Ay 124 Winter 2014 – HOMEWORK #4

Due Friday, Feb 28, 2014 by 5pm, in Steidel's mailbox in 249 Cahill

Problem 1 The giant elliptical M87 near the center of the Virgo cluster is orbited by a few thousand globular clusters. M87 has a distance modulus of $m - M = 31$. There are roughly equal numbers of globular clusters in every octave of projected radius, out to ~ 400 arc seconds from the nucleus of M87. Measurements of the radial velocities of the globular clusters show that they have a radius–independent line of sight velocity dispersion of 390 km s^{-1}, and a radius–independent rotation velocity of 110 km s^{-1}.

a) The surface brightness of M87 falls off as R^{-2} at projected radii $R > 60$ arc seconds from the center. How does the mass to light ratio scale with radius for $60'' < R < 400''$?

b) Calculate the total mass of M87 within 400 $''$. State any assumptions you make in obtaining your answer. The V-band luminosity internal to that radius is $7.6 \times 10^{10}\ L_{V_\odot}$. What is the mass-to-light ratio within that radius?

Problem 2 Assuming the observational $M_{\rm BH} - \sigma$ relation $M_{\rm BH} = 10^8(\sigma/200{\rm kms}^{-1})^4$, calculate for $M_{\rm BH} = 10^{5,6,7,8,9}$ the following:

a) the orbital period (in years) of stars whose semi-major axes are equal to r_h, the radius where the black hole begins to dominate the galaxy potential

b) the relaxation time (in years) at r_h

c) the radius from which a 100 $M_{\rm solar}$ black hole would sink to the center of the galaxy in 10^{10} years- you may assume that the galaxy density profile has $\rho \propto r^{-2}$ everywhere inside and outside r_h.)

Problem 3 One of the tools used by astronomers to assess whether a population of objects is distributed uniformly with distance is called the V/V_{max} test. For each object in a sample, one forms V_i/V_{max} by considering the ratio of the spherical volumes enclosed by the actual observed distance of object i, divided by the volume enclosed by the *maximum* distance at which the same object could be placed and still appear in the sample. For uniformly distributed objects, $\langle V/V_{max}\rangle$=0.5.

Suppose the density of galaxies falls off like the inverse 1.8 power of distance about an observer as the correlation function ξ suggests it will, on average. What mean value of V/V_{max} will observers determine for galaxies in a magnitude–limited sample? How many galaxies will be required in a sample to demonstrate that the galaxy distribution is not uniform using the mean V/V_{max} test?

Problem 4

a) Neglecting cosmological redshift effects, show that any stellar system with V-band surface brightness given by μ mag arcsec^{-2} has a surface luminosity density in projection on the sky of

$$\Sigma = 10^{0.4(26.4-\mu)}\ {\rm L_{V\odot}\ pc^{-2}}\ ,$$

so that, for example, $\mu = 26.4$ V-mag arcsec^{-2} equals 1 $\rm L_{V\odot}$ pc^{-2}.

b) From the photograph of the Sbc galaxy (D= 8.0 Mpc) NGC 5055 appended below, estimate the angle of inclindation i between the disk plane and the plane of the sky.

c) Photometry in the V band along the major axis of NGC 5055 gives $\mu_V = 20.1$ at $1'$, 21.1 at $2'$, 22.7 at $4'$, and 25.25 at $8'$, where μ_V has units of V mag arcsec^{-2}. If the stellar disk, as viewed face-on, has an exponential surface brightness $\Sigma_V = \Sigma_0 \exp(-r/R_s)$, use your fit to the major axis photometry to calculate R_s, Σ_0, and the total luminosity of the disk in solar luminosities (the sun has $M_V = 4.83$).

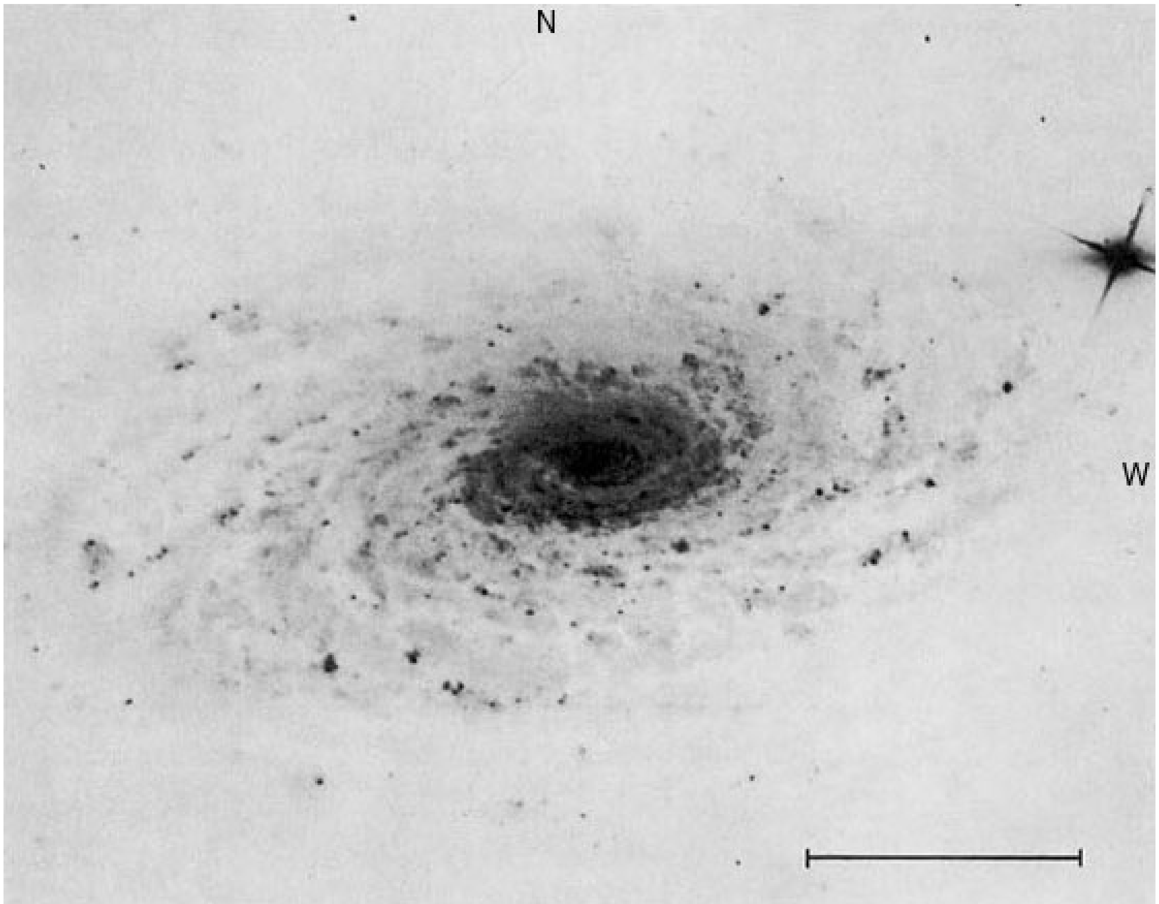

Figure 1: Photographic image of NGC 5055 from the revised Shapley-Ames catalog of nearby galaxies.

Problem 5 The famous "Toomre Criterion" (also known as "Toomre's Q") for the local stability of a differentially rotating thin disk (of gas or stars – the criteria are nearly identical) to axisymmetric perturbations is described in §6.2.3 of Binney and Tremaine (beginning on p. 494 in the 2008 edition). Compute Toomre's Q for stars in a Milky Way-like disk with a scale length $R_s \simeq 3$ kpc, and disk velocity dispersion. At what galactocentric radius would you predict the stellar disk would becomes Toomre unstable? What is wavelength of the "most-unstable" mode? How does the result change for a MW-like gaseous disk, and for the young stars that form from it? How would the answer change if you were to consider a gaseous disk whose surface density is entirely dominated by gas (i.e., $\Sigma_{\rm gas} >> \Sigma_{\rm stars}$ as might be found in a very young galaxy beginning its star-forming life)?