# Class #13: Chemical enrichment and chemical evolution

Structure and Dynamics of Galaxies, Ay 124, Winter 2009

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*Material in this class is taken mostly from Binney & Merrifield §5.2 & §5.3.*

## 1 Synthesis of Chemical Elements

Elements heavier than helium are produced almost exclusively as a result of stellar evolution. The chemical composition of stars in galaxies is therefore of interest for two reasons:

1. The metallicity of stars affects their luminosities and colors and so is important to understand if we want to extract physical information about galaxies from photometric measures;
2. Since stars of different masses produce different combinations of elements a detailed knowledge of the chemical composition of a galaxy can tell us about its star formation history.

### 1.1 Nuclear Physics

Each isotope can be described as a bound state of $Z$ protons and $N$ neutrons. An element corresponds to a column of fixed $Z$ in the $(Z, N)$ plane. We usually describe an isotope by the one or two letter label for the element plus the atomic number $A = Z + N$, so that $^{16}$O has $Z = 8$ and $N = 8$. Stable nuclides lie in the *stability band* in the $(Z, N)$ plane which runs parallel to but slightly above the $N = Z$ diagonal. Many of these states can undergo spontaneous nuclear decay and thereby move in the $(Z, N)$ plane (see Fig. 1).

The binding energy per nucleon, $\epsilon$, varies systematically across the $(Z, N)$ plane. Along a line of constant $A = Z + N$ (which cuts across the stability strip) the absolute value of $\epsilon$ first increase and then decreases. If we plot $\epsilon$ as a surface in the $(Z, N)$ plane we would therefore see a valley running along the stability band (known, not surprisingly, as the *stability valley*). The valley itself slopes downwards from each end (low and high $A$) to its lowest point in the middle at $^{56}$Fe (8.79 MeV below H at the low end of the valley and 1.21 MeV below $^{238}U$ at the upper end). Stars get the most of their energy by moving nucleons through this valley to lower values of $\epsilon$.

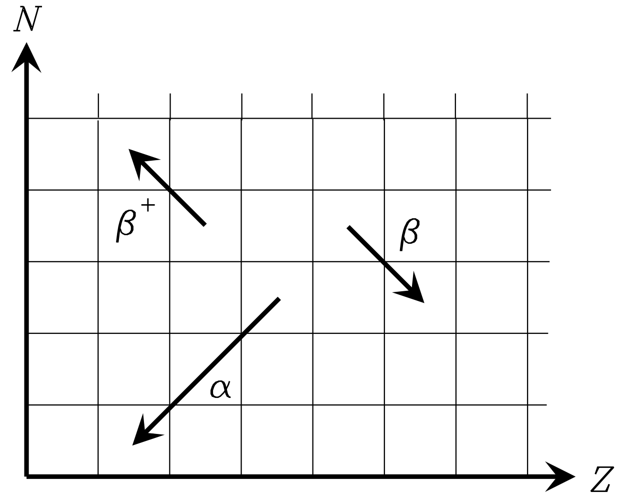


Figure 1: Motion in the $(Z, N)$ plane due to radioactive decay.

There are a few groups of nuclides in the $(Z, N)$ plane which are worth knowing about:

**$\alpha$-nuclides** This group consists of $^{20}$Ne, $^{24}$Mg, $^{28}$Si, $^{32}$S, $^{36}$A and $^{40}$Ca and can all be formed by adding 2,3,... $\alpha$ particles to $^{16}$O. They can be formed during C- and O-burning through

$$\begin{aligned} ^{12}\mathrm{C} + ^{12}\mathrm{C} &\rightarrow ^{20}\mathrm{Ne} + ^{4}\mathrm{He} \\ ^{16}\mathrm{O} + ^{16}\mathrm{O} &\rightarrow ^{28}\mathrm{Si} + ^{4}\mathrm{He}, \end{aligned} \tag{1}$$

and through capture of $\alpha$ particles in reactions such as

$$^{20}\mathrm{Ne} + ^{4}\mathrm{He} \rightarrow ^{24}\mathrm{Mg} + \gamma. \tag{2}$$

The $\alpha$-nuclides are all *primary nuclides* since they can be produced in a star which starts out with only H and He. Secondary nuclides require the presence of other primary nuclides before they can be synthesized. The very direct formation channels for these elements means that they form readily and are much more abundant than their immediate neighbors in the $(Z, N)$ plane for this reason. Their abundance also decrease smoothly with increasing atomic number (since each nuclide requires the formation of the preceeding nuclide before it can be formed).

**Iron peak nuclides** This group consists of elements near Fe with $40 < A < 65$ and incldues Sc, Ti, V, Cr, Mn, Fe, Co, Ni and Cu. The floor of the stability valley is quite flat in this region so that these nuclides all have similar binding energies. The iron-peak nuclides are primary nuclides which form late in the evolution of stars when the core is extremely hot. In fact, the core is usually so hot that these elements are constantly formed and splitting apart rapidly enough that the system comes into thermal equilibrium. The relative number of each species is then proportional to $\exp(-E/kT)$, where $E$ is the configuration's energy. The result is that many different nuclides are present in significant number.

**s-process nuclides** This group consists of elements with $A$ greater than that of the iron-peak nuclides. The 's' stands for slow and abundant s-process nuclides include $^{88}$Sr, $^{89}$Y, $^{90}$Zr,

$^{138}$Ba, $^{139}$La, $^{140}$Ce, $^{141}$Pr, $^{208}$Pb and $^{209}$Bi. Various reactions which occur in stars produce free neutrons. For example, during C-burning

$$^{12}\mathrm{C} +^{12}\mathrm{C} \rightarrow^{23}\mathrm{Na} + \mathrm{n}. \tag{3}$$

The nucleus (being neutral) can easily be absorbed by another nucleus nearby, forming a heavier isotope of the same element. For example, it could be absorbed by $^{56}$Fe to form $^{57}$Fe. This process will continue until an unstable isotope, $^{59}$Fe, is reached, which will then $\beta^-$ decay into $^{59}$Co. The process repeats, making heavier isotopes of Co until an unstable isotope is reached which then $\beta^-$ decays into Ni. This process can gradually build up elements of larger and larger $A$. The core of a star will contain isotopes throughout this entire sequence at any time and the relative number of each isotope will be inversely proportional to its cross section for absorbing a neutron (the larger the cross section, the quicker it absorbs a neutron and becomes a different isotope).

s-process nuclides are secondary nuclides because they need primary iron-peak nuclides to start from. Since a star only produces iron-peak elements at the end of its life (after neutron-producing C and O-burning) this can only happen if some iron-peak nuclides existed when the star formed. The abundance of secondary elements should scale (roughly) in proportion to the square of the abundance of primary nuclides (since we first have to make the primaries before they can be converted into secondaries).

**r-process nuclides** This group, consisting of nuclides such as $^{81}$Br, $^{84}$Kr, $^{128,130}$Te, $^{127}$I, $^{192}$Os, $^{193}$Ir and $^{196,198}$Pt, are also formed through neutron capture, but by a *rapid* capture process. The "rapid" here means that the time between neutron captures is significantly shorter than the timescale for $\beta^-$ decay. Then, as a nuclide absorbs neutrons to form heavier and heavier isotopes even unstable isotopes can not decay because they absorb another neutron before they have chance to. Eventually, as rapid absorption moves the nuclide far up the wall of the stability valley the decay timescale becomes sufficiently short and the nuclide must undergo $\beta^-$ decay, falling back down into the valley and coming to rest as an r-process element. The distrinction between r- and s-process elements is not a sharp one as many elements can be made through both processes.

## 1.2 Metal Production Below $M_{\mathrm{up}}$

Stars with initial masses below $M_{\mathrm{up}} \approx 8M_{\odot}$ do not result in supernovae explosions but instead become white dwarfs. Since white dwarfs have masses of approximately $1M_{\odot}$ we know that these stars must typically lose much of their mass in the late stages of stellar evolution (resulting in planetary nebulae). In these mass loss events it is the outer layers of the atmosphere that are ejected. Since we're typically only interested in elements produced in the star and which are returned to the ISM (since these can be used in future generations of star formation unlike the elements locked up in the core) we might therefore expect that such stars only enrich the ISM in He. However, observations clearly show that the atmospheres of stars in this mass range can become enriched in carbon indicating that they must dregde up significant amounts of enriched material from their cores. This C (and N and O) will be ejected during the plantary nebula phase, enriching the ISM. The degree of enrichment is not too well known—observationally it's difficult to measure the enhancement in these elements in planetary nebulae over the general abundance of the ISM and theoretically it depends on the details of dredge up which is complicated to model.

However, the best evidence suggests that these stars must play a significant role in enriching the ISM, particularly at early times when the ISM is quite metal poor.

### 1.3 Supernovae

Supernovae explosions occur either when a massive ($> 8M_\odot$) star undergoes core collapse or when an accreting C/O white dwarf approaches the Chandrasekhar limit and nuclear burning is triggered explosively in its core. While the internal structure of massive stars prior to core collapse is relatively easy to calculate, the details of the explosion and element production during the explosion are not. What is clear is that there is significant r-process production from the iron-peak elements in the pre-collapse core due to nuclear reactions triggered by the outgoing shockwave. New elements are also synthesized during a white dwarf (Type Ia) supernovae—almost the entire UV and optical luminosity of a Type Ia comes from radioactive decay of $^{56}$Ni.

Despite significant uncertainty in the numbers, it's clear that SNe contribute significantly to chemical enrichment of all types of nuclides.

## 2 Chemical Enrichment Models

Since each generation of stars pollutes the ISM with metals which are then incorporated into the next generation of stars we should be able to compute how the metallicity of stars in a galaxy evolves with time. We'll consider three different simple models which differ in how the galaxy interacts with its surrounding environment.

### 2.1 Closed-box Model

This is the simplest possible model and can be applied to a small region of a galaxy (e.g. the local region around the Sun). We assume that no material enters or leaves this region, that the region is initially entirely gaseous (no stars) and metal free and that turbulence keeps the region well-mixed and homogeneous. The gas will gradually be turned into stars and metals produced.

Suppose that at any time the mass of heavy elements in interstellar gas is $M_\mathrm{H}$ and the mass of interstellar gas is $M_\mathrm{g}$. The *metallicity* of the gas is defined to be

$$Z \equiv \frac{M_\mathrm{H}}{M_\mathrm{g}}. \tag{4}$$

The metallicity of the Sun is about $Z_\odot = 0.02$. Suppose that at this time the total mass of stars is $M_\mathrm{s}$ and that a new mass of stars, $\delta' M_\mathrm{s}$ forms. We make an *instananeous recycling approximation* in which we neglect the time between the formation of a generation of stars and the time at which those stars return enriched material to the ISM. This is a reasonable approximation for enrichment by core collapse SNe where the intervening time is a few million years (short compared to the timescales for evolution in galaxies), but breaks down for enrichment from Type Ia SNe and stars of only a few Solar masses. The theory can be refined to account for these delays (if you're happy to deal with lots of convolution integrals...). Suppose that the mass of these newly formed stars that remains after the massive stars have died is $\delta M_{\rm s}$. and let the mass of heavy element produced by these stars and returned to the ISM be $p\delta M_{\rm s}$, where $p$ is known as the *yield* of the stellar population. The net change in the heavy element content of the interstellar gas is then

$$\delta M_{\rm H} = p\delta M_{\rm s} - Z\delta M_{\rm s} = (p - Z)\delta M_{\rm s}. \tag{5}$$

The metallicity of the interstellar gas therefore changes by an amount

$$\delta Z = \delta \left( \frac{M_{\rm H}}{M_{\rm g}} \right) = \frac{1}{M_{\rm g}} (\delta M_{\rm H} - Z\delta M_{\rm g}). \tag{6}$$

Since mass must be conserved, $\delta M_{\rm s} = -\delta M_{\rm g}$ and so

$$\delta Z = -p \frac{\delta M_{\rm g}}{M_{\rm g}}. \tag{7}$$

If the yield of each generation of stars is the same[1] then we can integrate this to find

$$Z(t) = -p \ln \left[ \frac{M_{\rm g}(t)}{M_{\rm g}(0)} \right]. \tag{8}$$

If this model is correct that we'd expect that if we plot the measured metallicity in a galaxy as a function of the logarithm of the local gas fraction we'd get a straight line with slope $p$. We can also use this model to predict the metallicity distribution of the stars. The mass of stars with metallicity less than $Z$ is

$$\begin{aligned} M_{\rm s}[< Z(t)] &= M_{\rm s}(t) = M_{\rm g}(0) - M_{\rm g}(t) \\ &= M_{\rm g}(0) \left\{ 1 - \exp[-Z(t)/p] \right\}. \end{aligned} \tag{9}$$

This shows that stars should be widely distributed in metallicity with a significant fraction of stars having metallicities below, for example, one third of that of currently forming stars. The fraction of stars with metallicity less than a fraction $\alpha$ of that of currently forming stars is

$$\begin{aligned} \frac{M_{\rm s}[< \alpha Z(t)]}{M_{\rm s}[< Z(t)]} &= \frac{1 - \exp[-\alpha Z(t)/p]}{1 - \exp[-Z(t)/p]} \\ &= \frac{1 - x^{\alpha}}{1 - x}, \end{aligned} \tag{10}$$

where $x = M_{\rm g}(t)/M_{\rm g}$ is the gas fraction. In the Solar neighborhood, $x \approx 0.1$ so $M_{\rm s}(Z/3) \approx 0.51 M_{\rm s}(Z)$. Observationally, there are many fewer metal poor stars in the Solar neighborhood, suggesting that our simple closed box model is incorrect.

[1] This is approximately true if each generation has the same IMF. There is some dependence on the metallicity of the generation, but it's reasonably weak.

## 2.2 Leaky-box Model

For reasons not entirely understood star formation in molecular clouds is quite inefficient. As a result, SNe explosions and strong stellar winds can begin to occur from the first generation of stars to form in a molecular cloud before all of that cloud has turned into stars. The large energy and momentum associated with these winds and explosions can drive outflows of material from the cloud. This clearly isn't a closed box. We can model this as a *leaky box* by assuming that SNe drive out gas from the region at a rate proportional to the star formation rate:

$$\dot{M}_{\rm g} = -c\dot{M}_{\rm s}, \tag{11}$$

where $c$ is some constant which will depend on the nature of the region in question (in particular, it might depend on the depth of the potential well—deeper wells will be more difficult to eject from). The total (gas plus stars) mass in our region will then be $M_{\rm t}(t) = M_{\rm t}(0) - cM_{\rm s}(t)$, which implies a gas mass of $M_{\rm g}(t) = M_{\rm t}(0) - (1+c)M_{\rm s}(t)$. Repeating our analysis for the metallicity results in

$$\frac{{\rm d}Z}{{\rm d}M_{\rm s}} = \frac{p}{M_{\rm g}} = \frac{p}{M_{\rm t}(0) - (1+c)M_{\rm s}}. \tag{12}$$

Integraing this equation (assuming zero initial mass in stars) gives

$$M_{\rm s}(< Z) = \frac{M_{\rm t}(0)}{1+c}\left\{1 - \exp\left[-\frac{(1+c)Z}{p}\right]\right\}. \tag{13}$$

The only difference compared to the closed box model is that the yield is replaced by an *effective yield* $p/(1+c)$. This therefore won't change the fraction of stars below a fraction $\alpha$ of the metallicity of currently forming stars so cannot reconcile the observed lack of such low metallicity stars.

## 2.3 Accreting-box Model

Galaxies must be accreting gas from their surroundings (since they had to form from initially diffuse intergalactic gas). This accretion has an important consequence. Suppose a galaxy accretes at precisely the rate at which it forms stars: $\delta M_{\rm a} = \delta M_{\rm s}$. Then, the total gas mass in the galaxy will not change. Since the net effect of star formation is to remove a mass $Z\delta M_{\rm s}$ of metals from the ISM and to return a mass $p\delta M_{\rm s}$ we may expect that after some time an equilibrium will be reached at which $Z = p$ and the ISM does not become any more metal rich no matter how long we wait. After a sufficiently long time the majority of stars in the galaxy will have metallicities of $p$. This can help reduce the fraction of low metallicity stars and therefore reconcile the osbervations.

Using our previous expressions for the change in the metallicity of the ISM and the mass of metals produced we can derive an expression for the change in metallicity in terms of the change in the total mass $M_{\rm t}$:

$$\delta Z = \frac{1}{M_{\rm g}}[(p - Z)\delta M_{\rm t} - p\delta M_{\rm g}]. \tag{14}$$

Dividing by $\delta M_{\rm t}$ gives

$$\frac{{\rm d}Z}{{\rm d}M_{\rm t}} = \frac{1}{M_{\rm g}}\left[p - Z - p\frac{{\rm d}M_{\rm g}}{{\rm d}M_{\rm t}}\right]. \tag{15}$$

Defining $u \equiv \int \mathrm{d}M_\mathrm{t}/M_\mathrm{g}$ this becomes

$$\frac{\mathrm{d}Z}{\mathrm{d}u} + Z = p\left(1 - \frac{\mathrm{d}\ln M_\mathrm{g}}{du}\right), \tag{16}$$

which has solution

$$Z = p\left(1 - C\mathrm{e}^{-u} - \mathrm{e}^{-u}\int_0^u \mathrm{e}^{u'}\frac{\mathrm{d}\ln M_\mathrm{g}}{\mathrm{d}u'}\mathrm{d}u'\right). \tag{17}$$

A simple case is for zero initial metallicity and constant gas mass in which case

$$Z = p\left[1 - \exp\left(1 - \frac{M_\mathrm{t}}{M_\mathrm{g}}\right)\right], \tag{18}$$

so that $Z \approx p$ once $M_\mathrm{t} \gg M_\mathrm{g}$. The mass in stars more metal poor than $Z$ is just $M_\mathrm{t}(Z) - M_\mathrm{g}$ which implies

$$M_\mathrm{s}(< Z) = -M_\mathrm{g}\ln\left(1 - \frac{Z}{p}\right). \tag{19}$$

The yield can then be estimated from the local gas fraction and the current metallicity of interstellar gas to be $p \approx 0.02 \approx Z_\odot$. Then

$$M_\mathrm{s}(< 0.25Z_\odot) \approx 0.03M_\mathrm{t}. \tag{20}$$

So, only about 3% of long-lived stars should have metallicities less than one quarter Solar, which is in reasonable agreement with osbervations.