Star Track 2 - The Search for a Supermassive Black Hole

Early radio astronomers detected an immensely powerful source of radio waves towards the center of the Galaxy in the constellation Sagittarius; this mysterious object was designated SgrA*. More recently, infrared astronomers using adaptive optics have imaged individual stars near this object and tracked their motion with time. The observed orbit for one such star is plotted at right (also see CS-274 in the course reader; this star is "SO-2".)

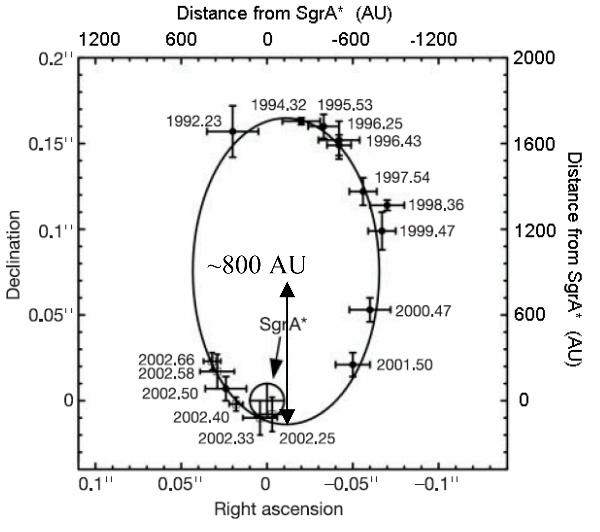

Figure modified from Schodel et al., *Nature* (419) p.694, Oct.2002

1. Make a *rough estimate* of the semimajor axis of the star's orbit in AU using the diagram. The semimajor axis is defined as the distance from the center of the ellipse to the edge along the ellipse's long axis. (This is an underestimate because the orbit is inclined about 45 degrees to the line of sight, but we'll ignore the inclination in this exercise.)

Using the scale on the right, we can estimate that the semimajor axis is about 800 AU.

2. Now estimate the period by comparing the year of the star's closest approach to SgrA* to the year it was furthest away (half an orbit). The year it was furthest away is not quite the top of the ellipse because of the inclination of the orbit.

The star SO-2 passed SgrA* in 2002.33, and was opposite it in about 1995.5 (half an orbit away).
So the period is (2002.3-1995.5) × 2 or about 13.6 years.

3. Before we do any calculations, stop and think a bit. Pluto has a semimajor axis of about 40 AU and a period of 250 years. But SO-2 has a huge semimajor axis and yet is whipping around the central object in only a few years. What does this tell us?

SO-2 is moving on a larger orbit than Pluto in less time, so it is moving much, much faster. To keep the star in a stable orbit, the gravity pulling on it must be incredibly strong, and the object much more massive than the Sun.

4. Kepler's 3rd law says that:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} R^3$$

Here P is the period, R is the semimajor axis, and m_1 and m_2 are the masses of the objects.

Use ratios to get rid of the constants, using the properties of the Earth's orbit around the Sun (P_E, R_E, and m_{Sun}). Then put the equation in terms of m_1/m_{Sun} (assume $m_2 << m_1$).

Assume $m_2 << m_1$: $$P^2 = \frac{4\pi^2}{G\,m_1} R^3$$

Compare to Earth/Sun: $$P_E^2 = \frac{4\pi^2}{G\,m_{Sun}} R_E^3$$

Use ratios: $$(P/P_E)^2 = \frac{(R/R_E)^3}{(m_1/m_{Sun})}$$

In terms of m: $$\frac{m_1}{m_{Sun}} = \frac{(R/R_E)^3}{(P/P_E)^2}$$

Original worksheet by D. Perley

5. Now calculate the mass of SgrA* in Solar masses using the formula above. Remember that $P_E = 1$ yr and $R_E = 1$ AU. (This won't exactly agree with the number given in class, but we're just doing a rough calculation.)

$$\frac{m_1}{m_{Sun}} = \frac{(R/R_E)^3}{(P/P_E)^2} = \frac{(800\ \text{AU} / 1\ \text{AU})^3}{(13.8\ \text{yr} / 1\ \text{yr})^2} = 2.7 \times 10^6$$

The object has a mass about 3 million times the mass of the Sun. (This is not too far off from the figure given in lecture of about 4.5 million Solar masses.)

6. The closest approach of SO-2 to SgrA* is only 120 AU, so SgrA* must be smaller than that. Calculate the *minimum* density of SgrA* (in solar masses per cubic AU) for it to be concentrated in this small region.

$$\begin{aligned}
\text{density} &= (\text{mass}) / (\text{volume}) \\
&= m / (4/3\ \pi\ R^3) \\
&\sim (3 \times 10^6\ m_{Sun}) / (4/3\ \pi\ (120\ \text{AU})^3) \\
&\sim (3 \times 10^6\ m_{Sun}) / (7.2 \times 10^6\ \text{AU}^3) \\
&\sim 0.4\ m_{Sun} / \text{AU}^3
\end{aligned}$$

7. Why do astronomers think this makes SgrA* a black hole, and not some other kind of object?

This is about half the mass of the sun per cubic AU – actually much less than the Sun's density. So in principle SgrA* "could" be some other object. However, this density is just a minimum, and we know of many reasons why it could be argued that this really is a black hole.

- There is no detectable optical or infrared emission from this source. We don't know of any practical way for millions of solar masses of material to not radiate optically fainter than an ordinary star other than a black hole.
- There is, however, powerful radio emission coming from the source, expected to come from around a black hold due to infalling material interacting with its magnetic fields.
- Physically, it is hard to see how so much matter could be concentrated into such a small region without collapsing into a black hole within a few thousand years at most – its own self-gravity would be overpowering.