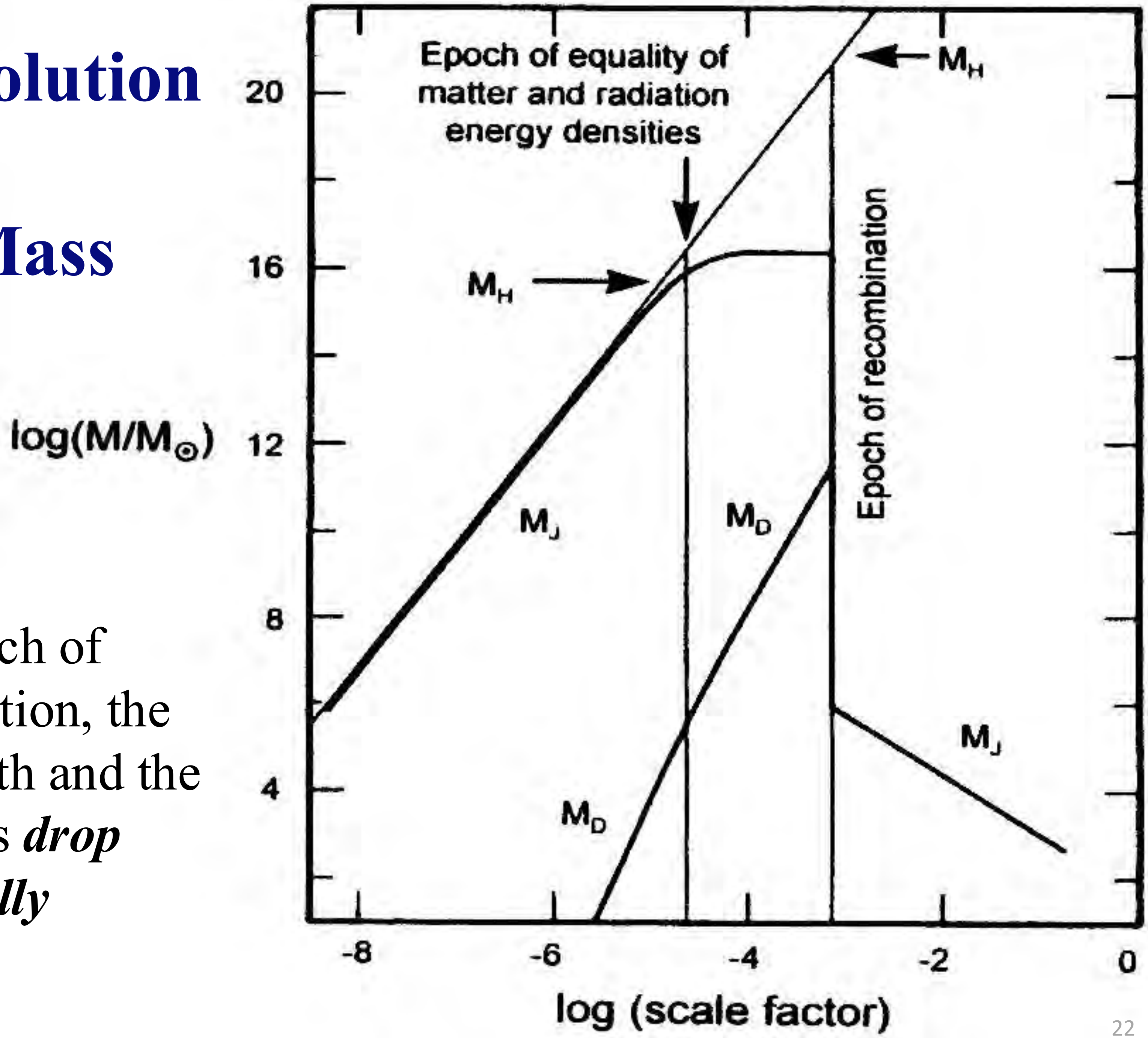
$$c^2_s = \frac{¶P}{¶r}$$

- ***Jeans Length*** is the scale at which sound waves can cross an object in about the time for gravitational collapse
- ***In a radiation dominated universe, Jeans Length is close to the horizon size***
- At matter-radiation equality the ***sound speed starts to drop***, and fluctuations can ***grow***

# The Evolution of Fluctuations



# The Evolution of the Jeans Mass

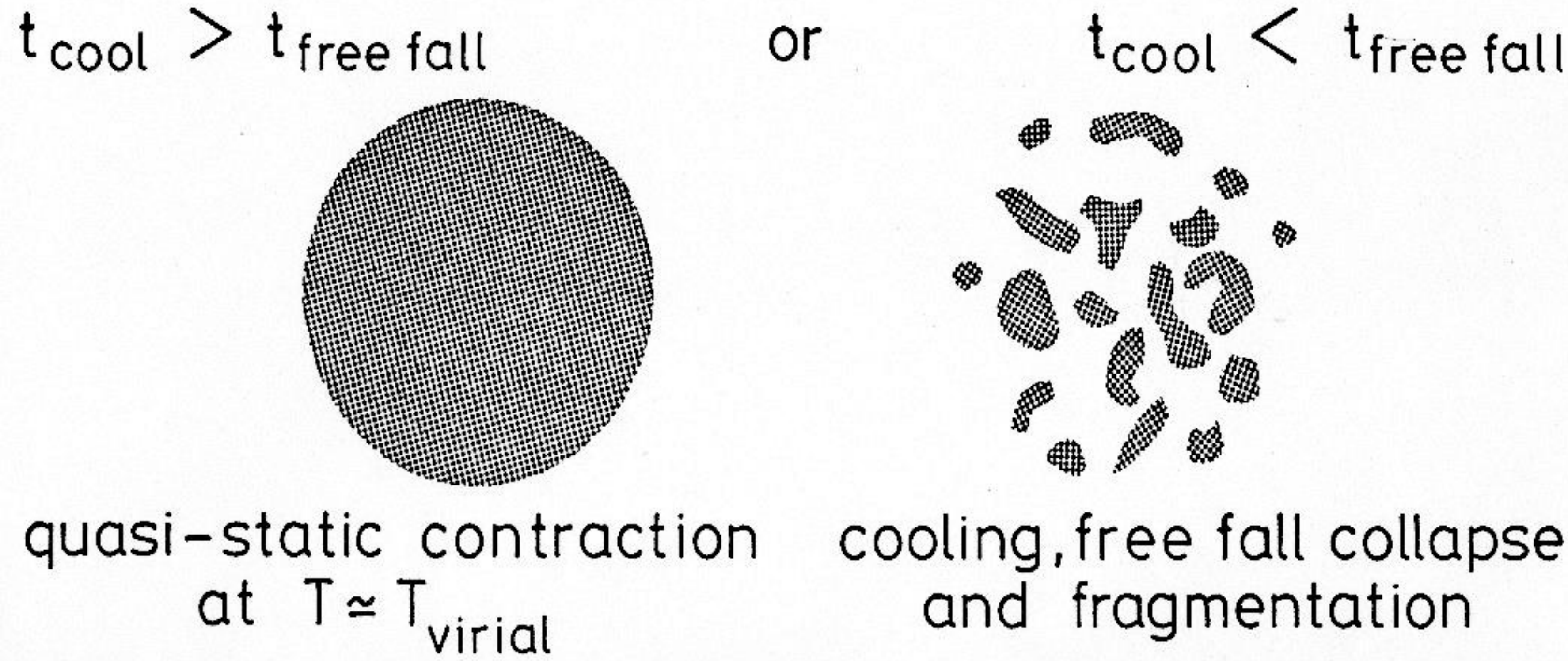



At the epoch of recombination, the Jeans length and the Jeans mass ***drop dramatically***

# Post-Recombination Universe

- Fluctuations continue to grow, and soon enter the *non-linear regime*, at progressively ever larger scales
- This evolution can then only be followed *numerically*
- Once the gas infalling into the DM potential wells (dark halos) is compressed enough, ***dissipative effects*** begin to play a significant role (shocks, star formation, feedback). These are difficult to simulate numerically!
- ***Energy dissipation accelerates collapse and leads to higher densities which cannot be achieved by the dissipationless collapse*** (a factor of 8…). This is often called "***cooling***"
- A good mechanism is inverse Compton scattering of CMB photons on hot electrons; this process is effective only at $z < 100$ or so, since the CMB is too hot at higher $z$' s

# Two Modes of Contraction




***Fluctuations which cool faster than they fall together under gravity alone are subject to Jeans instability and fragmentation***

That is what separates galaxies (smaller, cooling important) from the large-scale structure (larger, cooling not important)

# Cooling and Structure Formation

Define the ***cooling time*** as:

$$t_{cool} = E / |dE/dt| = 3\ n\ k\ T / [\ n^2\ \Lambda(T)\ ]$$

where ***n*** is the particle number density, ***k*** is the Boltzmann constant, and $\Lambda(T)$ is a cooling rate function (*not* cosmological constant!) which depends on the chemical composition

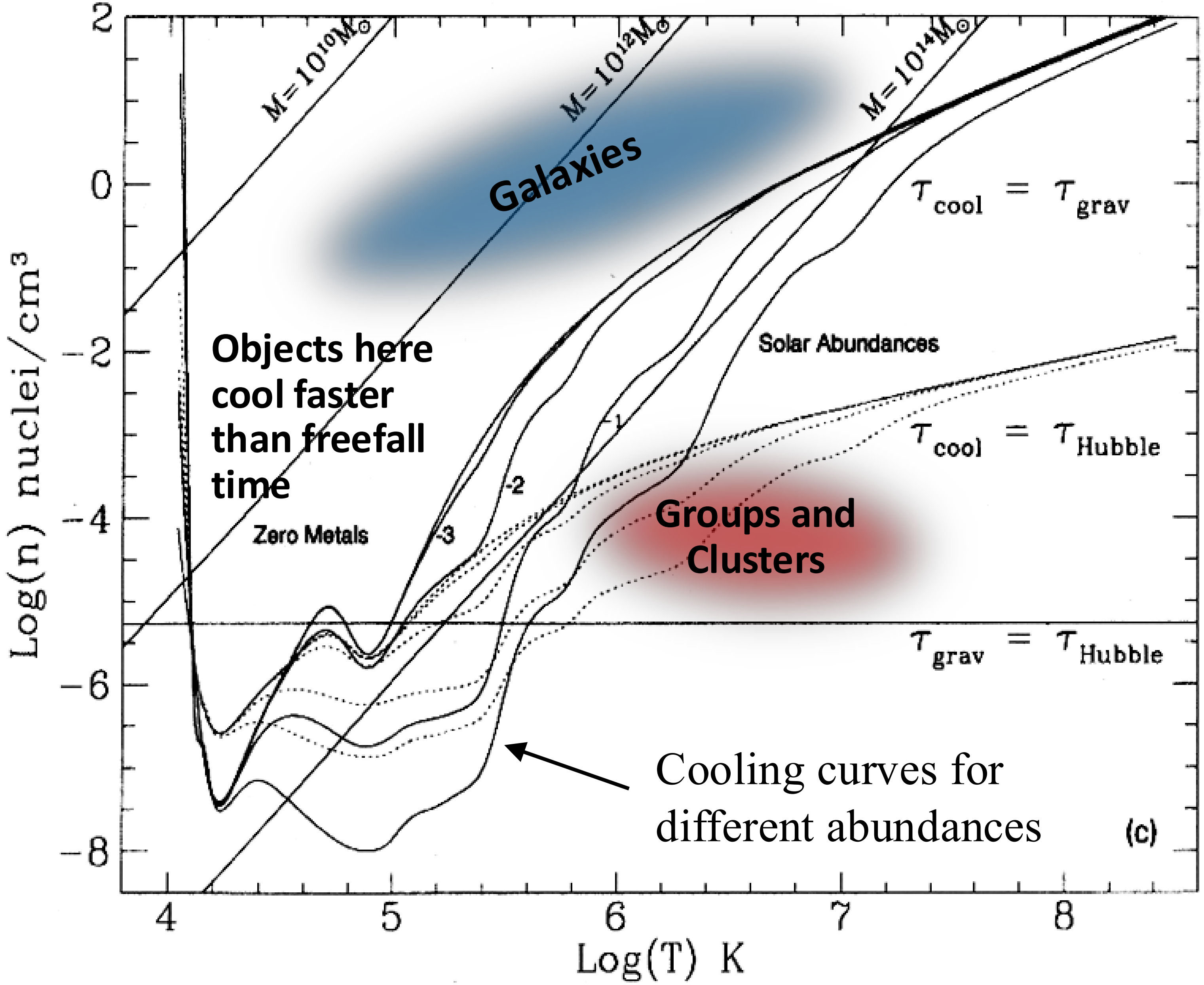
The ***key question*** is the relation between the free-fall time $t_{ff}$, cooling time, $t_{cool}$, and the Hubble time, $t_H$:

**If $t_{cool} < t_{ff}$** the cooling dominates the contraction, objects collapse faster, to smaller radii and higher densities; and v.v.

**If $t_{cool} > t_H < t_{ff}$** objects cannot form

The position of objects in the "cooling diagram" plane $\{T,n\}$ thus determines their fate! Note also that $M_{Jeans} = f(T,n)$

# The Cooling Diagram

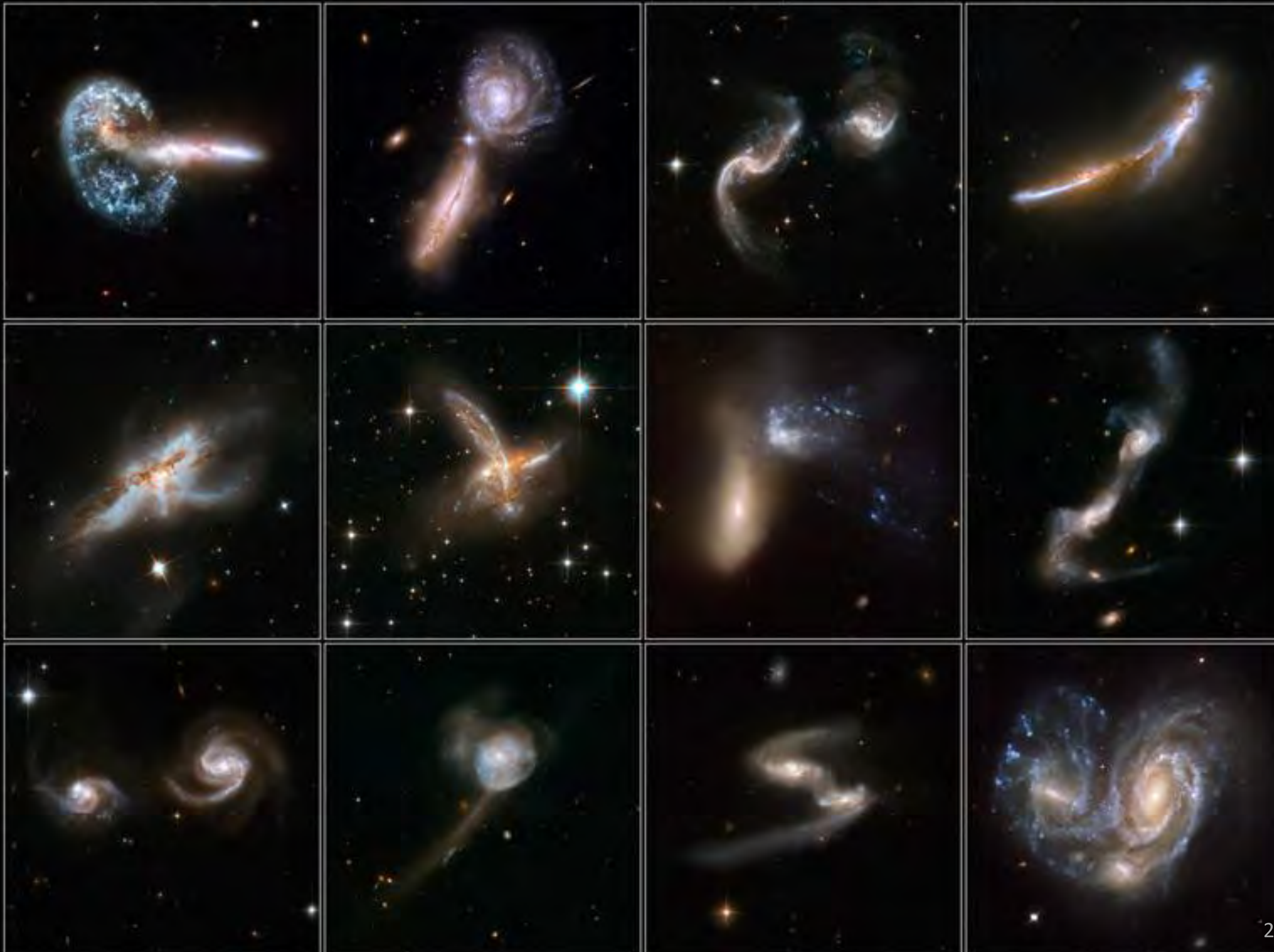


# Numerical Simulations in Cosmology

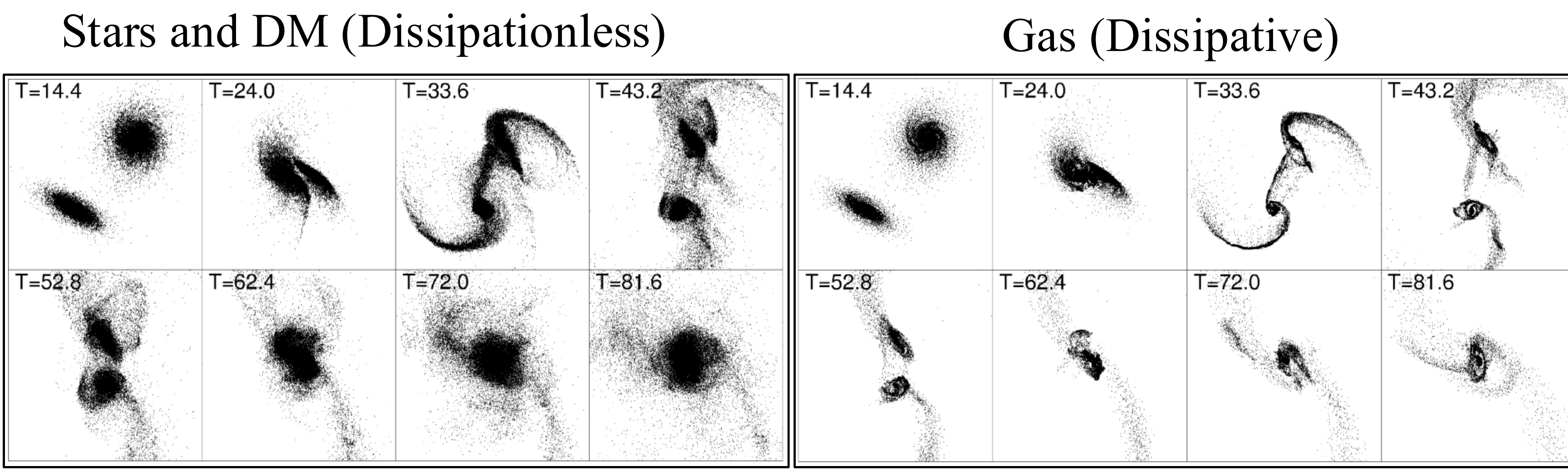
- Many problems in astrophysics generally *do not allow analytic solution*. A full 3D numerical treatment is required
- For the Newtonian gravity, there are no exact analytical solutions beyond the 2-body (Kepler) problem
- Numerical simulations thus play a key role in cosmology
- In cosmology know both the governing equations (Newtonian gravity, cosmological expansion) *and* can assume the initial conditions (primordial density field)
- Pure gravity simulations are dissipationless; adding gas and radiation makes the problem much harder, due to dissipation, hydrodynamical effects, radiation transfer…
- The output of simulations can be compared to observations

# Interacting Galaxies

*Hubble Space Telescope* • ACS/WFC • WFPC2



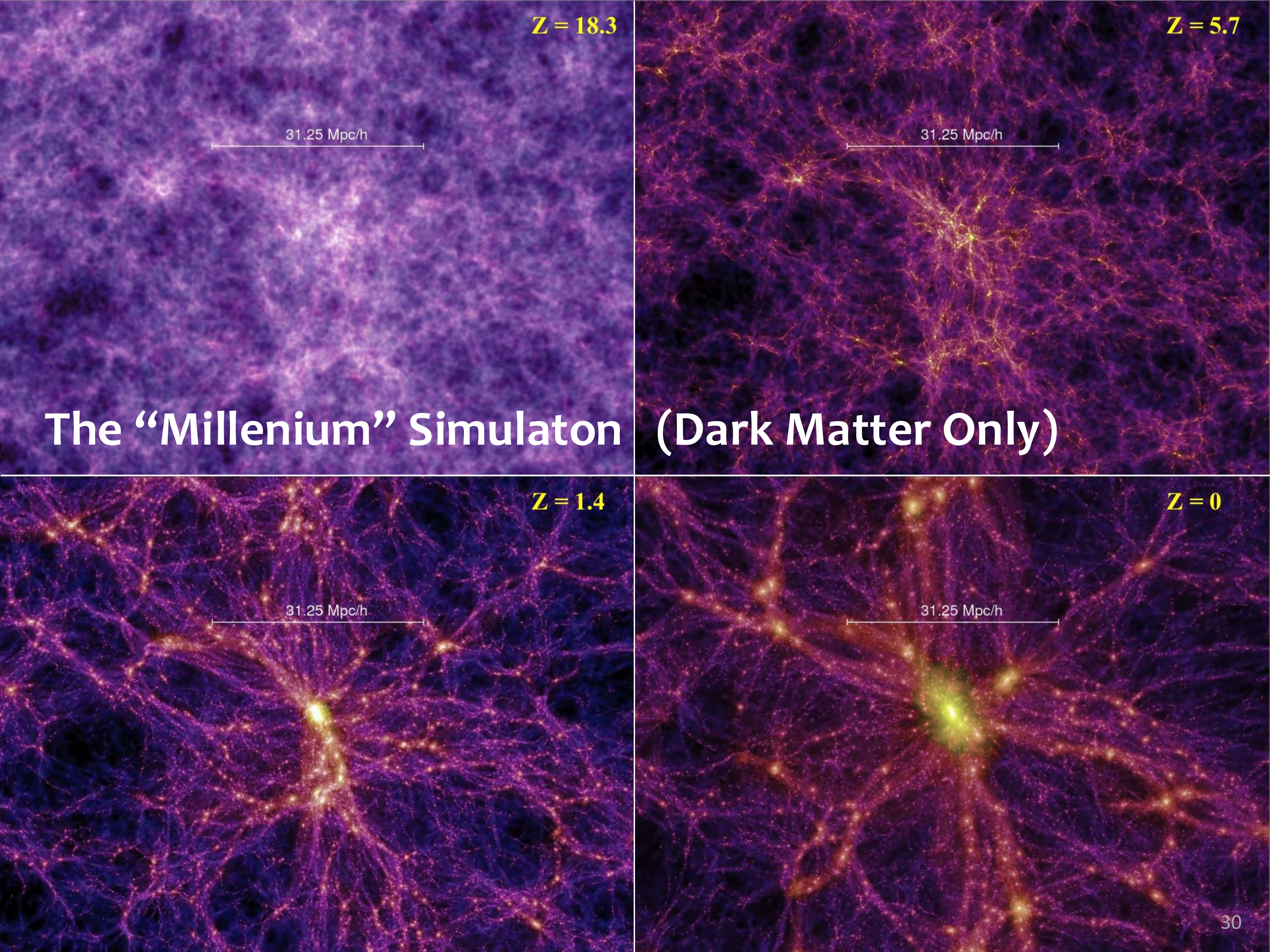
# Numerical Simulations of Galaxy Mergers



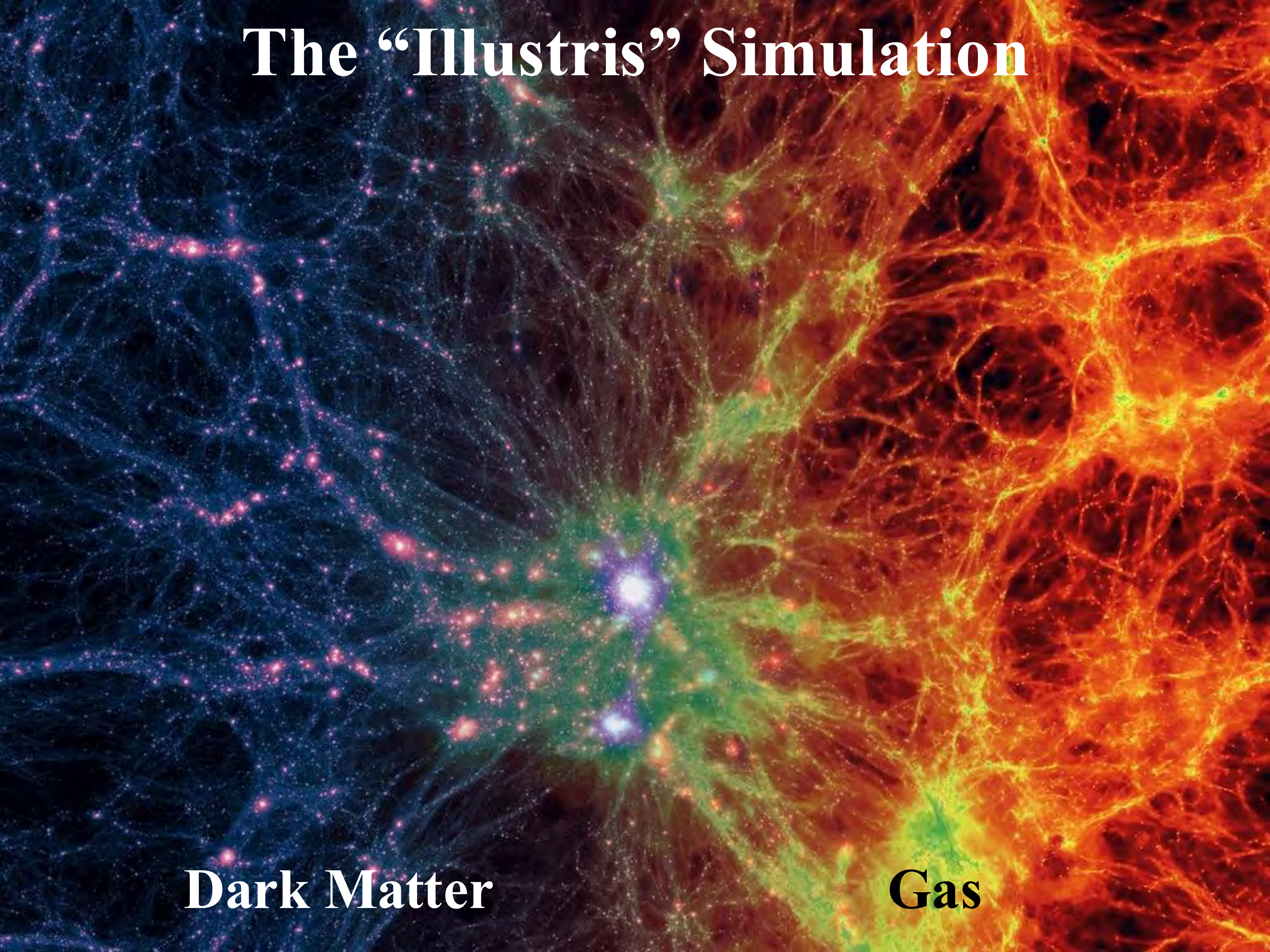

*Simulations by Barnes & Hernquist*

- Merge two nearly equal mass disk galaxies (or a lot of smaller galaxies); in a few dynamical times, ***the remnant looks just like an elliptical galaxy***
- In the same merger, ***gas quickly looses energy (since it is dissipative), and sinks towards the center***, where it can fuel a starburst, or an AGN if a massive black hole is present

# The "Millenium" Simulaton (Dark Matter Only)

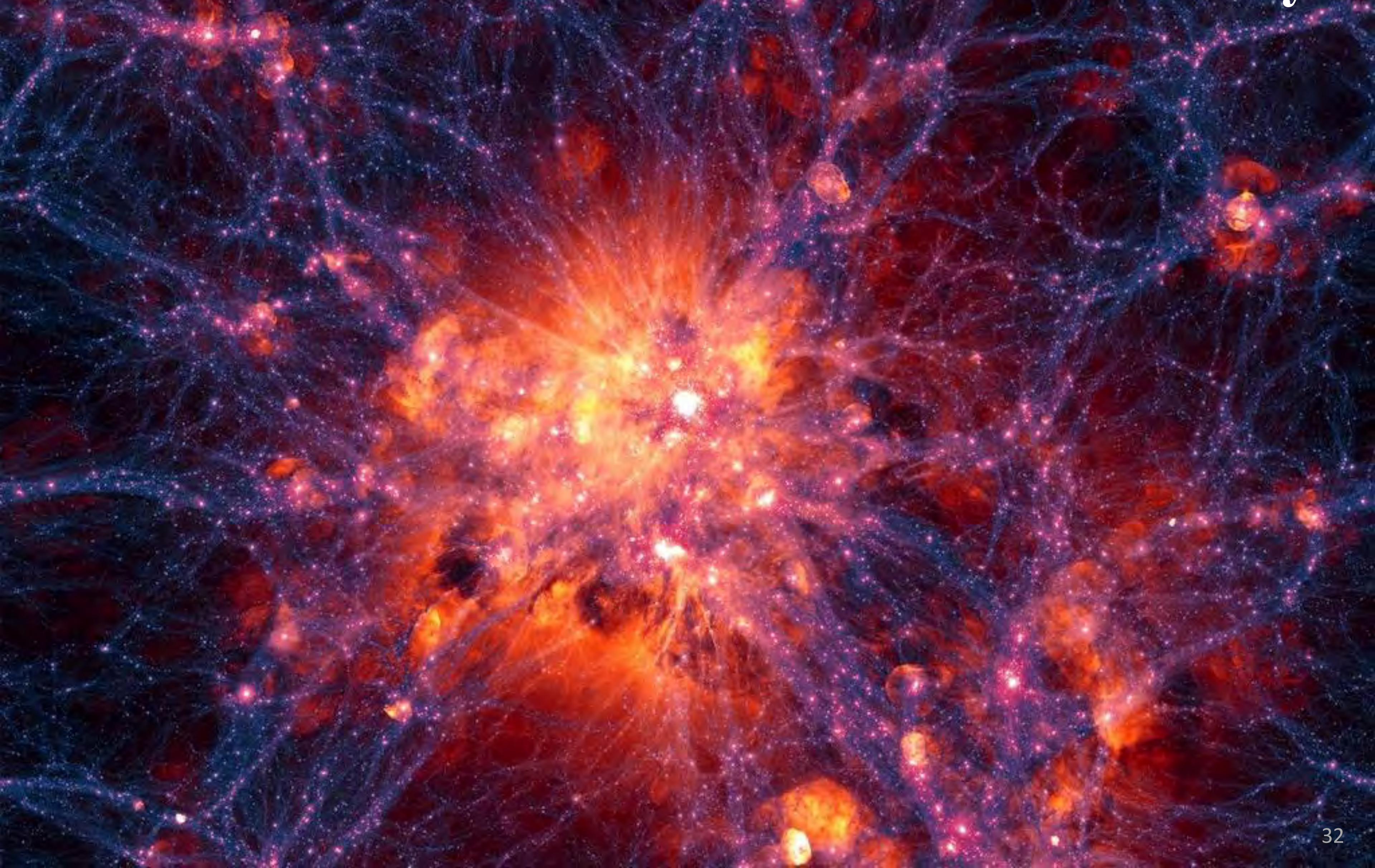


# The "Illustris" Simulation



# The “Illustris” Simulation

## Gas Velocities Overlaid on Dark Matter Density



# Structure Formation Theory: A Summary of the Key Ideas (1)

- Structure grows from ***initial density perturbations*** in the early universe, via gravitational infall and hierarchical merging
- ***Initial conditions*** described by the primordial density (Fourier power) spectrum $P(k)$, often assumed to be a power-law, e.g., $P(k) \sim k^n$, $n = 1$ is called a Harrison-Zeldovich spectrum
- ***Dark matter (DM) plays a key role***: fluctuations can grow prior to the recombination; after the recombination, baryons fall in the potential wells of DM fluc’s (proto-halos)
- ***Damping mechanisms erase small-scale fluctuations***; how much, depends on the nature of the DM: HDM erases too much of the high-freq. power, ***CDM fits all the data***
- Collapse occurs as blobs ? sheets ? filaments ? clusters

# Structure Formation Theory: A Summary of the Key Ideas (2)

- ***Pure (dissipationless) gravitational infall leads to overdensities of ~ 200*** when the virialization is complete
- ***Free-fall time scales*** imply galaxy formation early on ($t_{ff} \sim$ a few $\times 10^8$ yrs), clusters are still forming ($t_{ff} \sim$ a few $\times 10^9$ yrs)
- ***Characteristic mass for gravitational instability is the Jeans mass***; it grows before the recombination, then drops precipitously, from $\sim 10^{16}\ M_\odot$, to $\sim 10^5\ M_\odot$
- ***Cooling is a key concept***:
  - ***Galaxies cool faster than the free-fall time***: formation dominated by the ***dissipative*** processes, achieve ***high densities***
  - ***Groups and clusters cool too slowly***: formation dominated by ***self gravity, lower densities*** achieved
  - The cooling curve separates them