Physics 106a – Problem Set 2 – Due Oct 14, 2004

These problems cover the material on Newtonian mechanics of systems of particles in Section 1.3 of the lecture notes.

1. The center of gravity of a system of particles is the point about which external gravitational forces exert no net torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for the system of particles. (Review the discussion of angular momentum and torque on a system of particles and it should become clear how to approach this problem.)

2. Suppose that water drops are released from a point at the edge of a roof with a constant time interval Δt between one water drop and the next. The drops fall a distance l to the ground. If Δt is very short (so that the number of water drops falling through the air at any given instant is very large), find the height of the center of mass of the falling drops in terms of l.

3. Show that the center of mass of a uniform flat triangular plate is at the point of intersection of the lines drawn from the vertices to the midpoints of the opposite sides.

4. A two-stage rocket is to be built capable of accelerating a 100-kg payload to a velocity of 6000 m/sec in free flight in empty space (no gravitational field). Assume that the fuel is exhausted at a speed of 1500 m/sec, that the mass loss rate $\alpha = -\dot{m}$ is constant, and that the mass of an empty fuel container/rocket engine is 10% of the mass of fuel it can carry. Find the optimum choice of masses for the two stages such that the total take-off weight is minimized. Show that it is impossible to build a single-stage rocket that will provide the necessary final payload speed. Hints:

   - In terms of the formulae used in the lecture notes, the 10% relation says that if m_a is the first-stage payload mass, m_b is the empty fuel container/rocket mass, $m_1 = m_a + m_b$ is the mass of the first stage payload + empty rocket, and $m_0 = m_a + m_b + m_{f,1}$ is the initial mass of the first stage rocket including fuel, then $m_b = 0.1\, m_{f,1}$. And similarly for the second stage.
   - Be careful to understand which quantities have been provided and which remain as free parameters.

5. A rocket is fired vertically upward in the presence of a constant gravitational field. The initial mass is m_0, the exhaust speed u is constant, and the exhaust rate $\alpha = -\dot{m}$ is constant. After a total mass $\Delta\, m$ is expelled, the rocket has run out of fuel. Set up and solve the equation of motion to find the altitude as a function of time. Show that if m_0, u, and $\Delta\, m$ are fixed, then the larger the exhaust rate α, the greater the maximum altitude of the rocket.

6. A rope of length l is laid out across a frictionless table top with an end of length α hanging over the edge. The rope is released from rest. Find the time at which the rope leaves the

table. Solve the problem using both force and energy methods. Note: we have not discussed yet in this course second-order differential equations of the form $\ddot{x} - \alpha\, x = 0$. The generic solution to such an equation is $x(t) = C_1 \exp(\sqrt{\alpha}\, t) + C_2 \exp(-\sqrt{\alpha}\, t)$ with C_1 and C_2 to be determined by initial conditions.

7. A space ship of mass m_s, initial velocity $\vec{u}_s$ approaches the moon and passes by it. The distance of closest approach is R (measured from the center of the moon). The velocity $\vec{u}_s$ is perpendicular to the orbital velocity of the moon $\vec{u}_m$. Show that if the space ship passes behind the moon, it will gain energy, and calculate the increase in its kinetic energy as it leaves the vicinity of the moon as a function of the angle by which its path has been deflected in the center-of-mass frame. Do **not** attempt to calculate the deflection angle as a function of R. Assume $m_m \gg m_s$ where m_m is the mass of the moon. (Hint: Because you are not asked to determine the deflection angle, this is is not a gravity problem, it is a collision problem.)

8. A particle of mass m_1 with initial lab-frame velocity u_1 collides with a particle of mass m_2 at rest in the lab frame. The particle m_1 is scattered through angle ψ_1 in the lab frame and has final velocity v_1 where $v_1 = v_1(\psi_1)$. Find the surface such that the time of travel of the scattered particle 1 from the point of collision to the surface is independent of the scattering angle. After you have derived the general formula, consider the specific cases $m_2 = m_1$, $m_2 = 2\, m_1$, and $m_2 = \infty$.