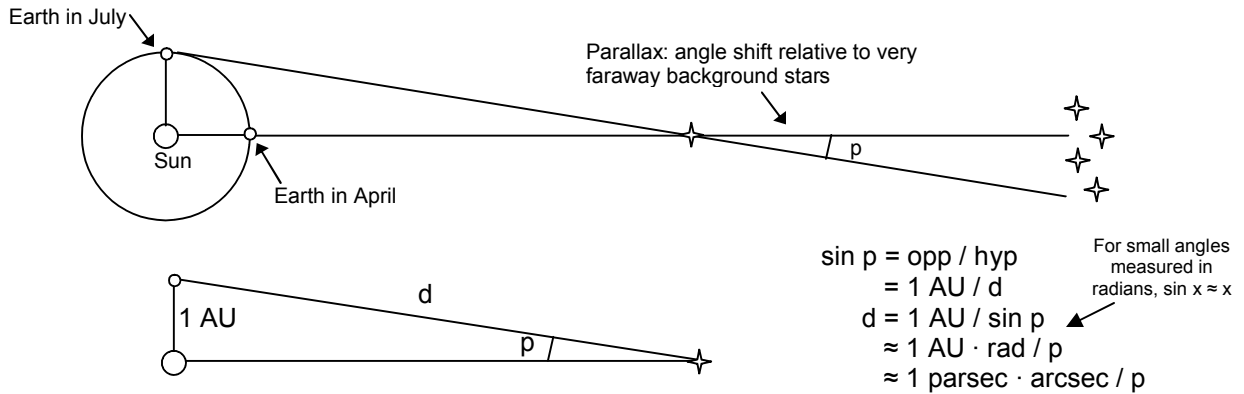# Measuring the Properties of Stars

Parallax – Calculating Distance

$$d = 1 / p$$
If distance is in parsecs
and parallax is in arcsec

We can determine the *distance* to stars by measuring parallax – the shift in the star's position in the sky over the course of the year (specifically, over 3 months) relative to other stars. The further away a star is, the smaller its shift in position – a relationship that can be easily determined by trigonometry:

Earth in July

Parallax: angle shift relative to very faraway background stars

Sun

Earth in April

p

$$\sin p = \text{opp} / \text{hyp}$$
$$= 1\ \text{AU} / d$$
$$d = 1\ \text{AU} / \sin p$$
$$\approx 1\ \text{AU} \cdot \text{rad} / p$$
$$\approx 1\ \text{parsec} \cdot \text{arcsec} / p$$

For small angles measured in radians, $\sin x \approx x$

1 AU

d

p

Astronomers define a convenient quantity known as the *parsec* to simplify parallax calculations – one parsec is the distance to a star with a parallax of one arcsecond (an arcsecond is 1/3600 of a degree), which works out to 206265 AU, 3.09×10^{13} km, or 3.261 light years. When we use this unit, the distance (in parsecs) is simply equal to one over the parallax angle (in arcseconds).

This technique only works out to a limited distance – for stars that are too far away, the parallax angle is too small to measure (this is the case for the "background stars" we use to measure parallax angles). At the present the smallest parallax angle we can measure is about 0.001 arcsec, or 1000 parsecs (about 3300 light years). This is enough for most of the stars we see in the sky, but not most of the stars in the Galaxy (which is over 20000 parsecs across).

Inverse Square Law – Calculating Luminosity

$$b = L / (4\pi d^2)$$

A quick glance at the night sky will tell you that different stars have different brightnesses. But how much of that effect is due to the fact that some stars are further away, and how much is due to certain stars being intrinsically brighter?

If we know the distances from parallax, we can remove the effect of distance and calculate the intrinsic brightness, or *luminosity* – the total energy the star emits per second. Imagine building a gigantic shell centered around the star, giving that shell a radius d equal to the distance between the star and Earth. A detector placed on the inside of the shell receives a certain amount of energy per second – this is the brightness *b* we measure, in ergs per second per square centimeter. But while a single detector only receives a tiny fraction of the star's energy, if we were to cover the *entire* shell in detectors, the shell would receive all the energy from the star. Since the surface area of the shell is $4\pi d^2$, and each unit of area receives *b* units of energy per second, the total luminosity (the energy per second captured by the whole sphere) is equal to **$L = b \times 4\pi d^2$**.

Be careful – luminosity is normally given in watts (joules per second) or ergs per second, or in solar luminosities, while the parallax formula gives d in units of parsecs. You have to convert the distance to meters or centimeters before plugging in – unless you use ratios (as you almost always should in this class).

Wien's Law – Calculating Surface Temperature

To a reasonably good approximation, stars are blackbodies – so if we graph a star's spectrum, we can calculate the temperature of its surface using Wien's Law. As given earlier in the semester, the peak wavelength (λ_{peak}, the wavelength at which the star emits the most energy) is given by $\lambda_{peak} = 3 \times 10^7$ Å·K / T. Rearrange terms to get a simple expression for determining the temperature: **T = 3 × 10^7 Å·K / λ_{peak}**.
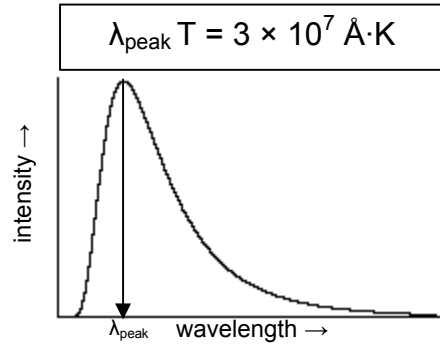
Note that this only gives the surface temperature (the temperature of the star's photosphere). The interior parts of the star are much, much hotter.

$$\lambda_{peak} T = 3 \times 10^7 \text{ Å·K}$$

Stefan-Boltzmann Law – Calculating Radius

As given in class, the Stefan-Boltzmann law indicates the amount of energy per second (power) emitted by a blackbody *per unit of surface area* as determined by its temperature. So if we know the *total* power emitted by a star (its luminosity), and we know its temperature from Wien's law, we can calculate its total surface area – and determine its size.

$$\varepsilon = \sigma T^4$$
$$L = 4\pi R^2 \sigma T^4$$
$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

To do this in practice, first find the energy per second emitted by each unit of surface area, ε, using the relation given in class ($\varepsilon = \sigma T^4$). The luminosity of a star is given by the power emitted per unit area times its surface area (L = ε S); for a sphere, S = $4\pi R^2$, so L = ε S = (σT^4)($4\pi R^2$). Expressing this in terms of radius, **R = (L / ($4\pi \sigma T^4$))$^{1/2}$**.

Never confuse the radius of the star R with the distance to the star d! Although both are measures of length that appear in our equations squared and multiplied by 4π, they are extremely different!

In addition, keep in mind that the Stefan-Boltzmann law tells you the total luminosity in *all* wavebands, including UV, infrared, and others – not just visible light. For most stars, the visible spectrum represents only a small fraction of this total energy, and the brightness you detect (and therefore the luminosity you derive) from your eye, or with an ordinary camera, cannot be compared to the Stefan-Boltzmann luminosity without making corrections.

Star Calculation Flowchart
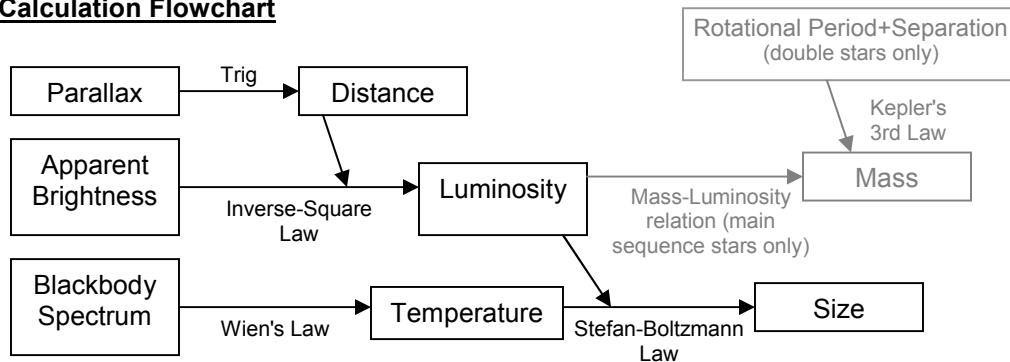
Grey indicates properties and rules not described in this worksheet.