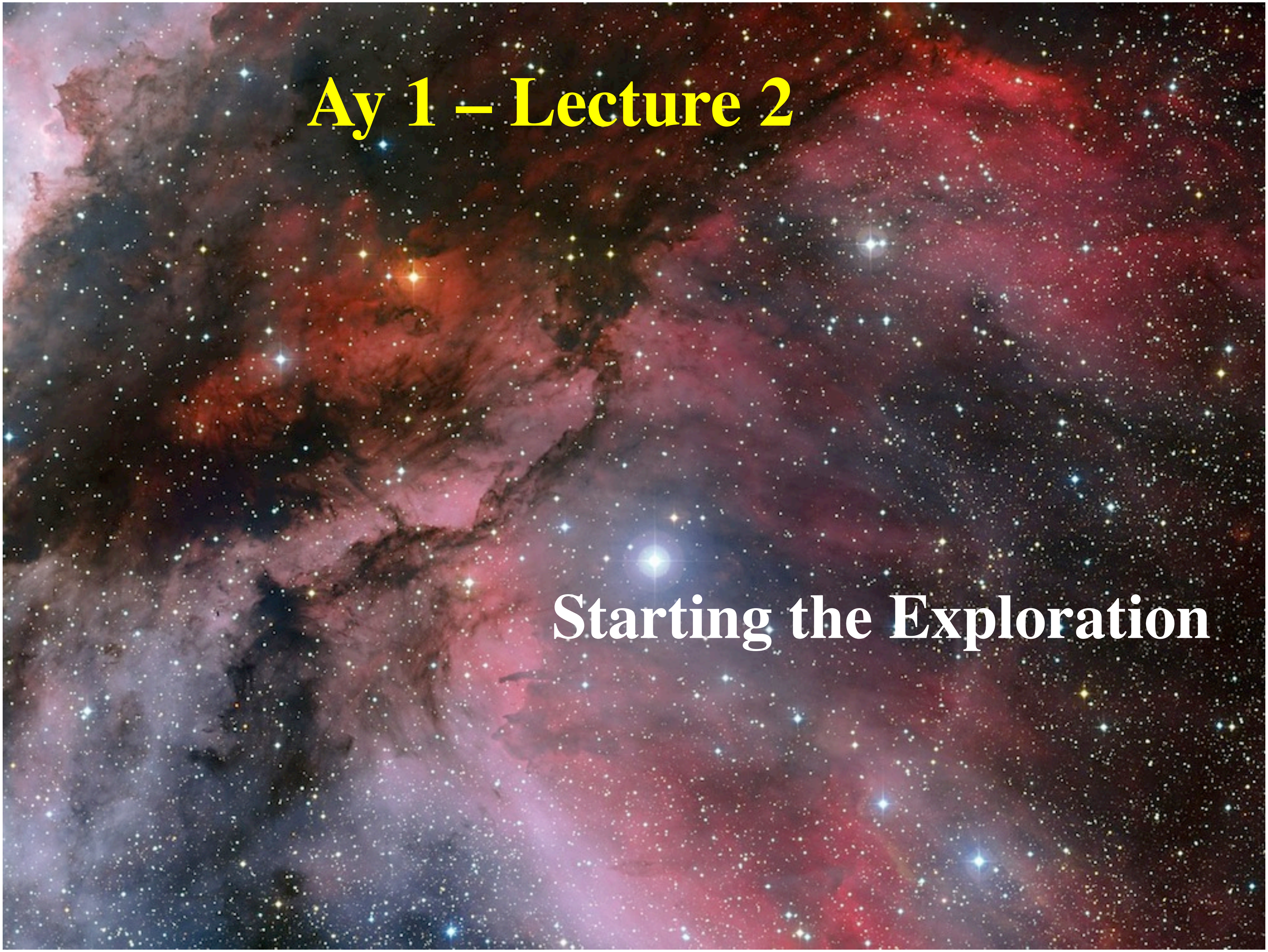
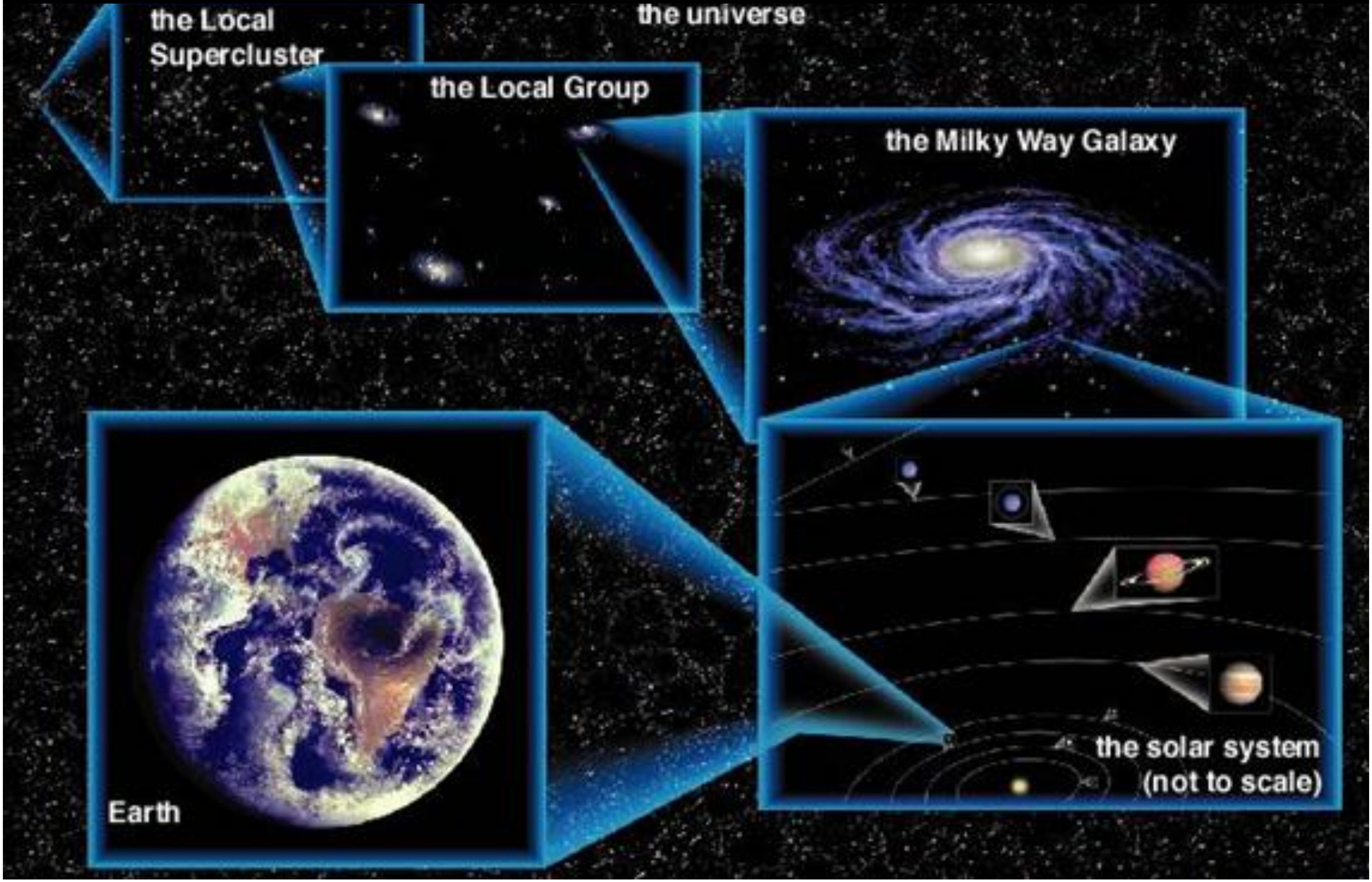
Ay 1 – Lecture 2



Starting the Exploration

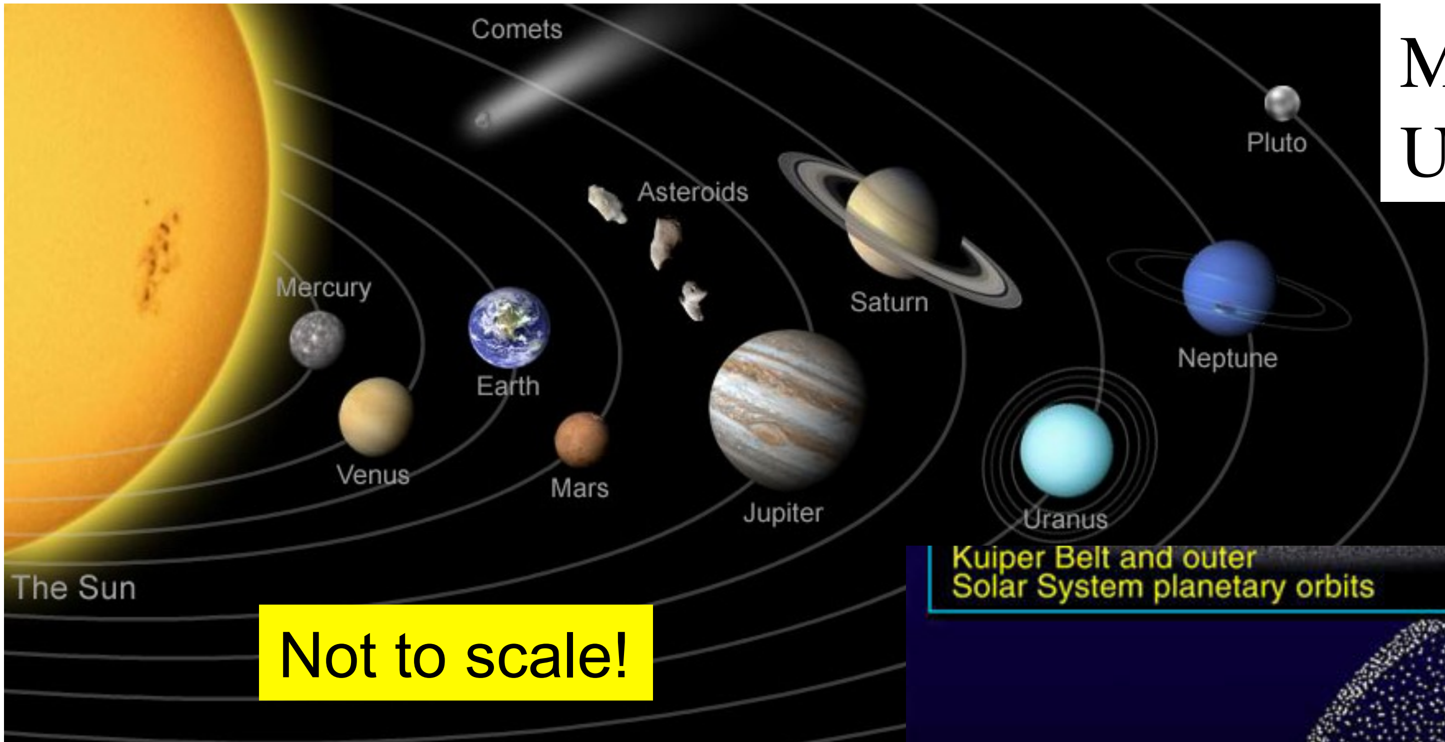
2.1 Distances and Scales



Some Commonly Used Units

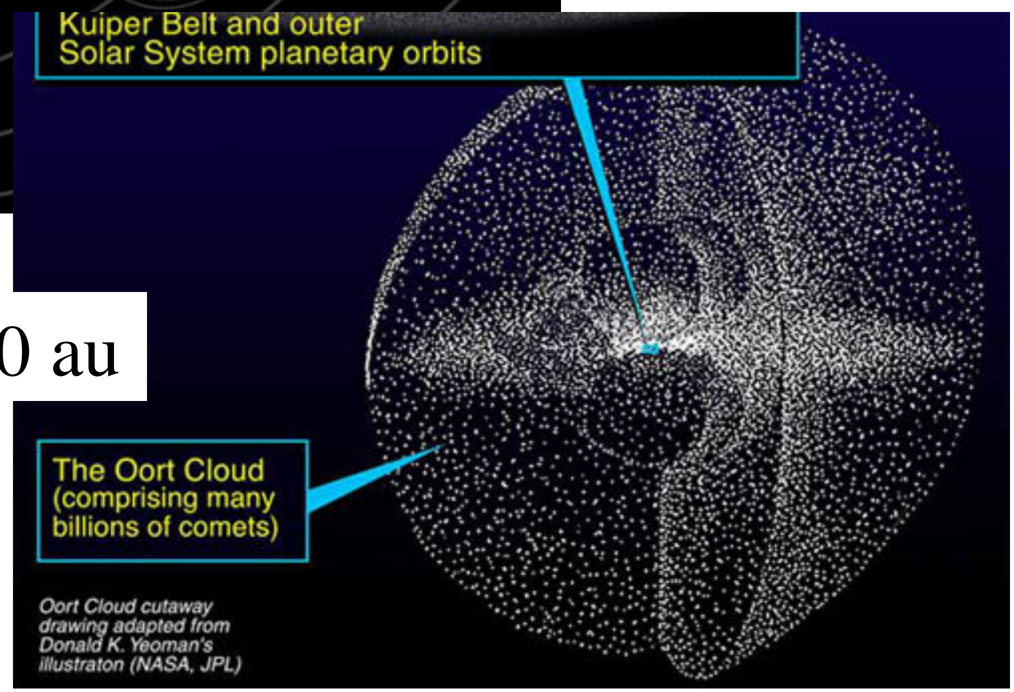
- **Distance:**
  - Astronomical unit: the distance from the Earth to the Sun, $1\ \text{au} = 1.496 \times 10^{13}$ cm $\sim 1.5 \times 10^{13}$ cm
  - Light year: $c \times 1$ yr, $1\ \text{ly} = 9.463 \times 10^{17}$ cm $\sim 10^{18}$ cm
  - Parsec: the distance from which 1 au subtends an angle of 1 arcsec,
    $1\ \text{pc} = 3.086 \times 10^{18}$ cm $\sim 3 \times 10^{18}$ cm
    $1\ \text{pc} = 3.26$ ly ~ 3 ly
    $1\ \text{pc} = 206{,}264.8$ au $\sim 2 \times 10^{5}$ au
- **Mass and Luminosity:**
  - Solar mass: $1\ M_{\odot} = 1.989 \times 10^{33}$ g $\sim 2 \times 10^{33}$ g
  - Solar luminosity: $1\ L_{\odot} = 3.826 \times 10^{33}$ erg/s $\sim 4 \times 10^{33}$ erg/s

The Scale of the Solar System

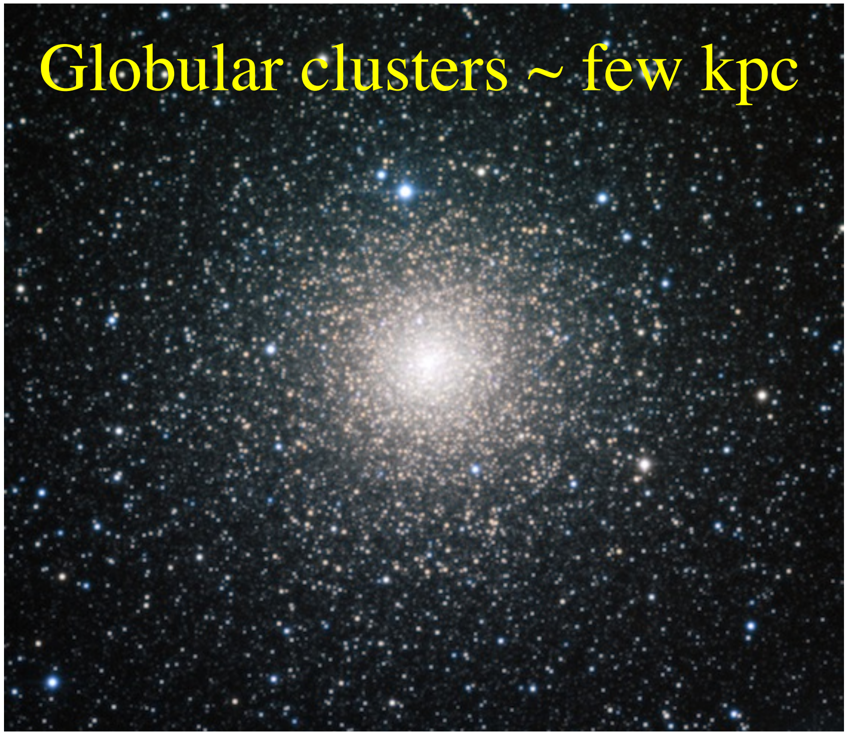


Major planets:
Up to ~ 50 au

The Oort cloud: ~ 1000 au

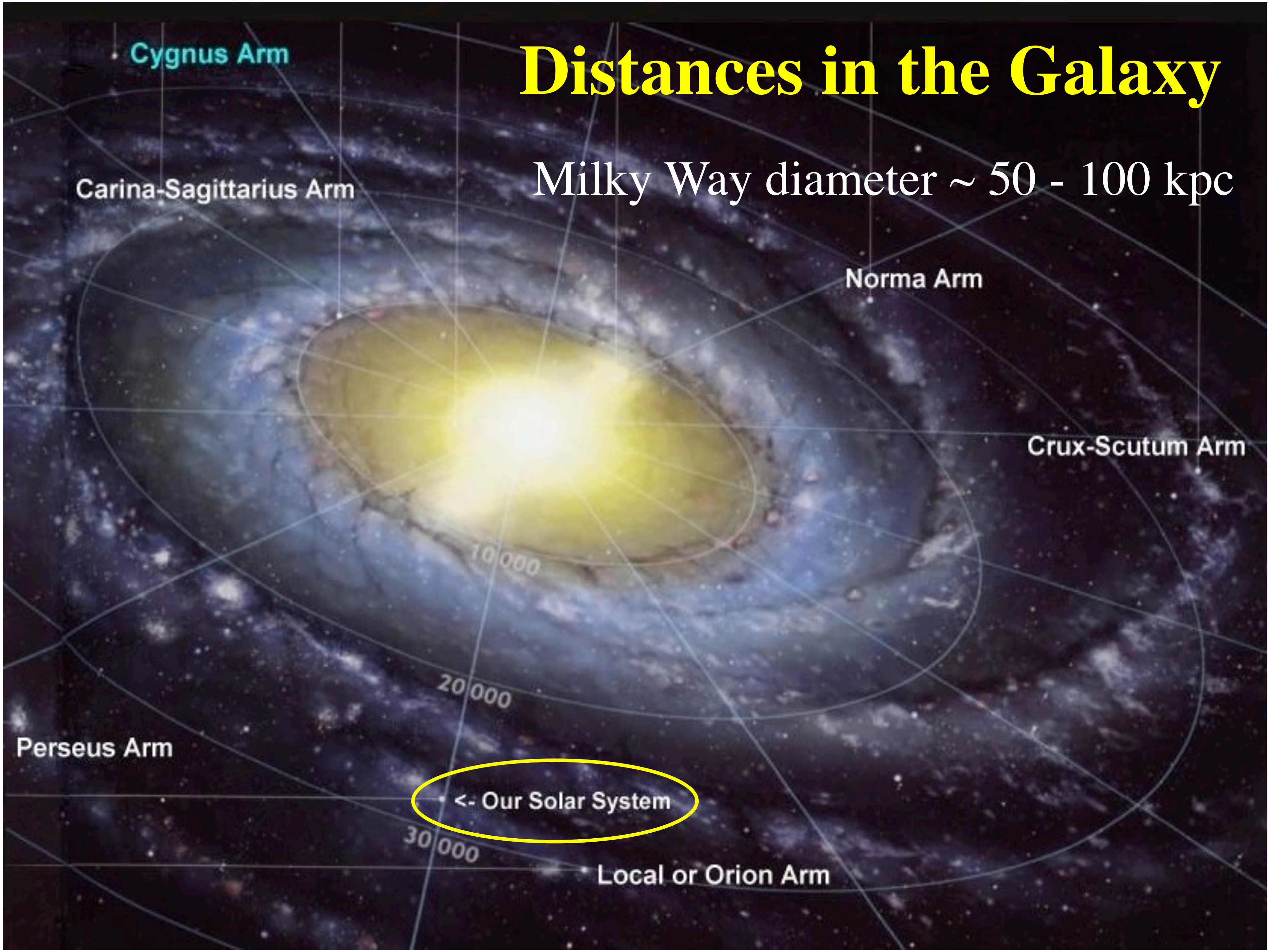


Stellar Distances

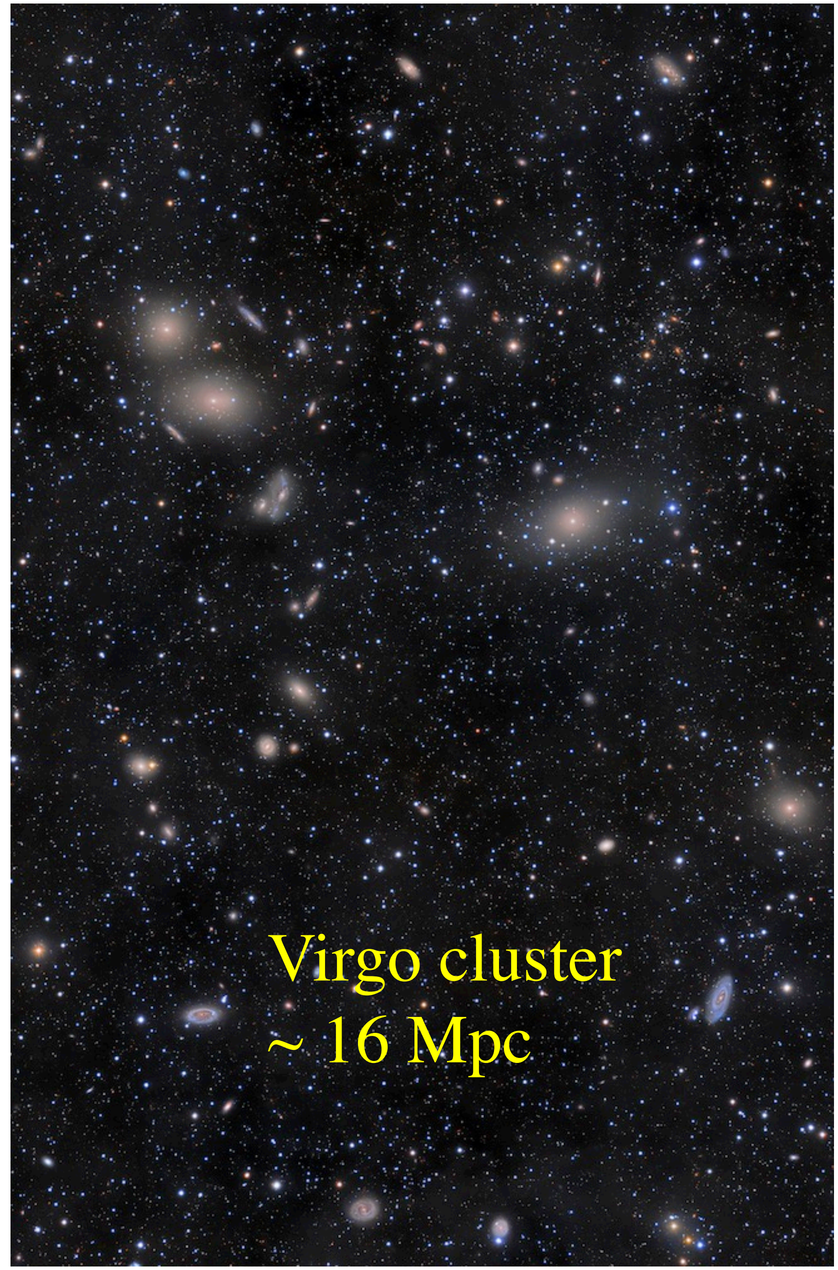


Distances in the Galaxy

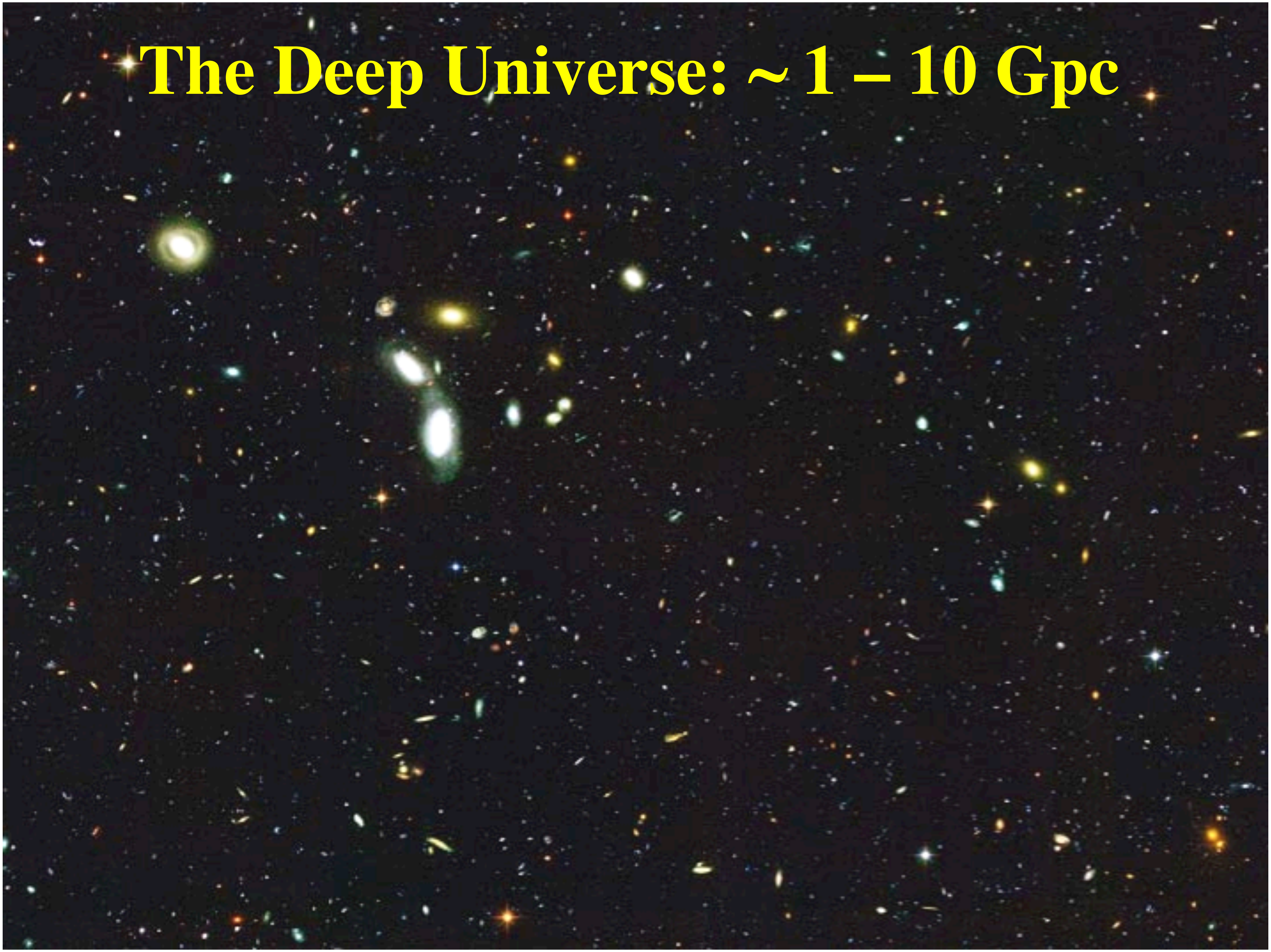
Milky Way diameter ~ 50 - 100 kpc



Our Extragalactic Neighborhood



The Deep Universe: ~ 1 – 10 Gpc

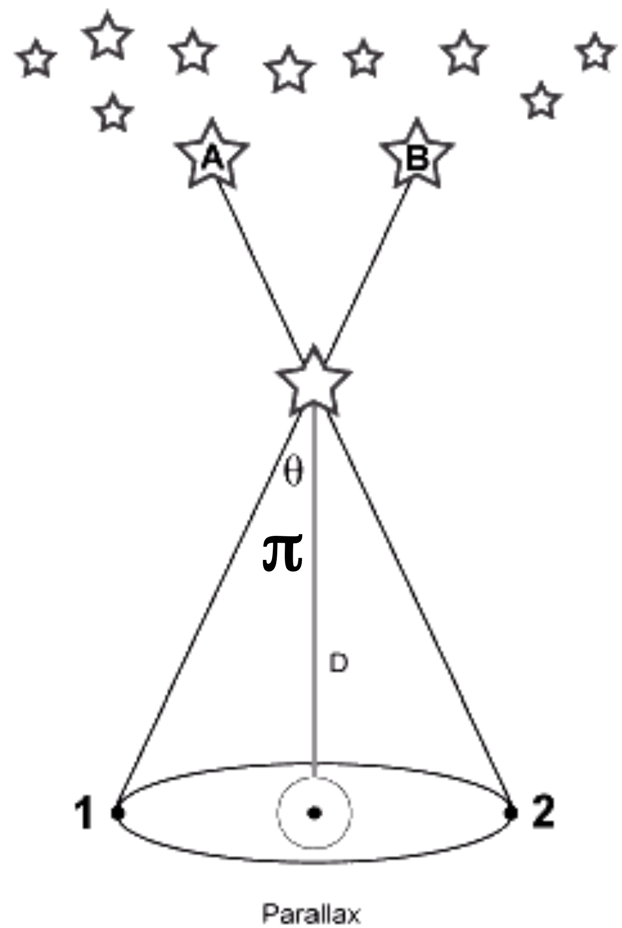


Distances and Parallaxes

- Distances are necessary in order to convert apparent, measured quantities into absolute, physical ones (e.g., luminosity, size, mass…)
- Stellar parallax is ***the only*** direct way of measuring distances in astronomy! Nearly everything else provides relative distances and requires a basic calibration
- Small-angle formula applies:

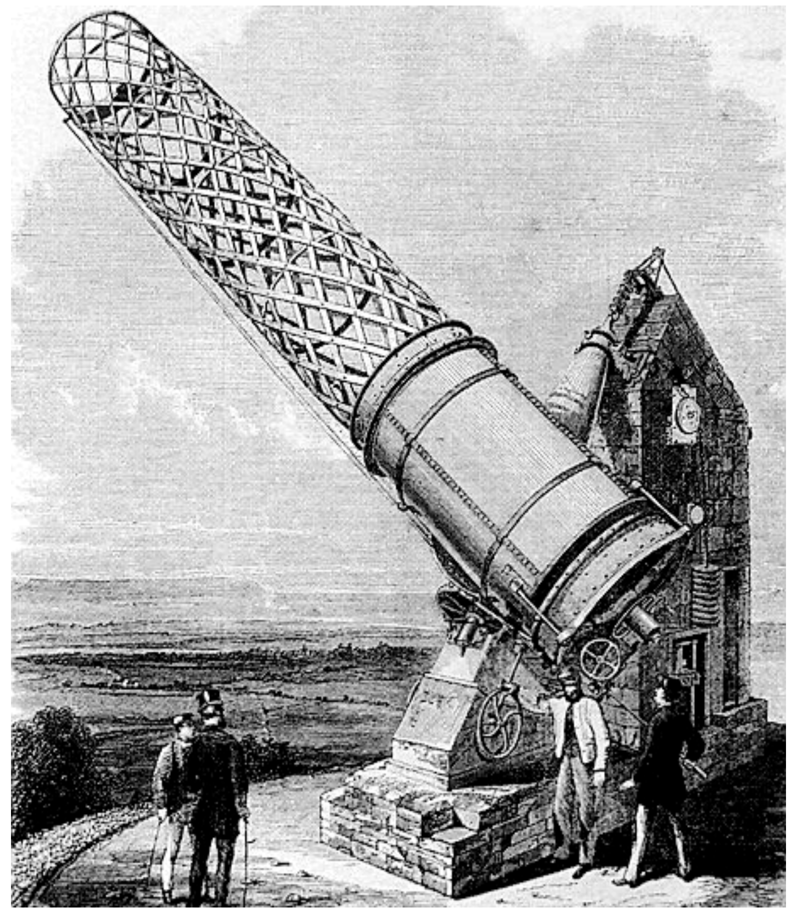
  $$\mathbf{D}\ [\text{pc}] = \mathbf{1} / \pi\ [\text{arcsec}]$$

- Limited by the available astrometric accuracy (~ 1 mas, i.e., D < 1 kpc or so, now)



How Far Can We Measure Parallaxes?

Since nearest stars are > 1 pc away, and ground-based telescopes have a seeing-limited resolution of ~1 arcsec, measuring parallaxes is hard.



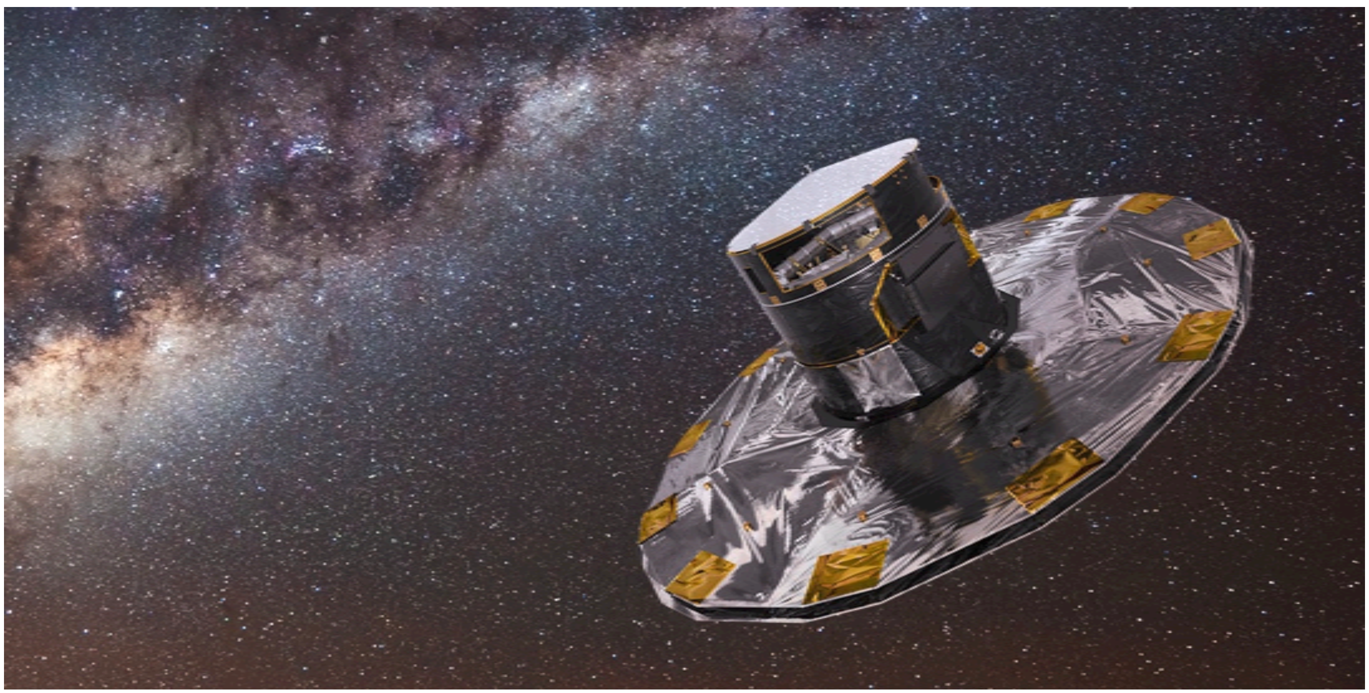
1838: Bessel measured $\pi = 0.316$ arcsec for star 61 Cyg (modern value $\pi = 0.29$ arcsec)

Current ground-based: best errors of ~ 0.001 arcsec

How Far Can We Measure Parallaxes?

Hipparcos satellite: measured ~10^5 bright stars with errors also of ~0.001 arcsec

GAIA satellite: will measure positions of ~10^9 stars with an accuracy of micro-arcsecs - this is a reasonable fraction of *all* the stars in the Milky Way!



Currently: measure D accurately to ~ a few $\times 100$ pc

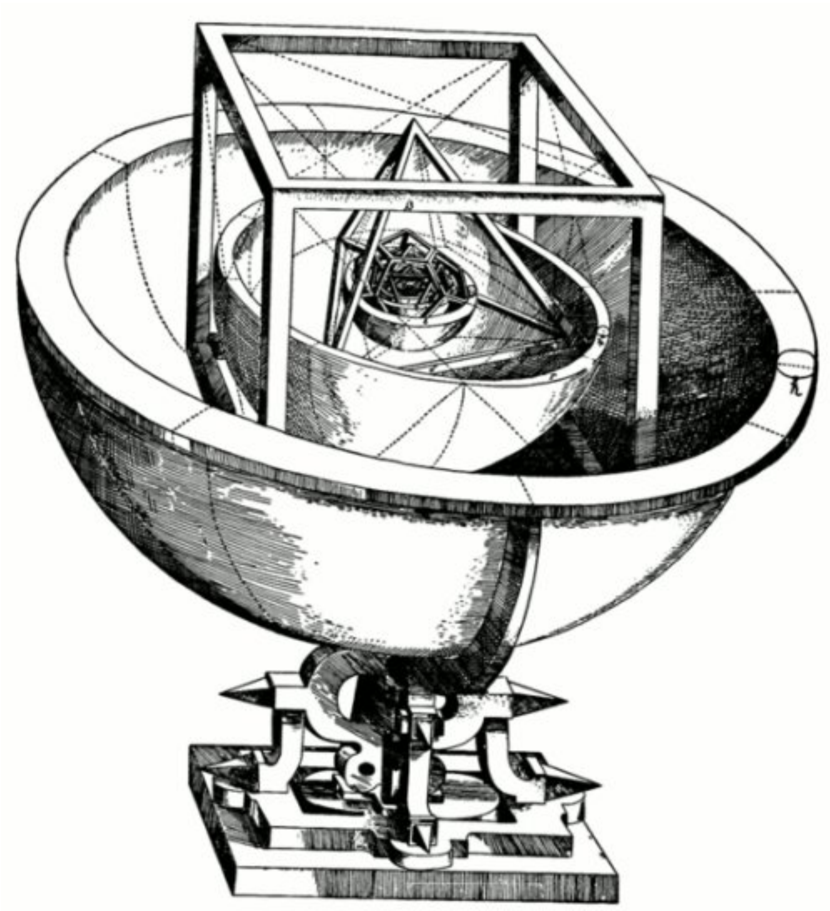
A parsec is…

A. Radius of the Earth's orbit
B. About 10^{27} cm
C. Angle corresponding to the size of the Earth's orbit from 1 light year away
D. About 3×10^{18} cm
E. About 200,000 astronomical units

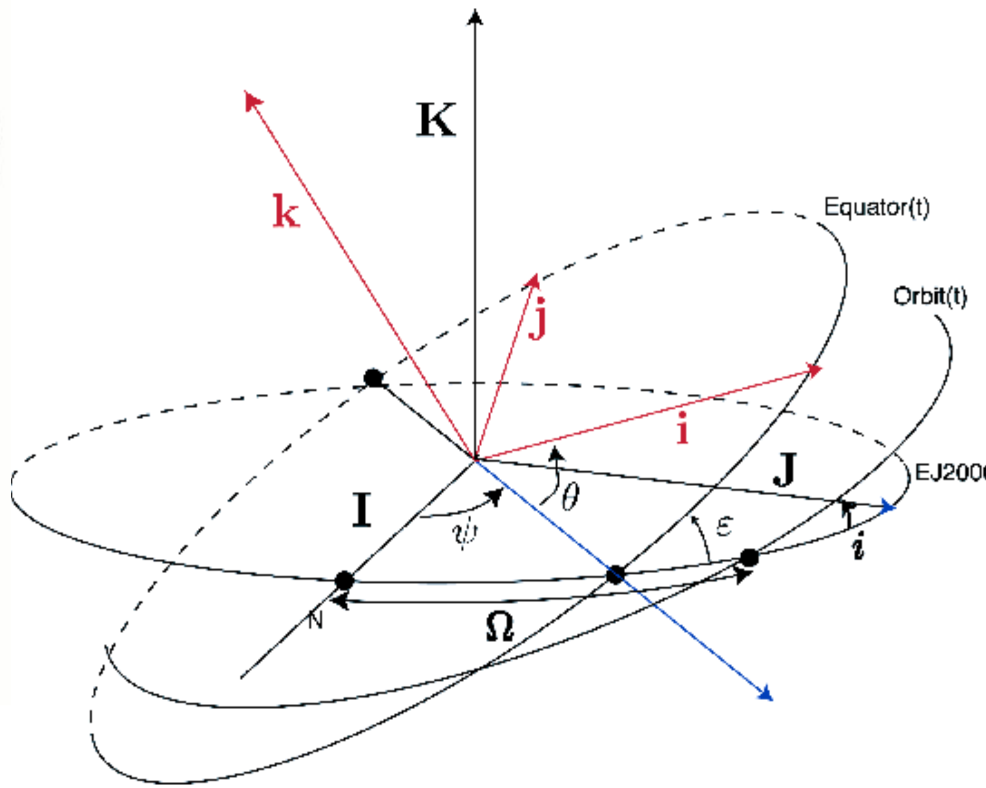
Distances to stars in our Galaxy range

A. From ~ 0.001 to ~ 50 kpc
B. From ~ 10^{18} cm to ~ 10^{23} cm
C. From ~ 1 to ~ 700 kpc
D. From ~ 1,000 to ~ 50,000 astronomical units

2.2 Kepler's Laws, Newton's Laws, and Dynamics of the Solar System



Kepler's nested Platonic solids



Kepler's Laws:

1. The orbits of planets are elliptical, with the Sun at a focus
2. Radius vectors of planets sweep out equal areas per unit time
3. Squares of orbital periods are proportional to cubes of semimajor axes:

$$P^{2}\,[\mathrm{yr}] = a_{pl}^{\;3}\,[\mathrm{au}]$$

- Derived empirically from Tycho de Brahe's data
- Explained by the Newton's theory of gravity

Newton's Laws

1. Inertia…
2. Force: $F = m\,a$
3. $F_{action} = F_{reaction}$

} → Conservation laws (E, $\boldsymbol{p}$, $\boldsymbol{L}$)

e.g., for a circular motion in grav. field:

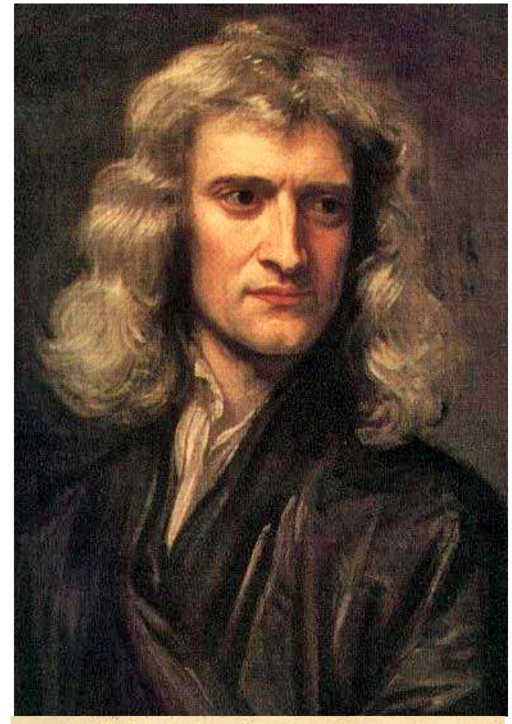
centifugal force = centripetal force

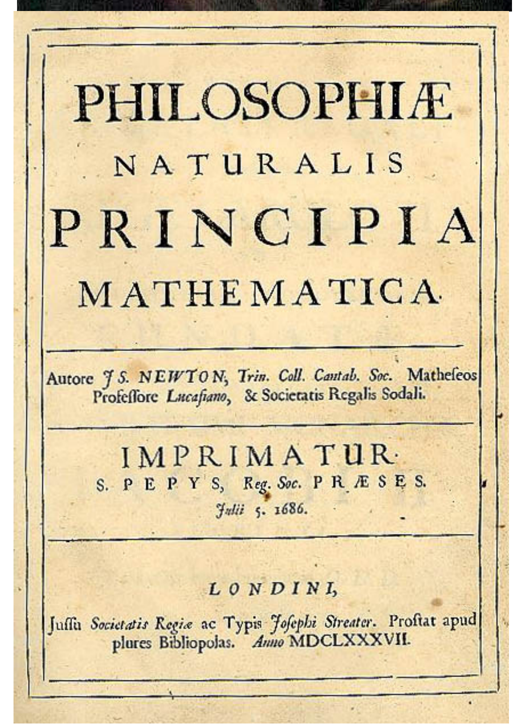
$$\frac{m\,V^2}{R} = G\,\frac{m\,M}{R^2}$$

- The law of gravity: $$F = G\,\frac{m_1\,m_2}{r^2}$$
- Energy: $E_{total} = E_{kinetic} + E_{potential}$

$$\frac{m\,V^2}{2} \qquad \frac{G\,m\,M}{R} \text{ (gravitational)}$$

- Angular momentum: $L = m\,V\,R$ (point mass)




PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

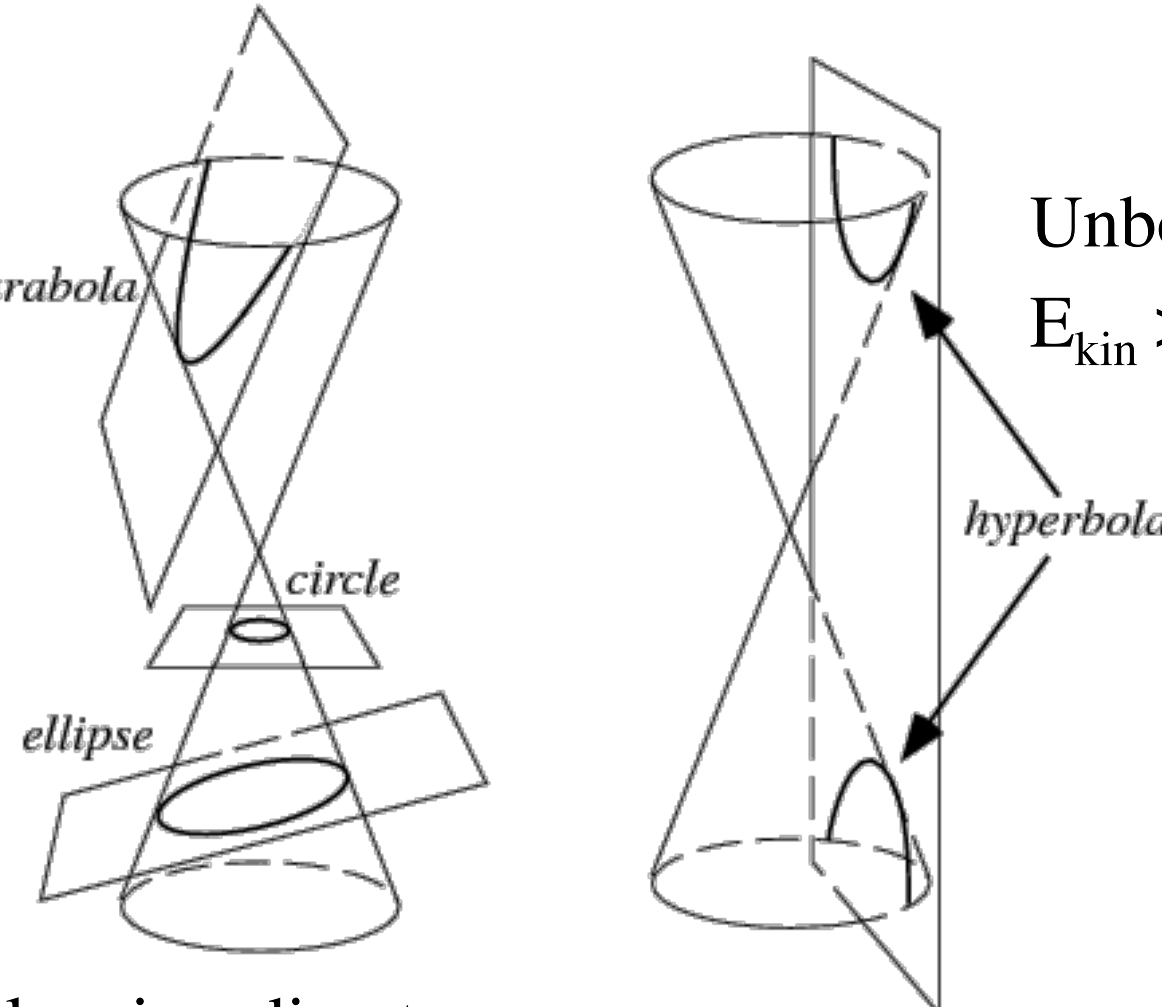
LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Motions in a Gravitational Field

- Motions of two particles interacting according to the inverse square law are conic sections:

Marginally bound:
$E_{kin} = |E_{pot}|$

Bound:
$E_{kin} < |E_{pot}|$



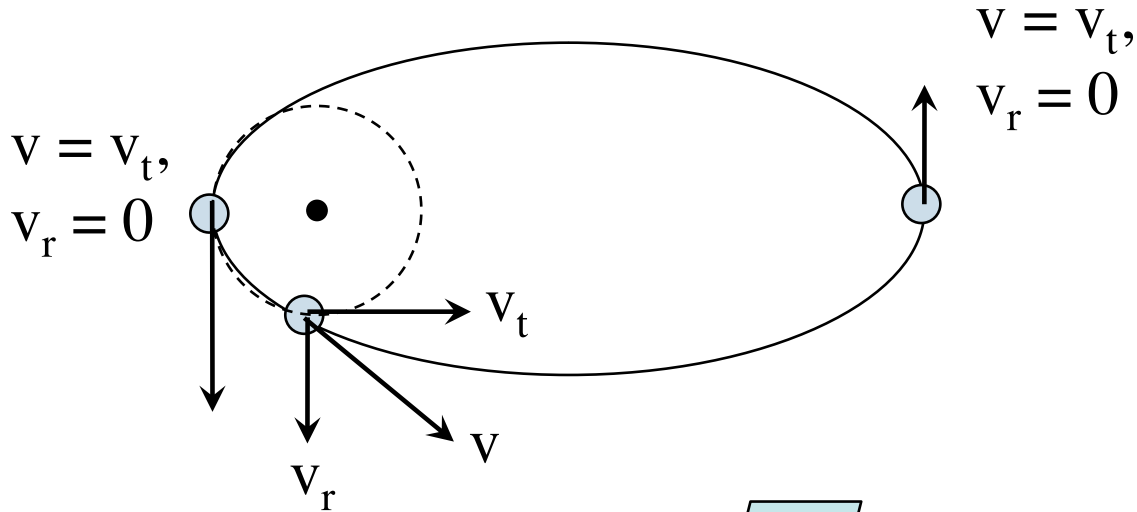
Unbound:
$E_{kin} > |E_{pot}|$

- Kepler's 1st law is a direct consequence

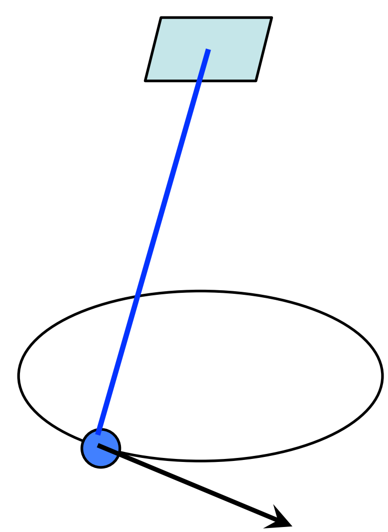
Why Ellipses?

A rigorous derivation (in polar coordinates) is a bit tedious, but we can have a simple intuitive hint:

Decompose the total velocity v into the radial (v_r) and tangential (v_t) components

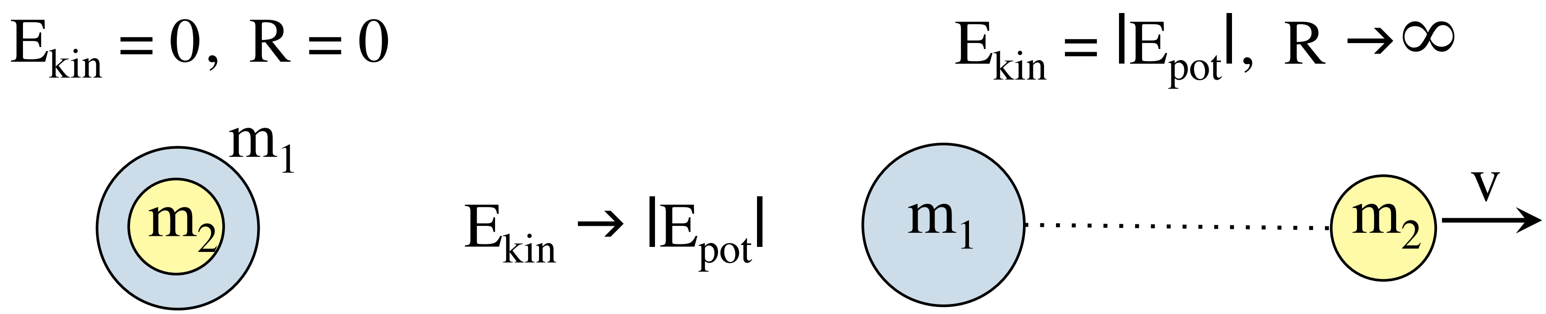


Consider the total motion as a synchronous combination of a radial and circular harmonic oscillator (recall that the period does not depend on the amplitude)



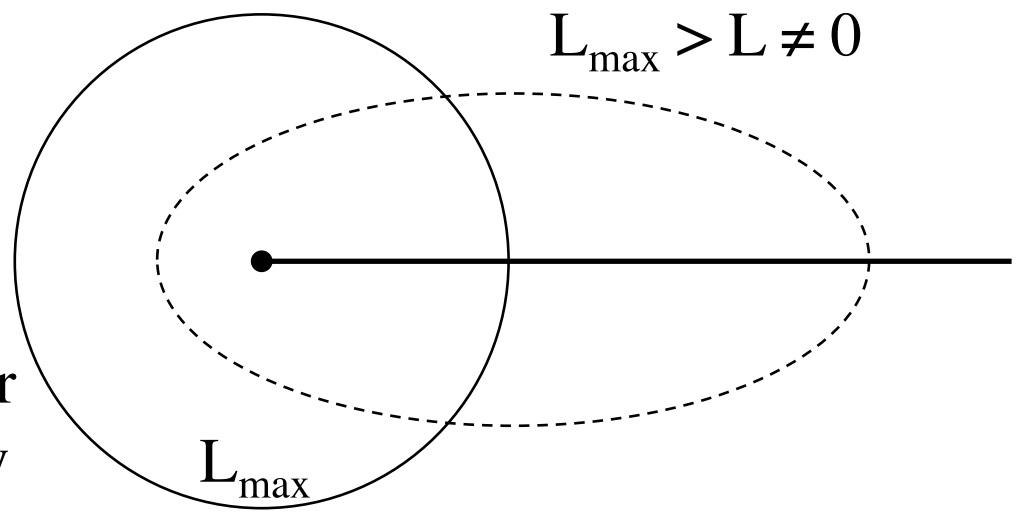
Orbit Sizes and Shapes

- For bound (elliptical) orbits, the *size* (semimajor axis) depends on the total energy:



- The *shape* (eccentricity) of the orbit depends on the angular momentum:

Circular orbit: maximum angular momentum for a given energy



Radial orbit: zero angular momentum

$L = 0$

Kepler's 2nd Law: A quick and simple derivation

Angular momentum, at any time: $L = M_{pl}\, V\, r = const.$

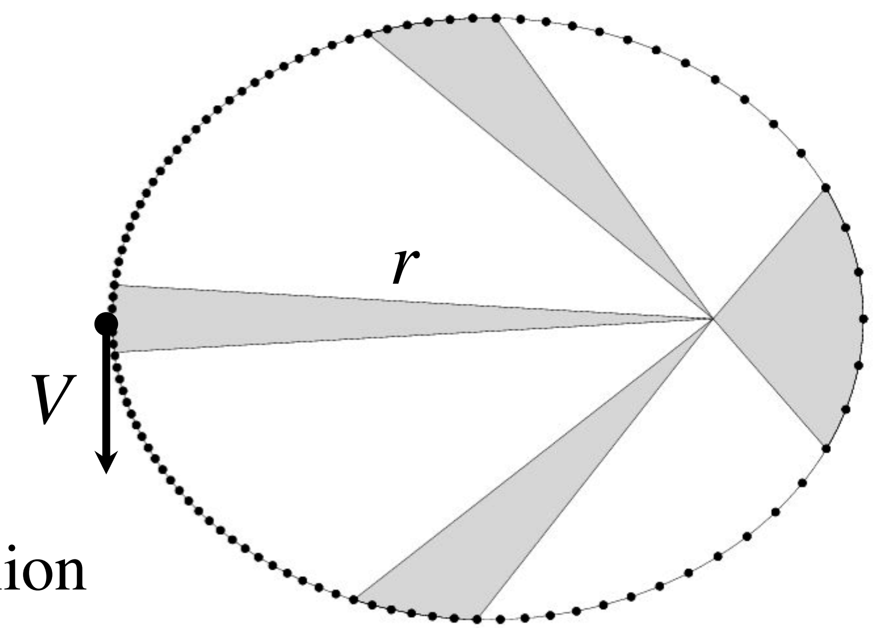
Thus: $V\, r = const.$ (this is also an "adiabatic invariant")

Element of area swept: $dA = V\, r\, dt$

Sectorial velocity: $dA/dt = V\, r = const.$

Independent of M_{pl} !

It is ***a consequence of the conservation of angular momentum.***



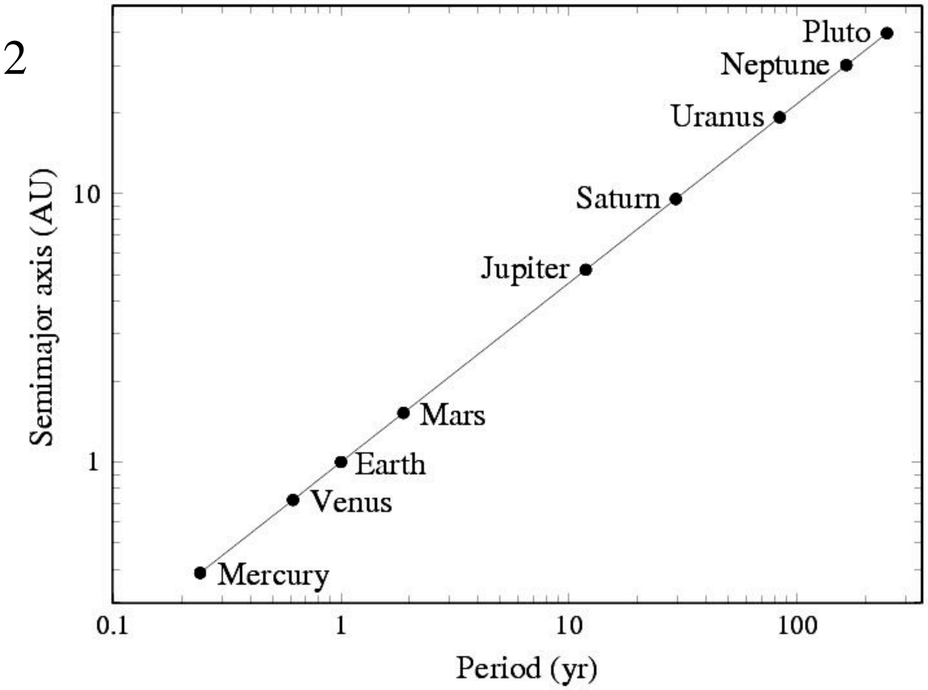
Planets move slower at the aphelion and faster at the perihelion

Kepler's 3rd Law: A quick and simple derivation

$F_{cp} = G\, M_{pl}\, M_{\odot} / (a_{pl} + a_{\odot})^{2}$

$\approx G\, M_{pl}\, M_{\odot} / a_{pl}^{2}$

(since $M_{pl} << M_{\odot}$, $a_{pl} >> a_{\odot}$)



$F_{cf} = M_{pl}\, V_{pl}^{2} / a_{pl}$

$= 4\,\pi^{2}\, M_{pl}\, a_{pl} / P^{2}$

(since $V_{pl} = 2\,\pi\, a_{pl} / P$)

$F_{cp} = F_{cf}$ ➞ $\boxed{4\,\pi^{2}\, a_{pl}^{3} = G\, M_{\odot}\, P^{2}}$ (independent of M_{pl} !)

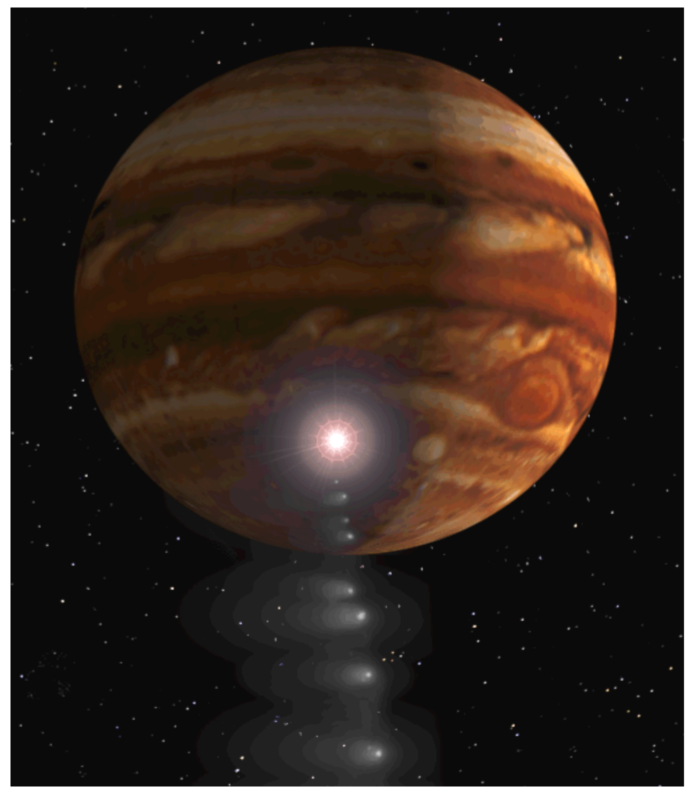
Another way: $E_{kin} = M_{pl} V_{pl}^{2} / 2 = E_{pot} \approx G\, M_{pl} M_{\odot} / a_{pl}$

Substitute for V_{pl} : $4\,\pi^{2}\, a_{pl}^{3} = G\, M_{\odot}\, P^{2}$

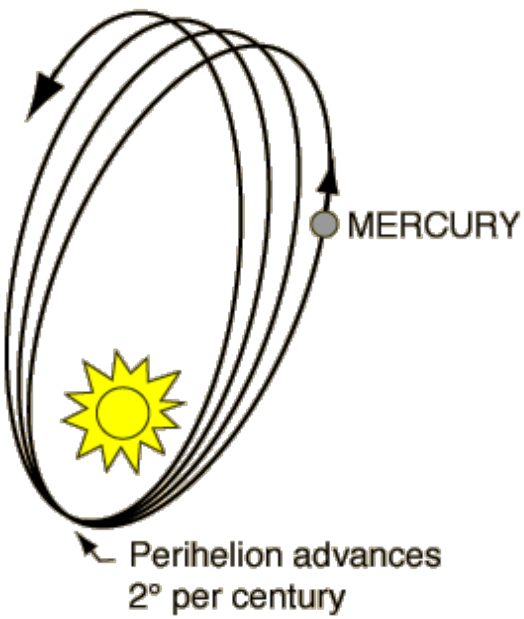
➞ It is ***a consequence of the conservation of energy***

It Is Actually A Bit More Complex …

- Kepler's laws are just an approximation: we are treating the whole system as a collection of isolated 2-body problems
- There are *no analytical solutions* for a general problem with > 2 bodies! But there is a good ***perturbation theory,*** which can produce very precise, but always approximate solutions
  - – Discovery of Neptune (1846)
  - – Comet impacts on Jupiter

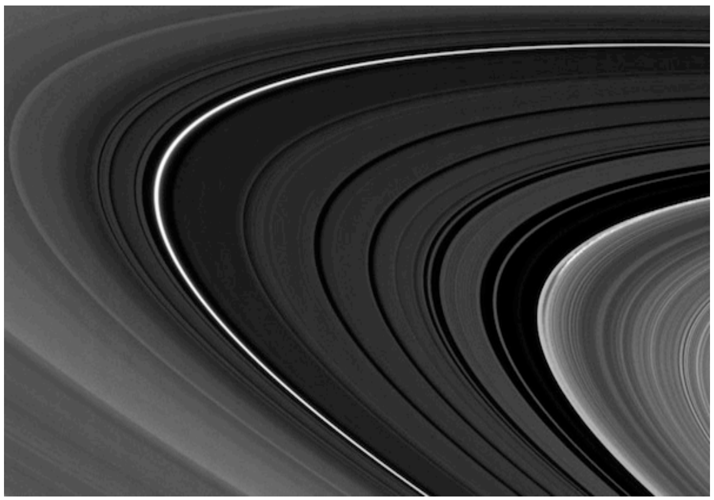


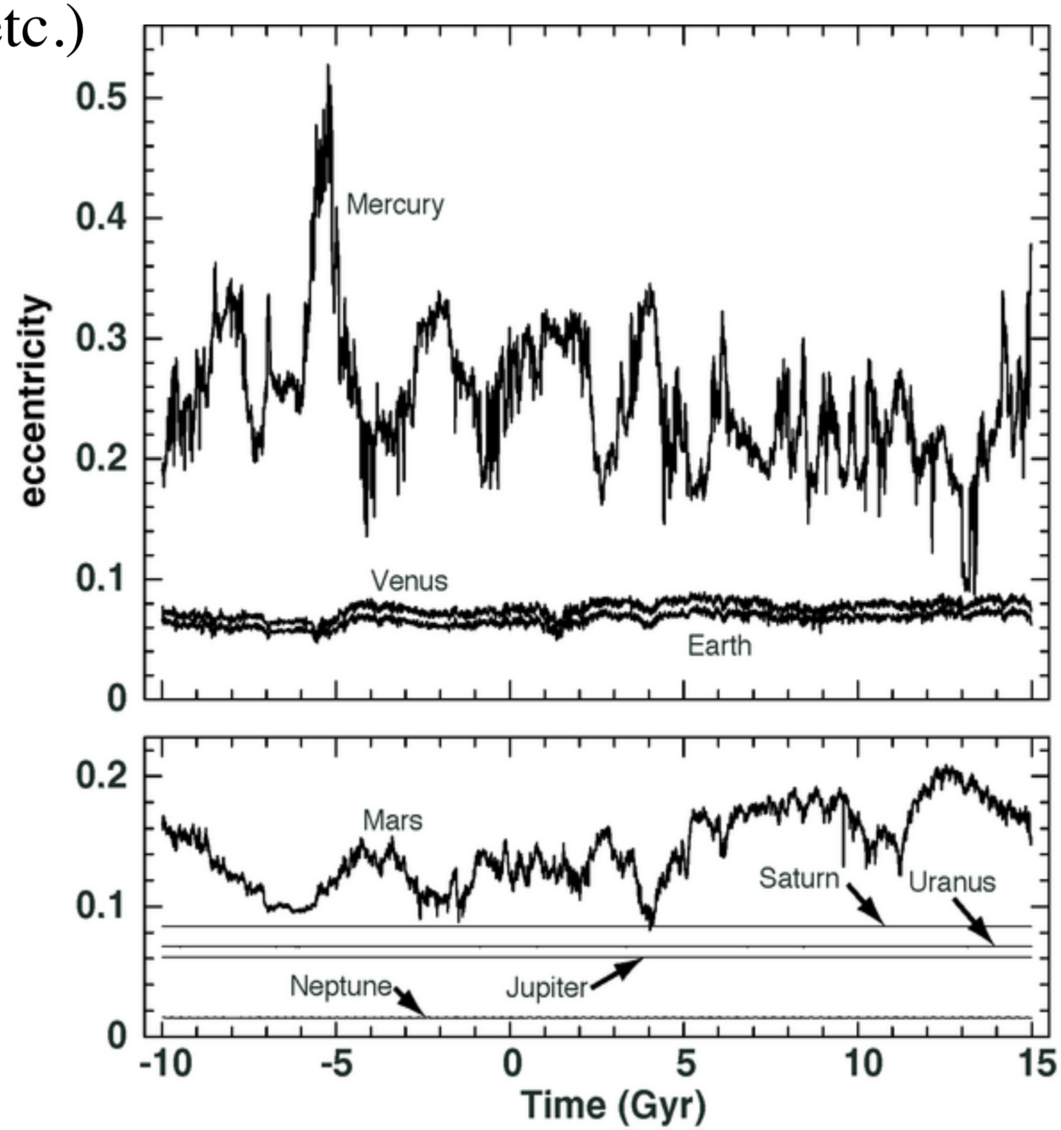
- Relativistic effects can be used to test theory of relativity (e.g., precession of Mercury's orbit



It Is Actually A Bit More Complex …

- Dynamical resonances can develop (rotation/revolution periods, asteroids; Kirkwood gaps; etc.)





- If you wait long enough, more complex dynamics can occur, including dynamical chaos

(Is Solar System stable?)

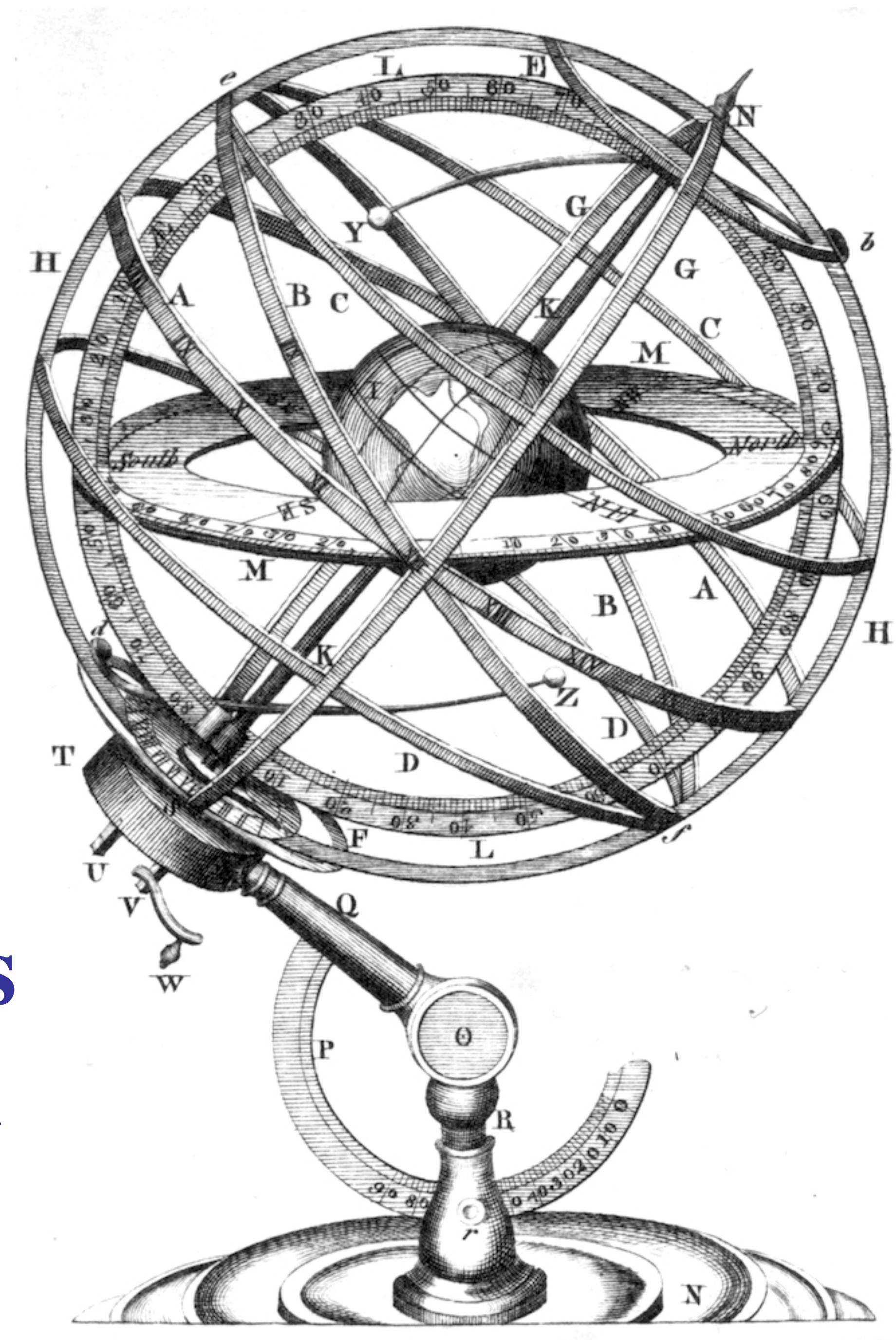
Kepler's 3rd law is…

A. Cubes of orbit sizes ~ squares of orbital periods
B. Squares of orbit sizes ~ cubes of orbital periods
C. A consequence of the conservation of energy
D. A consequence of the conservation of angular momentum

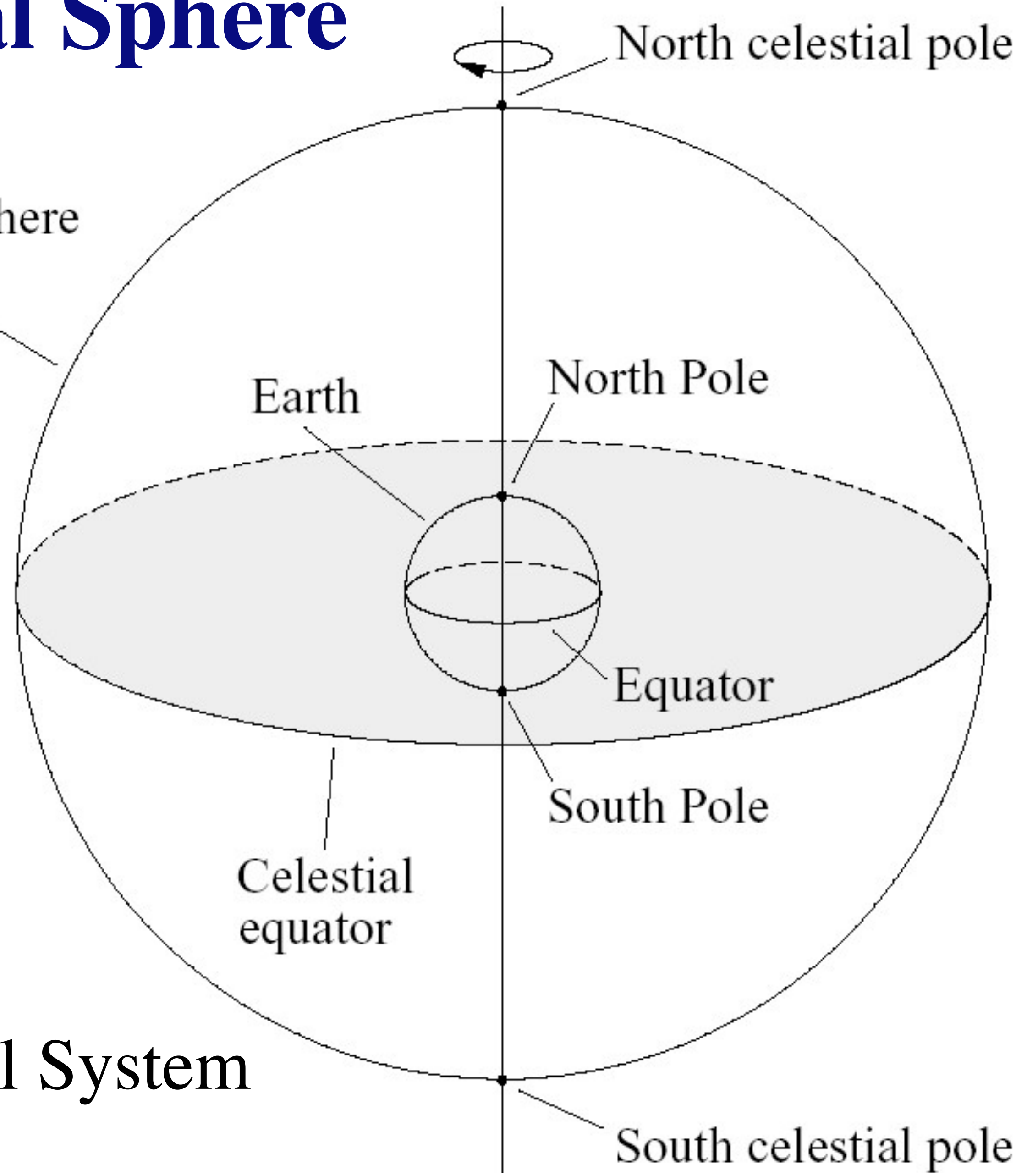
The shape of a closed orbit depends on

A. Total energy
B. Total angular momentum
C. Angular momentum for a given energy
D. None of the above

2.3 Celestial Coordinate Systems Time Systems, and Earth's Rotation



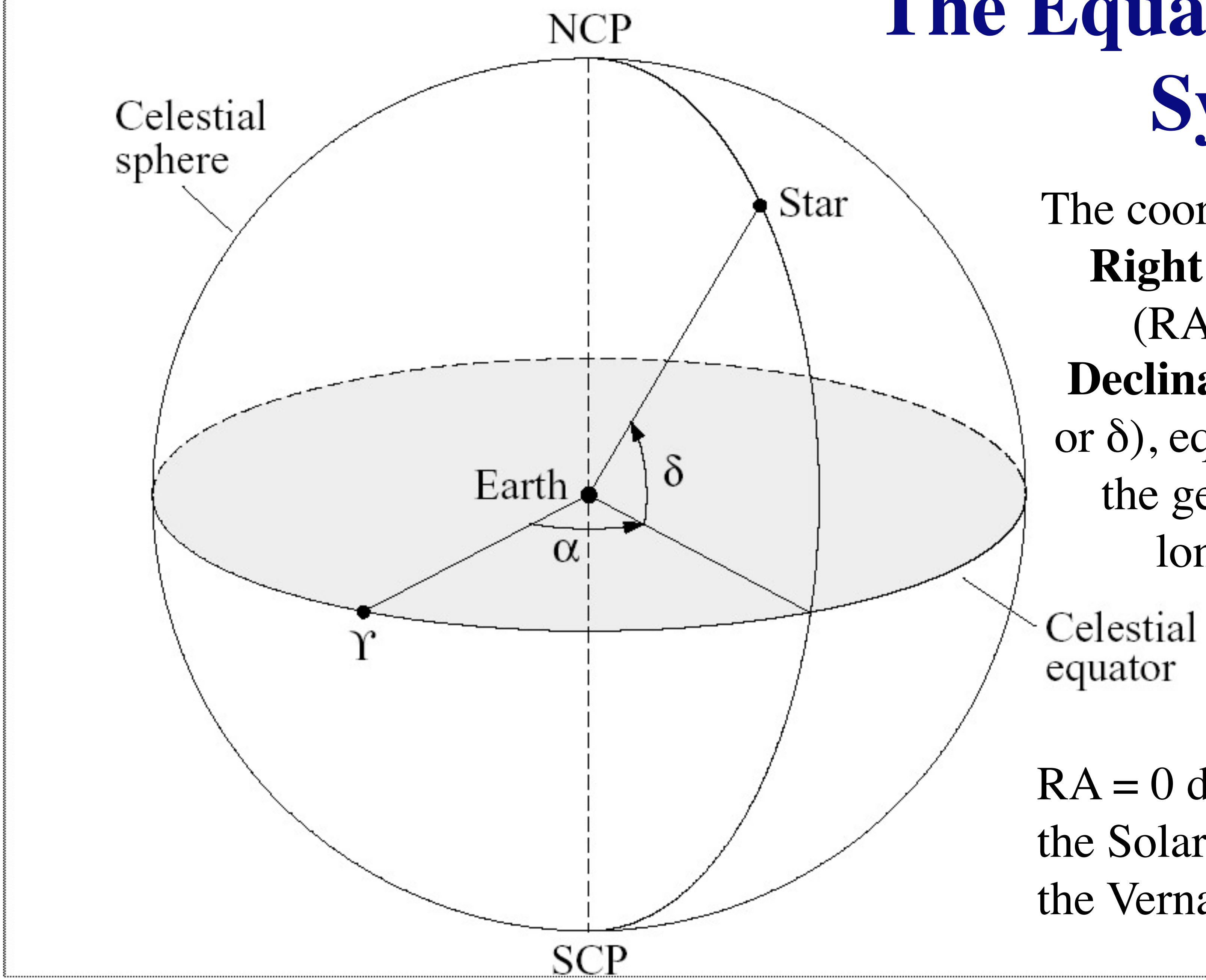
The Celestial Sphere



Think of it as an outward projection of the terrestrial long-lat coordinate system onto the sky

→ the Equatorial System

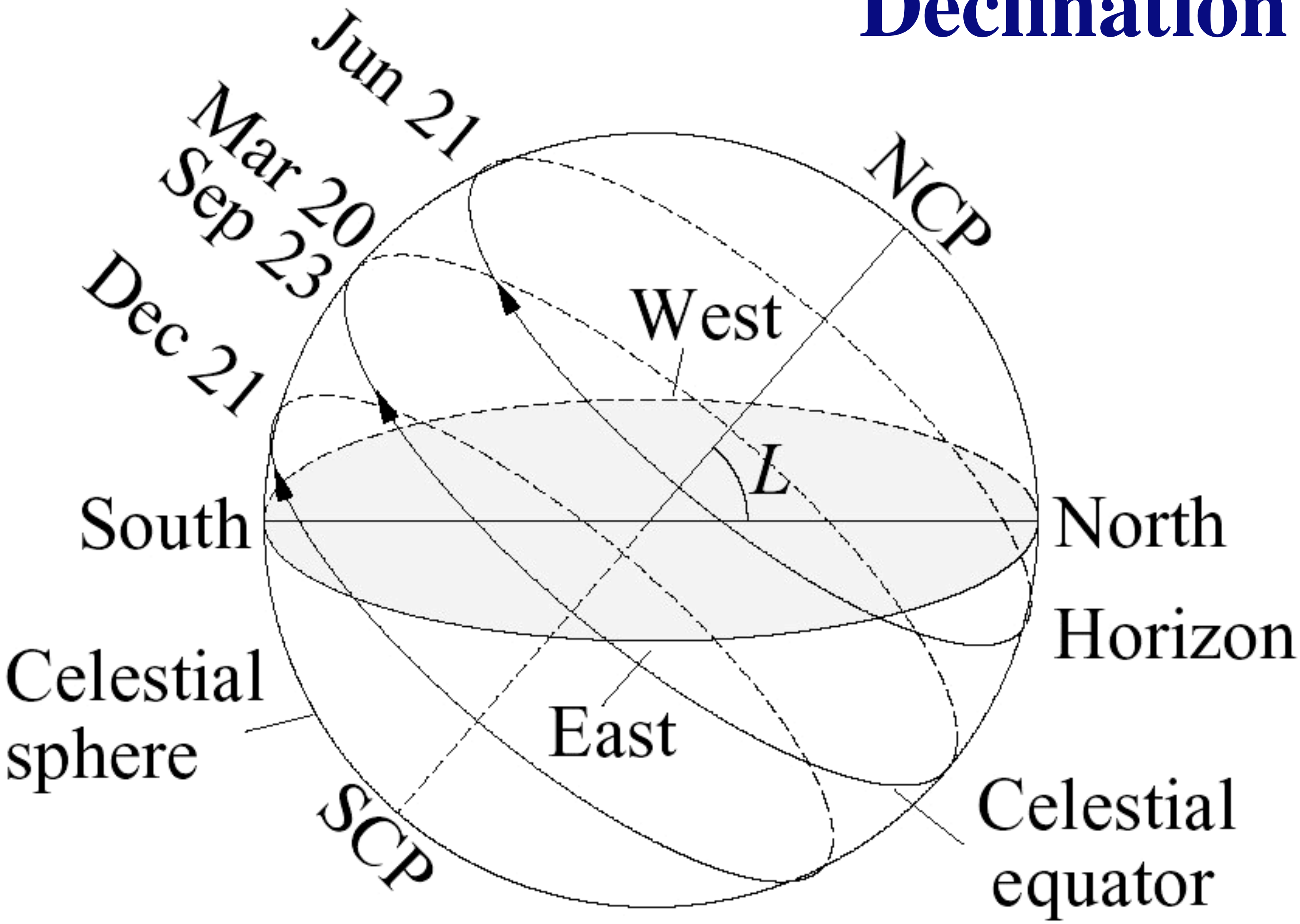
The Equatorial System



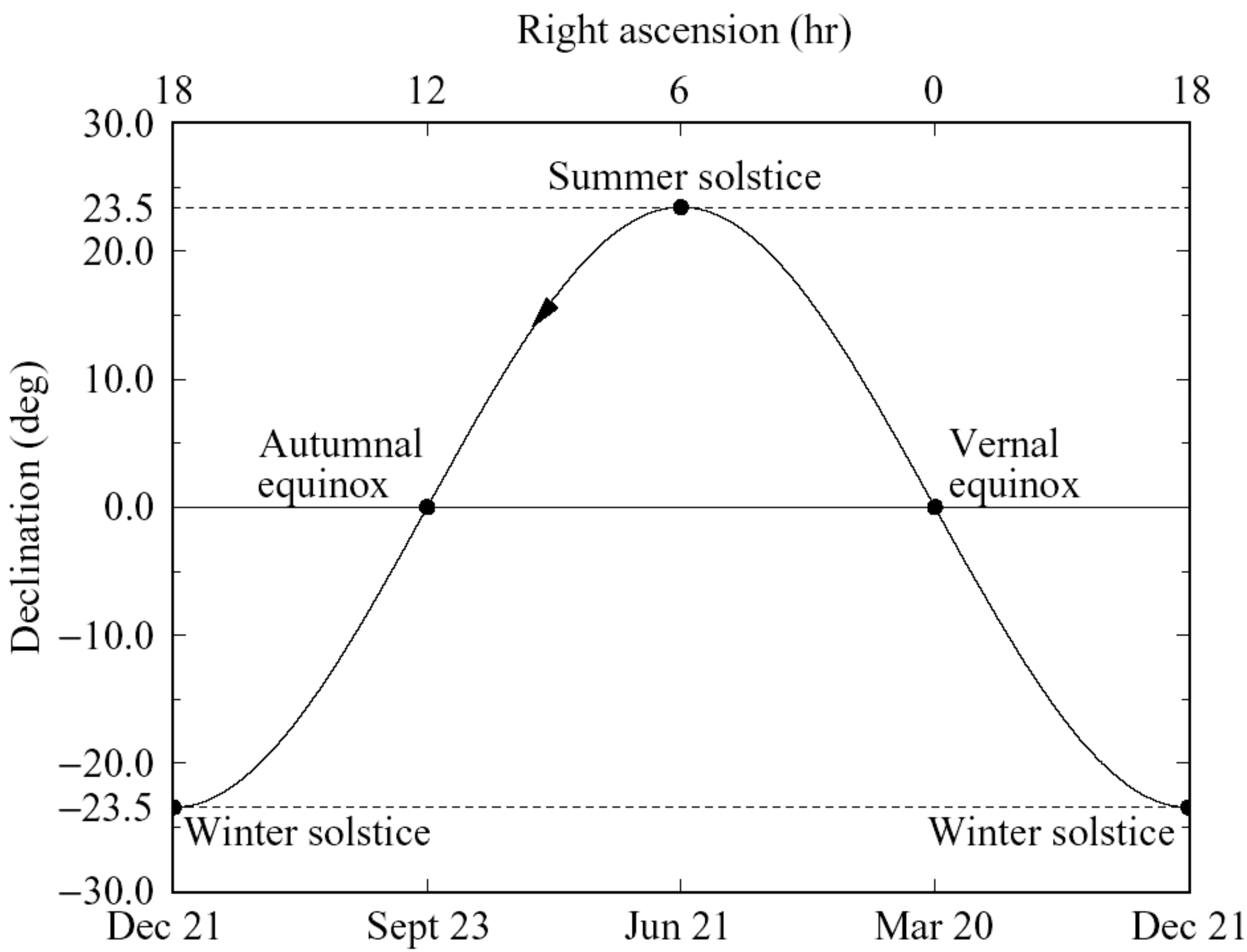
The coordinates are **Right Ascension** (RA, or α) and **Declination** (Dec, or δ), equivalent to the georgaphical longitude and latitude

RA = 0 defined by the Solar position at the Vernal Equinox

The Seasonal Change of the Solar Declination



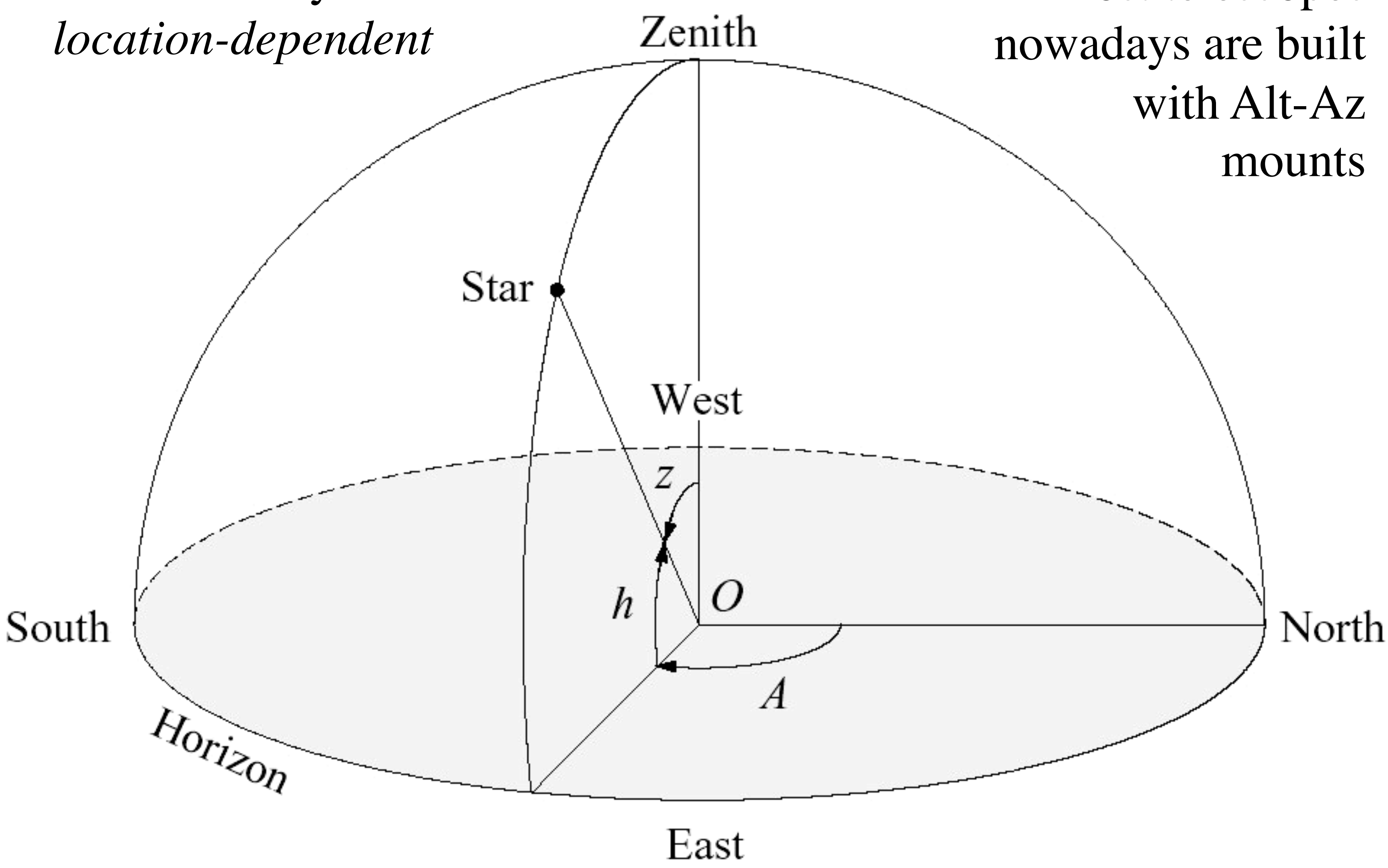
Annual Solar Path



The Alt-Az Coordinate System

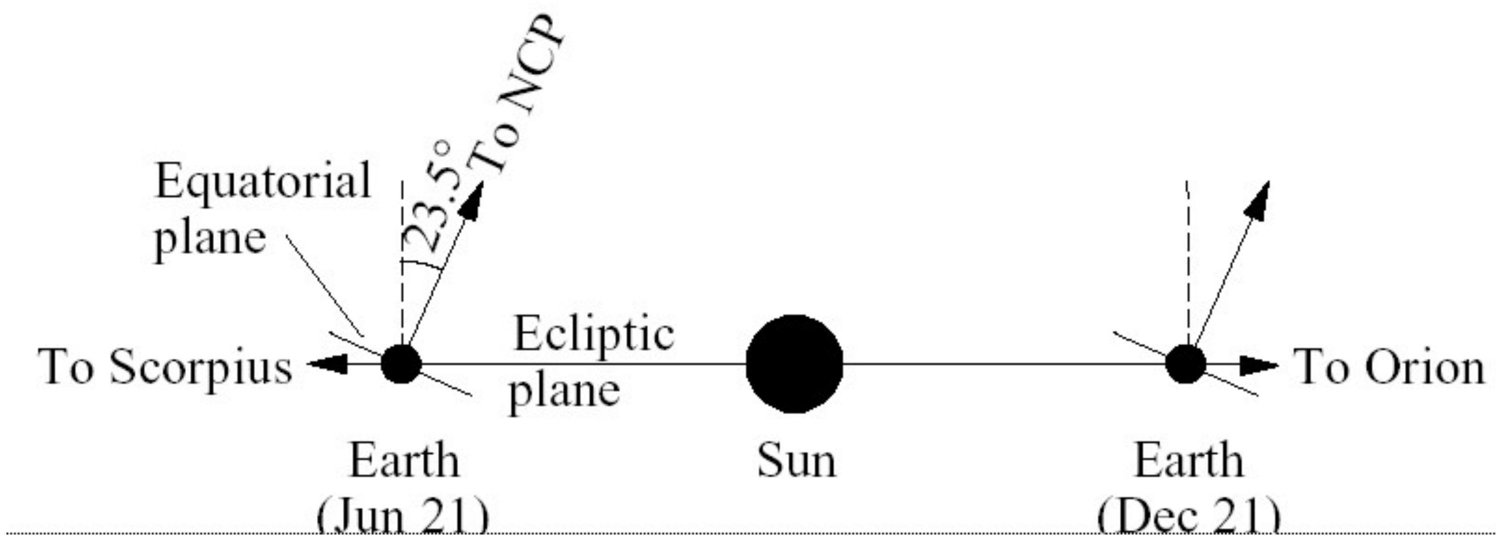
It is obviously *location-dependent*

Most telescopes nowadays are built with Alt-Az mounts



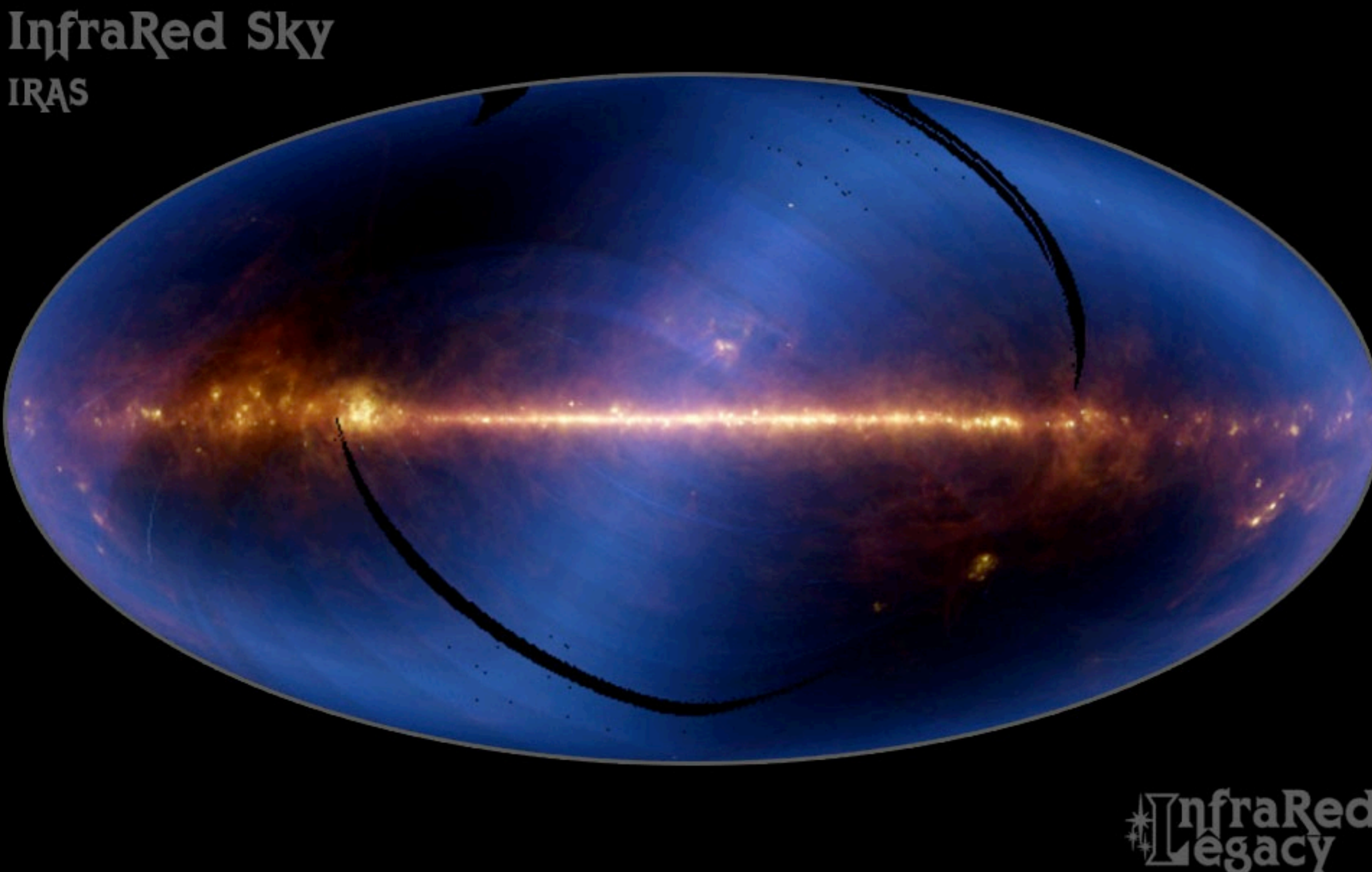
Other Common Cellestial Coordinate Systems

Ecliptic: projection of the Earth's orbit plane defines the Ecliptic Equator. Sun defines the longitude = 0.



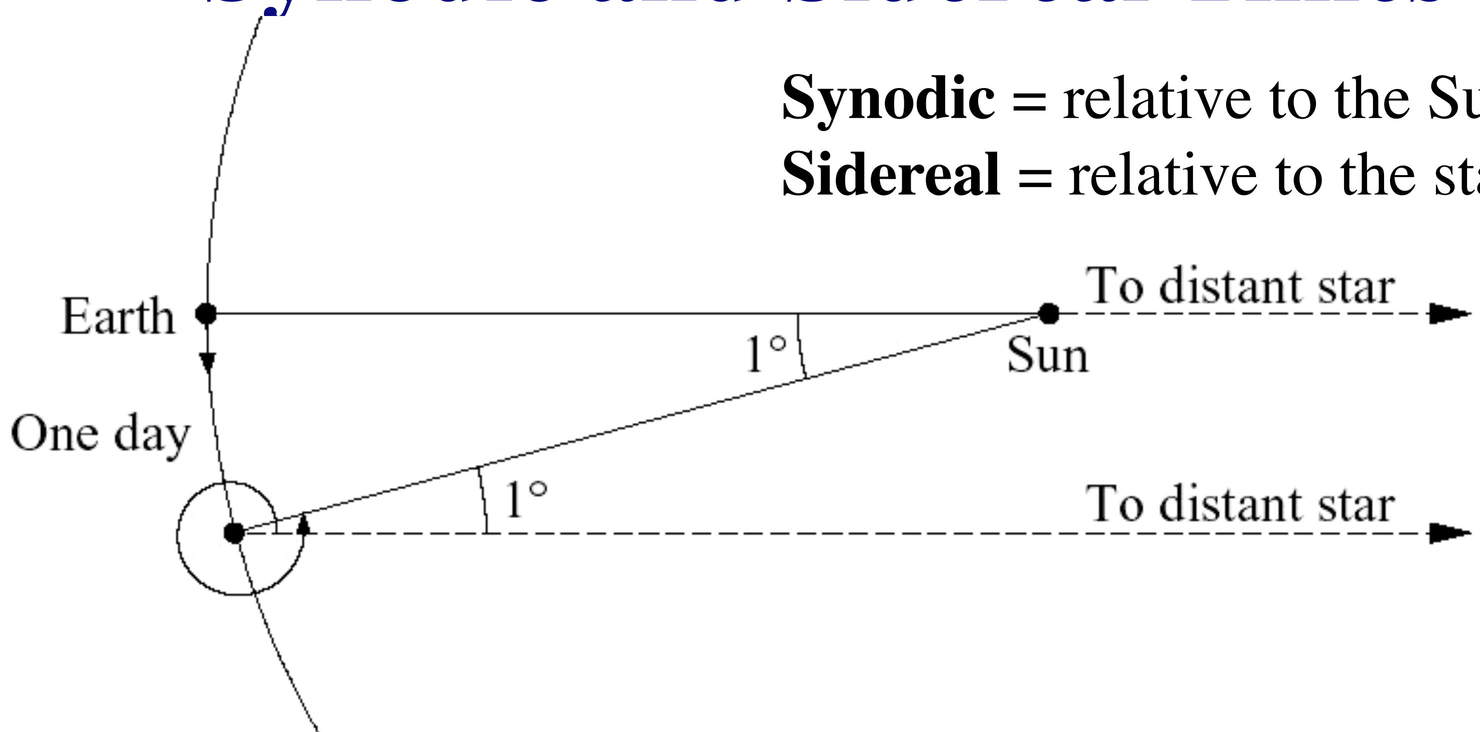
Galactic: projection of the mean Galactic plane is close to the agreed-upon Galactic Equator; longitude = 0 close, but not quite at the Galactic center. $(\alpha,\delta) \rightarrow (l,b)$

Ecliptic (Blue) and Galactic Plane (Red)



Synodic and Sidereal Times

Synodic = relative to the Sun
Sidereal = relative to the stars



As the Earth goes around the Sun, it makes an extra turn. Thus:

Synodic/tropical year = 365.25 (solar) days

Sidereal year = 366.25 sidereal days = 365.25 solar days

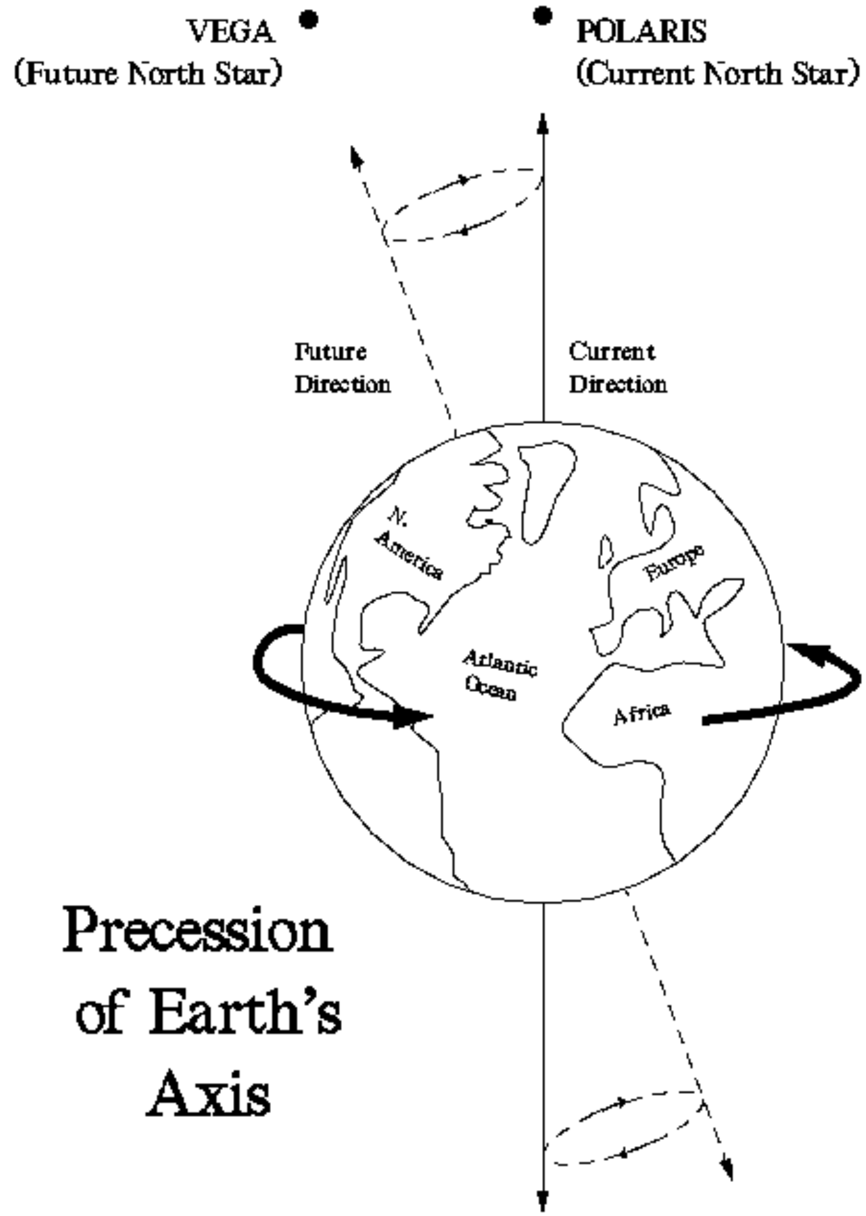
Universal time, UT = relative to the Sun, at Grenwich

Local Sidereal Time (LST) = relative to the celestial sphere

= RA now crossing the local meridian (to the South)

The Precession of the Equinoxes

- The Earth's rotation axis precesses with a period of ~ 26,000 yrs, caused by the tidal attraction of the Moon and Sun on the the Earth's equatorial bulge
- There is also *nutation* (wobbling of the Earth's rotation axis), with a period of ~ 19 yrs
- Coordinates are specified for a given **equinox** (e.g., B1950, J2000) and sometimes **epoch**

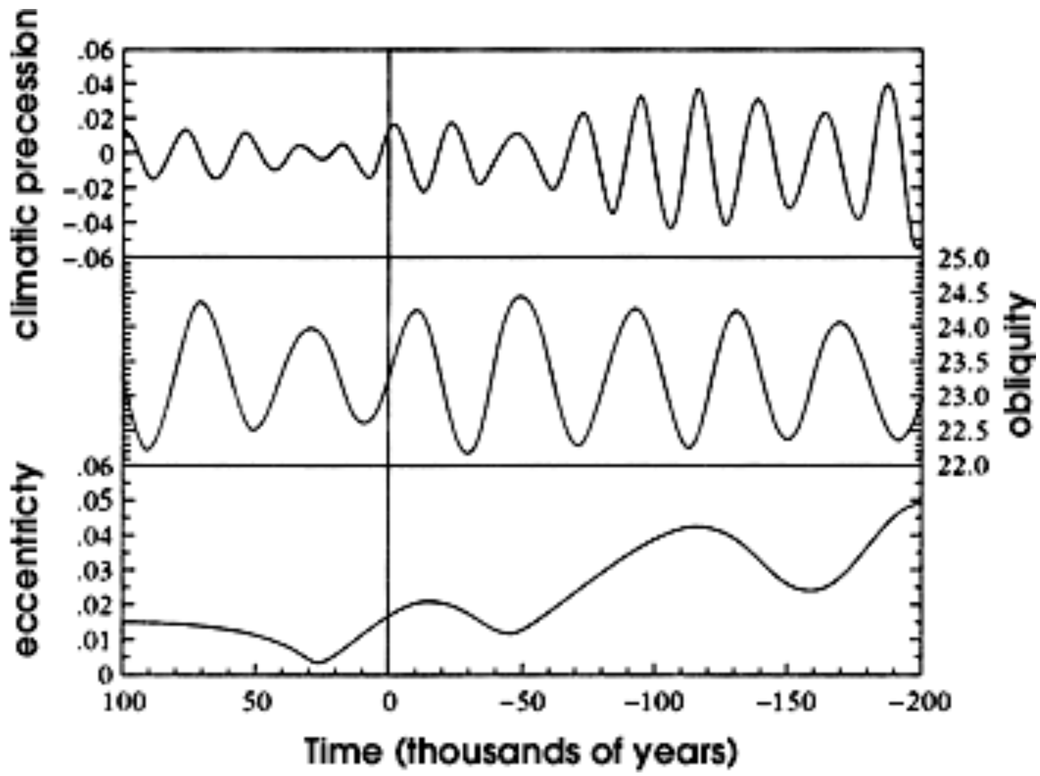


Earth's Orbit, Rotation, and the Ice Ages

Milankovich Theory: cyclical variations in Earth-Sun geometry combine to produce variations in the amount of solar energy that reaches Earth, in particular the ice-forming regions:

1. Changes in obliquity (rotation axis tilt)
2. Orbit eccentricity
3. Precession

These variations correlate well with the ice ages!



The change of seasons is due to…

A. The tilt of the Earth's rotation axis relative to the celestial equator
B. The tilt of the Earth's rotation axis relative to the plane of the ecliptic
C. Eccentricity of the Earth's orbit
D. Precession of the equinoxes
E. Human sacrifices

Johannes Kepler