Physics 106a/196a – Problem Set 2 – Due Oct 14, 2005

These problems cover the material on dynamics of systems of particles in Section 1.3 of the lecture notes. Problems 1 and 2 are for 106a students only, 3, 4, and 5 for 106a and 196a students, and 6 and 7 for 196a students only.

1. (106) Find the center of mass of a system consisting of a uniformly solid cone of base diameter $2\,a$ and height h and a solid hemisphere of radius a where the two bases are touching. You do not need to rederive the center of mass for the solid hemisphere, which was done in class.

2. (106) Astronaut Stumblebum wanders too far way from the space shuttle orbiter while repairing a broken communication satellite. Stumblebum realizes that the orbiter is moving away from him at 3 m/s. Stumblebum and his maneuvering unit have a mass of 100 kg, including a pressurized tank of mass 10 kg (including fuel mass). The tank includes only 2 kg of gas that is used to propel him in space. The gas escapes with a constant velocity of 100 m/s.

   (a) Will Stumblebum run out of gas before he reaches the orbiter?

   (b) With what velocity will Stumblebum have to throw the empty tank away to reach the orbiter?

   You may neglect the amount of time it takes for the gas to be released from the tank.

   Any apparent implications about the health of our nation's manned spaceflight program are entirely coincidental.

3. (106/196) A uniform rope of total length $2\,a$ hangs in equilibrium, over a smooth nail. A very small impulse causes the rope to slowly roll off the nail. Find the velocity of the rope as it just clears the nail. Assume the rope is prevented from lifting off the nail and is in free fall. Do the problem by both force and energy methods. Hint: you do not need to obtain a solution as a function of time (though you can do the problem that way); the problem only asks for the velocity at a certain *position*.

4. (106/196) Ballistic pendulum: A wooden block of mass M hangs by a massless, rigid rod of length l from a support point. A bullet of mass m shot at the block with speed v becomes embedded in the block, causing the block and rod to swing like a pendulum. Why is this an inelastic collision? Is energy liberated or absorbed, and how much? What is the total impulse that the bullet experiences? What about the impulse felt by the block? Why are these impulses related in the way they are? Find the maximum angle to which the block swings. No small-angle approximations are necessary in this problem.

5. (106/196) A neutral particle of unknown momentum and direction produces a reaction in a bubble chamber in which two charged particles of masses m_3, m_4 emerge with momenta p_3, p_4. The angle between their tracks is α. Find the direction and momentum of the incident particle. If the mass m_1 of the incident particle is known or suspected, find the energy Q

released in the reaction. This type of event corresponds to neutral kaon decay to two charged pions, the reaction by which the kaon was discovered – see

http://hepweb.rl.ac.uk/ppUKpics/POW/pr_971217.html

6. (196) A water droplet falling in the atmosphere is spherical. Assume that as the droplet passes through a cloud, it acquires mass at a rate equal to $k\,A$ where k is a positive constant and A is its cross-sectional area. Consider a droplet of initial radius r_0 that enters a cloud with a velocity v_0. Assume no resistive force and show

   (a) that the radius increases linearly with the time

   (b) that if r_0 is negligibly small then the speed increases linearly with the time within the cloud.

7. (196) The Compton scattering of x-rays can be interpreted as the result of elastic collisions between photons and free electrons. According to quantum theory, a photon of wavelength λ has kinetic energy $h\,c/\lambda$ and linear momentum of magnitude h/λ, where h is Planck's constant and c is the speed of light. In the Compton effect, an incident x-ray of known wavelength λ_i is scattered by an electron initially at rest. The outgoing photon makes an angle ψ_1 relative to the incoming photon direction and has longer wavelength λ_f. Assuming this process can be treated as an elastic collision, set up the equations expressing conservation of energy and momentum, using relativistic expressions for the electron energy and momentum. Show that the change in x-ray wavelength is

$$\lambda_f - \lambda_i = \frac{h}{m\,c}\,(1 - \cos\psi_1)$$

and that the angle the outgoing electron momentum makes with the incoming photon momentum is

$$\tan\psi_2 = \frac{\sin\psi_1}{\left(1 + \frac{h}{\lambda_i\,m\,c}\right)(1 - \cos\psi_1)}$$

The relativistic expressions for the momentum and kinetic energy of the electron are

$$p = \gamma\,m\,v$$
$$T = (\gamma - 1)\,m\,c^2$$

with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where v is the electron velocity. (We will derive these relativistic expressions later this term and early next term.)

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