Ay 124 Winter 2014 – HOMEWORK #3

Due Friday, Feb 14, 2014 by 5pm, in Steidel's mailbox in 249 Cahill

Problem 1 The nearest spiral galaxy to the Milky Way, M31, has a very concentrated nucleus. At a projected radius of 1 arcsec, stars in the nucleus have a line of sight velocity dispersion of 150 km s^{-1}, and are also rotating about the nucleus at 150 km s^{-1}. The total luminosity from within 1 arc second is $3 \times 10^6 L_{V_\odot}$.

a) Calculate the total mass, and the mass to light ratio, in the inner arcsecond of M31.

Assuming that the density in the nucleus of M31 scales as $\rho \propto r^{-2}$ for radii larger and smaller than that of your estimate in (a), and that the mass is dominated by 1 $M_\odot$ main sequence stars and $1 M_\odot$ white dwarfs and neutron stars (i.e., no massive black hole), estimate the radius in the nucleus at which:

b) The 2-body relaxation time equals the age of M31 ($\sim 10^{10}$ years)

c) A constant density core of that radius would core collapse in the age of M31.

d) The time for $10 M_\odot$ black hole remnants of massive stars to sink into the center of M31 equals the age of M31.

e) The time for a main sequence star to have a disruptive physical collision equals the age of M31.

Problem 2 The Plummer potential-density pair describes a spherical star cluster of mass M and characteristic radius r_0. It has the gravitational potential

$$\Phi(r) = \frac{-GM}{\sqrt{r^2 + r_0^2}}.$$

a) Use Poisson's equation to show that the associated mass density is

$$\rho(r) = \frac{3M}{4\pi r_0^3} \frac{1}{[1 + (r/r_0)^2]^{5/2}}.$$

b) Show that the surface mass density projected on the sky (M/L times the observed surface brightness, if the cluster were made of identical stars) as a function of projected radius R from the cluster center is

$$\Sigma(R) = \frac{M}{\pi r_0^2} \frac{1}{[1 + (R/r_0)^2]^2}.$$

c) Assume the cluster is made of identical stars. Compute, in units of r_0,

1) the *core radius* r_c, at which the surface density falls to half its central value.

2) the *half-light radius,* within which half of the total light is *projected.*

3) the *half-mass radius,* r_h, the radius of the 3-D sphere which contains half the cluster's total mass.