Physics 125b – Problem Set 14 – Due Mar 11, 2008

Version 1 – Mar 5, 2008

Many basic problems in QM can be found in textbooks – there are only so many solvable elementary problems out there. Please refrain from using solutions from other textbooks. Obviously, you will learn more and develop better intuition for QM by solving the problems yourself. We are happy to provide hints to get you through the tricky parts of a problem, but you *must* learn to set up and solve these problems from scratch by yourself.

1. Shankar 17.2.3 — Perturbation of hydrogen atom by finite-radius nucleus

2. Shankar 17.2.4 — Thomas-Reiche-Kuhn sum rule

3. Shankar 17.3.3 — Zeeman effect (though Shankar does not say it!). Hint: remember that the constant of proportionality between the $\vec{S}$ and its magnetic moment is different than between $\vec{L}$ and its magnetic moment.

4. The spin-orbit interaction in two-electron atoms gives three distinct energies in the triplet series. What are the values of $\left\langle \vec{L} \cdot \vec{S} \right\rangle$ if one electron is an s electron and the other one is a d electron?

5. One of the first highly unstable particles discovered was the π meson predicted by Yukawa. This particle, which plays an important role in nuclear forces, comes in three charge states, π^+, π^0, and π^-. It was found to have spin 0, so the question arose as to whether the spatial wavefunction of the π meson (pion), was odd or even under reflection, assuming that the known nuclear particles, the proton and the neutron, had positive intrinsic parity (wavefunction even under reflection). The following experiment was suggested.

   Consider the capture of π^- by a deuteron (a deuterium nucleus, consisting of a proton and a neutron). A slow pion in liquid deuterium loses energy by a variety of mechanisms, until finally it slows down and ends up in a Bohr orbit of the nucleus, and then is captured by the nucleus through the action of nuclear forces. In the nuclear reaction

   $$\pi^- + d \rightarrow n + n \tag{1}$$

   the angular momentum is 1; the pion has zero spin, the orbital angular momentum is 0 in the lowest Bohr state, so the only contribution is the angular momentum of the deuteron, which is 1. The two neutrons must therefore be in an angular momentum 1 state. If the total spin of the two neutrons is 0, then the total orbital angular momentum must be 1. If the total spin of the two-neutron state is 1, then orbital angular momentum 0, 1, 2 are possible. However, a singlet state of two identical fermions must have even angular momentum, and is thus excluded. A triplet state must have odd orbital angular momentum, and this is possible if the orbital angular momentum is 1. Such a state, however, has odd parity, and hence the pion must have odd parity.

Suppose the π^- had spin 1, but was still captured in a $l = 0$ orbital state. What are the possible two-neutron final states? Which states are allowed if the π^- had negative parity?

Suppose the π^- has spin 0 and negative parity, but is captured in the above reaction from a $l = 1$ orbital state. Show that the two neutrons must be in a singlet state.