

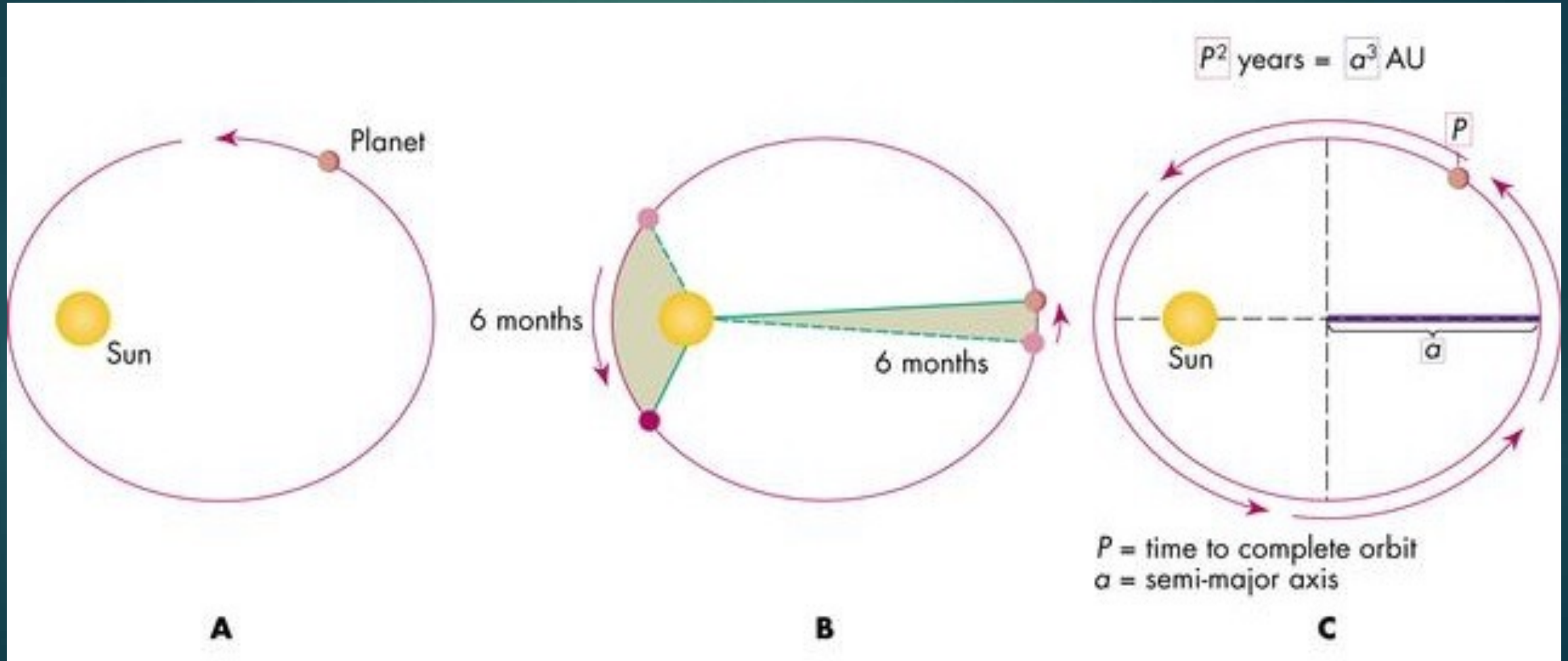
Johannes Kepler: The Unity and Harmony of the Cosmos

Power Laws, Scale Independence,
and the Perfect Fifth

Kepler's Triumph's

- ▶ The Third Law
- ▶ Humankind's first empirically established power law
- ▶ Scale independence
- ▶ Mass of planets, stars, galaxies, black holes, clusters of galaxies

Kepler's Three Laws of Planetary Motion



- ▶ Celestial objects do not follow perfect, circular orbits around the earth.
- ▶ The second law challenges the belief that a celestial object moves with unchanging speed. Furthermore, this is a quantifiably testable prediction. Their speed is greatest closest to the sun and slowest at aphelion. This is a daring prediction based on scant data
- ▶ The third law reveals Kepler's belief that mathematics and music are the language of God
- ▶ It's beautiful in its simplicity

$$a^3 = P^2$$

Kepler's epiphany of the Third Law was on 15 May, 1618


- ▶ *and if you want the exact moment in time, it was conceived mentally on 8th March in this year one thousand six hundred and eighteen, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the **15th of May** and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labour of seventeen years on the observations of Brahe and the present study, which conspired together, that at first I believed I was dreaming, and assuming my conclusion among my basic premises. But it is absolutely certain and exact that "the proportion between the periodic times of any two planets is precisely the sesquialterate [3/2] proportion of their mean distances ..." (Harmonice mundi Book 5, Chapter 3, trans. Aiton, Duncan and Field, p. 411).*

Why did Kepler choose $3/2$ to test with Tycho's data?

- ▶ As a student of music theory, he would have known that the perfect fifth is the most consonant of all intervals which can be formed. Many people believe this interval is the basis of harmony. The fifth is also one of the easiest intervals for humans to learn to sing, meaning it is a strong and easily-learned musical interval that most people can learn to hear relatively quickly.
- ▶ He was a good enough mathematician to be able to test various exponents, using the newly discovered logarithms of Napier. How many attempts we have no way of knowing. We don't know how soon he chose the musical interval known as Perfect Fifth to test. The Perfect Fifth consists of two notes that have a frequency ratio of $3:2$. The violin, viola, and cello are tuned by Perfect Fifths separating each string. Because it is the most consonant interval, it is the most pleasing to the ear. The strings of a violin, for example, are tuned to G, D, E, A, each separated by Perfect Fifths.
- ▶ The Perfect Fifth is the most prominent of the Pythagorean intervals, which are all built on 3 and 2. Each of these musical intervals involves a power of 2 and a power of 3.

All western stringed instruments use the Perfect Fifth for tuning.

Violin	Frequency	Ratio of upper frequency to lower frequency
E5	659.3	1.4984
A4	440	1.4981
D4	293.7	1.4985
G3	196	
Viola		
A4	440	1.4981
D4	293.7	1.4985
G3	196	1.4985
C3	130.8	
Cello		
A3	220	1.4986
D3	146.8	1.4980
G2	98	1.4985
C2	65.4	



▶ Whenever string players tune their instruments using the perfect fifth, they are in resonance with the harmony of the planets. Even when the violin is silent, the strings remain tuned to the perfect fifth, containing the pattern of the Third law. The planets continue following that pattern, quietly circling without end or beginning along their elliptical pathways.

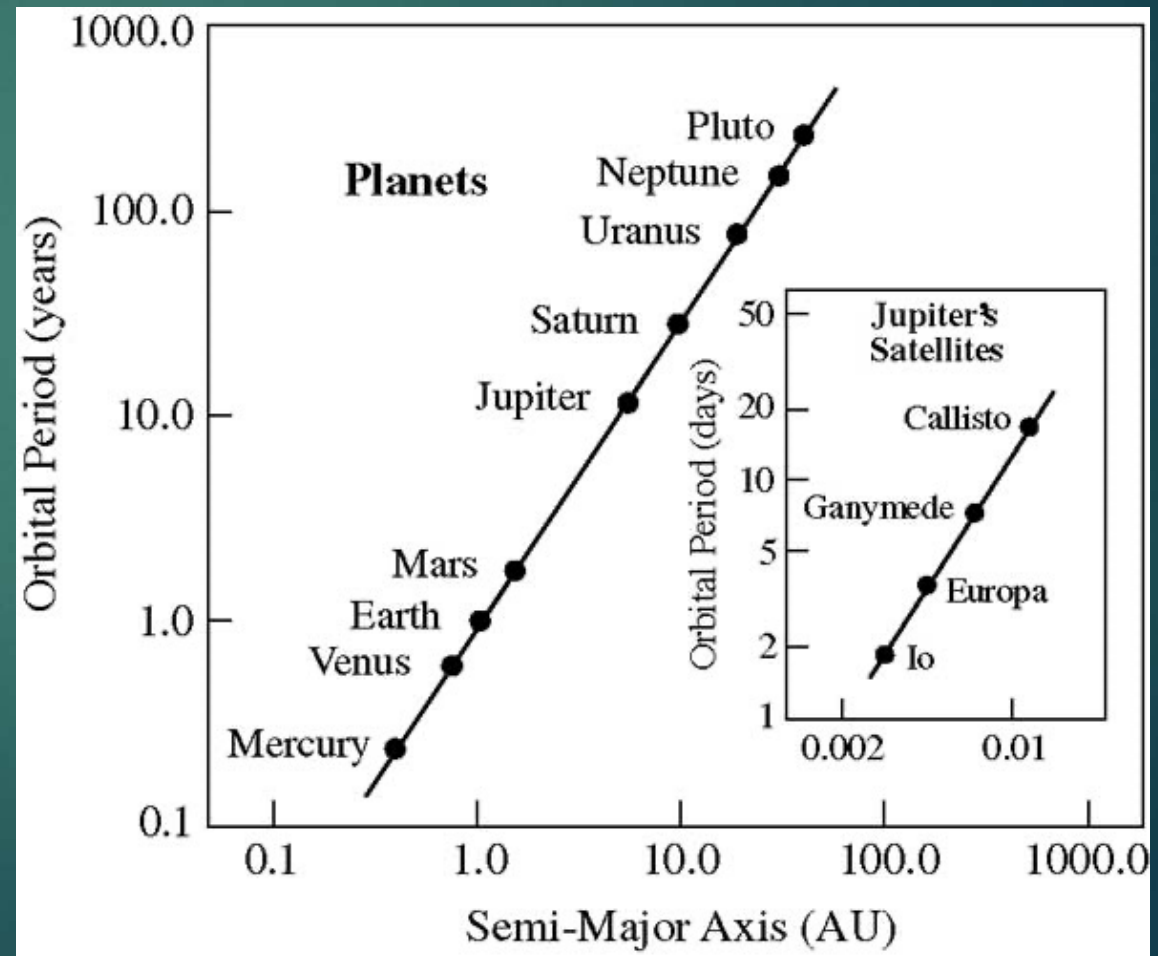
- ▶ Perhaps this is what the poet thought:
- ▶ *Only by the form, the pattern,*
- ▶ *can words or music reach*
- ▶ *The stillness, as a Chinese jar still*
- ▶ *Moves perpetually in its stillness.*
- ▶ *Not the stillness of the violin, while the note lasts,*
- ▶ *Not that only, but the co-existence,*
- ▶ *Or sat that the end precedes the beginning,*
- ▶ *And the end and the beginning were always there*
- ▶ *Before the beginning and after the end.*
- ▶ T. S. Eliot, *Burnt Norton*

Tycho Brahe's data provided the data-driven test of the Perfect Fifth

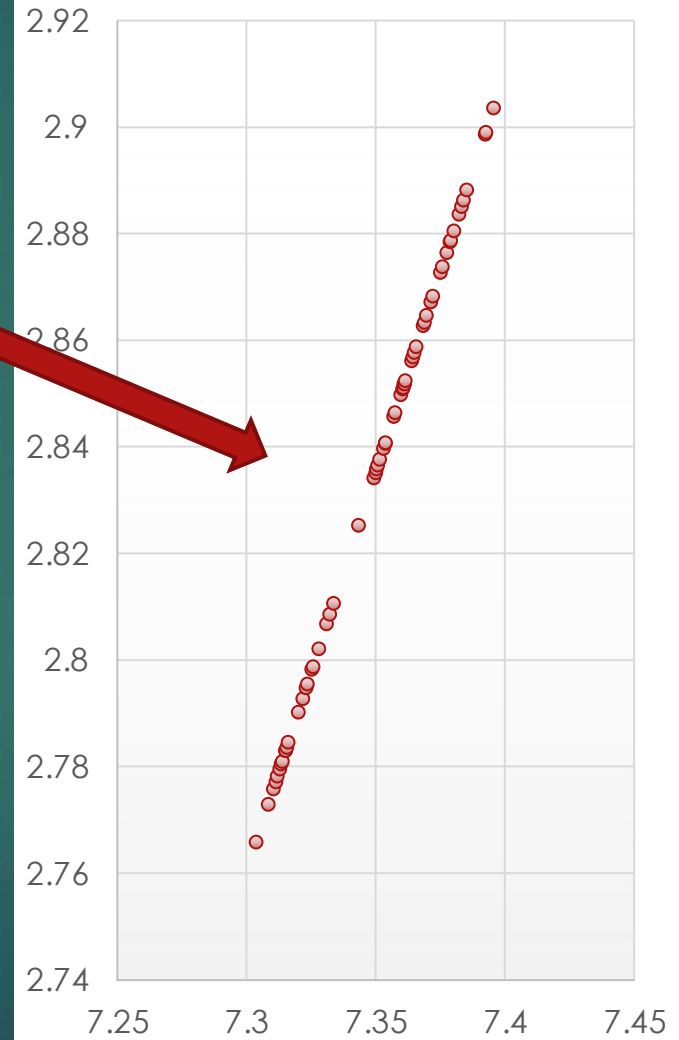
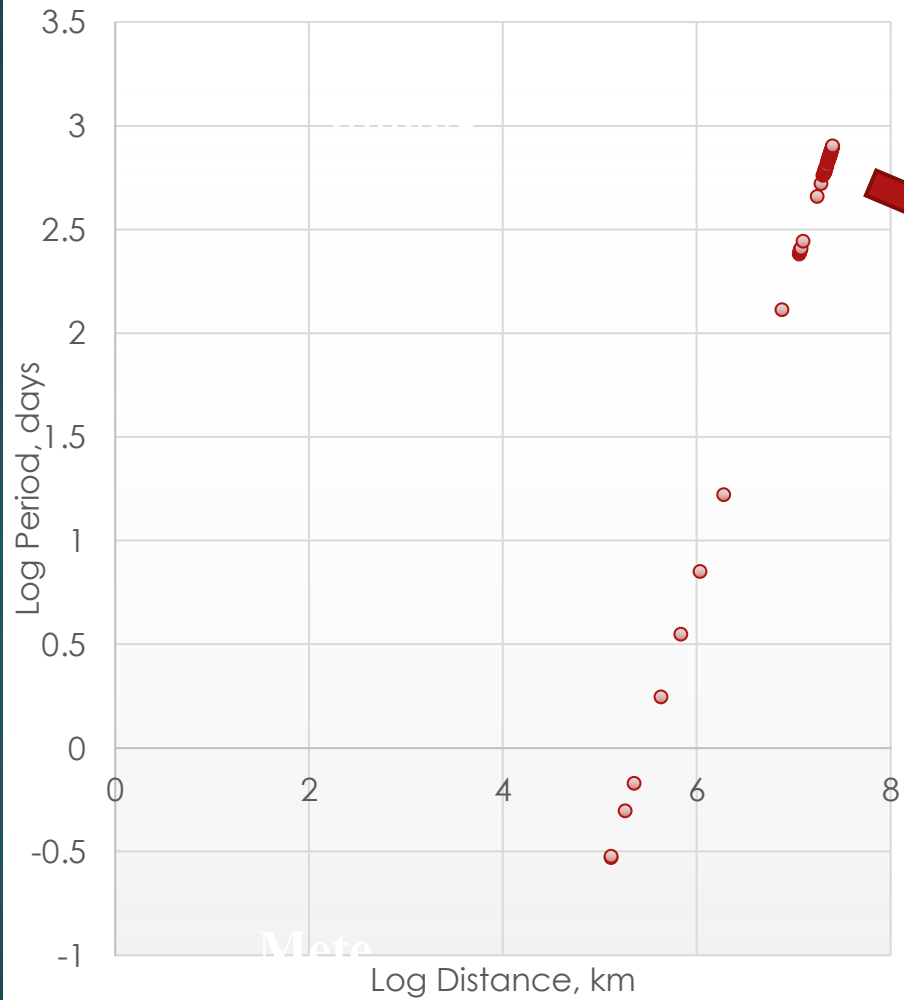
Planet	a (AU)	P (days)	Perfect Fifth: $a^3/P^2 \times 10^{-6}$	Perfect Fourth: $a^4/P^3 \times 10^{-8}$	Lower imperfect fifth $a^{40}/P^{27} \times 10^{-71}$
M	.389	87.77	7.64	3.39	134
V	.724	224.7	7.52	2.42	78.6
E	1	365.25	7.50	2.05	64.6
M	1.524	686.95	7.49	1.66	52.8
Jupiter	5.2	4332.62	7.43	.899	28.0
Saturn	9.51	10759.2	7.43	.657	18.6

A characteristic of power laws is scale invariance, which is significant in astrophysics

- ▶ The same $3/2$ power law works regardless of the scale. There is no fundamental length!
- ▶ Kepler discovered that feature when he showed it worked for the moons of Jupiter which had been discovered by Galileo just 8-9 years earlier.



The Satellites of Saturn



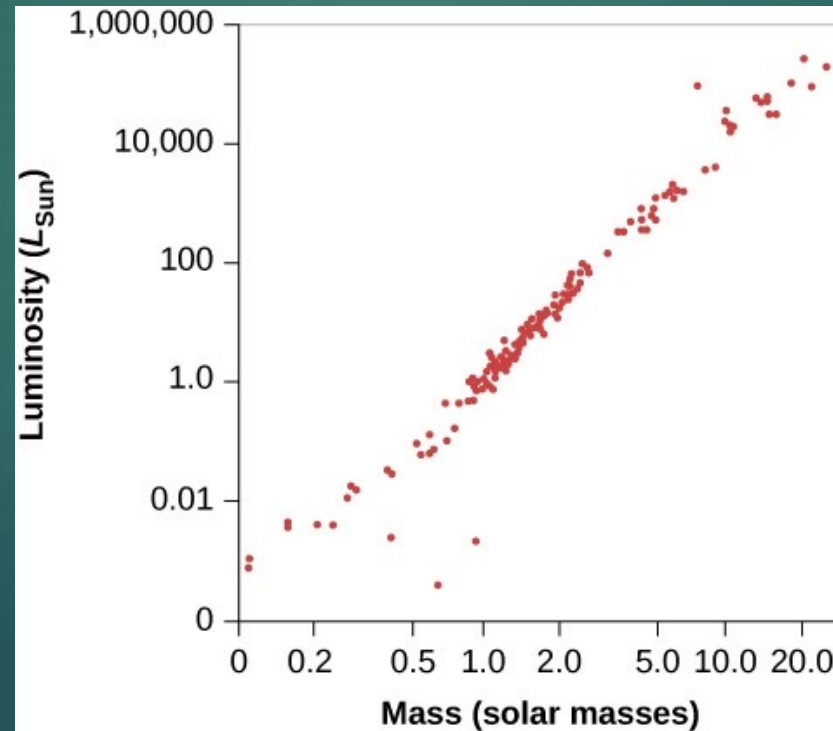
The next step was for Newton to utilize the third law to propose the law of gravity

“In the beginning of the year 1665, I found the method of approximating series and the Rule for reducing any dignity of any Binomial into such a series. The same year in ... November had the direct method of Fluxions, and in January had the Theory of Colours, and in May following I had entrance into the inverse method of Fluxions. And the same year I began to think of the orb of the Moon ... from Kepler’s Rule of the periodical times of the Planets ... I deduced the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve.”

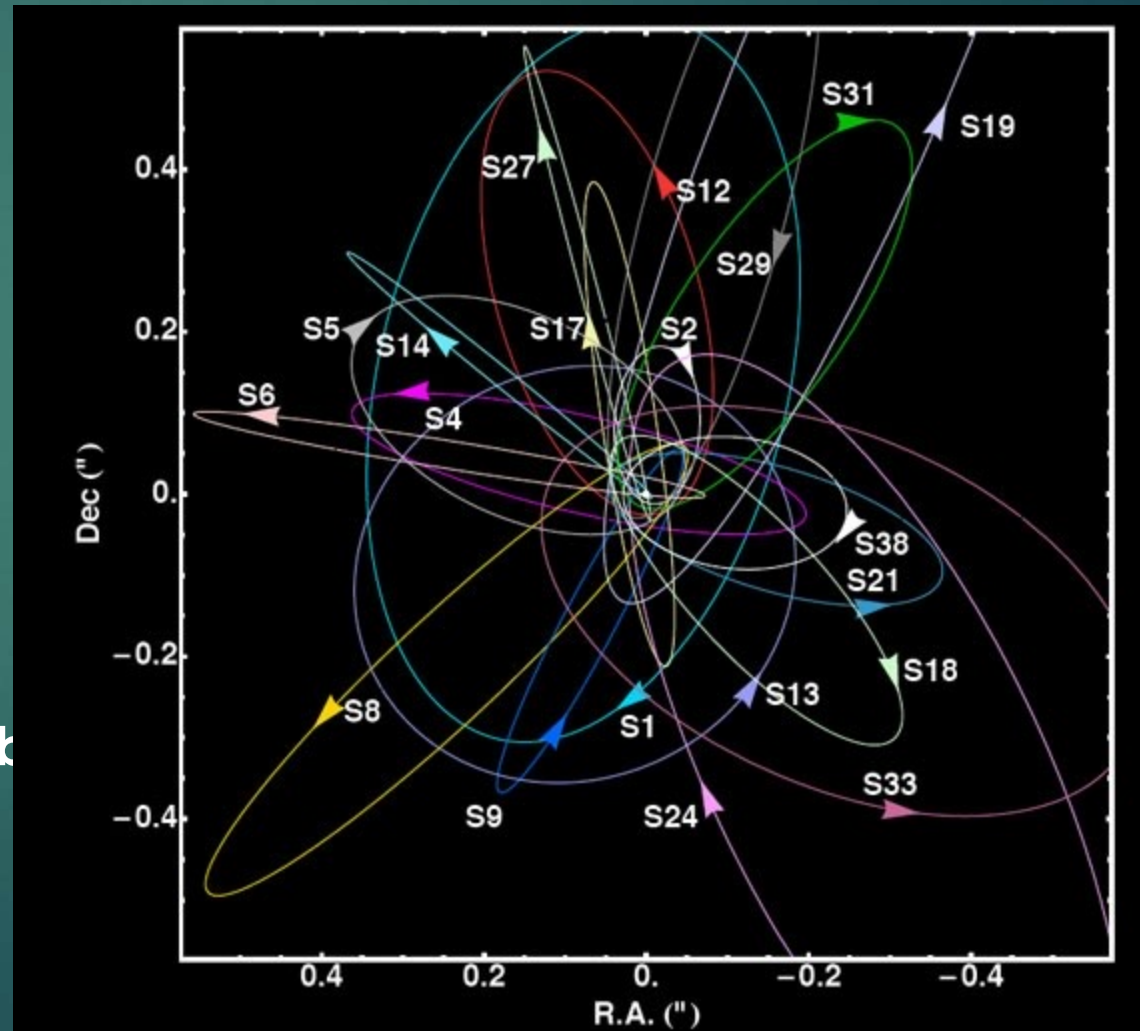
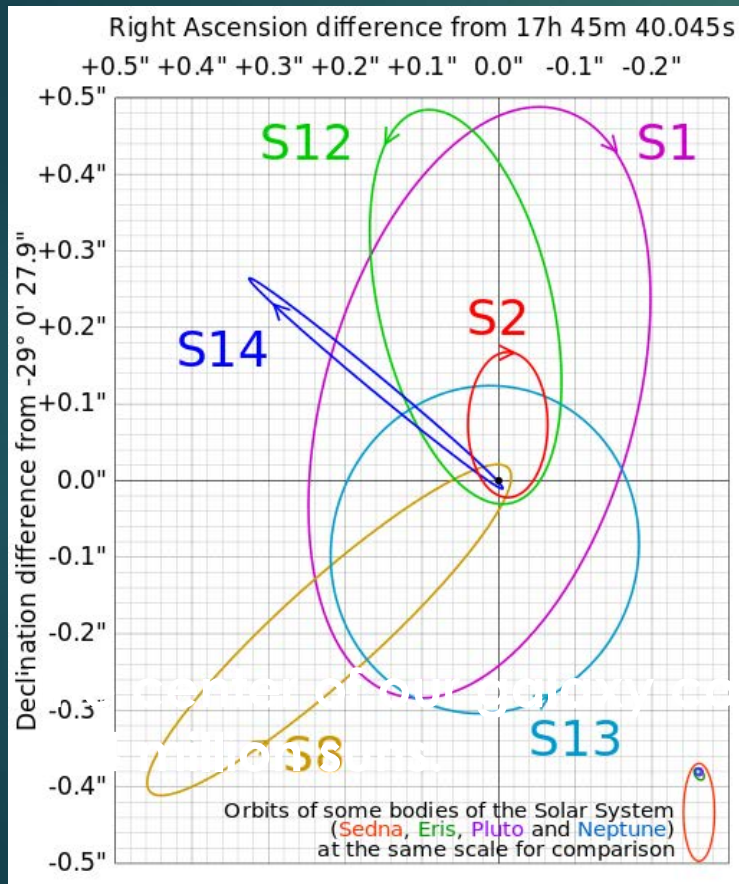
The result was a new form of
the power law

$$M = a^3/P^2$$

Which has been an essential tool of astronomy
and astrophysics ever since.

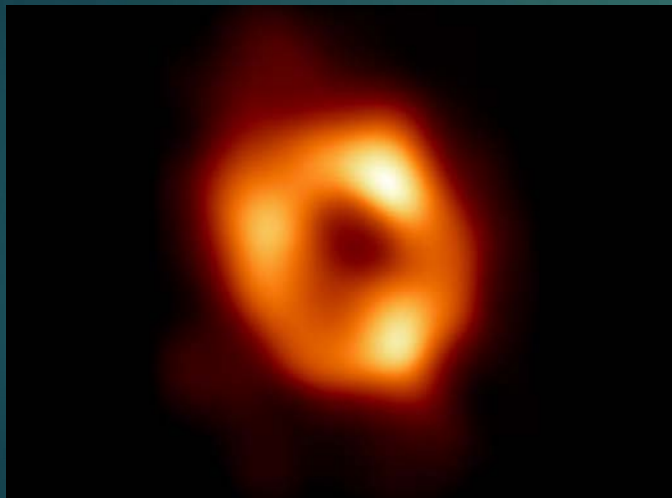


The “Keplerian” orbits of stars around the black hole SgrA*



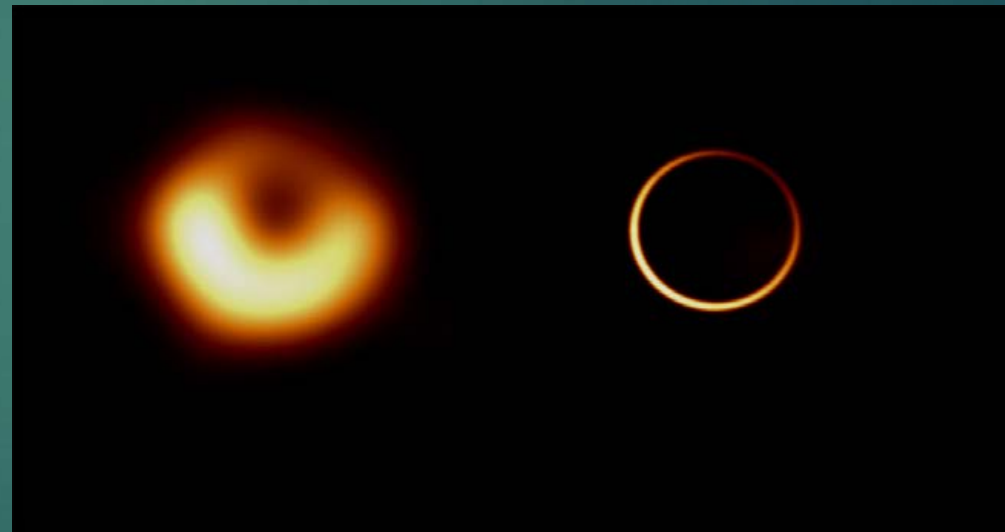
The crowning triumph of the third law and scale independence is found in the centers of M87 and the Milky Way galaxy. SgrA*

SgrA*



4 million solar masses

M87



6.5 billion solar masses

We have come full circle from 1618 and the power laws and scale independence discovered by Kepler. M87 is a thousand times more massive than Sgr* but it is a thousand times further away; consequently, these huge black holes have about the same size in the sky. That's scale independence *par excellence*!

