Physics 106a/196a – Problem Set 3 – Due Oct 20, 2006

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These problems cover the material on Lagrangian mechanics in Section 2.1 of the lecture notes, as well as in Hand and Finch Ch. 1. Problems 1 and 2 are for 106a students only, 3 and 4 for 106a and 196a students, and 5 and 6 for 196a students only.
v. 2: Hand and Finch 1-10, Eqn 1.95 *is* correct. My original error on the sign was a good example of misapplying the time derivative. I could have found the error quickly by trying a test case.

1. (106) Hand and Finch 1-3
2. (106) Hand and Finch 1-16. In addition, find the stable and unstable equilibria and explain briefly how and why these depend on d (good exercise in seeing through the math).
3. (106/196) Hand and Finch 1-22
4. (106/196) Hand and Finch 1-10. Equation 1.95 *is* correct. Part (b) should be clarified to read: "Use this fact to solve for $\dot{x}(t)$ in terms of x, $f(x)$, and the constant value of H, which you may call E." Part (c) is also a bit unclear. You should assume that the bead starts at rest, that the starting point is $y_0 = f(0)$ and that $y_0 > y_1$ (obviously, the bead will not slide down from y_0 to y_1 if $y_0 < y_1$), and that the solution will involve y_0. Why is the condition $y_0 > y_1$ mathematically (as opposed to physically) necessary? Note how the math enforces the physics.
5. (196) Consider the ladder example in Chapter 1 of Hand and Finch, Figure 1.5.
   (a) How many degrees of freedom are there if you ignore the constraint of the ladder resting against the wall? Define at least one set of coordinates that encompass the degrees of freedom.
   (b) Write down the constraints and define a set of generalized coordinates that accomodate the constraints.
   (c) Write down a virtual displacement in terms of the generalized coordinates and the original coordinates.
   (d) Write down the generalized forces corresponding to the generalized coordinates.
   (e) Write down the kinetic energy in terms of the generalized coordinates and velocities.
   (f) Find the generalized equation of motion.
   (g) Write down the potential energy and the Lagrangian and show the Euler-Lagrange equation gives the same result at the generalized equation of motion.

   You should obviously pick generalized coordinates that make the algebra as easy as possible.

6. (196) Two wheels of radius a are mounted on the ends of a common axle of length b such that the wheels rotate independently. The whole combination rolls without slipping on a plane. Show that there are two nonholonomic equations of constraint,

$$\cos\theta\, dx + \sin\theta\, dy = 0$$
$$\sin\theta\, dx - \cos\theta\, dy = \frac{1}{2} a\,(d\phi_1 + d\phi_2)$$

(where θ, ϕ_1, and ϕ_2 are similar in meaning as for the case of a simple disk rolling on a plane and x and y are the coordinates of a point on the axle midway beween the two wheels) and one holonomic equation of constraint

$$\theta = C - \frac{a}{b}\,(\phi_1 - \phi_2)$$

where C is a constant.