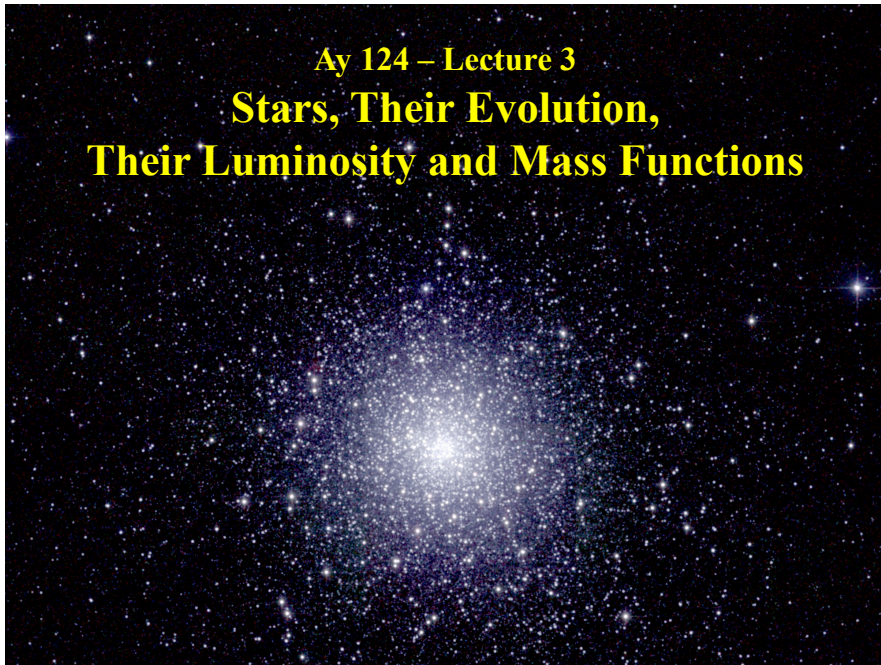


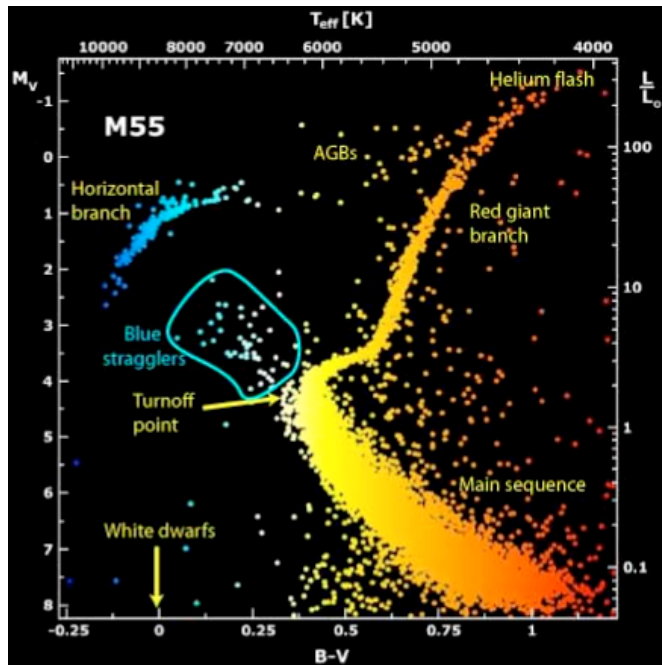
Ay 124 – Lecture 3
**Stars, Their Evolution,
 Their Luminosity and Mass Functions**



Understanding Stellar Populations

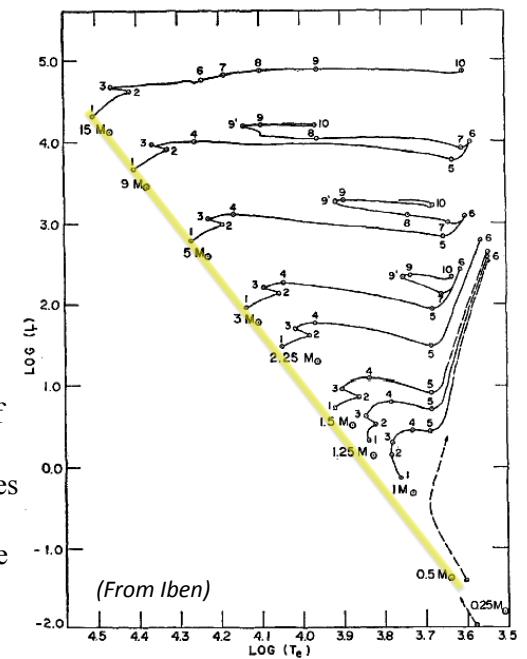
- The HRD (CMD) is the fundamental tool and framework for understanding of stars and their evolution
 - And thus, for the modeling and understanding of the evolution of galaxies and their stellar populations
 - It is also a distance indicator for stars and stellar systems
- *Mass is the dominant parameter* which determines where a star will fall on the HRD, and how it will evolve
 - It also determines the luminosities, lifetimes, evolutionary end products, chemical enrichment potential, etc.
 - Metallicity is a secondary parameter, and it shifts the stellar sequences at a given age
 - It also affects the spectra of stellar populations
- Stellar structure and evolution are now very well understood, and can be used to interpret the observations of galaxies near and far

Typical
 HRD for an
 old
 (evolved)
 stellar
 population,
 e.g., a
 globular
 cluster



**Stars of Different
 Mass Evolve Off
 the MS Along
 Different Paths in
 the HRD, but the
 Physics is Basically
 the Same**

Note that the nonlinearity of the mass-luminosity relation(s) for stars decouples the stellar population components which dominate the mass from those which dominate the light



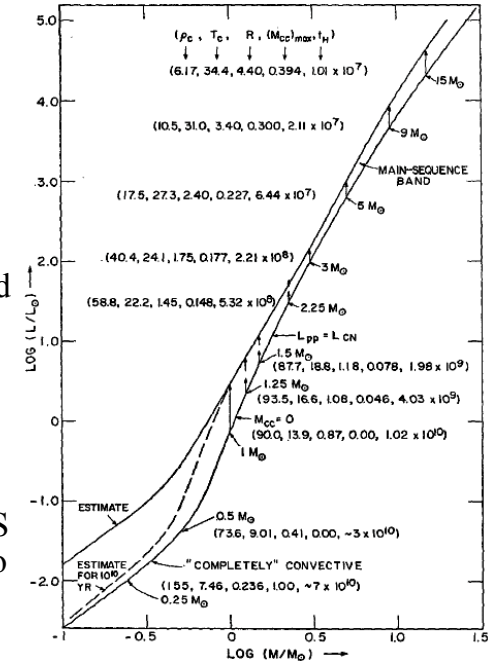
Main Sequence and the Range of Stellar Masses

- MS is the locus where stars burn H into He in their cores
- Objects which cannot reach the necessary $[T, \rho]$ to ignite this fusion, because of their **low mass** ($M_{\star} < 0.08 M_{\odot}$) are called **brown dwarfs** (however, they may burn the trace amounts of primordial deuterium)
- Not obvious why should stars form a (nearly) 1-dim. family of objects with the mass as the dominant parameter
- The **high-mass end** of the stellar family is set by the **Eddington limit**
 - Believed to be $\sim 100 M_{\odot}$ for \sim Solar metallicities
 - Current record: A1 in NGC3603 ($\sim 84 M_{\odot}$)
 - Zero-metallicity stars could have been more massive



Life on the MS

- Burning H into He changes the chemical composition and lowers the pressure
- Core shrinks, heat up and burns H at a higher rate
- Luminosity increases, which causes the outer envelope to swell up
- The MS stars begin on the lower edge of the MS band and move up and to the right



Stellar Lifetimes on the Main Sequence

- The duration of a star's MS lifetime depends on the amount of hydrogen in its core (fuel supply) and the rate at which it is consumed, i.e., luminosity
- Core H burning ends when the star converts a certain fraction of its total mass into He, the so-called **Schönberg-Chandrasekhar limit**,

$$(M_{\text{core}}/M_{\star}) \approx 0.37 (\mu_{\text{e}}/\mu_{\text{core}})^2 \approx 0.08 - 0.1$$

- Since the M-L relation is so non-linear, $L \sim M^{\alpha}$, with $\alpha \sim 3 - 5$, **the more massive a star, the shorter is its MS lifetime:**

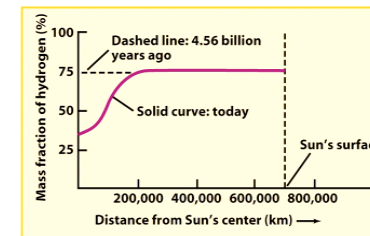
$$L \propto M^{3.5} \rightarrow t_{\text{ms}} \propto M^{-2.5}$$

- The same principle applies to post-MS stages of stellar evolution
- Generally, later stages of stellar evolution last shorter, because there is less fuel to burn, and because of the non-linearity of the M-L relation, which may be driven by the highly non-linear dependence of the TNR rates on $[T, \rho]$

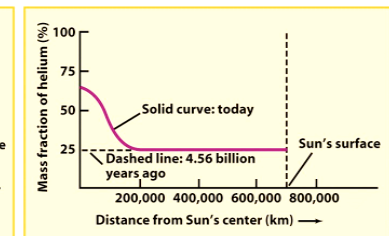
Stellar Lifetimes on the Main Sequence

Mass (M_{\odot})	Surface temperature (K)	Spectral class	Luminosity (L_{\odot})	Main-sequence lifetime (10^6 years)
25	35,000	O	80,000	4
15	30,000	B	10,000	15
3	11,000	A	60	800
1.5	7000	F	5	4500
1.0	6000	G	1	12,000
0.75	5000	K	0.5	25,000
0.50	4000	M	0.03	700,000

The main-sequence lifetimes were estimated using the relationship $t \propto 1/M^{2.5}$ (see Box 21-2).



(a) Hydrogen in the Sun's interior



(b) Helium in the Sun's interior

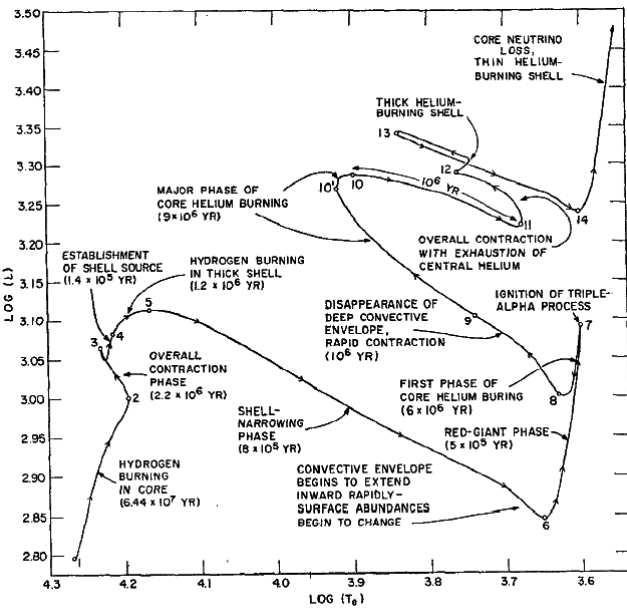
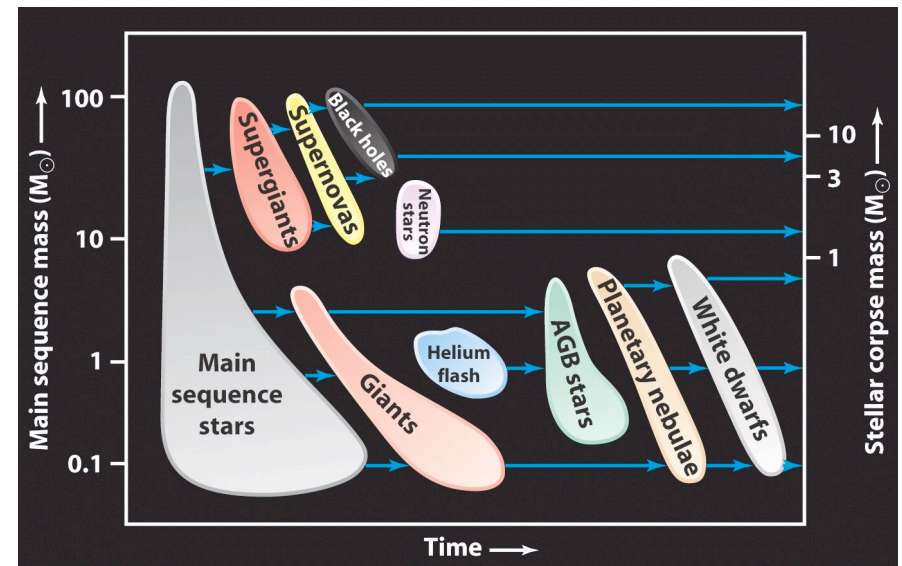


FIG. 1. The path of a metal-rich $5M_{\odot}$ star in the Hertzsprung-Russell diagram.

Stellar Evolution
is a
Sequence of Different Energy Production (Nuclear Fusion) Mechanisms

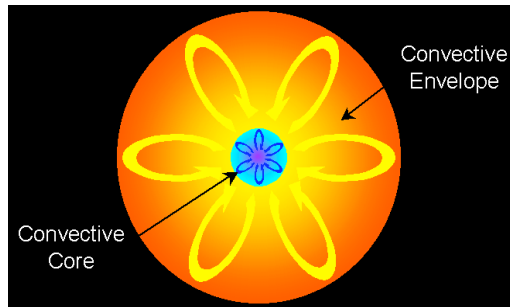
(From Iben)

Pathways of Stellar Evolution



Very Low Mass Stars: Red Dwarfs

- Mass $< 0.4 M_{\odot}$
- Their structure is all convection zone, H and He is mixed throughout the star
- Just burn H slowly
- Will never build up a He core, and never ignite He
- Could perhaps survive on the MS for a 100 Gyr!
- Then just fade as a WD / Black Dwarf
- Unimportant contributor to the total luminosity of a galaxy, but contain most of the stellar mass

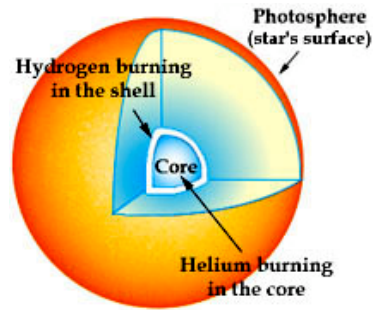


The End of the MS Phase

- On the MS, a star is in a hydrostatic equilibrium, and its core is sufficiently hot to fuse H into He
- Now the star has two chemically distinct zones, a core of inert He surrounded by an H envelope - the core of a MS star is not sufficiently hot for He burning
- When the core becomes pure He, a new evolutionary phase starts - the ascent to the **Red Giant Branch (RGB)**
- Without energy generation, the core cannot support itself against gravitational collapse and so it begins to shrink; as it collapses it heats up
- This heat is transferred to a thin shell of H around the core which reaches a temperature in which H fusion can occur
- This He flash ends the ascent to the RGB, star descends to HB

Red Giants

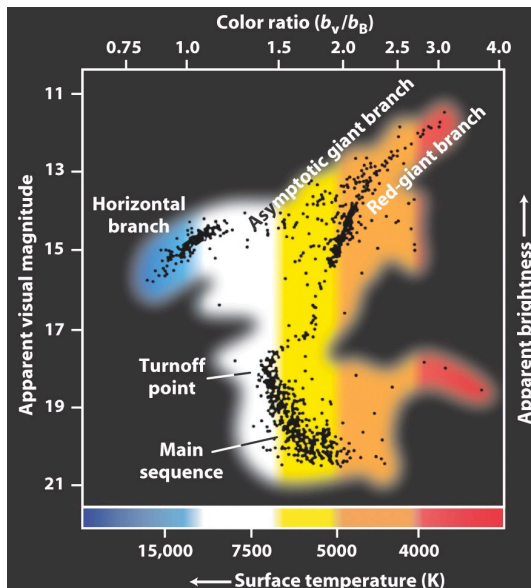
- As the core continues to collapse, the temperature in the H fusing shell continues to rise and thus the luminosity in the shell increases as does the pressure
- The entire star is no longer in a hydrostatic equilibrium, and the envelope begins to expand
- As they expand these outer layers cool - the star becomes redder, while its luminosity increases: the star slowly ascends the RGB
- When the core has heated to $T \sim 10^8$ K, it starts the fusion of He into C (He flash), which dominates the total energy production
- The star is in a quasi-static equilibrium. The lifetime of a star as a Red Giant is about 10% of its MS lifetime



Helium Flash: The End of the RGB

- H fusion leaves behind He ash in the core of the star which cannot begin to fuse until the core reaches temperature of $\sim 10^8$ K. How a star begins He fusion depends on its mass:
 - $M > 3 M_{\odot}$ stars contract rapidly, their cores heat up, and He fusion begins gradually
 - Less massive stars evolve more slowly and their cores contract so much that degeneracy occurs in the core
 - When the temperature is hot enough He fusion begins to make energy and the T rises, but pressure does not increase due to degeneracy
- Higher T increases He fusion even further resulting in a runaway explosion: the **Helium Flash** which for a few minutes can generate more luminosity than an entire galaxy. The flash does not destroy the star: the envelope absorbs the energy
- The star then sheds its convective envelope, and the radiative core settles on the **Horizontal Branch** (HB), the He main sequence

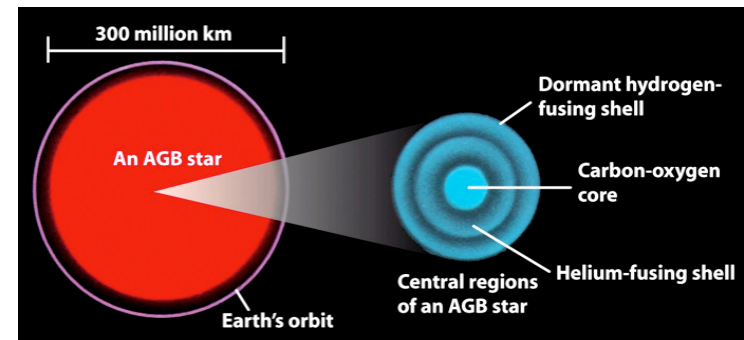
Low-mass stars go through two distinct red giant stages



A low-mass star becomes

- A red giant (RG) when shell H fusion begins
- A horizontal branch (HB) star when core He fusion begins
- An **asymptotic giant branch** (AGB) star when the He in the core is exhausted and shell He fusion begins
- AGB stars shedding their envelopes are the major sources of interstellar dust

Dredge-ups bring the products of nuclear fusion to a giant star's surface

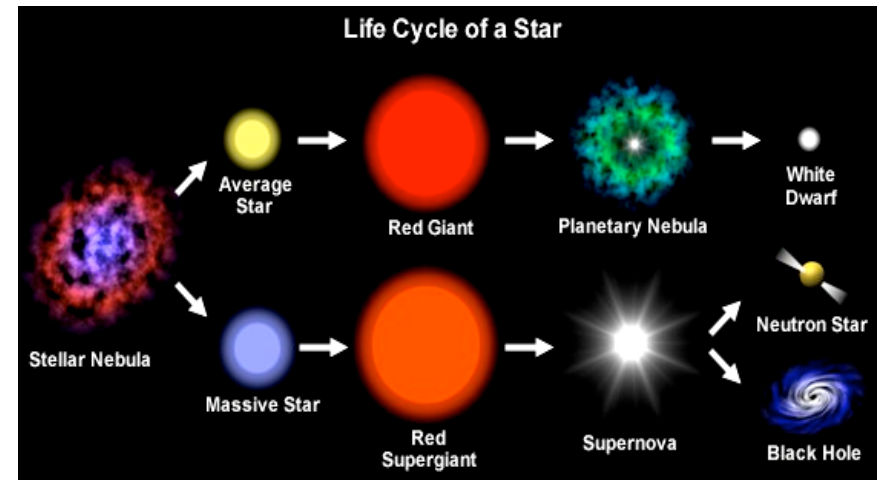


- As a low-mass star ages, convection occurs over a larger portion of its volume
- This takes heavy elements formed in the star's interior and distributes them throughout the star
- When the star loses its envelope at the end, the ISM is enriched

The End Phases of Stellar Evolution

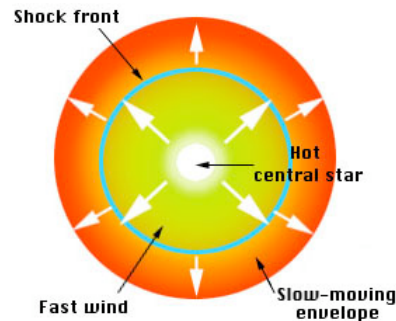
- The evolution and eventual fate of stars in the late stages of their lives are critically dependent on the amount of matter they have at birth:
- Stars with initial masses of *less* than $\sim 8 M_{\odot}$ end their lives as *white dwarfs* (WD). The star sheds its RG envelope, which becomes a *planetary nebula* (PN), and the inert, degenerate core cools passively, and it photoionizes the PN
- Stars with initial masses *greater* than $\sim 8 M_{\odot}$ end their lives by exploding as *supernovae* (SNe, generally type II). The stellar remnants are *neutron stars* (NS) or *black holes* (BH), depending on the progenitor mass
 - Note: this is not the only way to produce a SN; type Ia SNe come from accreting WD
 - SNe are the main agents of the chemical enrichment of the ISM and subsequent generations of stars

The Life and Death of Stars

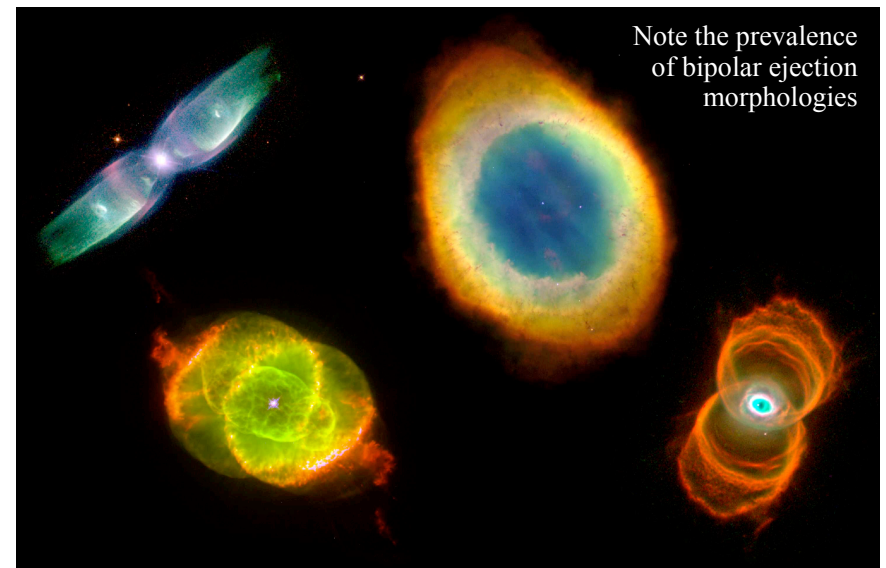


Planetary Nebulae

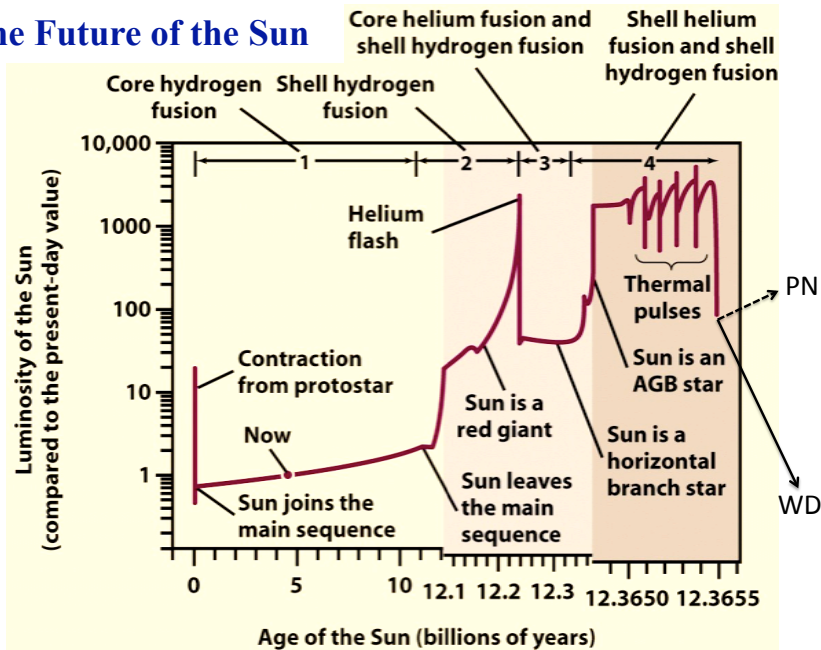
- A RG brightens by a factor of $\sim 10^3 - 10^4$. The outer, H-rich envelope swells up to a few au, with $T \sim 2,000 - 3,000$ K
- A strong stellar wind begins to blow and it carries away most of the H envelope
- During the final shedding of its envelope, when the mass loss is the greatest, the star becomes unstable and pulsates, with periods \sim few months to > 1 yr. Such stars are called *long-period variables* (LPVs)
- The envelope material ejected by the star forms an expanding shell of gas that is known as a *planetary nebula (PN)*
- PNe expand with $V \sim 10 - 20$ km/s, and plow into the surrounding ISM, contributing to its chemical enrichment



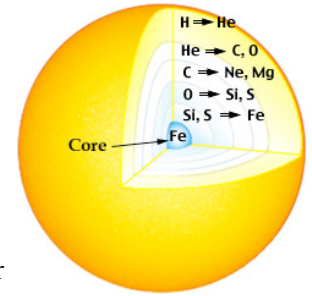
Planetary Nebulae From HST



The Future of the Sun



High-mass stars create heavy elements in their cores

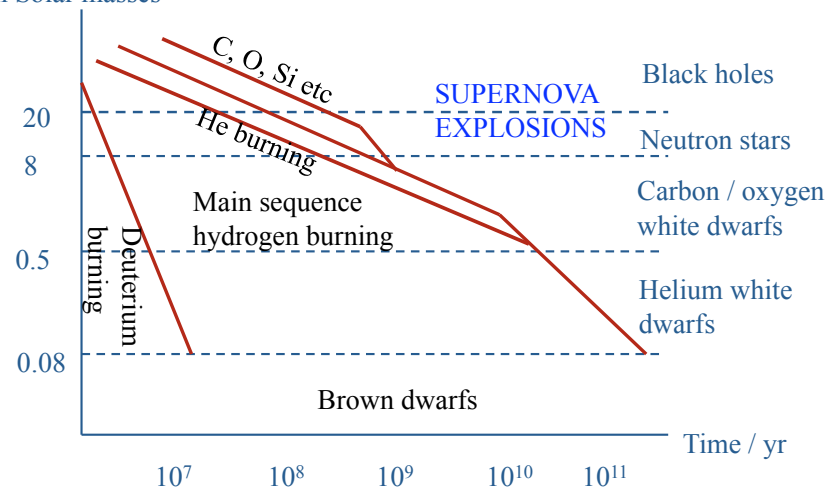


- A high mass star undergoes an extended sequence of thermonuclear reactions in its core and shells: He to C, N, O, then to Ne, Mg, then to Si, S, all the way to Fe
 - Beyond Fe, TNRs are endothermal; heavier elements are produced in SN explosions

Stage	Core temperature (K)	Core density (kg/m ³)	Duration of stage
Hydrogen fusion	4×10^7	5×10^3	7×10^6 years
Helium fusion	2×10^8	7×10^5	7×10^5 years
Carbon fusion	6×10^8	2×10^8	600 years
Neon fusion	1.2×10^9	4×10^9	1 year
Oxygen fusion	1.5×10^9	10^{10}	6 months
Silicon fusion	2.7×10^9	3×10^{10}	1 day
Core collapse	5.4×10^9	3×10^{12}	¼ second
Core bounce	2.3×10^{10}	4×10^{15}	milliseconds
Explosive (supernova)	about 10^9	varies	10 seconds

Stellar Lifecycle Summary

Initial stellar mass in Solar masses

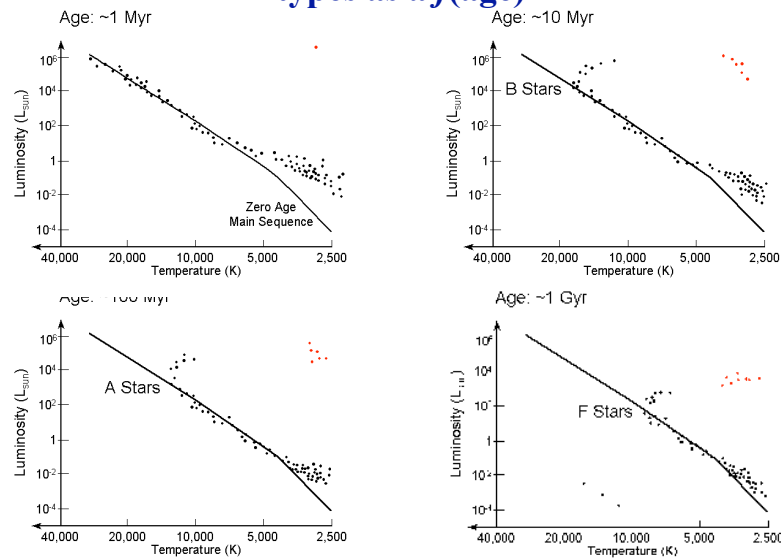


(From P. Armitage)

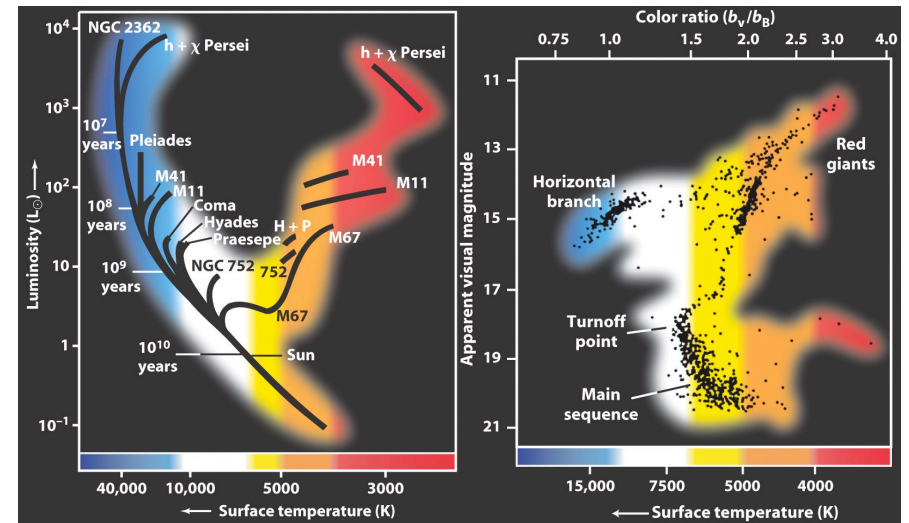
Testing Stellar Evolution

- The problem: stellar evolution happens on Gigayear time scales
- The solution: use H-R Diagrams of star clusters with a wide range of ages and stellar masses
 - Open clusters contain $\sim 10^2 - 10^4$ stars with a broad mass range; globular clusters contain $\sim 10^4 - 10^6$ stars
 - All stars are at the same distance, so it is easy to measure their relative luminosities
 - They (generally) have the same age, have the same chemical composition, i.e., they are "simple stellar populations"
- Each cluster thus provides a snapshot of what stars of different masses look like at the same age and composition (coeval populations)
- Lines describing stellar population loci at a given age are called isochrones; they are metallicity-dependent

The RGB peels off from the MS at different spectral types as a $f(\text{age})$



HRDs of open clusters with a range of ages



Stellar Masses

- The most important physical property of stars: determines everything else (L , T , evolution ...)
- Mass-luminosity relation is a key concept
- It is only from binary stars that we can accurately determine the masses of individual stars***
- A key uncertainty is the orbit inclination
- Eclipsing binaries are the most useful in that the masses and radii of the individual stars can be determined from the light curve and radial velocity data, using the Kepler's laws:

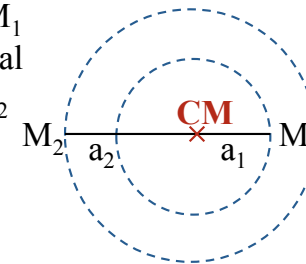
$$M_1 + M_2 = \text{const.} \times a^3 / P^2$$

(Note: exactly the same approach is used in radial velocity searches for extrasolar planets)

Stellar Masses From Binaries

Kepler's Law: consider for simplicity circular orbits

Stars mass: M_1 and M_2 , orbital radii a_1 and a_2



In orbit around the center of mass (CM) of the system

From definition of center of mass: $M_1 a_1 = M_2 a_2$

Let total separation: $a = a_1 + a_2$


Then:
$$a_2 = \frac{M_1}{M_1 + M_2} a$$

(From P. Armitage)

Apply Newton's law of gravity and condition for circular motion to M_2 : $\frac{GM_1M_2}{a^2} = M_2a_2\Omega^2$ Ω is angular velocity of the binary

Substitute for a_2 : $\Omega = \sqrt{\frac{G(M_1 + M_2)}{a^3}}$
 $P = \frac{2\pi}{\Omega}$

Visual binary: see each orbit so know immediately a_2 / a_1 :

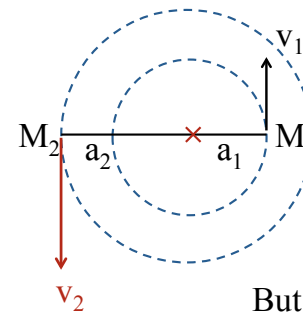
 determines *ratio* of masses M_1 / M_2

If we know distance, then angular separation + d gives a , which with period P determines *sum* of masses $M_1 + M_2$

(From P. Armitage)

So this is enough information to get both M_1 and M_2 ...

Now consider spectroscopic binaries with circular orbits (often a good approximation because tides in close binaries tend to circularize the orbits)



Velocities are constant around the orbit:

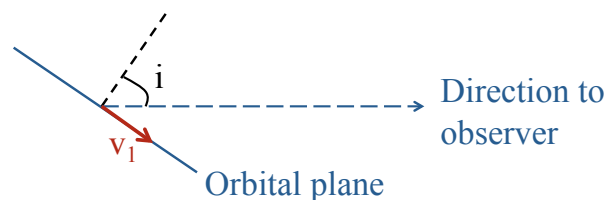
$$Pv_1 = 2\pi a_1$$

$$Pv_2 = 2\pi a_2$$

But alas, there are projection effects...

(From P. Armitage)

We don't observe v_1 and v_2 - only the component of those velocities along our line of sight:



Maximum component of velocity along the line of sight is:

$$v_{r1} = v_1 \cos(90 - i) = v_1 \sin i$$

$$v_{r2} = v_2 \sin i$$

\uparrow
 Radial velocities are the observables i is the inclination angle of the binary system

(From P. Armitage)


Ratio of maximum observed radial velocities is:

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2 \sin i}{v_1 \sin i} = \frac{2\pi a_2 / P}{2\pi a_1 / P} = \frac{a_2}{a_1} = \frac{M_1}{M_2}$$

Ratio of masses can be found if we see spectral lines from both stars (a "double-lined" spectroscopic binary), without knowing the inclination. To find the sum of the masses, note:

$$a = a_1 + a_2 = \frac{P}{2\pi} (v_1 + v_2)$$

Use Kepler's law again: $P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)} = \frac{P^3 (v_1 + v_2)^3}{2\pi G (M_1 + M_2)}$

 $M_1 + M_2 = \frac{P}{2\pi G} (v_1 + v_2)^3$

(From P. Armitage)

Replace v_1 and v_2 with the observable radial velocities:

$$M_1 + M_2 = \frac{P}{2\pi G} \frac{(v_{r1} + v_{r2})^3}{\sin^3 i}$$

So... we can determine sum of masses (and hence the Individual masses M_1 and M_2) **only** if the inclination i can be determined.

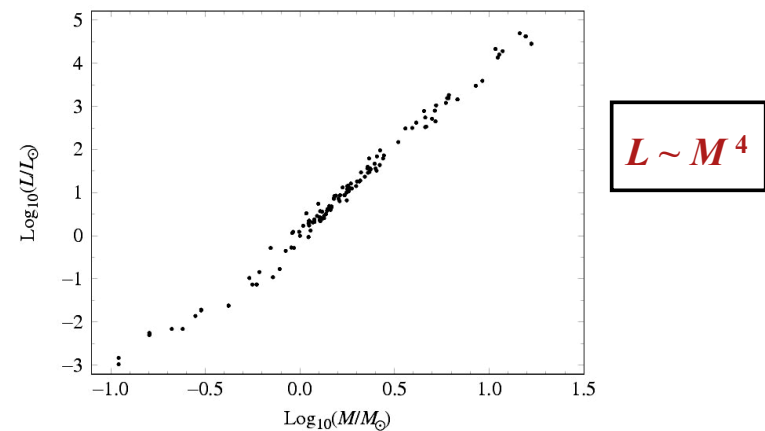
Requires that the stars are also eclipsing:

- Detailed shape of lightcurve gives i
- Obviously must be close to $i = 90^\circ$ to see eclipses!

Rare binaries are main source of information on stellar masses...

(From P. Armitage)

Stellar Mass-Luminosity Relation



- A key relation for understanding of stellar physics
- Main Sequence stars follow this relation, but giants, supergiants, & white dwarfs do not (different slopes)

Stellar Luminosity and Mass Functions

- Basic statistical descriptors of stellar populations: probability distribution for stellar luminosities (a function of the bandpass) and masses
- Most important: stellar **Initial Mass Function (IMF)** = mass function at the formation time
- Key in understanding and modeling star formation and galaxy evolution
- Needed in order to estimate stellar (baryonic?) mass content of galaxies (and other stellar systems) from the observed luminosities
- Very, very hard to do - depends on having lots of very reliable, well-understood data and calibrations

Determining the IMF is a tricky business...

- Observed star counts
 - Understand your selection effects, completeness
 - Get the distances
 - Estimate the extinction
 - Correct for unresolved binaries
- Get the Present-Day Luminosity Function (PDLF)
 - Assume the appropriate mass-luminosity relation
 - It is a function of metallicity, bandpass, ...
 - Theoretical models tested by observations
- Get the Present-Day Mass Function (PDMF)
 - Assume some evolutionary tracks, correct for the evolved stars (also a function of metallicity, ...)
 - Assume some star formation history
- Get the Initial Mass Function (IMF)!

Luminosity Functions

Suppose we measure the distances and apparent magnitudes m of all stars within some limiting distance d_{max} (a “volume limited sample” - easier in theory than in practice!)

Convert from apparent magnitudes to absolute magnitudes M using the known distance d to each star and the definition:

$$M = m - 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$$

↑
absolute magnitude in some waveband, e.g. in the visual M_V

Finally count the number of stars with M between $(M-0.5)$ and $(M+0.5)$, and divide by the volume $\frac{4}{3}\pi d_{max}^3$ surveyed. This gives the **luminosity function**

(From P. Armitage)

More formally:

$$\Phi(M)\Delta M = \text{number density (stars per pc}^3\text{) of stars with absolute magnitude } M \text{ between } M \text{ and } M+\Delta M$$

↑
luminosity function

Identical concept applies to galaxies (though typically measure numbers of galaxies per Mpc^3 rather than per pc^3)

Can be hard to measure $\Phi(M)$:

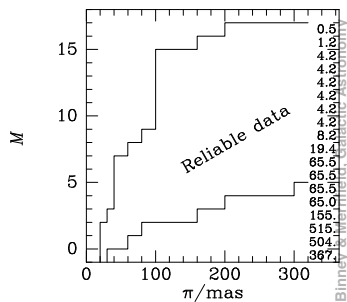
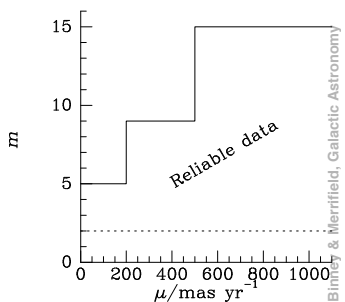
- for very low mass stars (M large), which are dim unless very close to the Sun
- for massive stars (M small), which are rare

Luminosity function is the basic observable for studying a *population* of stars

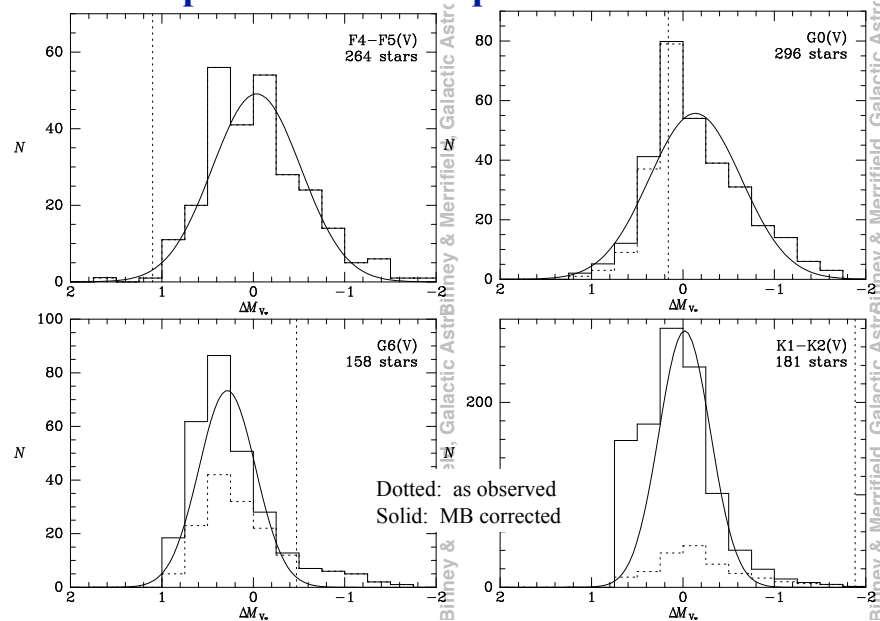
(From P. Armitage)

Distances to Large Samples of Stars

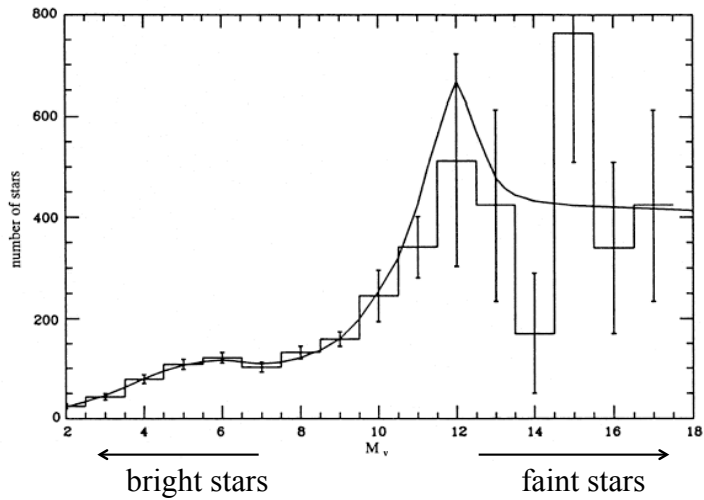
- In any survey volume, fainter objects will be missing at the progressively larger distances: the **Malmquist Bias**
- Errors in parallax measurements will bring some stars “closer” than they really are, but some stars in the other direction, so they will be missing from the sample; this asymmetry is the **Lutz-Kelker Bias**
- One way to select samples of nearby stars is through proper motions, which are much easier to measure than parallaxes



Examples of the Malmquist Bias Corrections

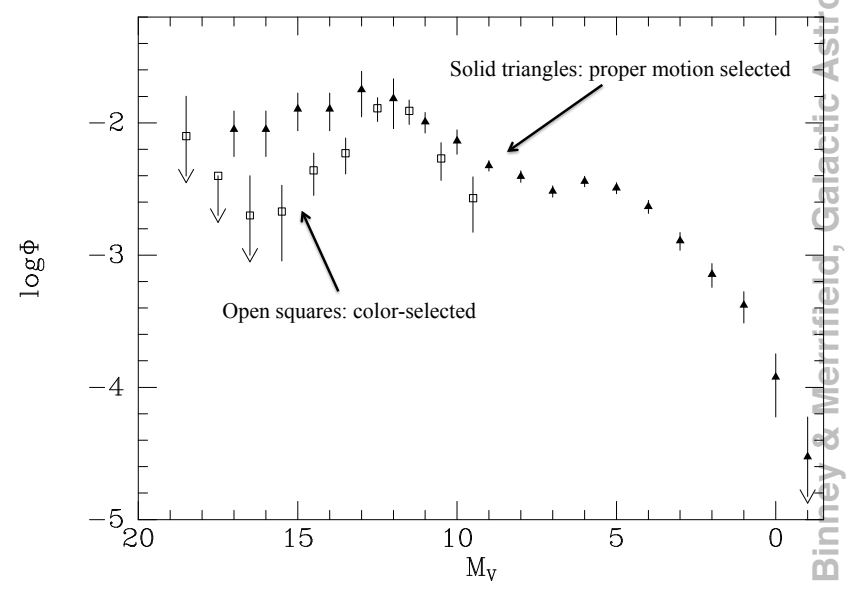


Local luminosity function (stars with $d < 20$ pc) for the Milky Way measured by Kroupa, Tout & Gilmore (1993):



(From P. Armitage)

Local V Band Luminosity Function



Binney & Merrifield, Galactic Ast...

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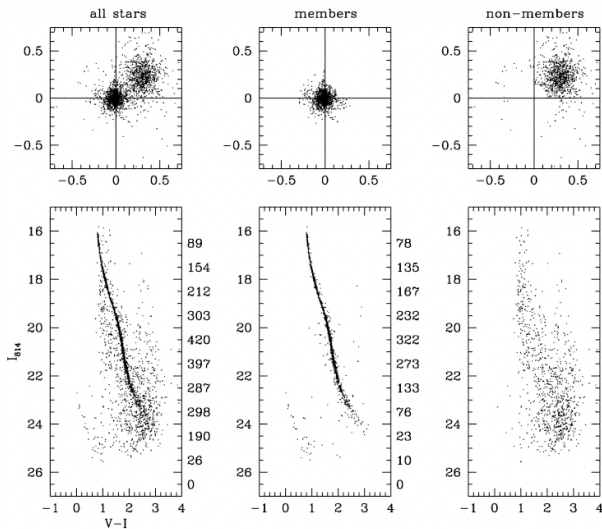


Fig. 1.— Proper-motion distributions (top) and color-magnitude diagrams (bottom). The scale of the proper motions is displacement in Wide Field Camera (WFC) pixels over the 32 month time baseline; a full WFC pixel of displacement would correspond to 37.5 mas yr⁻¹. Since all reference stars were cluster members, the zero point of motion is the mean motion of cluster stars. Left: The entire sample; center: stars within the proper-motion region described in the text; right: stars outside this region. Numbers at right are stars per unit-magnitude bin.

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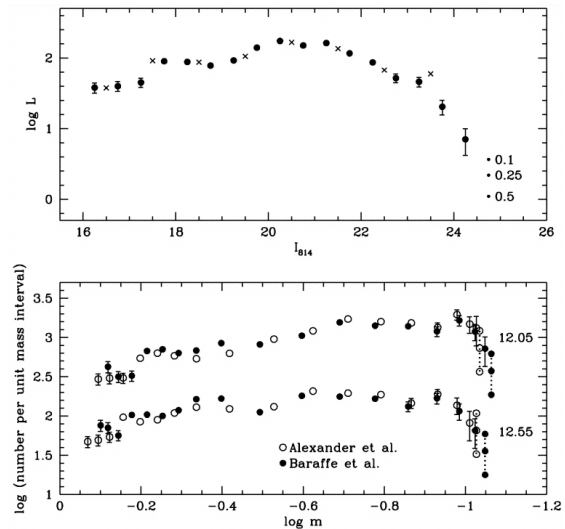


Fig. 4.— Top: Our new luminosity function of NGC 6397, with Poisson error bars (plotted only when they are larger than the sizes of the symbols). The vertical array of three small circles is explained in the text. The crosses are the LF given by CPK, converted to the present field size. Bottom: Mass functions, as derived from each of the two MLRs indicated. For clarity, the sets of three points representing the empty bin have been connected with lines. The error bars arise from those of the log LF points. The MFs are shown for two different assumed distance moduli, as labeled.

Initial Mass Function (IMF)

Starting from the observed luminosity function, possible to derive an estimate for the Initial Mass Function (IMF). To define the IMF, imagine that we form a large number of stars.

Then:

$$\xi(M)\Delta M = \text{the number of stars that have been born with initial masses between } M \text{ and } M+\Delta M \text{ (careful not to confuse mass and absolute magnitude here)}$$

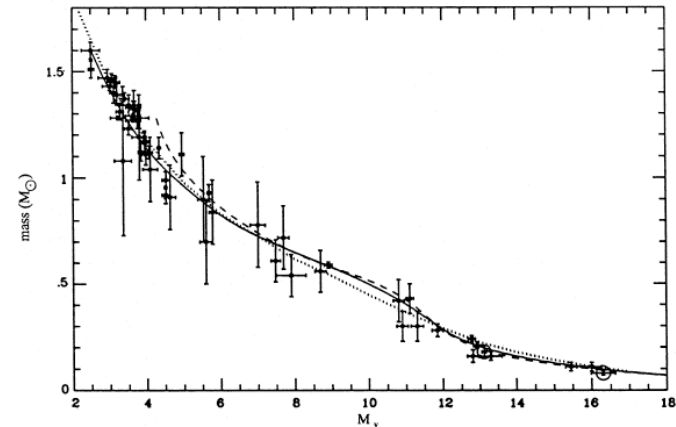
this is the Initial Mass Function or IMF

The IMF is a more fundamental theoretical quantity which is obviously related to the star formation process. Note that the IMF only gives the **initial** distribution of stellar masses immediately after stars have formed - it is **not** the mass distribution in, say, the Galactic disk today (PDMF)

(From P. Armitage)

In practice: several obstacles to getting the IMF from the LF:

1. Convert from absolute magnitude to mass

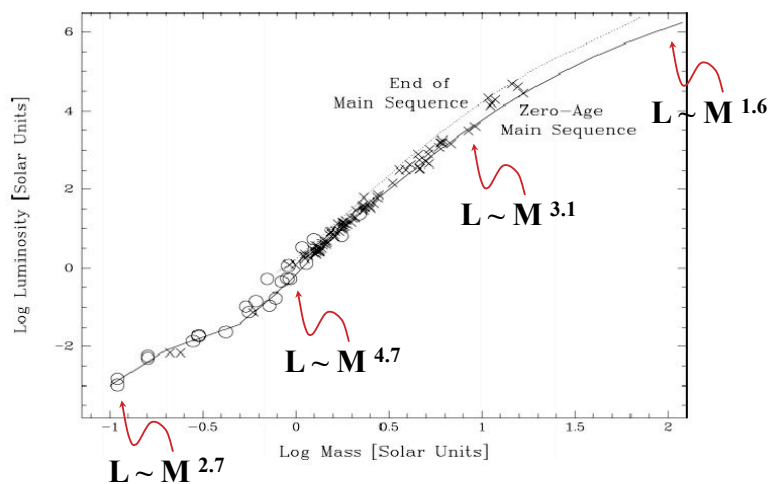


Need stellar structure theory, calibrated by observations of eclipsing binaries

(From P. Armitage)

The Mass-Luminosity Relation

Use it to convert stellar luminosities into masses:



And, of course, it is a function of bandpass ...

A problem: Massive stars have short lifetimes

Suppose we observe the luminosity function of an old cluster. There are **no** very luminous main sequence stars. But this does not mean that the IMF of the cluster had zero massive stars, only that such stars have ended their main sequence lifetimes

More generally, we need to allow for the differing lifetimes of different stars in deriving the IMF. If we assume that the star formation rate in the disk has been constant with time, means we need to weight number of massive stars by $1/t_{ms}$, where t_{ms} is the main sequence lifetime.

Massive stars are doubly rare - few are formed plus they don't live as long as low mass stars... Thus, poor statistics

(From P. Armitage)

Another problem at the high mass end: Mass loss

For massive stars, mass loss in stellar winds means that the present mass is smaller than the initial mass

A problem at the low mass end: Completeness

These faint stars cannot be seen very far, and are easy to miss

* * *

All these difficulties mean that although the local IMF is well determined for masses between $\sim 0.5 M_{sun}$ and $\sim 50 M_{sun}$, but:

- not well determined at the very low mass end (mainly because the relation between luminosity and mass is not so well known)
- for very massive stars - simply too rare
- in other galaxies, especially in the distant Universe

(From P. Armitage)

Salpeter Mass Function

The Initial Mass Function for stars in the Solar neighborhood was determined by Salpeter in 1955:

$$\xi(M) = \xi_0 M^{-2.35}$$

constant which sets the local stellar density

Using the definition of the IMF, the number of stars that form with masses between M and $M+\Delta M$ is: $\xi(M)\Delta M$

To determine the total number of stars formed between M_1 and M_2 , integrate the IMF between these limits:

$$N = \int_{M_1}^{M_2} \xi(M) dM = \xi_0 \int_{M_1}^{M_2} M^{-2.35} dM$$
$$= \xi_0 \left[\frac{M^{-1.35}}{-1.35} \right]_{M_1}^{M_2} = \frac{\xi_0}{1.35} [M_1^{-1.35} - M_2^{-1.35}]$$

(From P. Armitage)

Can similarly work out the total **mass** in stars born with mass $M_1 < M < M_2$:

$$M_* = \int_{M_1}^{M_2} M \xi(M) dM$$

Properties of the Salpeter IMF:

- most of the stars (by number) are low mass stars
- most of the **mass** in stars resides in low mass stars
- following a burst of star formation, most of the **luminosity** comes from high mass stars

Salpeter IMF must fail at low masses, since if we extrapolate to arbitrarily low masses the total mass in stars tends to infinity!

Observations suggest that the Salpeter form is valid for roughly $M > 0.5 M_{sun}$, and that the IMF flattens at lower masses. The exact form of the low mass IMF remains uncertain

(From P. Armitage)

What is the origin of the IMF?

Most important unsolved problem in star formation. Many theories but no consensus

Observationally, known that dense cores in molecular clouds have a power-law mass function rather similar to the IMF. So the IMF may be determined in part by how such cores form from turbulent molecular gas

Is the IMF universal?

Most theorists say no. Predict that fragmentation is easier if the gas can cool, so primordial gas without any metals should form more massive stars (Pop III)

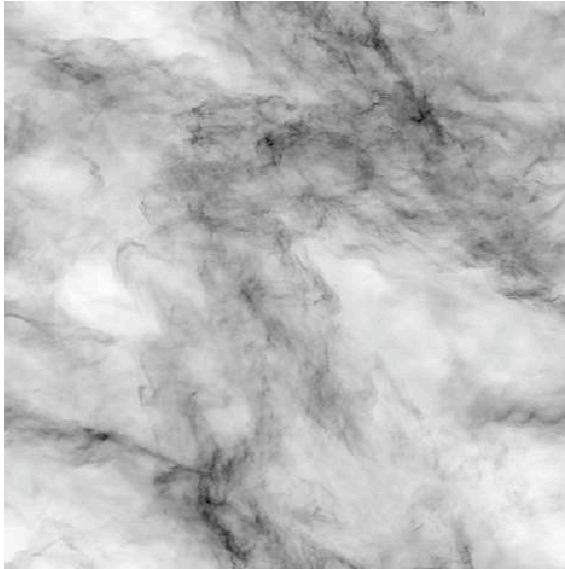
Observationally, little or no evidence for variations in the IMF in our Galaxy or nearby galaxies, but it is not excluded

(From P. Armitage)

The Physical Origin of the IMF

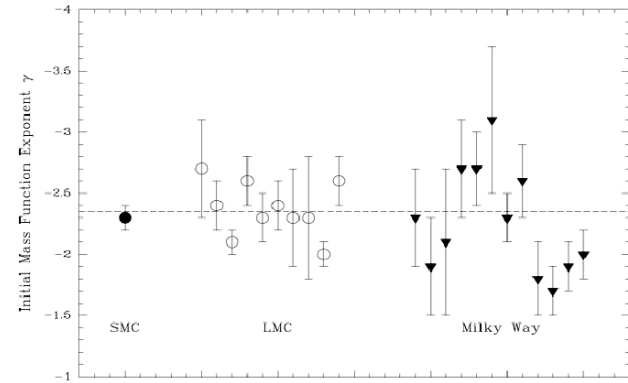
Not yet well understood ...

Interstellar turbulence
 ↓
 Power-law scalings
 ↓
 Protostellar cloud fragmentation spectrum
 ↓
 Power-law IMF?



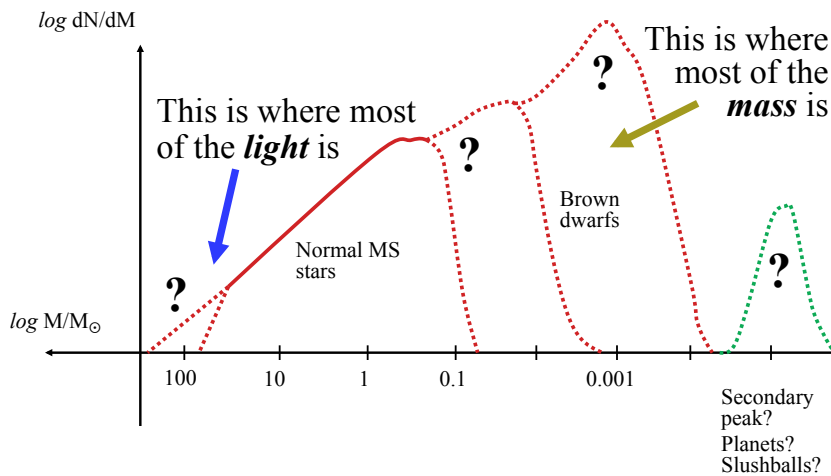
Numerical simulation of ISM turbulence →

The Universality of the IMF?



Looks fairly universal in the normal, star-forming galaxies nearby. However, it might be different (top-heavy?) in the ultraluminous starbursts (if so, the inferred star formation rates would be wrong). It might be also a function of metallicity: top-heavy for the metal-poor systems (including the primordial star formation)

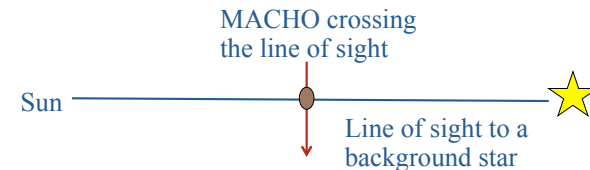
Where are the Baryons?



Gravitational Microlensing

Lensing event occurs as a Massive Compact (Halo) Object, MACHO (could be a main sequence star, white or brown dwarf, neutron star or black hole, or ... ?), passes within an angular distance θ_E of a background star:

- background star initially brightens
- eventually fades as the alignment is lost



Since the cross section for a strong lensing is small compared to interstellar separations, such events must be exceedingly rare

Expected Gravitational Microlensing Lightcurves:

The **peak magnification** depends on the lens alignment (impact parameter)

The **event duration** depends on the lens velocity

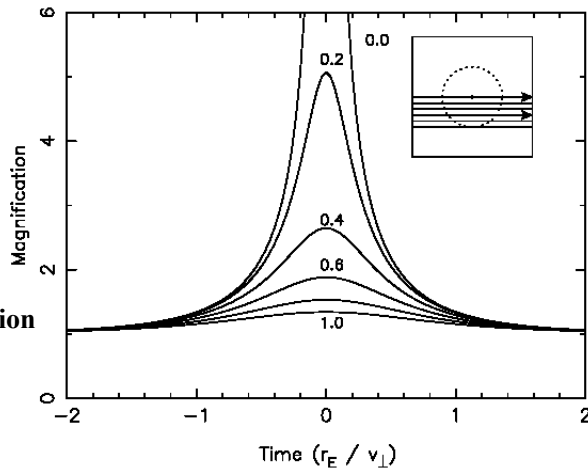
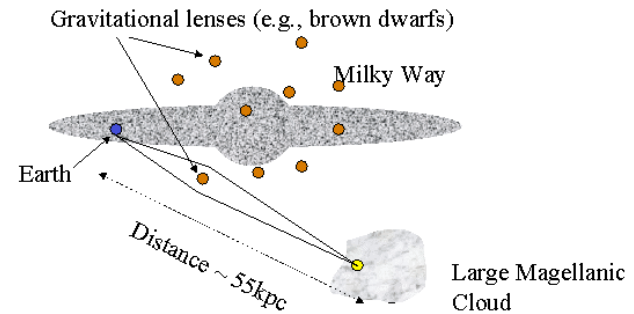


Figure 2. Microlensing event lightcurves (magnification versus time) for six values of the impact parameter $u_{\min} = 0.0, 0.2, \dots, 1.0$ as labelled. Time is in units of the Einstein radius crossing time r_E/v_{\perp} . The inset illustrates the Einstein ring (dotted circle) and the source paths relative to the lens (dot) for the six curves.

How Can We Detect MACHOs ?

- Problem: a probability of a distant star being lensed is maybe $\sim 10^{-7}$ per year
- Solution: monitor $\sim 10^7$ stars simultaneously!
Typically in the LMC or the Galactic Bulge



Microlensing Experiments



Several experiments have searched for microlensing events:

- toward the Galactic Bulge (lenses are disk or bulge stars)
- toward the Magellanic Clouds (lenses could be stars in the LMC / SMC, or halo objects)

MACHO (Massive Compact Halo Object):

- observed 11.9 million stars in the Large Magellanic Cloud for a total of 5.7 years

OGLE (Optical Gravitational Lensing Experiment):

- ongoing experiment
- presently monitor ~ 33 millions stars in the LMC, plus ~ 170 million stars in the Galactic Bulge

The First MACHO Event Seen in the LMC Experiment →

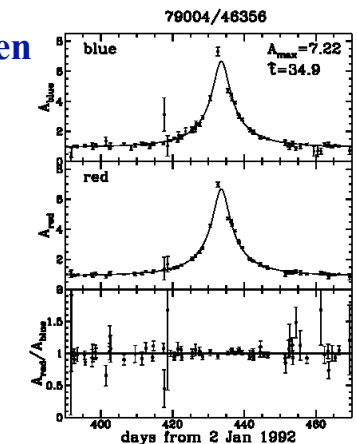
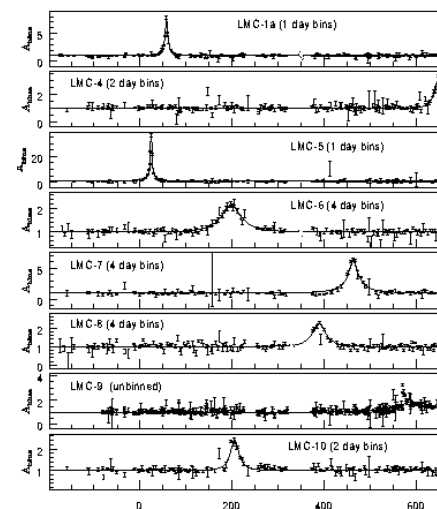


Figure 3. The first LMC microlensing candidate from the MACHO project. (Expanded view: 6 yr of constant data are

To date, hundreds (or more) of microlensing events have been detected by various groups

The Einstein radius for a single lens of mass M , at distance d_L , observer-source distance is d_S , lens-source distance is $d_{LS} = d_S - d_L$

$$\theta_E = \frac{2}{c} \sqrt{\frac{GMd_{LS}}{d_L d_S}}$$

Probability that this lens will magnify a given source is:

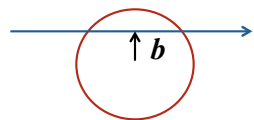
$$P \propto \theta_E^2 \propto \left(\frac{d_{LS}}{d_L d_S} \right) \times M \quad \text{directly proportional to the mass of the lens}$$

Same is obviously true for a population of lenses, with total mass M_{pop} - just add up the individual probabilities. Conclude:

- The fraction of stars that are being lensed at any one time measures the **total mass** in lenses, independent of their individual masses
- Geometric factors remain - we need to know **where** the lenses are to get the right mass estimate

For each event, there are only two observables:

- Duration τ - if we know the location of the lens along the line of sight this gives the lens mass directly
- Peak amplification A : this is related to how close the line of sight passes to the center of the Einstein ring



$$\text{Define } u = \frac{b}{d_L \theta_E}$$

$$A = \frac{u^2 + 2}{u \sqrt{u^2 + 4}}$$

Note: amplification tells us nothing useful about the lens!

Additionally, observing many events gives an estimate of the probability that a given source star will be lensed at any one time (often called the *optical depth to microlensing*). This measures the **total mass** of all the lenses, if their location is known.

Lensing time scale: equals the *physical* distance across the Einstein ring divided by the relative velocity of the lens:

$$\tau = \frac{2d_L \theta_E}{v_L}$$

$$\tau = \frac{4}{v_L c} \sqrt{\frac{GMd_L d_{LS}}{d_S}}$$

Time scale is proportional to the square root of the individual lens masses

Put in numbers appropriate for disk stars lensing stars in the Galactic bulge:

- $d_S = 8$ kpc, $d_L = d_{LS} = 4$ kpc
- $M = 0.3 M_\odot$
- $v_L = 200$ km s⁻¹

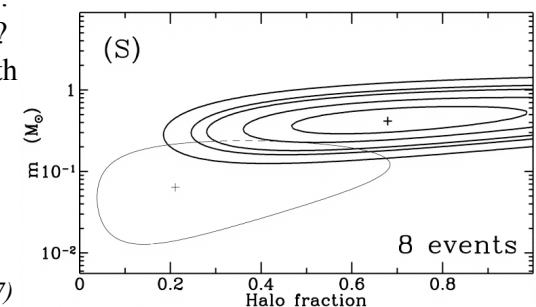
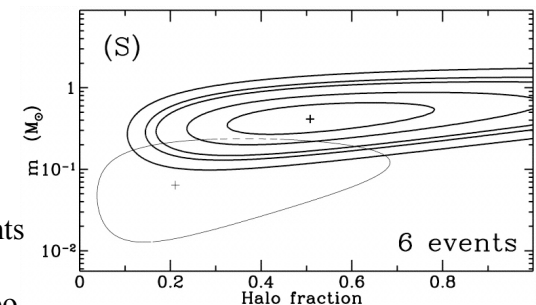
$$\rightarrow \tau \approx 40 \sqrt{\frac{M}{0.3 M_{sun}}} \text{ days}$$

Events with $\tau \sim 1$ day: $M < \text{Jupiter mass}$ ($\sim 10^{-3} M_\odot$)

Events with $\tau \sim 1$ year: $M \sim 25 M_\odot$ (e.g. stellar black holes)

What Are MACHOs?

Analysis of the LMC microlensing experiments suggest that MACHO masses are $\sim 0.5 M_\odot$: too heavy for brown dwarfs. Old halo white dwarfs?? (There are problems with that...)



(Alcock et al. 1997)

MACHO Results

Based on the number and duration of MACHO events, *if the lenses are objects in the Galactic Halo:*

- 20% of the mass of the Galactic halo (inferred from the Galactic rotation curve) is in the form of MACHOs; the idea that *all* the mass in the halo is MACHOs is definitely ruled out
- Typical mass is between $0.15 M_{\odot}$ and $0.9 M_{\odot}$

One interpretation of these results is that the halo contains a much larger population of white dwarf stars than suspected. This poses other problems: requires a major epoch of early star formation to generate these white dwarfs - but what about the corresponding metals?

Ambiguity in the distance to the lenses is the main problem!

Distance ambiguity can be resolved in a few special cases:

- a) If distortions to the light curve caused by the motion of the Earth around the Sun can be detected (parallax events)
- b) If the lens is part of a binary system. Light curves produced by binary lenses are much more complicated, but often contain sharp spikes (caustic crossings) and multiple maxima.

—————→
This provides more information about the event. (This one was close to the SMC)

