Would like to know the time evol'n of
the composition of I.S. gas in galaxies, and the
resultant time evol'n of stellar abundances.
In reality, this is an enormously complicated question-
let's see how far we can get with some simplifying
assumptions.

1. Region under study has ISM of uniform composition
2. Gains or loses mass only through gas flows

For more complicated situations (e.g., abundance
gradients in galactic disks or halos) we divide
the galaxy into zones.

[Assumption 2 is, e.g., known not to be true for
early phase of the Galactic halo]

Total mass $M$ changes:
\[
\frac{dM}{dt} = f = \text{net rate of infall or accretion of gas}
\]

Mass of stars changes as a result of star formation
and mass loss from evolved stars
\[
\frac{dM_s}{dt} = \dot{M}_s = \dot{M}_\text{ejection} - \dot{M}_s \text{star formation rate}
\]
Gas mass changes via SF, ejection, and net inflow or
outflow,

\[ \frac{dM_g}{dt} = -Y + E + f \]

note \( M(t) = M_s(t) + M_g(t) \)

Gas fraction in a region:

\[ \frac{M_g}{M} \quad ; \quad M_s = (1-Y)M \]

Now, \( E \) depends on IMF and SFR. We'll assume that
each star undergoes all mass loss after a time \( T_m \)
that it lives on the M.S. If a star dies at time \( t \),
it was born \( (t - T_m) \). Let \( \omega_m \) be the
remnant mass (as a function of \( m \)).

\[ E(t) = \int_{m_t}^{\infty} (m - \omega_m) \psi(t - T_m) \varnothing(m) \, dm \]

where \( m_t \) is the mass at the top of the M.S.
at time \( t \) and \( \varnothing(m) \) is the IMF at time \( t - T_m \) (not time
now).

Generally, we lump all heavy elements other than
\( H, He \) into "metals". Assume:

1. Production of metals is only a fn. of stellar initial
   and not of initial metallicity

2. Metals are mixed "instantaneously" into the ISM
   [not too bad an approximation, since most metals are
   produced by very short-lived massive stars]

\[ \Rightarrow \text{ "Instantaneous Recycling Approximation"} \]
Mass of metals $ZM_g$ in ISM (gas phase)

\[
\frac{d}{dt} (ZM_g) = \int \gamma Z \Psi + E_Z + Z_f \Psi
\]

rate of change in stars total ejection rate of metals from stars metallicity

Now, let $p_{Zm}$ be the mass fraction of a star of mass $m$ that is converted to metals and ejected.

rate of new metals from stars at time $t$ is

\[
\int \int \int m p_{Zm} \Psi(t-T_m) \phi(m) dm
\]

Unprocessed material occupies a mass $(m-W_m-m p_{Zm})$ of the ejected part of a star of mass $m$ and the metal abundance in the unprocessed ejected gas is $Z(t-T_m)$.

So, the total ejection rate of old and new metals is

\[
E_Z(t) = \int \int \left[ (m-W_m-m p_{Zm}) Z(t-T_m) + m p_{Zm} \right] \Psi(t-T_m) \phi(m) dm
\]

The mean metallicity of stars ever formed $Z_s$ is

\[
M_g Z_s + Z_s M_g = \int \int \int m p_{Zm} \Psi(t-T_m) \phi(m) dt' dm
\]

Some quantities depend only on stellar evolution, not on $\Psi(t)$.
1) The “returned fraction” \( R \). The turnoff mass at \( t=\text{now} \) can be taken to be \( \approx 1 \, M_\odot \). \( R \) is the fraction of the mass put into stars at a given time in the past, that is returned to the ISM by now.

\[
R = \int_{m_\odot}^{\infty} (m - m_\odot) \Phi(m) \, dm
\]

\( m_\odot = 1 \, M_\odot \)

E.g., if all stars in a galaxy formed in a rapid initial burst at \( t=0 \), \( 10^{10} \) yrs later the total mass in stars would be \( M_\odot (1-R) \).

2) The “yield” of heavy elements \( y \) — the mass of new metals ejected (eventually) when \( 1 \, M_\odot \) is turned into stars.

\[
y = \frac{1}{(1-R)} \int_{m_\odot}^{\infty} m \rho_m \Phi(m) \, dm
\]

\( m_\odot = 1 \, M_\odot \)

E.g., if a mass \( M_\odot \) formed in a single initial burst, then by now \( (10^{10} \) yrs later) the mass of new metals ejected would be \( y (1-R) M_\odot \).

Most heavy elements are produced by massive stars, so \( R \) and \( y \) don’t depend strongly on \( m_\odot \) at the present epoch.

Some approximations are now possible:
\[ R \equiv \text{returned fraction} \quad R = \int_{m_0}^{\infty} \frac{(m - m_0) \varphi(m) \, dm}{M_0 (1 - R)} \]

\[ \text{e.g., if all stars formed in a burst at } t=0, 10^{10} \text{ yr later,} \]

\[ \approx 20\% \text{ of mass was returned to ISM} \]

\[ y \equiv \text{"yield"} \equiv \text{mass of new metals ejected (eventually) when } 1 \text{ MO is formed into stars} \]

\[ y = \frac{1}{(1 - R)} \int_{m_0}^{\infty} m \varphi(m) \, dm \quad \text{for } m_0 = 1 \text{ MO} \]

\[ \text{e.g., if } M_0 \text{ formed at } t=0, \text{ by } 10^{10} \text{ yr later mass of new metals ejected would be} \]

\[ y \left[(1 - R)M_0\right] \quad (y \approx 1.5 \text{ solar}) \]
Consider “Instantaneous Recycling” again:

Stars are divided into 2 classes

1. Those that never eject material \((m < m_2)\), \(T_m = \infty\)
2. Those that eject mass as soon as they are born: \(T_m \to 0\)

So, lower mass limit in integrals now taken to be \(m_1(t)\) and \(t - T_m \to t\)

\[ E(t) = R \psi(t) \]

\[ E_Z(t) = R Z(t) \psi(t) + y (1 - R) [1 - Z(t)] \psi(t) \]

where \(t = \) current time, e.g. now

In practice, \(Z \ll 1\) \((\approx 0.02\) now\) so

\[ E_Z(t) \approx R Z(t) \psi(t) + y (1 - R) \psi(t) \]

\[ \Rightarrow \text{w/ these simplifications} \]

\[ \frac{dM_g}{dt} = (1 - R) \psi \] \(\text{net infall/outflow} \)

\[ \frac{dM_g}{dt} = - (1 - R) \psi + f \]

\[ \frac{d(Z M_g)}{dt} = - Z (1 - R) \psi + y (1 - R) \psi + Z_f f \]

\[ \Rightarrow M_g \frac{dZ}{dt} = y (1 - R) + (Z_f - Z) f \]

\[ \text{difference in metallicity between infalling gas and ISM} \]
**Constant IMF**

In the case that $R, y$ are constants, but not $f(t)$

So,

$$M_3(t) = (1-R) \int_0^t \Psi(t') dt' = (1-R) \bar{\Psi} t$$

with recycling and constant IMF,

$$Z_s M_3 + Z M_G = y M_3$$

so,

$$Z_s = y - \frac{y}{1-y} Z$$

($$y = \frac{M_G}{M}$$)

"Gas fraction"

\[ \Rightarrow Z_s \to y \quad \text{as} \quad y \to 0 \]

i.e., when no gas left, all the metals ever made and ejected ($y M_3$) are incorporated into later generations of stars. An estimate of the yield for the solar neighborhood where $y=0.05$ is $y=0.02 \approx 20 \quad (Z_0 \approx 0.015)$

**CLOSED BOX SYSTEM**

Assume there is no inflow or outflow, $f=0$, $M=\text{const.}$

$$M_y^o = M_{\text{tot}}, \quad M_3^o = 0, \quad Z_o = Z_s^o = 0$$

$$\frac{d(ZM_G)}{dM_G} = Z - y \quad \text{(combine 2 eqn's to eliminate t)}$$

$$M_G \frac{dZ}{dM_G} + Z = Z - y \quad \Rightarrow \quad M_G \frac{dZ}{dM_G} = -y$$

\[ \Rightarrow \quad Z = y \ln \left( \frac{M_{\text{tot}}}{M_G} \right) \quad \text{"Simple Closed Box Model"} \]
Infall Balanced by Star Formation

Assume SF just keeps up w/rate of infall plus mass ejection from stars:

\[ \Psi = f + R \Psi \]

so, \( m_0 = \text{const} = M_0 \); \( m_0^0 = 0 \), \( z_0 = 0 \)

Assume \( z_f = 0 \) (not necessarily true!)

Can show that \( M_0 \frac{dz}{dm} = y - z \)

We solve this in terms of \( \nu \), the ratio of the accreted mass to the initial mass

\[ \nu \equiv \frac{M - M_0}{M_0} = \frac{M - M_0}{M_0} = y - 1 \]

and \( z = y (1 - e^{-\nu}) \)

Since \( z_0 = y - \frac{1}{1-y} z = y - \frac{1}{1-y} y (1 - e^{-\nu}) \)

In this case, as \( y \to 0 \), \( z \to y \); whereas in closed box model \( z \to 1 \)

Represents an equilibrium between infall and rate of enrichment by evolved stars.
General Statements

For instantaneous recycling and constant IMF,

1. $Z \propto y$. True for "primary" elements whose production in stars independent of initial composition.

2. $Z \propto y$ depends mostly on current properties of the system.

3. $Z \propto y$ does not depend strongly on $y$, $\Psi/f \Rightarrow$ regions of galaxies w/ very different $M/\psi$ may have similar $Z$.

Comparison w/ Observations


- Over $10^9$ yrs, the abundances in the Galactic disk have only changed from $Z/4$ to $Z$.

"The G-Dwarf Problem"

Basically, there are far too few metal-poor (disk) stars compared to expectation of simple models (first realized in 1962!)

Under a closed box model, one can work out the cumulative metallicity distribution of stars still on the main sequence (e.g. G, K, M class)
We saw previously that
\[ M_S = (1 - y) M_{\text{tor}} \quad y \equiv \frac{M_g}{M} \]

The fraction of all stars made while gas fraction was \( \geq y \) is
\[ \frac{M_S}{M_S'} = \frac{1 - y}{1 - y'} \quad M_S', y' \text{ are present values} \]

These stars formed when gas metallicity was \( Z \leq y \ln(y') \)

\[ \Rightarrow \text{fraction of stars of metallicity } \leq Z \text{ is} \]
\[ S(Z) = \frac{M_S}{M_S'} = \frac{1 - \exp(-Z/y)}{1 - y'} \]

We can eliminate the yield \( y \) using \( y = \frac{Z}{y'} \ln(y') \)
and the assumption that the IMF (and \( \varpi \)) have been constant

\[ S_0, \quad S(Z) = 1 - \frac{Z_0}{1 - y'} \quad \text{(Subscripts \( 0 \) indicate present values)} \]

\[ S(Z) = \begin{cases} \frac{Z_0}{1 - y'} & \text{for } Z < Z_0 \\ \frac{Z}{1 - y'} & \text{for } Z \geq Z_0 \end{cases} \]

\[ \ln \frac{Z}{Z_1} = 2.2 \quad y' = 0.05 \]

Disk Star Metallicity

Histogram
Possible Solutions to G-Dwarf Problem

1. extreme infall - disk grows by accretion, slowly, beginning w/ small amount of gas

   \[ \Rightarrow \text{distribution strongly peaked at } Z = y \]
   \[ \Rightarrow \text{too few stars with } Z < Z_1 \]

2. pre-enrichment of disk gas - can explain radial abundance gradients

   Gas that formed disk had been enriched prior to infall - can reduce to

   \[ Z = Z_0 + y \ln \left( \frac{Z}{Z_0} \right) \]
   \[ Z_0 \text{ = initial metallicity of infalling gas} \]
   \[ Z_1 \text{ = present metallicity} \]

   \[
   S(Z) = \frac{1 - y_1}{1 - y_1} \left( \frac{Z - Z_0}{Z_1 - Z_0} \right) \]

   This actually works quite well w/ \( Z_0 = 0.2 Z_1 \)

   CF: HVC recent data!!

3. Variable IMF - "top-heavy" IMF at early times \( \Rightarrow \) rapid enrichment w/o producing low mass stars

   not a crazy sol'n

4. "Metal Enhanced" S.F.

   Suppose stars form preferentially in regions w/ higher \( Z \)
   (e.g., more efficient cooling?) NOT A CRAZY IDEA

AGE DATING - The AGE of the Milky Way

- nuclear cosmochronology
- stellar evol'n MS timescales (GC's)
- W.D. Cooling curve
Stellar Age-Dating of the Galaxy

1. The White Dwarf Cooling curve

Search for a cutoff in the W.D. luminosity function at the cool (faint end). This cutoff indicates, when combined with evolutionary models, that

\[ t_{\text{disk}} \approx 9.3 \pm 2.0 \text{ Gyr} \]

\[ \log N \]

\[ \text{NOTE SOLAR NUBAD!} \]

This is very difficult in practice, because cool W.D.'s are intrinsically very faint (the cutoff in the LF occurs at \( M_V \sim 16-17 \)), i.e.

\[ \log \left( \frac{L_{\text{wd}}}{L_\odot} \right) \approx -4.5 \]

\[ D_{\text{max}} \approx 250 \text{ pc} \quad \text{for } m-M=7 \]

Cool W.D.'s are also redder than hot W.D.'s and therefore more difficult to distinguish from disk dwarfs. Depends also on models for cooling-systematics?

2. Nuclear Cosmochronology

Examine abundances of radioactive isotopes with \( \Delta t/2 \) on the order of the age of the Universe.

Best numbers are for Th/Eu (date solar system at \( 4.5 \text{ Gyr} \)).


CS 22892-052 extremely metal poor star

\( \text{Th/Eu} = 0.219 \quad \text{as compared to } \text{S.S. value 0.463} \)

\[ \Rightarrow T(\text{CS}) \gg T(\text{SS}) \Rightarrow \text{Radioactive decay age } 15.2 \pm 3.7 \text{ Gyr} \]

(a lower limit)