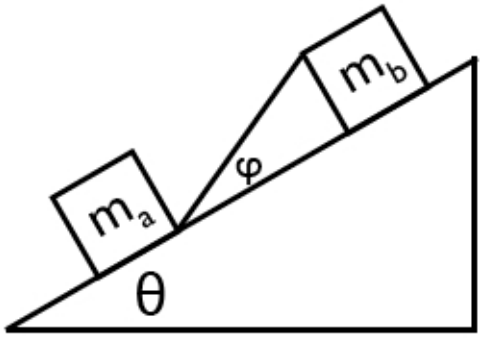
Physics 106a – Problem Set 1 – Due Oct 7, 2004

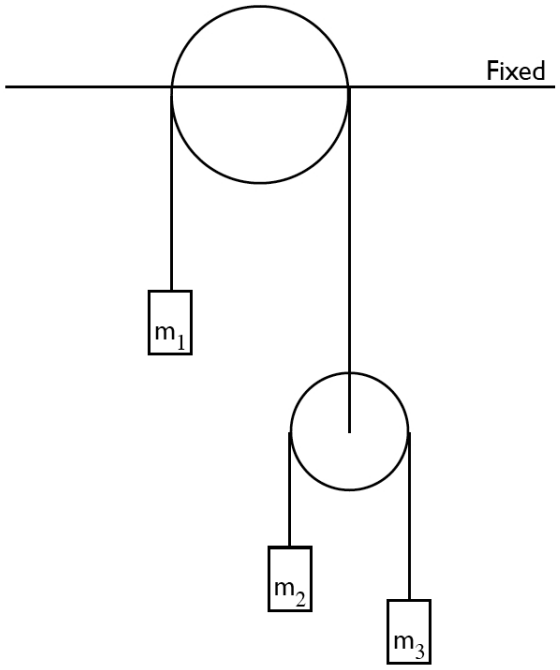
These problems cover the material on Newtonian mechanics and gravitation in Sections 1.1 and 1.2 of the lecture notes.

1. Two blocks of masses m_a and m_b are sliding down an inclined plane making an angle θ with the horizontal. The leading block has a coefficient of kinetic friction μ_k; the trailing block has a coefficient of kinetic friction $2\,\mu_k$. A string connects the two blocks; this string makes and angle ϕ with the ramp. Find the acceleration of the blocks and the tension in the string.



Problem 1

2. A mass m_1 hangs from one end of a string passing over a frictionless massless pulley. A second frictionless, massless pulley hangs from the other end of the string. Masses m_2 and m_3 hang from a second string passing over this second pulley. Find the acceleration of the three masses, and find the tensions in the two strings.



Problem 2

3. A circle of rope of mass m and radius r is spinning about its center so that each point of the rope has a speed v. Calculate the tension in the rope. (Hint: consider the force on an infinitesimal section of the rope that subtends an angle $d\theta$ at the center.)

4. A particle of mass m slides down an inclined plane under the influence of gravity. The plane makes an angle θ with horizontal. If the motion is resisted by a force $|\vec{F}| = kv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}\left(e^{kd/m}\right)}{\sqrt{\frac{k\,g}{m}\,\sin\theta}}$$

5. A car is moving at constant speed u. A person in the car fires an arrow straight ahead with a speed v with respect to the car. What is the kinetic energy gain of the arrow as measured by a person in the car? By a person standing by the side of the road? How much work is done by the bow shooting the arrow? And by the car?

6. A particle of mass m moving in one dimension has potential energy $U(x) = U_0\left[2(x/a)^2 - (x/a)^4\right]$ where U_0 and a are positive constants.

   - Find the force $F(x)$ that acts on the particle.
   - Sketch $U(x)$. Find the positions of stable and unstable equilibrium.

- What is the minimum speed the particle must have at the origin to escape to infinity?
- At $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed. Find $x(t)$ and sketch the result.

Note: in order to ensure a level playing field, be aware that this is Thornton and Marion problem 2.52, with the results provided in the solutions appendix to that text. Of course, you must show your work, so looking at the answers will only confirm that you got the correct result (and might help you find math errors).

7. A planet of density ρ_1 (constant density, spherical core, radius R_1) with a thick spherical cloud of dust (constant density ρ_2, radius R_2) is discovered. What is the force on a particle of mass m placed within the dust cloud?

8. A thin disk of mass M and radius R lies in the (x, y) plane with the z-axis passing through the center of the disk. Calculate the gravitational potential $\Psi(z)$ and the gravitational field $\vec{g}(z) = \vec{\nabla}\Psi(z) = -\hat{z}\, d\Psi(z)/dz$ along the z axis *only*. You will find the problem extremely difficult if you go off the z axis.