The Big Mama in M104

The biggest black holes are found in other galaxies, especially large ellipticals. The Doppler technique is the method of choice, but its implementation here is slightly different from the method used on exoplanets and binary stars. Below is a Hubble telescope spectrum (in the region containing the emission lines from ionized oxygen and nitrogen) of the center of the galaxy M104, the "Sombrero Galaxy". The top spectrum is a region around the very center, while the lower two spectra are regions just flanking the center.

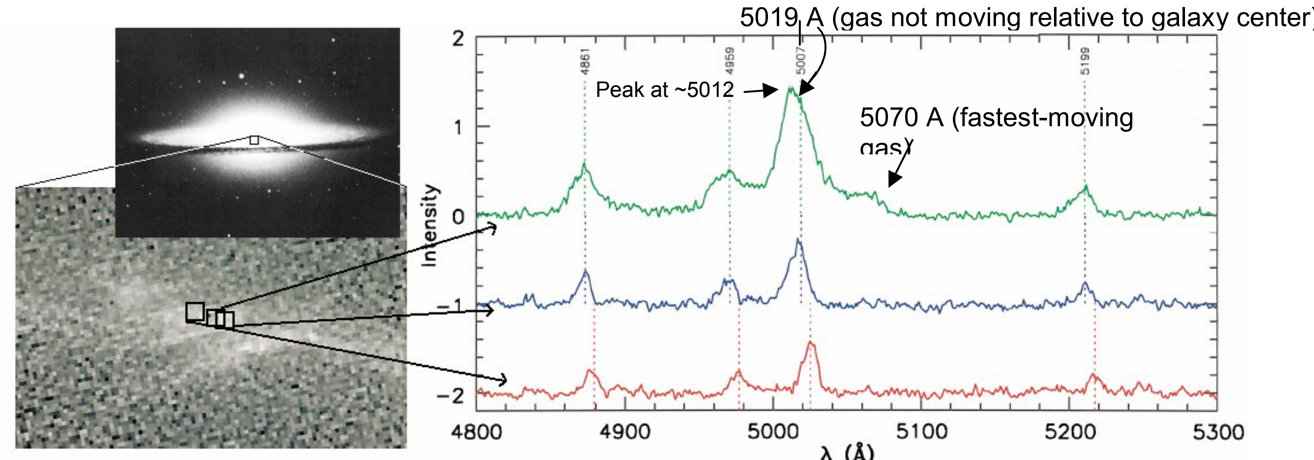

Figure modified from Kormendy et al., *Astrophysical Journal* (493) p.L91, Dec 1996

1. First of all, notice that *all* the lines are shifted somewhat to longer wavelengths. The numbers on top of the dotted lines indicate the rest (laboratory) wavelength in Angstroms, all of which are somewhat less than the observed line wavelengths. This is because, like most galaxies, M104 is receding from us due to the expansion of the universe. Use the peak wavelength of the *top* spectrum to calculate the wavelength shift $\Delta\lambda$ of an emission line of your choice due to the galaxy's overall recession.

The center of the line corresponds to the gas that is not moving (radially) relative to the center of the galaxy, which itself isn't moving relative to the galaxy as a whole. This "average" (mean) of the emission line is given by the dotted lines – for the spectrum centered around the Galaxy's center (the "green spectrum") this average wavelength is 5019 Angstroms, corresponding to a shift for the whole galaxy of $\Delta\lambda$ = 12 Angstroms. (The big emission line is somewhat asymmetric, and its peak is actually a bit offset from this – although I suggested you use the peak in the directions, the average is a better estimate in this case.)

2. Use this to calculate the redshift, $z = \Delta\lambda / \lambda$.

$z = \Delta\lambda / \lambda = (12\ A) / (5007\ A) = 0.0024$

3. Now calculate the velocity in km/s using the Doppler formula, $v / c = \Delta\lambda / \lambda$. (c = 300,000 km/s.)

$v / c = 0.0024$
$v = 0.0024c \sim 700$ km/s

4. Use Hubble's law, $v = H_o d$, to estimate the distance to M104 in megaparsecs. Hubble's constant is about 70 km/s/Mpc.

$v = H_o d$
$d = v / H_o$
~ 700 km/s / (70 km/s/Mpc)
~ 10 Mpc

5. Now we want to calculate the *internal* motion of the gas in M104 that is generating these emission emission lines (that is, the motion of the gas relative to the center of M104, rather than relative to us). Using any one of the three spectra above, find the wavelength shift $\Delta\lambda$ of the *fastest-moving* gas in the part of the galaxy that was imaged.

There are a few ways to approach this. Note that the "blue spectrum", from the right of the center of the galaxy has a peak to the left (shorter wavelengths – a blueshift) , and the "red spectrum", from the left of the center, has a peak to the right (longer wavelengths – a redshift). This is because the gas is rotating around a spot in the center (the "green spectrum" at top.)

Therefore, the green spectrum encloses the black hole, but because our spectrum combines all the light from some small region of this space, this includes gas from *right* next to the black hole, as well as further from the black hole, and on both the left and right sides. So different parts of the gas have different velocities and different Doppler shifts, which all blend together to make the big fat emission line in the diagram.

However, we are concerned about the *fastest-moving* gas – that is, the gas with the largest redshift or blueshift. As you can see, the brightest line of the spectrum has emission out to 5070 Angstroms. This is a shift of 51 Angstroms relative to the nearby, non-moving gas.

6. Convert this to a measure of the orbital velocity of this gas.

Use the Doppler formula again:

$$v / c = \Delta\lambda / \lambda$$
$$= 63 / 5019$$
$$= 0.0126$$
$$v = 0.0126\, c$$
$$= 3800 \text{ km/s}$$

7. The slit that took the spectrum above covered a region of M104 corresponding to about 500,000 AU in diameter, so we know the gas from #6 is *at least* this close to the central object. Using this knowledge, and the velocity above, we can get the mass. Kepler's third law says:

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} R^3$$

But we also know that for circular orbits, $2\pi R = vP$. Using CS-92 as a guide and assuming $m_1 << m_2$, rearrange some terms to come up with an expression for the mass m_2 in terms of v and R.

Assume $m_1 << m_2$: $$P^2 = \frac{4\pi^2}{G\, m_2} R^3$$

Substitute P = 2πR/v $$(2\pi R/v)^2 = \frac{4\pi^2}{G\, m_2} R^3$$

Factors of $4\pi^2$ cancel: $$\frac{R^2}{v^2} = \frac{R^3}{G\, m_2}$$

In terms of m: $$m_2 = \frac{R\, v^2}{G}$$

8. Use ratios, and the properties of the Earth's orbit around the Sun (v_E, R_E, and m_{Sun}), to get rid of the nasty constants in this expression.

Compare to Earth/Sun: $m_{Sun} = \frac{R_E\, v_E^2}{G}$ (also see CS-92)

Use ratios: $(m/m_{Sun}) = (R/R_E)\,(v/v_E)^2$

9. Earth's velocity around the Sun is v_E = 30 km/s or so, and R_E is 1 AU. Using these numbers and the equation from #8, calculate the approximate mass of the object at the center of M104 in solar masses.

$$(m/m_{Sun}) = (R/R_E)\,(v/v_E)^2$$
$$(m/m_{Sun}) = (500{,}000 \text{ AU} / 1 \text{ AU})\,((3800 \text{ km/s}) / (30 \text{ km/s}))^2$$
$$= 8.04 \times 10^9$$

The black hole is about 8 billion times the mass of the Sun. (This is a bit of an overestimate because we used a too-large value for R – a more precise analysis says it is about 1 billion solar masses. That's still a lot!)