Class #9: Modeling the Evolution of Stellar Populations

Structure and Dynamics of Galaxies, Ay 124, Winter 2009

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Material in this class is taken mostly from Binney & Merrifield §5.1 & §5.4.

1 Stellar Evolution & Color Magnitude Diagram

This class is about galaxies. Unfortunately, the primary means that we have to study galaxies is by their emitted light. Since this comes primarily from stars we need to know about how stars evolve and what we can learn from their emitted light. Fortunately, stars are relatively simple systems (unless you're a stellar astrophysicist...)—for once the spherical approximation actually works!

The simplicity of stars is highlighted by the fact that they occupy only well-defined regions of the color-magnitude (CMD) diagram (see Fig. 1). The main features of the CMD can be reasonably well explained by assuming stars to be spherical, in hydrostatic equilibrium and to be powered by nuclear fusion in their cores. The main sequence corresponds to the period of nuclear hydrogen burning (p-p chain and CNO cycle), with stars along the main sequence differing in mass.

Once a star runs out of hydrogen fuel its structure if forced to change and it will move away from the main sequence. By studying the CMD we can therefore infer details of stellar evolution. In practice, this is all much more difficult: stars can have convective regions, can experience mass loss in winds, can undergo periods of pulsation and have nuclear burning rates which depend on the details of how rapidly helium sinks towards their centers (among other things). As a result, models of stellar evolution are still uncertain—they generally become more uncertain the further along in a stars lifetime they proceed (due to accumulating errors).

1.1 Modelling Stellar Colors and Magnitudes

Stellar structure models predict things such as the bolometric luminosity and effective temperature of a star. Observationally, what we can typically measure is the absolute magnitude and color of a star. To connect theory to experiment we therefore need to know the spectral energy distribution (SED) of the model stars. There are essentially two ways to do this, each with their own problems:

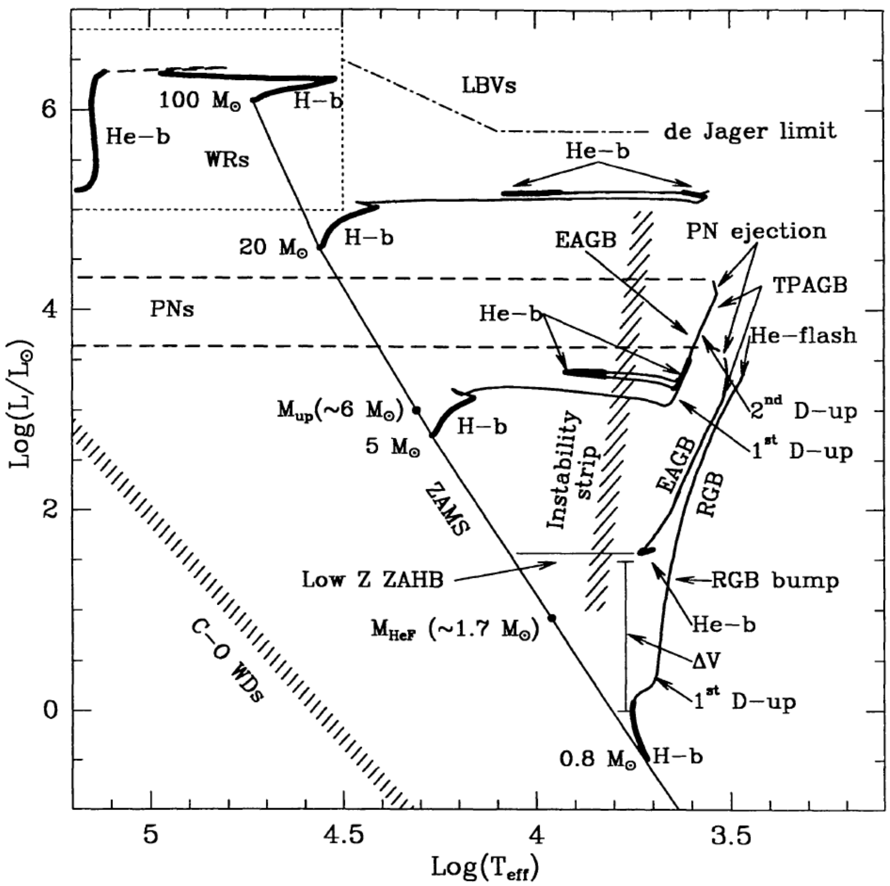

Figure 1: Main features of the color-magnitude diagram for composition $Y = 0.28$, $Z = 0.2$.

1. Create a library of observed stellar SEDs for stars of known bolometric luminosity and effective temperature. Then, for each model star, find the closest matching star (in terms of luminosity and temperature) in the library and adopt that SED. Magnitudes and colors are then found by integrating the SED through the relevant filters. The problem with this approach is that it's limited by how extensive the library is—it's difficult to find very low or very high metallicity stars near the Sun for example, so such libraries usually don't contain many such stars.

2. Write a stellar atmosphere code which handles the details of the radiative transfer to predict the SED from a given stellar structure model. Since millions of atomic and molecular lines appear in the SED this requires the use of very sophisticated codes and accurate and extensive data on these lines. Cooler stars (for which there are more lines) are more difficult to handle.

1.2 Features of the CMD

We can understand the main features of the CMD relatively easily (see Fig. 1). Qualitatively, a stars evolution is governed by the fight between the gravothermal catastrophe (in which as a star loses energy by radiating it must contract and therefore heat up, so radiates faster, and contracts faster. . .) and nuclear burning which provides an energy source to counteract the contraction. Once a star runs out of hydrogen to burn, gravity wins until the star has contracted to the point where helium burning can begin, a balance between gravity and nuclear burning once again exists. Once helium runs out, the star must contract again until C can burn and so on. The periods of equilibrium during stable nuclear burning are long-lived and so we expect stars to "pile up" in these phases, leading to high density in the corresponding region of the CMD.

Main Sequence (MS) The period of H to He burning is the longest stage of a stars life. Therefore, we expect most stars we see to be in this phase and so should see this as a high density in the CMD. Stellar structure models identify this period of stable H burning with the Main Sequence. Stellar mass varies along the MS (more massive stars at higher luminosity) and the chemical composition of stars determines the locus of the MS.

Subgiant branch (SGB) (Just below the RGB in Fig. 1.) Corresponds to stars which are just leaving the MS and are transitioning from burning H in the cores to a shell around the core. (Quite a short lived phase, so is sparsely populated and seen most clearly in globular clusters.)

Red giant branch (RGB) After moving over to H shell burning stars move onto the RGB and develop fully convective envelopes.

Horizontal Branch (HB) Once the core of star becomes hot enough to burn helium it moves onto the horizontal branch. During the helium core burning phase the luminosity of a star gradually increases so it will move slowly upwards from the HB.

Asymptotic Giant Branch (AGB) After helium is exhausted in the core, a star begins to burn He in a shell (and maybe C in the core) and moves rapidly up the AGN (which parallels the RGB but is slightly bluer). Evolution happens very rapidly from this point and significant mass loss begins to occur at a high rate. As a star approaches the tip of the AGB it begins to pulsate (due to an interplay between the He and H burning shells) which can result in a convective "dredge up" of carbon from the center of the star. Pulsation near the tip of the

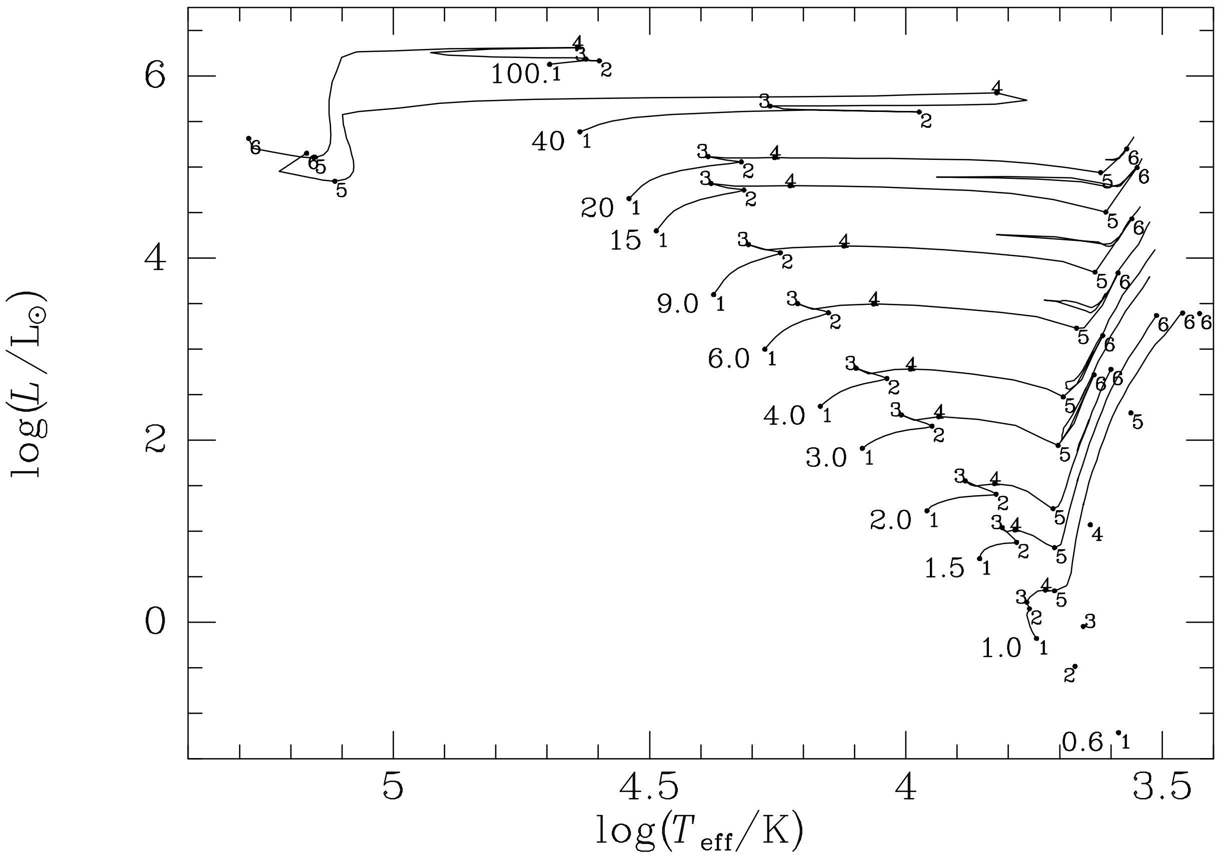

Figure 2: Evolutionary tracks for Solar metallicity stars from $0.6M_\odot$ to $100M_\odot$ as indicated.

AGB causes stars to quickly shed their outer hydrogen layers. This leaves the hot, blue core, causing the star to move rapidly to the left in the CMD. The UV light can cause the shed H to fluoresce producing a planetary nebula.

Instability Strip A region of instability due to the presence of a He^+ ionization zone (see below). Gives rise to periodically pulsating stars (RR Lyraes, Cepheids).

White Dwarf Sequence Once He burning stops the star fades and cools moving to the far left on the CMD. With no heat source, the star has collapsed and become a white dwarf, supported by degeneracy pressure (in which the gravothermal instability does not apply). It will slowly cool.

1.3 Important Initial Masses

The primary variable that affects how a star evolves is its mass (see Fig. 2). A few key masses are worth knowing about:

Brown Dwarfs An initial mass of at least $M_{\rm H} \approx 0.08M_\odot$ is required to get on to the main sequence and thereby be classed as a star. Objects less massive are never able to contract sufficiently to ignite hydrogen burning in their cores, instead reaching a point where they become supported by electron degeneracy pressure. These "brown dwarfs" can burn some light elements (such as D and Li). Jupiter is a low mass example of this class.

Convective Cores The next significant mass on the MS is $M_{\rm conv} \approx 1.1 M_\odot$. Stars more massive than this have convective cores (at least, part of the core is convective, the core becomes fully convective at around $1.5 M_\odot$). These stars are hot enough to burn via the highly temperature sensitive CNO cycle which tends to concentrate energy production in the core. Radiation cannot transport the energy out rapidly enough and so a convective core forms.

Helium Flash Stars initially less massive than $M_{\rm HeF} \approx 1.8$–2.2 (exact value depending on chemical composition) experience a "helium flash". As these stars move up the RGB their cores become degenerate. Since the pressure in degenerate matter depends only on density (and not temperature), once a temperature sufficient for helium ignition is reached the helium burning dumps heat into the core of the star. Since the pressure does not increase the core cannot expand to adjust to this new state. As a result, the temperature in the core rises rapidly, resulting in a higher rate of fusion and increased luminosity (see Fig. 3).

Supernovae Stars more massive than $M_{\rm up} \approx 8 M_\odot$ can ignite C quiescently in their cores and continue burning all the way up to Fe. Since this is the most stable nucleus further fusion can't release any more energy. As a result, the core contracts, raising the Fermi energy of the electrons and heating the core. This leads to catastrophic gravitational collapse because of a) capture of electrons by nuclei and b) fission of nuclei into α-particles by energetic photons. Both reduce the pressure in the core (by taking energy from the electron and photon fields) causing the core to contract. This speeds up those processes and runaway collapse results. The result of the core collapse is a supernovae, which leaves either a neutron star or black hole remnant.

Pair-production Instability In stars more massive than $M \approx 60 M_\odot$ Fe production never occurs. Instead, when the core is O dominated the temperature reaches $T_{\rm e} = 2 m_{\rm e} c^2 / k \approx 2 \times 10^7$K and production of $\rm e^- e^+$ pairs is possible. The rapid increase in the density of pairs with temperature implies a large specific heat of the vacuum[1] which destabilizes the star (since it results in $\gamma \equiv c_P / c_V < 4/3$ in which case the thermal energy of the star does not increase as rapidly as the gravitational energy as the star contracts). What happens next is not entirely certain but depends on the mass of the star. What's clear is that O burning will result in strong pulsations and mass loss. This may result is eventual core collapse, may entirely disrupt the star or, for stars more massive than around $300 M_\odot$ the star may collapse directly to a black hole without any pulsation.

1.4 Effects of Metallicity

While the gross structure of the CMD is the same for stars of any composition, the details do depend on the values of X, Y and Z (the mass fractions of hydrogen, helium and "metals"). For the Sun, $Y = 0.28$ and $Z = 0.02$, while for primordial stars $Y = 0.23$ and $Z = 0.0004$. Lower

[1] Specific heat capacity, $c_V \equiv |\partial U / \partial T|_V$. If $U \sim NkT$ and N is a strongly increasing function of T then c_V gets a large contribution from a ${\rm d}N/{\rm d}T$ term.

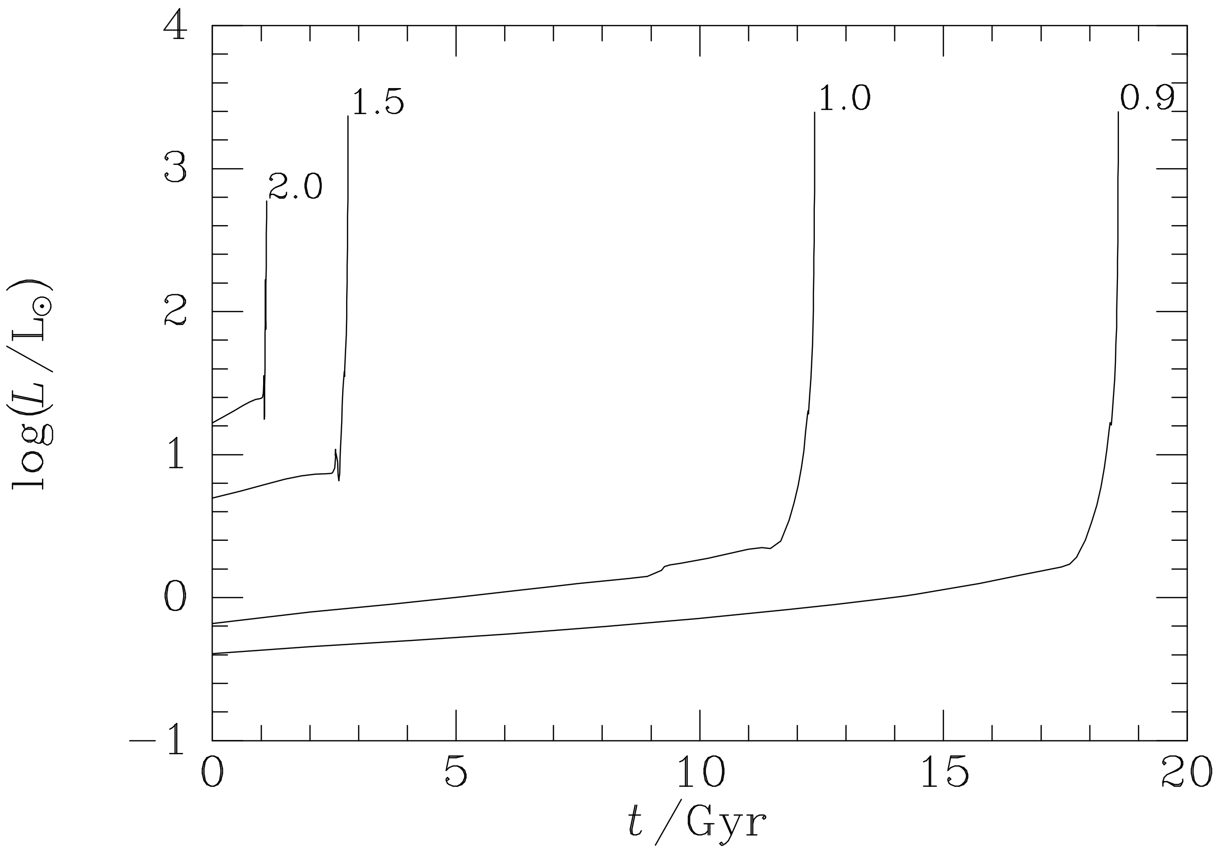

Figure 3: Luminosity of stars evolving from the MS to the He flash. Curves are labelled by stellar mass in $M_\odot$.

metallicity tends to make stars brighter and hotter (due to differences in structure and radiative transfer through their atmospheres).

1.5 Star Formation and Initial Mass Function

Star formation is much more complicated than stellar structure because i) while the star is forming we cannot assume hydrostatic equilibrium, ii) the interstellar medium from which stars form has a complicated, turbulent structure making the initial conditions poorly understood and iii) magnetic fields probably play an important role in the initial collapse and magnetohydrodynamics is difficult!

What we do know is that young stars tend to be found in clusters associated with dense interstellar clouds (often in spiral arms). This implies that stars actually form from such clouds (maybe after collapse is triggered by a passing shock front). The internal dynamics of these clouds is not well understood but probably involves a balance between gravity (trying to collapse them) and magnetic fields and turbulence (resisting gravity). In the densest regions of the clouds the ionization level can become small and magnetic fields decouple from the gas allowing gravity to win. As the core collapses the gas becomes more neutral, magnetic fields less coupled and the process can continue. This is a *protostar*.

As a protostar collapses in near free-fall it must dissipate the released gravitational energy. Initially, it's optically thin so this is easy (it radiates in the infrared), so the protostar contracts, stays at about 10–20K and increases in luminosity. Eventually, the material becomes dense and optically thick, trapping radiation and allowing the core to heat—the increased pressure stops collapse. As the core continues to heat it reaches 2000K at which point H_2 is dissociated into H—this energy sink allows the core to collapse again, heating the core until H is ionized. The increase in free electrons provides a lot of optical depth, trapping photons and arresting the collapse again. The star's luminosity at this point is sufficiently high that it becomes convective and settles onto a "Hayashi line" in the CMD. At this points it's close to hydrostatic equilibrium and evolves along the Hayashi line slowly (with some burning of D and Li). Eventually, the core temperature is high enough for H burning and the star has reached the main sequence. The whole process is quite rapid ($\sim$75Myr for a $1M_\odot$ star).

2 Evolution of Stellar Populations

A galaxy consists of stars spanning a variety of masses, ages and metallicities. Usually what we measure for a galaxy is the combined light from all of these stars. It's therefore useful to understand how a "stellar population" evolves. While we can't expect to reconstruct all details of the CMD from a spectrum of a stellar population, we might be able to figure out, e.g. the mean metallicity or age of that population.

The study of the evolution of stellar populations was kicked of by Beatrice Tinsley in the 1970's (during visits to Caltech). To simplify the problem we can think of a "simple stellar population"—a set of stars with different masses but all formed at the same time and with the same chemical composition. (We can build more complicated populations later by adding these simple populations together.)

For a simple stellar population a few insights can be made. Firstly, once stars more massive than about $2M_\odot$ have died the light of the population is dominated by the stars on the RGB, HB and AGB. The effective temperature of these doesn't depend very strongly on stellar mass. So, as stars of different masses evolve off of the main sequence and onto these branches we'll see little change in the color of the population. Net result: populations older than about 1.5Gyr should have slowly evolving colors.

The luminosity of the population will continue to change though due to the differing number of stars in these branches at any given time. To see this, we begin by defining an *initial mass function*, $\Phi(M)$ such that the number of stars in the population with masses in the range M to $M + \mathrm{d}M$ is $\mathrm{d}N(M) = M_{\rm pop}\Phi(M)\mathrm{d}M$ where $M_{\rm pop}$ is the total initial mass in the population (you'll see different normalizations of $\Phi(M)$—this one means that $\Phi(M)$ is normalized to unit stellar mass). The time spent on the giant branch is much less than spent on the MS. Therefore, if a star releases energy $E_{\rm GB}$ on the giant branch the luminosity of the population is

$$L \approx \left(E_{\rm GB}\frac{\mathrm{d}N}{\mathrm{d}M}\right)_{M_{\rm GB}}\left|\frac{\mathrm{d}M_{\rm GB}}{\mathrm{d}t}\right|, \tag{1}$$

where $M_{\rm GB}$ is the mass of a star just entering the giant branch at a given time. The MS lifetime of a star is approximately

$$\tau_{\rm MS} \approx 10{\rm Gyr}\left(\frac{M}{M_\odot}\right)^{-2.5}, \tag{2}$$

so that

$$M_{\rm GB}(t) \approx (\frac{t}{10{\rm Gyr}})^{-0.4}M_\odot, \tag{3}$$

and

$$\frac{\mathrm{d}M_{\rm GB}}{\mathrm{d}t} \approx -0.4\left(\frac{M_{\rm GB}}{M_\odot}\right)^{3.5}\left(\frac{M_\odot}{10{\rm Gyr}}\right). \tag{4}$$

The initial mass function can be fit reasonably well by

$$\frac{\mathrm{d}N}{\mathrm{d}M} = K\left(\frac{M}{M_\odot}\right)^{-\alpha}, \tag{5}$$

with $\alpha \approx 2.5$ for $M \lesssim M_\odot$ and K is a normalization constant. Putting this all together

$$L = \frac{KM_\odot E_{\rm GB}(M_{\rm GB})}{25{\rm Gyr}}\left(\frac{M_{\rm GB}}{M_\odot}\right)^{3.5-\alpha}. \tag{6}$$

Differentiating this gives

$$\begin{aligned}\frac{\mathrm{d}\ln L}{\mathrm{d}\ln t} &= \left[\frac{\mathrm{d}\ln E_{\rm GB}}{\mathrm{d}M_{\rm GB}} + (3.5-\alpha)\right]\frac{\mathrm{d}\ln M_{\rm GB}}{\mathrm{d}\ln t} \\ &= 0.4\alpha - \left(1.4 + 0.4\frac{\mathrm{d}\ln E_{\rm GB}}{\mathrm{d}\ln M_{\rm GB}}\right).\end{aligned} \tag{7}$$

$E_{\rm GB}$ is probably only weakly dependent on $M_{\rm GB}$ ($0 < \mathrm{d}\ln E_{\rm GB}/\mathrm{d}\ln M_{\rm GB} < 1$) so unless $\alpha > 3.5$ the luminosity of the simple population should be a decreasing function of time.

Since Tinsely's early work, people (Bruzual, Charlot, Worthy, Bressan. . .) have built much more accurate and elaborate models of the evolution of simple stellar populations. (They still have their limitations though, due both to limitations of stellar evolution calculations and of stellar atmosphere models/libraries.) Using these simple stellar populations we can construct the SED of a galaxy assuming that we know its star formation rate and metallicity as a function of time. Suppose, for example, that a galaxy has a star formation rate $\dot{\psi}(t)$ and that the metallicity of forming stars is $Z(t)$[2]. If $l_\lambda(\lambda;\tau,Z)$ is the SED of a simple stellar population of age τ and metallicity Z then the SED of the galaxy at some time t is given by the convolution integral

$$L(\lambda;t) = \int_0^t l_\lambda(\lambda;t-t',Z[t'])\dot{\psi}(t')\mathrm{d}t'. \tag{8}$$

In practice it turns out that by using this type of modelling you can estimate the total stellar mass of a galaxy (particularly if you have measurements in the near infrared which are sensitive to the lower mass, long-lived stars). Estimating the mean age and metallicity of the stars is more difficult as the two are degenerate—a young stellar population has similar colors to an older but less metal rich population (both tend to look quite blue). Thinking about galaxies, we have a couple of additional problems:

1. We expect a range of ages and metallicities and the SED is some non-linear function of these, making it a less than ideal way to get at these quantities;
2. Even if we do measure a mean stellar age, this is not a unique meaning of "age" for a galaxy. The stars in a galaxy could have formed long ago in small, progenitor galaxies which only recently merged together to form the galaxy we see. The age of the stars and the age of the galaxy (i.e. the time since it was assembled) can then be very different.

[2] We'll discuss how metallicity evolves in a galaxy in a later class.