Ay 124 Winter 2016 – HOMEWORK #2

Due Friday, Jan 29, 2016 by 5pm, in Denise's mailbox in 249 Cahill

Problem 1

A galaxy forms stars at a constant rate with an initial mass function

$$\frac{dN_0}{dM} \propto M^{-(1+x)}.$$

Assume that stars have luminosities $L \propto M^4$. In parts (a) and (b) consider only stars more massive than $1M_\odot$, whose lifetimes are shorter than the age of the galaxy.

a) Find the slope x such that an observer in a homogeneous, isotropic region counts, at every apparent bolmetric magnitude, equal numbers of stars in each octave of luminosity. What type of star dominates the counts if x is flatter than this critical value?

b) Find the slope x such that an observer in an infinite thin disk of stars counts, at every apparent bolometric magnitude, equal numbers of stars in each octave of luminosity. What type of star dominates the counts if x is steeper than this in value?

c) In the Milky Way, Salpeter thought $x = 1.35$. Using your results from (a) and (b) and the scale heights of stars in the Galaxy given in the included table, describe how the fractions of stars of different luminosities should vary in all-sky samples of various magnitudes. Below what apparent bolometric magnitude will stars less massive than $1M_\odot$ start to dominate the all-sky samples?

d) What spectral type is the typical $m_V = 18$ star in the Galactic plane? Would you expect it to be significantly reddened?

Table 4-16. Scale Heights β_S in the Direction Perpendicular to the Galactic Plane and Surface Density Σ_S for Various Objects

Object	β_S(pc)	$\Sigma_S\left(\frac{\text{stars}}{\text{pc}^2}\right)$	$\Sigma'_S\left(\frac{\mathcal{M}_\odot}{\text{pc}^2}\right)$
O stars	50	1.5×10^{-6}	10^{-4}
Classical Cepheids	50	7.5×10^{-6}	5×10^{-5}
B stars	60	6×10^{-3}	6×10^{-2}
Galactic clusters	80	–	–
Interstellar dust and gas	120	–	
A stars	120	6×10^{-2}	0.1
F stars	190	0.6	0.6
Planetary nebulae	260	–	–
gK stars	270	1.2×10^{-3}	3×10^{-2}
Novae	300	–	–
dG stars	340	2	2
dK stars	350	3.5	2.5
dM stars	350	20	9
gG stars	400	6×10^{-2}	1.6×10^{-1}
White dwarfs	500	12.5	10
Long-period variables (M5–M8)	700		
RR Lyrae variables ($P < 0^d.5$)	900		
Long-period variables (M0–M4)	1000		
RR Lyrae variables ($P > 0^d.5$)	2000		
W Virginis variables (spheroidal-component Cepheids)	2000		
Subdwarfs	2000		
Globular clusters	3000		

SOURCE: Adapted from (A1, 247), (A1, 249), and (A1, 251), by permission

Problem 2. Chemical Evolution Scenarios

Modify the "closed box" model for chemical evolution by allowing a source/sink of gas; continue to assume a time-independent renormalized heavy-element yield y, complete mixing, instantaneous recycling and $Z \ll 1$. Let the box contain a gas mass $M_g(t)$, a stellar mass $M_s(t)$, and let gas with metallicity Z_f be be added to or ejected from the box at rate $\dot{M}_f$. Let $M_s(t=0)=0$ and $Z(t=0)=0$ (no stars or heavy elements in the primordial gas).

a) Show that the equations for the evolution of Z are

$$\dot{M}_g + \dot{M}_s = \dot{M}_f,$$

$$\dot{Z}M_g - y\dot{M}_s = (Z_f - Z)\dot{M}_f.$$

b) Outflow model: gas is blown out of the box at a rate which is a constant fraction η of the star formation rate $\dot{M}_s$: $\dot{M}_f = -\eta\dot{M}_s$. Show that when the gas fraction is $f_g = M_g/(M_g+M_s)$,

$$Z = \frac{y}{1+\eta}\ln\left[1 + (1+\eta)\left(\frac{1}{f_g} - 1\right)\right].$$

c) Inflow model: pristine ($Z=0$) gas flows into the box at a rate which is a constant fraction η of the star formation rate: $\dot{M}_f = +\eta\dot{M}_s$. Derive an expression analogous to the one in part (b) for Z as a function of the gas fraction f_g.

d) In the inflow model of part (c), derive an expression for the fraction of stars with metallicity less than Z at a time when the metallicity is Z_1 (the expression should involve only the variables f_g, Z, Z_1, and η). What value of η is required to solve the "G-dwarf problem" in the local solar neighborhood, i.e. give 2% of F,G dwarf stars with $Z < (1/6)Z_1$, ($Z_1 = 0.03 = 1.5Z_\odot$ is the present metallicity of gas in the solar neighborhood)? Are you uncomfortable with this solution? Would there be a solution if the fraction of stars with $Z < 1/6Z_1$ were much less than 2%?