Stellar Dynamics

- **Gravity** is generally the only important force in astrophysical systems (and almost always a Newtonian approx. is OK)
- Consider astrophysical systems which can be approximated as a self-gravitating "gas" of stars (~ point masses, since in most cases $R_\star \ll$ r.m.s.): ***open and globular star clusters, galaxies, clusters of galaxies***
- If 2-body interactions of stars are important in driving the dynamical evolution, the system is called ***collisional*** (star clusters); if stars are mainly moving in the collective gravitational field, it is called ***collisionless*** (galaxies)
  - Sometimes stars actually collide, but that happens only in the densest stellar systems, and rarely at that

Dynamical Modeling of Stellar Systems

- A stellar system is fully described by an evolving phase-space density distribution function, $f(\mathbf{r},\mathbf{v},t)$
  - Unfortunately, in most cases we observe only 3 out of 6 variables: 2 positional + radial velocity; sometimes the other 2 velocity comp's (from proper motions); rarely the 3rd spatial dimension
  - … And always at a given moment of *t*. Thus we seek families of stellar systems seen at different evolutionary states
- Not all of the phase space is allowed; must conserve integrals of motion, energy and angular momentum:

$$\frac{E}{m} = \frac{1}{2}v^2 + \Phi(r) \quad \text{and} \quad \frac{\vec{J}}{m} = \vec{r} \times \vec{v}$$

- The system is finite and $f(\mathbf{r},\mathbf{v},t) \geq 0$. The boundary conditions:

$$f \rightarrow 0 \quad \text{as either} \quad r \;\text{ or }\; v \rightarrow \pm\infty$$

Dynamical Modeling of Stellar Systems

- The evolution of $f(\mathbf{r},\mathbf{v},t)$ is described by the Bolzmann eqn., but usually some *approximation* is used, e.g., Vlasov (= collisionless Boltzmann) or Fokker-Planck eqn.
  - Their derivation is beyond the scope of this class …
- Typically start by assuming $f(\mathbf{v},t)$, e.g., a Maxwellian
- Density distribution is obtained by integrating $\rho(\mathbf{r}) = \int f(\mathbf{r},\mathbf{v})\, d\mathbf{v}$
- From density distribution, use Poisson's eqn. to derive the gravitational potential, and thus the forces acting on the stars:

$$\nabla^2\Phi(r) = 4\pi G\rho \quad \text{and} \quad F = -\nabla\Phi(r)$$

- The resulting velocities must be consistent with the assumed distribution $f(\mathbf{r},\mathbf{v})$
- The system can evolve, i.e., $f(\mathbf{r},\mathbf{v},t)$, but it can be usually described as a sequence of quasi-stationary states

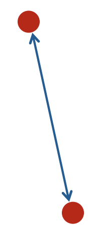
Dynamics of Stellar Systems

- The basic processes are acceleration (deflection) of stars due to encounters with other stars, or due to the collective gravitational field of the system at large
- Stellar encounters lead to dynamical ***relaxation***, wheby the system is in a thermal equilibrium. The time to reach this can be estimated as the typical star to change its energy by an amount equal to the mean energy; or the time to change its velocity vector by ~ 90 deg.
- There will be a few strong encounters, and lots of weak ones. Their effects can be estimated through Coulom-like scattering

Strong encounters

In a large stellar system, gravitational force at any point due to all the other stars is almost constant. Star traces out an orbit in the smooth potential of the whole cluster.

Orbit will be changed if the star passes very close to another star - define a strong encounter as one that leads to $\Delta \mathbf{v} \sim \mathbf{v}$.



Consider two stars, of mass m. Suppose that typically they have average speed V.

Kinetic energy: $\frac{1}{2} m V^2$

When separated by distance r, gravitational potential energy: $\frac{Gm^2}{r}$

(From P. Armitage)

By conservation of energy, we expect a large change in the (direction of) the final velocity if the change in potential energy at closest approach is as large as the initial kinetic energy:

Strong encounter

$$\frac{Gm^2}{r} \approx \frac{1}{2} m V^2 \Rightarrow r_s \equiv \frac{2Gm}{V^2}$$

↑ Strong encounter radius

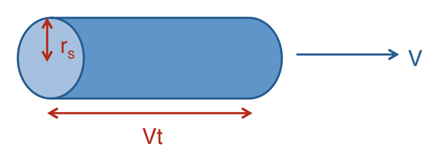
Near the Sun, stars have random velocities V ~ 30 km s^{-1}, which for a typical star of mass 0.5 M_{sun} yields r_s ~ 1 au.

Good thing for the Solar System that strong encounters are very rare…

(From P. Armitage)

Time scale for strong encounters:

In time t, a strong encounter will occur if any other star intrudes on a cylinder of radius r_s being swept out along the orbit.



Volume of cylinder: $\pi r_s^2 V t$

For a stellar density n, mean number of encounters: $\pi r_s^2 V t n$

Typical time scale between encounters:

$$t_s = \frac{1}{\pi r_s^2 V n} = \frac{V^3}{4\pi G^2 m^2 n}$$

(substituting for the strong encounter radius r_s)

Note: more important for **small** velocities.

(From P. Armitage)

Plug in numbers (being careful to note that n in the previous expression is stars per cubic cm, not cubic parsec!)

$$t_s \approx 4\times 10^{12}\left(\frac{V}{10\ \mathrm{km\ s^{-1}}}\right)^{3}\left(\frac{m}{M_{sun}}\right)^{-2}\left(\frac{n}{1\ \mathrm{pc^{-3}}}\right)^{-1}\ \mathrm{yr}$$

Conclude:

- stars in the disks of galaxies ($V \sim 30$ km s^{-1}, $n \sim 0.1$ pc^{-3} near the Sun), never physically collide, and are extremely unlikely to pass close enough to deflect their orbits substantially.

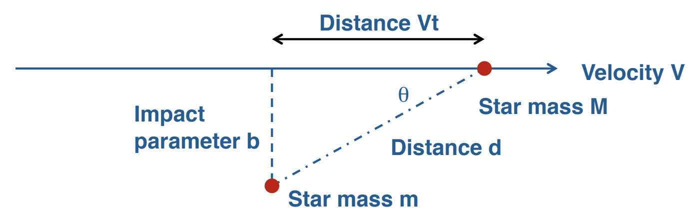
- in a globular cluster ($V \sim 10$ km s^{-1}, $n \sim 1000$ pc^{-3} or more), strong encounters will be common (i.e. one or more per star in the lifetime of the cluster).

(From P. Armitage)

Weak encounters

Stars with impact parameter $b \gg r_s$ will also perturb the orbit. Path of the star will be deflected by a very small angle by any one encounter, but cumulative effect can be large.

Because the angle of deflection is small, can approximate situation by assuming that the star follows *unperturbed* trajectory:



Define distance of closest approach to be b; define this moment to be t = 0.

(From P. Armitage)

Force on star M due to gravitational attraction of star m is:

$$F = \frac{GMm}{d^2} = \frac{GMm}{b^2 + V^2t^2} \qquad \text{(along line joining two stars)}$$

Component of the force **perpendicular** to the direction of motion of star M is:

$$F_\perp = F\sin\theta = F\times\frac{b}{d} = \frac{GMmb}{\left(b^2+V^2t^2\right)^{3/2}}$$

Using F = mass x acceleration:

$$F_\perp = M\frac{dV_\perp}{dt}$$

← Velocity component perpendicular to the original direction of motion

Integrate this equation with respect to time to get final velocity in the perpendicular direction.

Note: in this approximation, consistent to assume that $V_\parallel$ remains unchanged. Whole calculation is OK provided that the perpendicular velocity gain is *small.*

(From P. Armitage)

Final perpendicular velocity is:

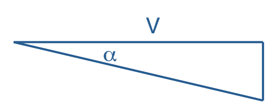
$$\Delta V_\perp = \int_{-\infty}^{\infty}\frac{dV_\perp}{dt}dt$$

$$= \frac{1}{M}\int_{-\infty}^{\infty}F_\perp(t)dt$$

$$= \frac{1}{M}\int_{-\infty}^{\infty}\frac{GMmb}{\left(b^2+V^2t^2\right)^{3/2}}dt = \frac{2Gm}{bV}$$

← As before, small V leads to larger deflection during the flyby

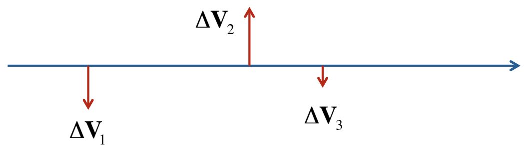
Deflection *angle* is:



$$\Delta V_\perp \qquad \alpha \approx \tan\alpha = \frac{\Delta V_\perp}{V} = \frac{2Gm}{bV^2}$$

Setting V = c, see that this is exactly half the correct relativistic value for massless particles (e.g. photons).

(From P. Armitage)



If the star receives many independent deflections, each with a random direction, expected value of the perpendicular velocity after time t is obtained by summing the *squares* of the individual velocity kicks:

$$\langle \Delta V_\perp^2 \rangle = \Delta V_1^2 + \Delta V_2^2 + \Delta V_3^2 + \ldots$$

Writing this as an integral (i.e. assuming that there are very many kicks):

$$\langle \Delta V_\perp^2 \rangle = \int_{b_{\min}}^{b_{\max}} \left(\frac{2Gm}{bV} \right)^2 dN$$

Where dN is the expected number of encounters that occur in time t between impact parameter b and b + db.

(From P. Armitage)

Using identical reasoning as for the strong encounter case:

$$dN = n \times Vt \times 2\pi b db$$

- Number density of perturbing stars (n)
- Distance star travels in time t (Vt)
- Area of the annulus between impact parameter b and b + db ($2\pi b db$)

This gives for the expected velocity:

$$\langle \Delta V_\perp^2 \rangle = \int_{b_{\min}}^{b_{\max}} nVt \left(\frac{2Gm}{bV} \right)^2 2\pi b db$$

$$= \frac{8\pi G^2 m^2 nt}{V} \int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

$$= \frac{8\pi G^2 m^2 nt}{V} \ln\left[\frac{b_{\max}}{b_{\min}} \right]$$

Logarithm means in a uniform density stellar system, \`encounters' with stars at distances (b -> 10b) and (10b -> 100b) etc contribute equally to the deflection.

(From P. Armitage)

Relaxation time

After a long enough time, the star's perpendicular speed will (on average) grow to equal its original speed. Define this as the *relaxation time* - time required for the star to lose all memory of its initial orbit.

Set: $$V^2 = \langle \Delta V_\perp^2 \rangle = \frac{8\pi G^2 m^2 n t_{\text{relax}}}{V} \ln\left[\frac{b_{\max}}{b_{\min}} \right]$$

…and solve for t_{relax}: $$t_{\text{relax}} = \frac{V^3}{8\pi G^2 m^2 n \ln[b_{\max}/b_{\min}]}$$

Recall that the **strong** encounter time scale was: $$t_s = \frac{V^3}{4\pi G^2 m^2 n}$$

$$t_{\text{relax}} = \frac{t_s}{2\ln[b_{\max}/b_{\min}]}$$

Frequent distant interactions are more effective at changing the orbit than rare close encounters…

(From P. Armitage)

Relaxation time for a stellar cluster

The factor $\ln[b_{max} / b_{min}]$ depends upon the limits of integration. Usually take:

- $\mathbf{b_{min}}$ to be the strong encounter radius r_s (~1 au for the Sun). Approximations made in deriving the relaxation time are definitely invalid for $r < r_s$.
- $\mathbf{b_{max}}$ to be the characteristic size of the whole stellar system - for the Sun would be reasonable to adopt either the thickness of the disk (300 pc) or the size of the galaxy (30 kpc).

$$\ln[b_{\max}/b_{\min}] = 18 - 23$$

Because the dependence is only logarithmic, getting the limits exactly right isn't critical.

(From P. Armitage)

$$t_{\text{relax}} \approx \frac{2\times10^{12}}{\ln[b_{\max}/b_{\min}]}\left(\frac{V}{10\text{ km s}^{-1}}\right)^{3}\left(\frac{m}{M_{sun}}\right)^{-2}\left(\frac{n}{1\text{ pc}^{-3}}\right)^{-1}\text{ yr}$$

Evaluate the relaxation time for different conditions:

	Sun	Globular cluster	Open cluster
V / km s^{-1}	30	10	1
n / pc^{-3}	0.1	10^4	10
size / pc	1000	5	5
	3×10^{13} yr	~100 Myr	~100 Myr

Predict that clusters ought to evolve due to star-star interactions during the lifetime of the Galaxy.

(From P. Armitage)

Can use the virial theorem to write this result in an alternate form:

$$2\langle \text{KE}\rangle + \langle \text{PE}\rangle = 0$$

Average value of the kinetic energy

Average value of gravitational potential energy

For a cluster of N stars, each of mass m, moving at average velocity V in a system of size R:

- total mass M = Nm
- total kinetic energy: $\frac{1}{2}NmV^2$
- gravitational potential energy: $\sim \frac{GM^2}{R} = \frac{G(Nm)^2}{R}$

Applying the virial theorem: $V = \sqrt{\frac{GNm}{R}}$

(From P. Armitage)

Number density of stars = number of stars / volume:

$$n = \frac{N}{\frac{4}{3}\pi R^3}$$

Range of radii over which weak interactions can occur is:

$$\frac{b_{\max}}{b_{\min}} = \frac{R}{r_s} = \frac{RV^2}{2Gm} = \frac{R}{2Gm}\times\frac{GNm}{R} = \frac{N}{2}$$

Finally define the **crossing time** for a star in the cluster:

$$t_{\text{cross}} = \frac{R}{V}$$

(From P. Armitage)

Ratio of the relaxation time to the crossing time is:

$$\frac{t_{\text{relax}}}{t_{\text{cross}}} = \frac{\dfrac{V^3}{8\pi G^2 m^2 n \ln[b_{\max}/b_{\min}]}}{\dfrac{R}{V}} = \frac{V^4}{8\pi G^2 m^2 n R \ln[b_{\max}/b_{\min}]}$$

Substitute for V, n and $[b_{\max} / b_{\min}]$ and this simplifies to:

$$\frac{t_{\text{relax}}}{t_{\text{cross}}} = \frac{N}{6\ln[N/2]}$$

In a cluster, number of orbits a star makes before it is significantly perturbed by other stars depends only on the number of stars in the system.

Interactions are negligible for galaxy size systems, but very important for small clusters.

(From P. Armitage)

Consequences of relaxation

Evaporation: two-body relaxation allows stars to exchange energy amongst themselves. If at some moment a star becomes unbound (kinetic + potential energy > 0) then it will escape the cluster entirely.

Evaporation time $t_{evap} \sim 100\ t_{relax}$, and although long, it limits the cluster lifetime.

Evaporation is accelerated by **tidal shocks**, which implant additional kinetic energy to the stars:

For globular clusters, passages through the Galactic disk or bulge

For open clusters, passages of nearby giant molecular clouds (GMCs) or spiral density waves

(From P. Armitage)

Mass segregation: two-body relaxation tries to equalize the **kinetic energy** of different mass stars, rather than their velocity. Since:

$$\mathrm{KE} = \frac{1}{2} m V^2$$

…more massive stars tend to have smaller velocities and sink to the center of the cluster.

Core collapse: stars in the cluster core tend to have higher velocities. If they attempt to equalize kinetic energy with stars outside the core, they lose energy, and sink even further toward the center.

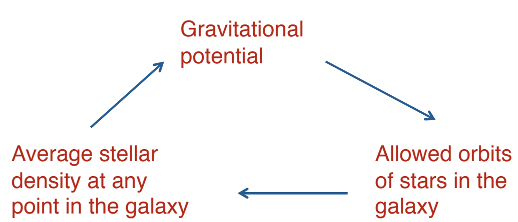
Limit of this process is called *core collapse* - eventually contraction is probably halted by injection of energy from binary stars.

(From P. Armitage)

Analysis suggests that large-N, roughly spherical systems are stable, long lived structures (elliptical galaxies, the bulges of spiral galaxies).

Does **not** mean that anything goes as far as galaxy shapes:

- most obviously, need to have consistency between the mean stellar density and the gravitational force:



- also more subtle issues - e.g. if the potential of the galaxy admits *chaotic* orbits, then even small perturbations can shift stars into qualitatively different orbits.

(From P. Armitage)

Dynamical Friction

Why does the orbit of a satellite galaxy moving within the halo of another galaxy decay?

Stars in one galaxy are **scattered** by gravitational perturbation of passing galaxy.

Stellar distribution around the intruder galaxy becomes asymmetric - higher stellar density downstream than upstream.

Gravitational force from stars produces a `frictional' force which slows the orbital motion.

(From P. Armitage)

Dynamical Friction

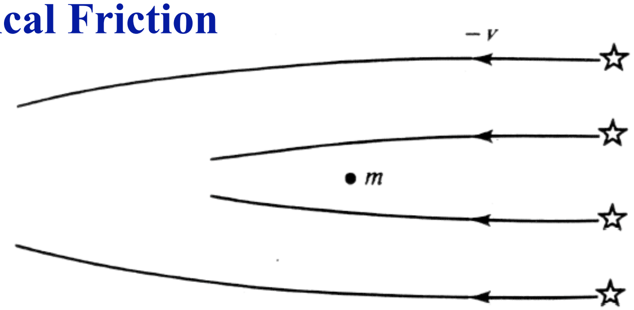
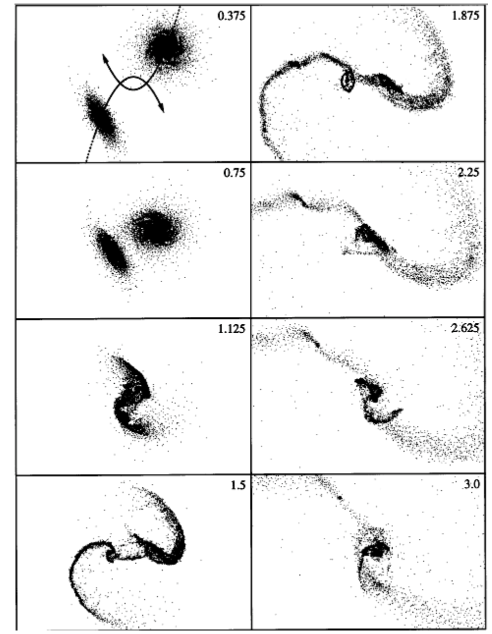


Figure 14.14. Dynamical friction arises when a mass m moves at velocity $\mathbf{v}$ relative to a distribution of stars which statistically have no mean motion. For simplicity, we have depicted the stars to be at rest (relative to the center of their galaxy), and we have drawn the situation as it appears to an observer at rest relative to m. Thus, mass m sees stars approach at velocity $-\mathbf{v}$, and be deflected by the gravity of m. This deflection produces a slight excess of mass behind M, since the stars will on the average be closer together after deflection than before. This mass excess pulls m in the direction of $-\mathbf{v}$, producing a net drag which tends to decrease $\mathbf{v}$.

(From F. Shu)

Numerical Simulation of Merging Disk Galaxies



(From Barnes & Hernquist)

Major Galaxy Mergers

- Direct consequence of dynamical friction
- Formation of tidal tails, bridges, etc.
- Stars, gas, and dark matter behave differently
- Generally lead to onset of starburst and nuclear activity

Major mergers between typical large galaxies are relatively rare, but **Minor mergers** between galaxies of very different masses are much more common.

Example: the Magellanic clouds, bound satellites orbiting within the extended halo of the Milky Way, ~50 kpc distance. Eventually will spiral in and merge into the Milky Way.

Sagittarius dwarf galaxy is another satellite which is now in process of merging…

This was much more common at high redshifts ➙ *galaxy evolution*

(From P. Armitage)

How quickly will the LMC merge with the Milky Way?

Simple estimate - dynamical friction time:

$$t_{friction} \approx \frac{V}{|dV/dt|} \approx \frac{V^3}{4\pi G^2 M n m \ln\Lambda}$$

200 km/s

~3

10^{10} Solar masses

Galactic density at LMC -
for flat rotation curve estimate
3×10^{-4} Solar masses / pc^3

With these numbers, estimate orbit will decay in ~3 Gyr
Close satellite galaxies will merge!

(From P. Armitage)