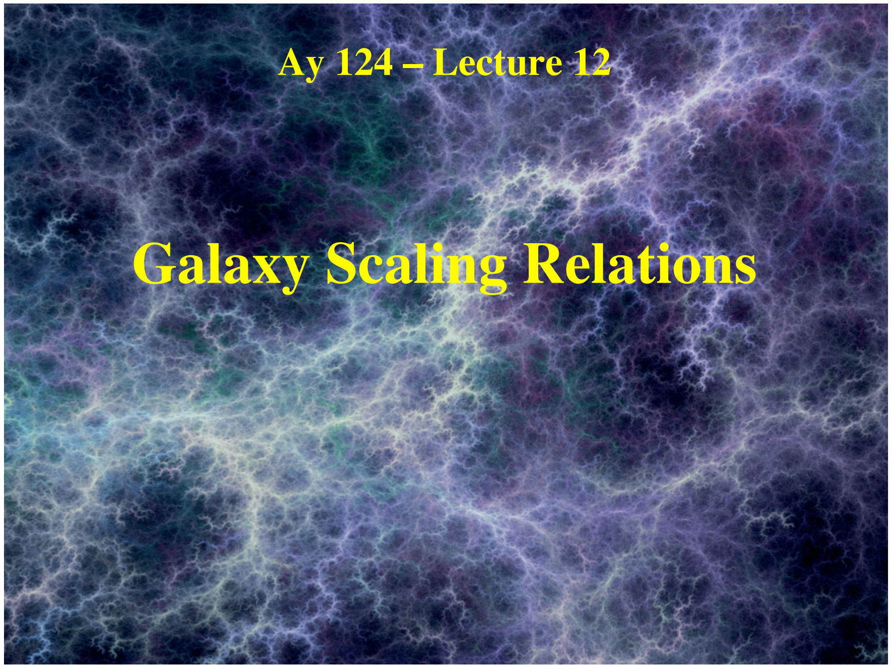


Galaxy Scaling Laws

- When correlated, global properties of galaxies tend to do so as power-laws; thus “scaling laws”
- They provide a quantitative means of examining physical properties of galaxies and their systematics
- They reflect the internal physics of galaxies, and are a product of the formative and evolutionary histories
  - Thus, they could be (and are) different for different galaxy families
  - We can use them as a fossil evidence of galaxy formation
- When expressed as correlations between distance-dependent and distance-independent quantities, they can be used to measure relative distances of galaxies and peculiar velocities: thus, it is really important to understand their intrinsic limitations of accuracy, e.g., environmental dependences
- Their origins are generally not yet well understood

Deriving the Scaling Relations

Start with the Virial Theorem: $$\frac{GM}{\langle R \rangle} = k_E \frac{\langle V^2 \rangle}{2}$$

Now relate the observable values of ***R, V*** (or σ), ***L***, etc., to their “true” mean 3-dim. values by simple scalings:

$$R = k_R \langle R \rangle \qquad V^2 = k_V \langle V^2 \rangle \qquad L = k_L I R^2$$

One can then derive the “virial” versions of the FP and the TFR:

$$R = K_{SR} V^2 I^{-1} (M/L)^{-1}$$

$$L = K_{SL} V^4 I^{-1} (M/L)^{-2}$$

Where the “structure” coefficients are:

$$K_{SR} = \frac{k_E}{2G k_R k_L k_V}$$

$$K_{SL} = \frac{k_E^2}{4G^2 k_R^2 k_L k_V^2}$$

Deviations of the observed relations from these scalings must indicate that either some ***k’***s and/or the ***(M/L)*** are changing

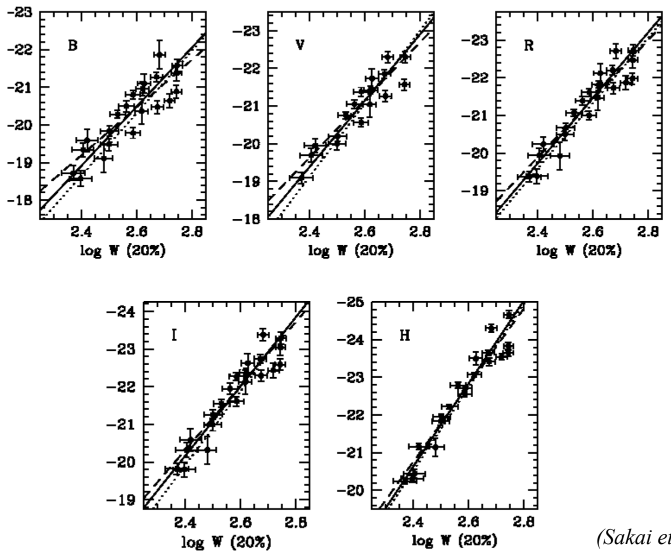
The Tully-Fisher Relation (TFR)

- A well-defined luminosity vs. rotational speed (often measured as a H I 21 cm line width) relation for spirals:

  $L \sim V_{rot}^{\gamma}$, $\gamma \approx 4$, varies with wavelength

  Or: $M = b \log(W) + c$, where:
  - M is the absolute magnitude
  - W is the Doppler broadened line width, typically measured using the HI 21cm line, corrected for inclination $W_{true} = W_{obs} / \sin(i)$
  - Both the slope b and the zero-point c can be measured from a set of nearby spiral galaxies with well-known distances
  - The slope b can be also measured from any set of galaxies with roughly the same distance - e.g., galaxies in a cluster - even if that distance is not known
- Scatter is ~ 10-20% at best, better in the redder bands

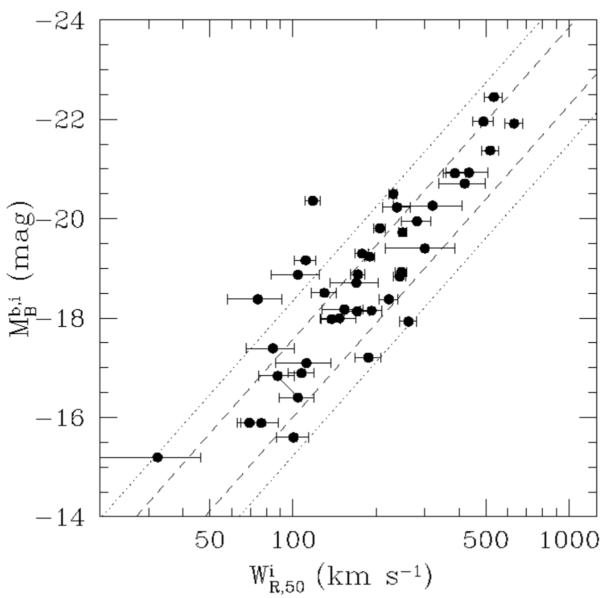
Tully-Fisher Relation in Diferent Bands



(Sakai et al. 1999)

Why is the TFR So Remarkable?

- Because it connects a property of the dark halo - the maximum circular speed - with the product of the net integrated star formation history, i.e., the luminosity of the disk
- Halo-regulated galaxy formation/evolution?
- The scatter is remarkably low - even though the conditions for this to happen are known not to be satisfied
- There is some important feedback mechanism involved, which we do not understand yet
- Thus, the TFR offers some important insights into the physics of disk galaxy formation



Low surface brightness galaxies follow the <u>same</u> TF law as the regular spirals: so it is really relating the baryonic mass to the dark halo

(Zwaan et al. 1995)

Deriving the Tully-Fisher Relation

In part, Tully-Fisher relation reflects dynamics of a disk galaxy. Estimate the luminosity and maximum circular velocity of an exponential disk of stars:

Empirically, disk galaxies have an exponential surface brightness profile:

$$I(R) = I(0)\ e^{-R/h_R}$$

Integrate this across annuli to get the total luminosity:

$$L \propto \int_0^\infty 2\pi R I(0) e^{-R/h_R} dR$$

$$L \propto I(0) h_R^2$$

If the mass of the exponential disk dominates the rotation curve, then the enclosed mass within radius ***R*** will be proportional to the enclosed luminosity:

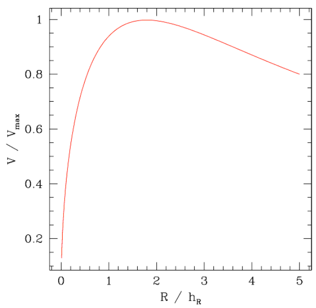
$$M(R) \propto L(R) \propto \int_0^R 2\pi R' I(0) e^{-R'/h_R} dR'$$

Approximately, use formula for spherical mass distribution to get ***V(R)***:

$$\frac{V^2(R)}{R} = \frac{GM(R)}{R^2}$$

$$V^2(R) \propto \left[\frac{h_R}{R} - \frac{h_R}{R} e^{-R/h_R} - e^{-R/h_R} \right] \times h_R$$

Dependence on $\boldsymbol{R}$ always occurs via the combination $\boldsymbol{R / h_R}$



Function in […] peaks at $\boldsymbol{R \sim 1.8\, h_R}$

Conclude that: $V_{\max} \propto \sqrt{h_R}$

Eliminate $\boldsymbol{h_R}$ using previous result: $L \propto V_{\max}^4$

But we assumed:

1. I(0) = const.
2. (M/L) = const.

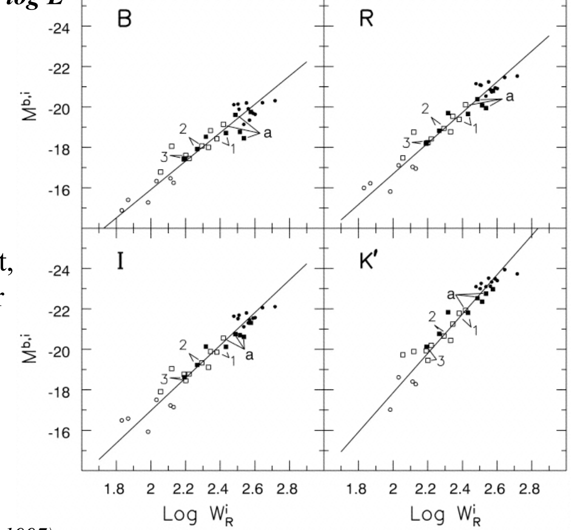
} Both are incorrect!

TFR for Normal and LSB Disks

TFR for normal disk galaxies (solid symbols) and for low surface brightness disks (open symbols)

The slope, the intercept, and the intrinsic scatter are the same for both samples!

log L

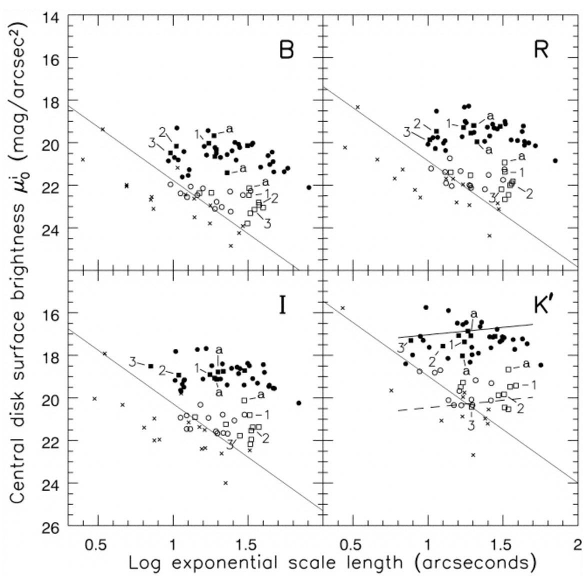


log Velocity width

(Figure from Tully & Verheijen 1997)

Examine the Ingredients of the TFR

Central surface brightness and disk scale lengths each span a dynamical range of about 2 orders of magnitude, and do not correlate at all



(Tully & Verheijen 1997)

A more detailed derivation of the TFR, gives 2 other alternative conditions:

$$L \sim V_m^4 X$$

Either: $X \sim (M/L)^{-1} \Sigma_h^{-1}$

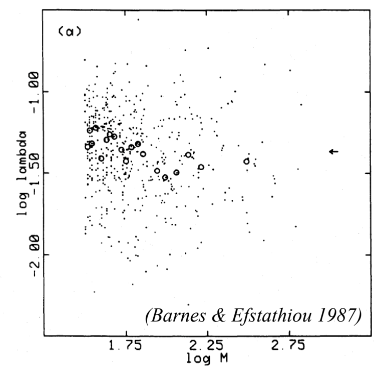
where Σ_h is the surface density of the dark halo,

Or: $X \sim (M/L)^{-2} I_0^{-1} \lambda^{-2}$

where λ is the "spinup parameter", a measure of the specific angular momentum:

$$\lambda \equiv J|E|^{1/2} G^{-1} M^{-5/2}$$

Simulations indicate a spread of an order of magnitude in λ, independent of galaxy's mass, or any other relevant quantity

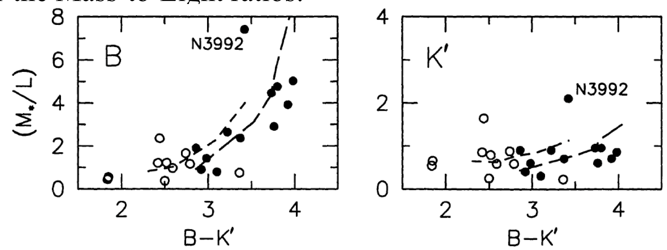


(Barnes & Efstathiou 1987)

The Conditions for the Existence of TFR

Now consider the Mass-to-Light ratios:

An order of magnitude spread!



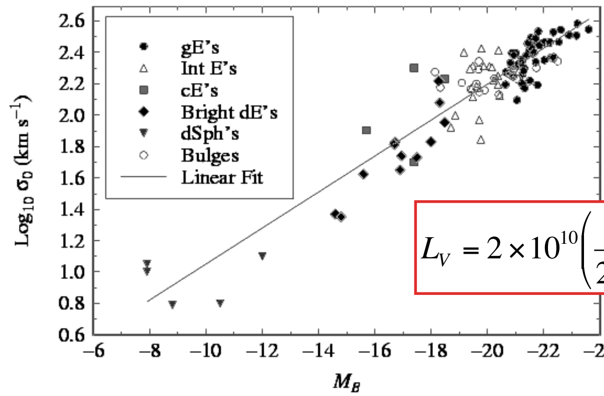
Recall the TFR formula: $$L = K_{SL} V^4 I^{-1} (M/L)^{-2}$$

So, each of the ingredients forming the proportionality coefficient (surface brightness, scale length, (M/L), spinup parameter) shows a huge spread, and they do not correlate …

… and yet, a nearly scatterless TFR results!

The Faber-Jackson Relation

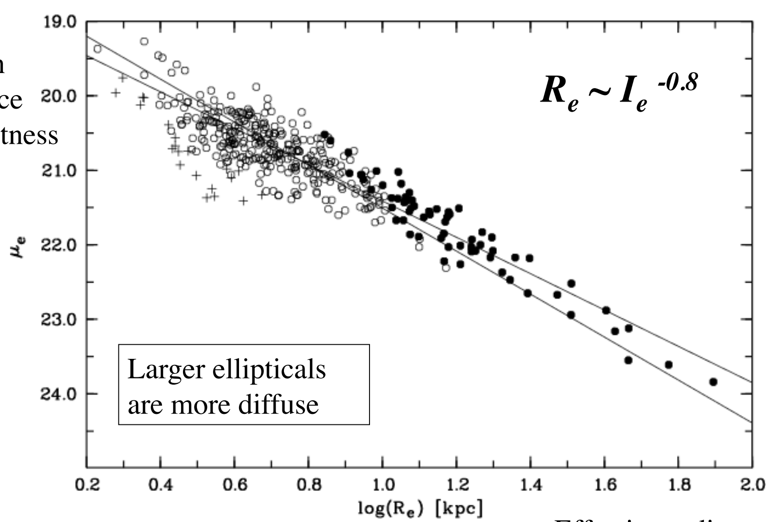
Analog of the Tully-Fisher relation for spirals, but instead of the peak rotation speed V_{max}, measure the velocity dispersion. This is correlated with the total luminosity:



$$L_V = 2 \times 10^{10} \left(\frac{\sigma}{200 \text{ kms}^{-1}} \right)^4 L_{sun}$$

The Kormendy Relation

Mean surface brightness



Effective radius

Can We Learn Something About the Formation of Ellipticals From the Kormendy Relation?

From the Virial Theorem, $m\sigma^2 \sim GmM/R$

Thus, the dynamical mass scales as $M \sim R\sigma^2$

Luminosity $L \sim I R^2$, where I is the mean surface brightness

Assuming $(M/L) = const.$, $M \sim I R^2 \sim R\sigma^2$ and $I R \sim \sigma^2$

Now, if ellipticals form via dissipationless merging, the kinetic energy per unit mass $\sim \sigma^2 \sim const.$, and thus we would predict the scaling to be $R \sim I^{-1}$

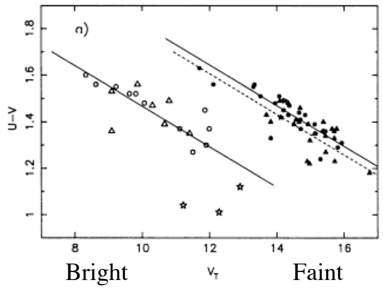
If, on the other hand, ellipticals form via dissipative collapse, then $M = const.$, surface brightness $I \sim M R^{-2}$, and thus we would predict the scaling to be $R \sim I^{-0.5}$

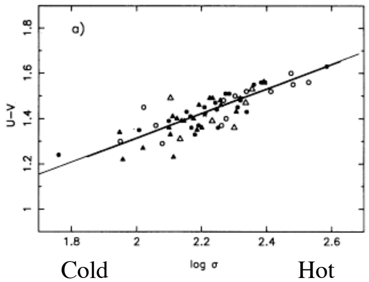
The observed scaling is $R \sim I^{-0.8}$. Thus, *both* dissipative collapse and dissipationless merging probably play a role

Metallicity-Luminosity Relation

also known as the Color-Magnitude Relation

There is a relation between the color (a metallicity indicator) and the total luminosity or velocity dispersion for E galaxies:





R
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B
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e

Brighter and dynamically hotter galaxies are redder. This could be explained if small E galaxies were younger or more metal-poor than the large ones. More massive galaxies could be more effective in retaining and recycling their supernova ejecta.

Towards the Discovery of the FP

There were two motivational streams:

1. ***How many statistically significant properties describe elliptical galaxies, and how are they related?*** Or: what is the "manifold of elliptical galaxies"?
   - The pioneering work by Brosche (1973), Brosche & Lentes (1982)
2. ***What is the "2nd parameter" in the F-J relation, so that it can be improved as a distance indicator for early-type galaxies?***
   - The Davis-Djorgovski-Kent mini-survey (1982/3) [6 parameters?]
   - The Fall & Efstathiou paper (1984) [*L*-σ-Mg plane]
   - Lauer's study of E-galaxy cores (1986) [almost!]

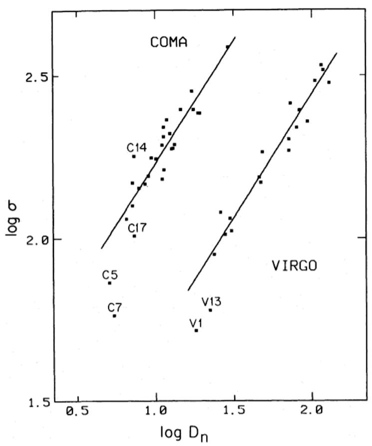
The actual discovery/realization:

Dressler et al. (1987) [the "7 Samurai"]: the D_n-σ relation

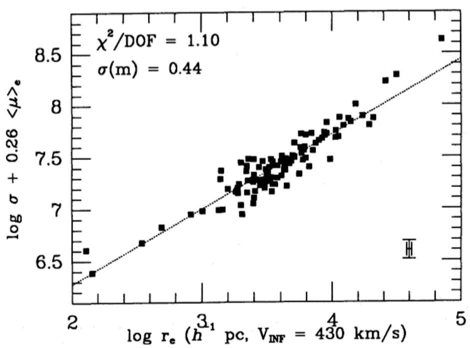
Djorgovski & Davis (1987): a plane in the *L*-*R*-σ-μ space

The Initial Renderings

D_n-σ, Dressler et al. 1997



R-σ-μ, Djorgovski & Davis (1987)



The "canonical form": $R \sim \sigma^A I^B$

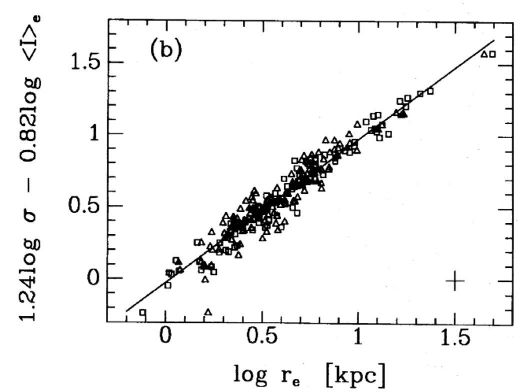
R = non-isophotal radius (r_e, r_η, …)

σ = central proj. velocity disp.

I = mean surf. br. in linear units

Fundamental Plane Relations

- ***A set of bivariate scaling relations for elliptical galaxies,*** including relations between distance dependent quantities such as radius or luminosity, and a combination of two distance-independent ones, such as velocity dispersion or surface brightness
- In a set of ~ 10 independently measured global parameters, there are only 2 statistically independent ones
- Scatter ~ 10%, but it could be lower?



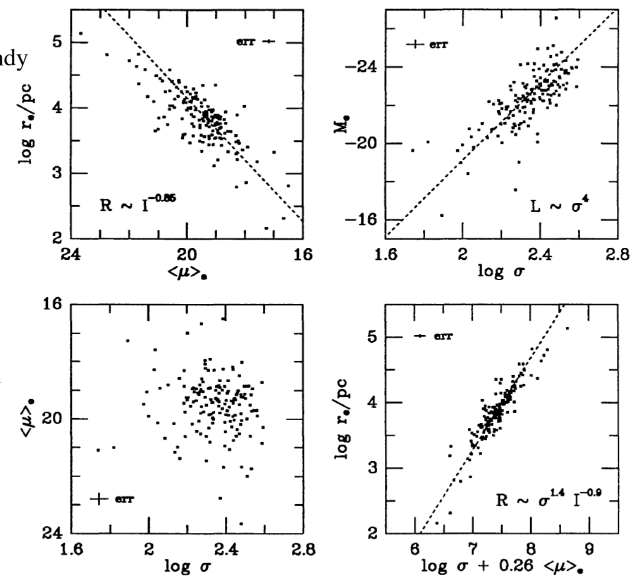
Scaling Relations for Ellipticals

Kormendy rel'n

Faber-Jackson rel'n

Cooling diagram

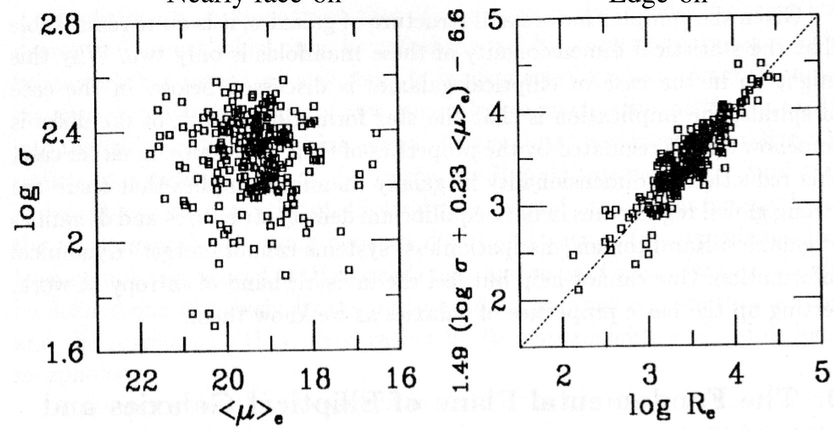
Fundam. Plane



Different Views of the FP

Nearly face-on

Edge-on

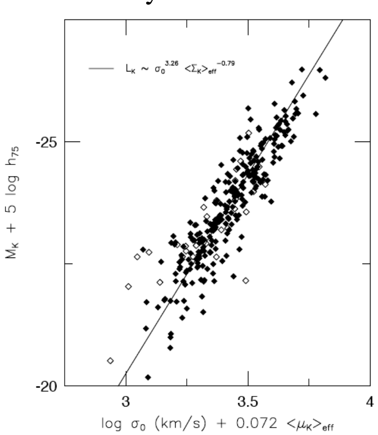


Commonly expressed as a bivariate scaling relation $R \sim \sigma^{1.4} I^{-0.8}$

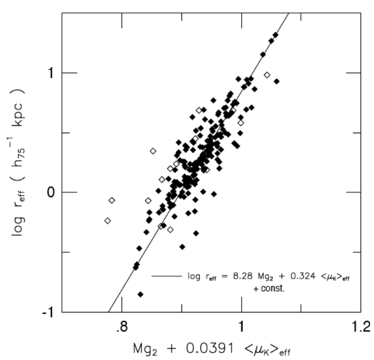
Where R is the radius, I the mean surf. brightness, σ the velocity disp.

Different Views of the FP

Luminosity instead of radius



Mg abs. line strength index (a measure of metallicity) instead of velocity dispersion



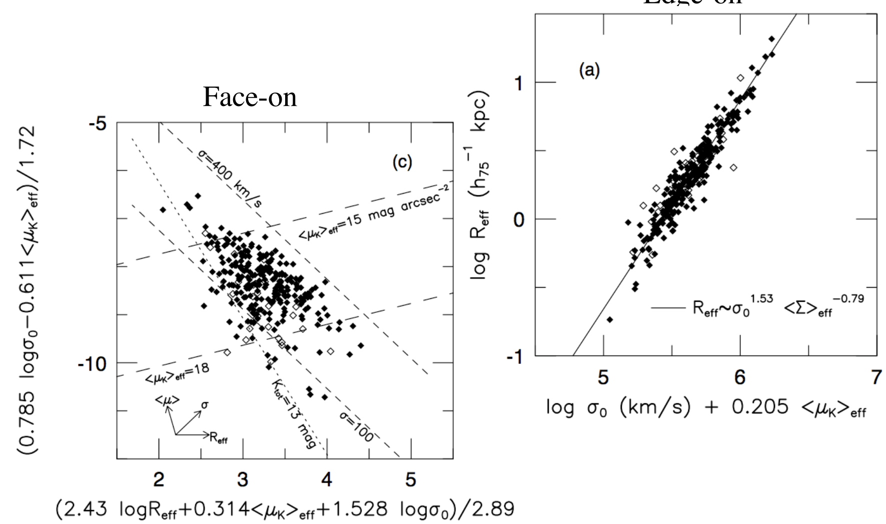
FP connects stellar populations and dynamical and structural parameters of ellipticals

FP in the K-Band (~ nearly bolometric)

(Pahre et al. 1988)

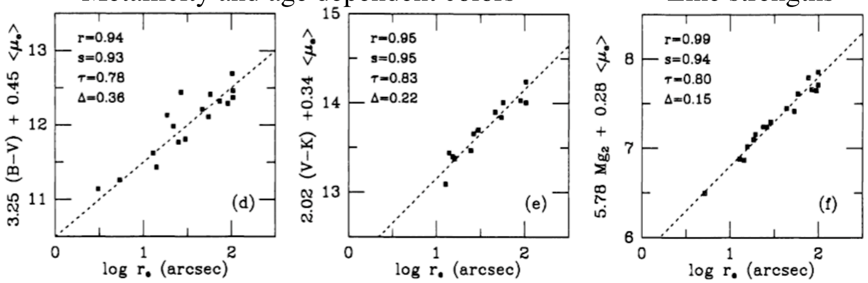
Face-on

Edge-on



Stellar Population Variables Also Participate in the FP (de Carvalho & Djorgovski 1989)

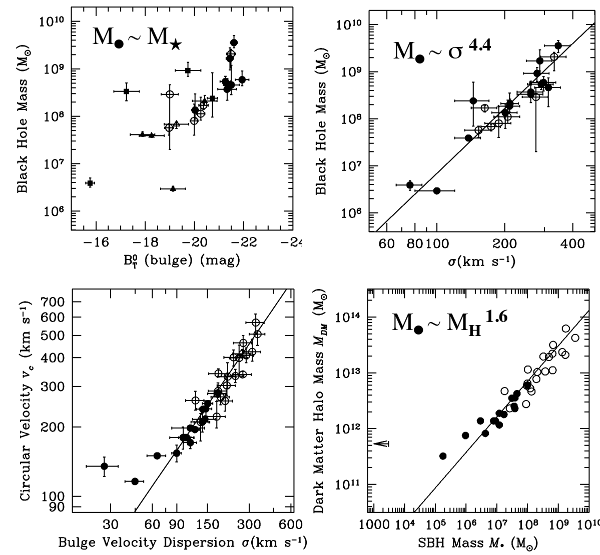
Metallicity and age dependent colors　　　　Line strengths



This implies that the chemical enrichment (and star formation?) histories of ellipticals are regulated by their global dynamical and structural parameters

… And so are their central SMBHs (many authors…)

The SMBH - Host Galaxy Correlations



Fundamental Plane and M/L Ratios

Write the FP scaling relation as: $R \sim \sigma^A I^B$

Where the observed values are ***A*** ~ 1.4, ***B*** ~ -0.8, uncertain by about 10%, and depending on the bandpass

Recall from Virial Theorem: $\langle R \rangle \sim \langle V^2 \rangle \langle I \rangle^{-1} (M/L)^{-1}$

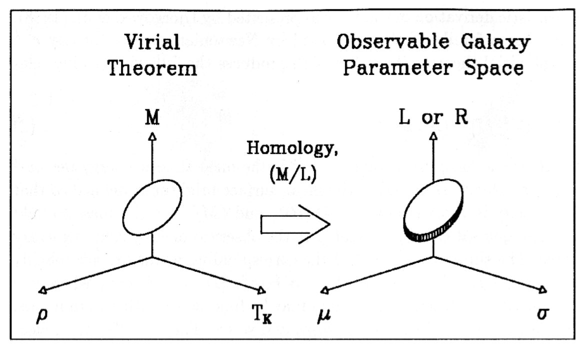
Then $k_V^{-1} \, k_I \, k_R \, (M/L) \sim \sigma^{A-2} \, I^{-B-1}$

If all ellipticals have the same structure, i.e., they are just scaled versions of each other (a homologous family), then all ***k_X = const.*** and all change must be in ***(M/L).*** Approximately,

$(M/L) \sim L^{\alpha}$, where ***α*** ~ 0.2 (visible) or ~ 0.1 (IR)

But we know that E's are not a homologous family, so the tilt of the FP must have complex reasons

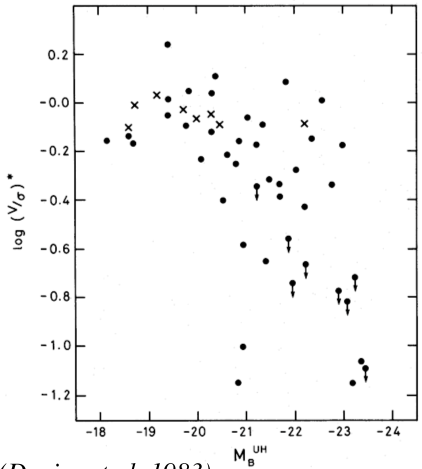
From Virial Theorem to FP



- Galaxies must be on a "Virial Theorem Plane" in the space of mass, mean density, and kinetic temperature
- ***If*** galaxies represent a homologous family of structures ***and*** had (M/L) = const., then they should follow the VTP: ***$R \sim \sigma^2 I^{-1}$***
- Since they don't, and the observed FP scaling is: ***$R \sim \sigma^{1.4} I^{-0.8}$***, either one or both of these assumptions must be broken

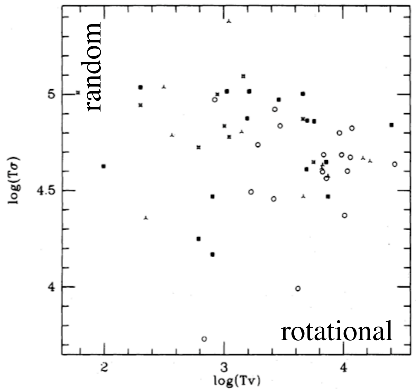
Breaking the Homology: Dynamics

More luminous/larger ellipticals are more radially anisotropic:



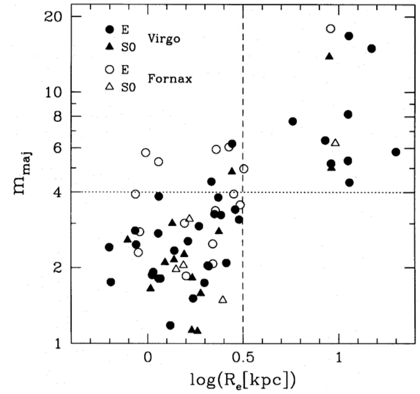
(Davies et al. 1983)

There is a wide spread in rotational contributions to the total kinetic energy:



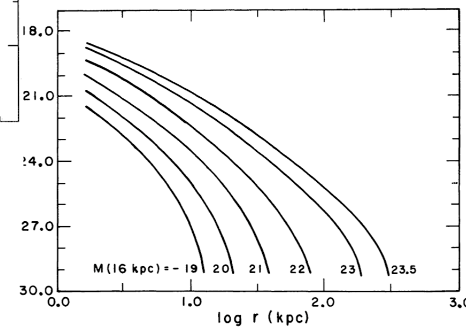
(Busarello et al. 1992)

Breaking the Homology: Density Profiles



← Sersic profile index ($r^{1/n}$) correlates with the galaxy size *(D'Onofrio et al. 1994)*

More luminous ellipticals have shallower profiles →
(Schombert 1986)

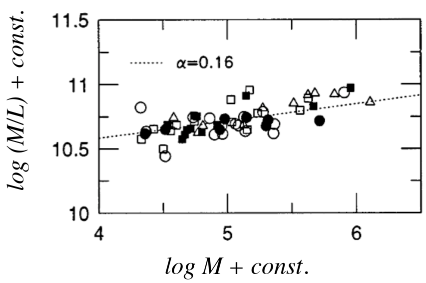


Fundamental Plane and M/L Ratios

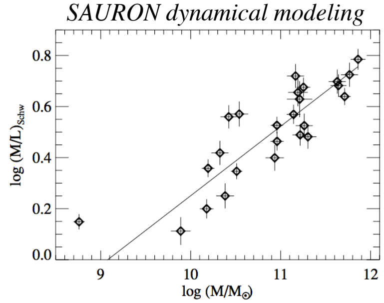
If we *assume* homology and attribute all of the FP tilt to the changes in ***(M/L)*** , $(M/L) \sim L^{\alpha}$, $\alpha \sim 0.2$ (vis) or ~ 0.1 (IR)

Possible causes: systematic changes in $M_{visible}/M_{dark}$, or in their relative concentrations; or in the stellar IMF

Pahre et al. 1995: K-band FP

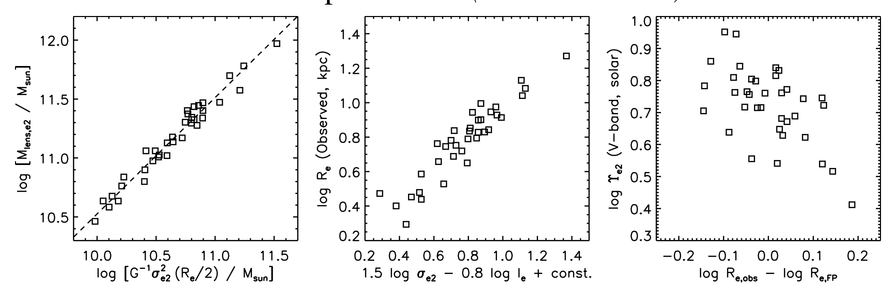


Cappellaro et al. 2006: SAURON dynamical modeling



Mass-Based Fundamental Plane

The use of lensing galaxies allows for the determination of their ***mass-based*** structural parameters *(Bolton et al. 2007)*



Traditional FP fit gives $R \sim \sigma^{1.4} I^{-0.8}$, consistent with other work. *Replacing* the surface brightness I with the *projected mass density* Σ gives a "mass plane" scaling: $R \sim \sigma^{1.8 \pm 0.2} \Sigma^{-1 \pm 0.2}$, consistent with the Virial Theorem, and with a smaller scatter!

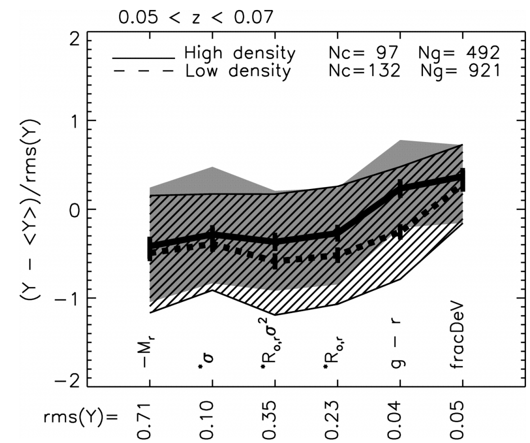
This implies *a homology of mass* (if not light) structures of E's

Environmental Dependence (?)

- FP intercepts (zero-points) are operationally interchangeable with peculiar velocities; zero-point variations would cause spurious V_{pec}'s
  - A highly controversial subject…
- Numerous spectroscopic studies find systematic differences between E's in different density environments
- However, ***no convincing evidence*** for cluster-to-cluster variations has been found by a number of studies
- Even if we assume that *all* FP-based V_{pec}'s are entirely spurious, due to environmental variations, that would imply the zero-point differences of at most ~ 10%; and clearly that is an overestimate
- ***Thus, we conclude that for the present-day (cluster) E's, the intercepts of the FP are universal to better than 10% (and could be 0%)***

Environmental Dependence

Numerous spectroscopic studies indicate that *E's in denser environments are systematically redder, older, more metal-rich, dimmer* - but it is not clear if coeval E-galaxy populations would have different FP zero points



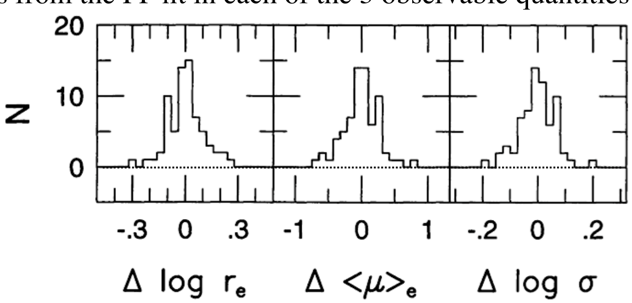
Probably the best study to date: Bernardi et al. (2006), from SDSS:

Implies ~ 0.075 mag difference between FPs for E's in low and high density environments

The Remarkably Small Scatter of the FP

Residuals from the FP fit in each of the 3 observable quantities

(Djorgovski et al. 1995, and consistent with other studies)



Total r.m.s.	0.085	0.23	0.054
Est. intrinsic	< 0.055	< 0.15	< 0.035

Thus, the intrinsic thickness of the FP is at most a few % (and could be zero) - despite the observed broad variety of kinematical and density profiles, projection effects, etc. etc.

For any elliptical galaxy today, big or small,

Just Two Numbers

determine *to within a few percent or less*:

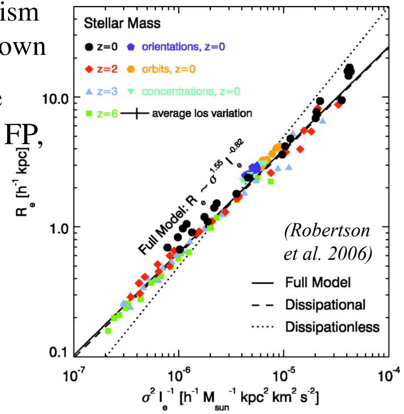
Mass, luminosity (in any OIR band),
Any consistently defined radius
Surface brightness or projected mass density
Derived 3-d luminosity, mass, or phase-space density
Central projected radial velocity dispersion
OIR colors, line strengths, and metallicity
Mass of the central black hole
… and maybe other things as well

And they do so regardless of the:

Star formation and merging formative/evolutionary history
Large-scale environment
Details of the internal structure and dynamics (including S0's)
Projection effects (direction we are looking from)

How Can This Be?

- The implication is that elliptical galaxies occupy only a small, naturally selected, subset of all dynamical structures which are in principle open to them
  - Maximum entropy states? But gravothermal entropy is notoriously difficult to define, and the mechanism to achieve this is completely unknown
- Numerical sim's can *reproduce* the observed structures of E's, and the FP, but they *do not explain* them
- Understanding of the origin of the small scatter of the FP (or, equivalently, the narrow range of their dynamical structures) is ***an outstanding problem***



(Robertson et al. 2006)

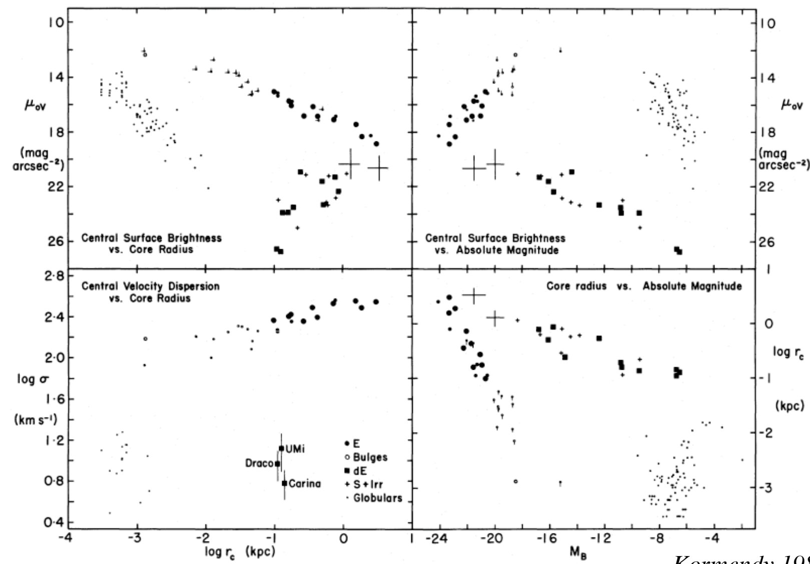
Fundamental Plane Summary

- The FP correlations are a set of bivariate scaling laws, connecting a number of fundamental properties of early-type galaxies
- They provide unique observational constraints on the structure, formation, and evolution of early-type galaxies
- Their formative processes tightly couple the dynamical structure, chemical enrichment (star formation) history, and growth of their central black holes, in a remarkably robust manner, with just two parameters accounting for many fundamental properties
- The small scatter of the FP implies that ellipticals cover only a very limited, standardized range of dynamical structures; the mechanism of this natural selection is not yet understood
- FP correlations are the sharpest tool in our observational arsenal to study the evolution of early-type galaxies

Dwarf Galaxies

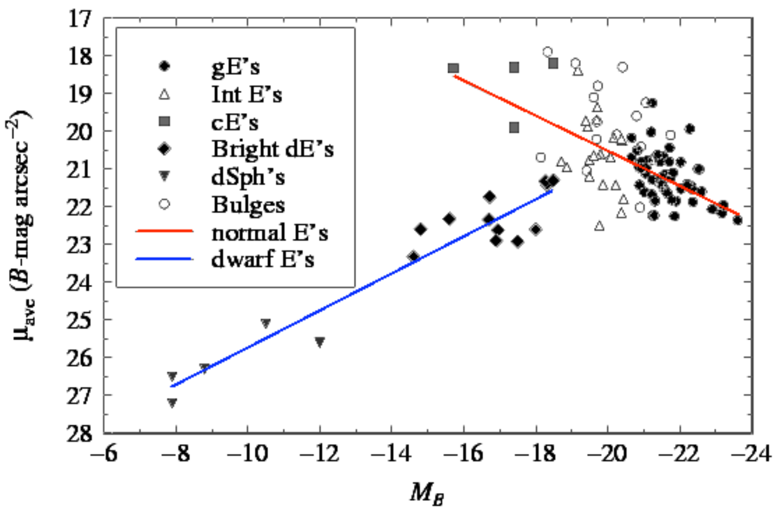
- Dwarf ellipticals (dE) and dwarf spheroidals (dSph) are a completely different family of objects from normal ellipticals - they are not just small E's
  - In fact, there may be more than one family of gas-poor dwarf galaxies …
- Dwarfs follow completely different correlations from giant galaxies, suggestive of *different formative mechanisms*
  - E.g., merging could be less important, but galactic winds more important for dwarfs
- They are generally dark matter (DM) dominated, especially at the faint end of the sequence
- One possible scenario is that SN winds can remove baryons from these low-mass systems, while leaving the DM
- This would naturally lead to a single-parameter family of objects

Parameter Correlations

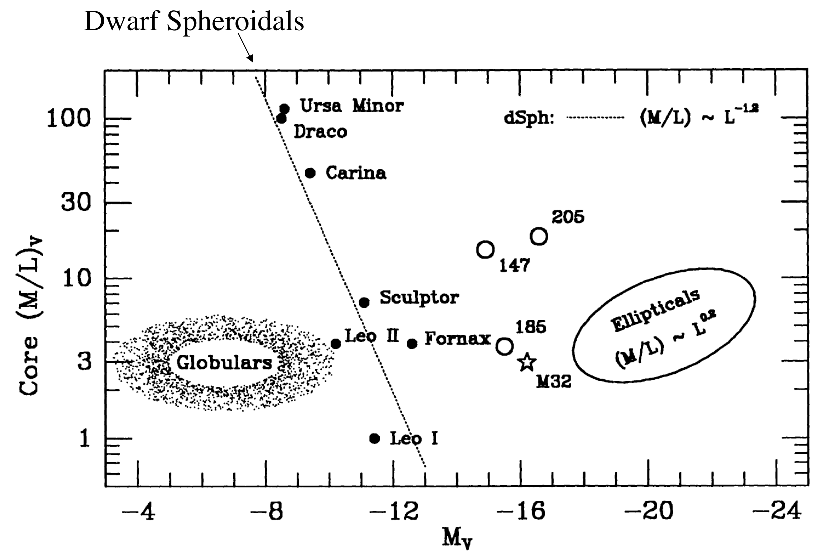


Kormendy 1985

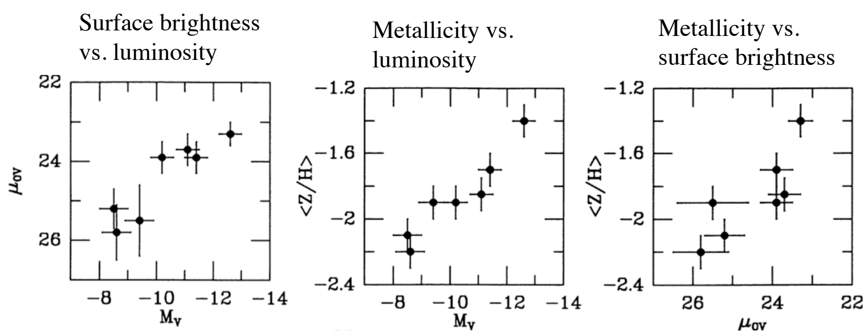
Mean Surface Brightness vs. Absolute Mag.



Mass to Light Ratios



Dwarfs: A Single-Parameter Family?



(de Carvalho & Djorgovski 1992)

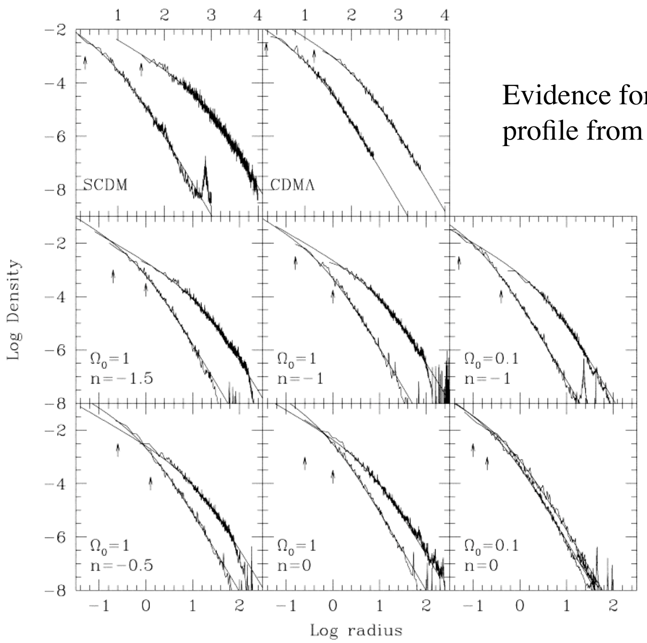
Confirmed by the more modern data and analysis, e.g., Woo et al. (2008)

The Dark Halos

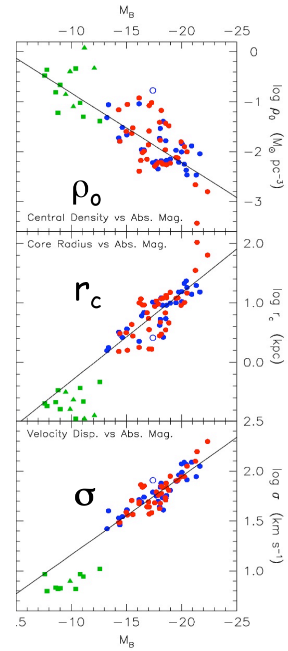
- Many of galaxy scaling relations may be driven by the properties of their dark halos
- It is possible to infer their properties from detailed dynamical profiles of galaxies and some modeling
- Numerical simulations suggest a universal form of the dark halo density profile (NFW = Navarro, Frenk & White):

$$\frac{\rho(r)}{\rho_{crit}} = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2}$$

(but one can also fit another formula, e.g., with a core radius and a finite central density)



Evidence for the NFW profile from simulations



Dark Halo Scaling Laws

$\rho_o \sim L_B^{-0.35}$

$r_c \sim L_B^{0.37}$ (fits to Sc-Im only)

$\sigma \sim L_B^{0.20}$

■ ▲ are dSph, dIrr

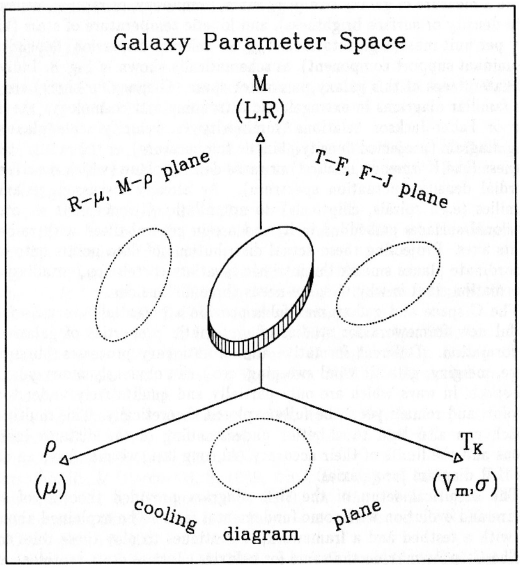
so expect the surface density

$\Sigma \sim \rho_o \, r_c$

to be ~ constant over this range of M_B, and it is

(Kormendy & Freeman 2003)

The Galaxy Parameter Space



A more general picture

Galaxies of different families form 2-dim. sequences in a 3+ dimensional parameter space of physical properties, much like stars form 1-dim. sequences in a 2-dim. parameter space of {L,T} - this is an equivalent of the H-R diagram, but for galaxies

Comments on the Scaling Relations

- Probably the most challenging thing to understand about these galaxy scaling relations is their ***thinness***: we can understand their slopes, but not why they are so sharply defined: intrinsic spread in many coefficients and/or (M/L) should thicken them considerably - but for some reason it does not. This is still a great mystery.
- Other stellar systems, from globular clusters to clusters of galaxies have fundamental scaling relations of their own
- We use these relations as distance indicators, assuming that they are universal; but small systematic variations in their slopes or intercepts, e.g., in different environments, would introduce systematic distance errors and spurious peculiar velocities
  - There is some evidence for that…

Galaxy Scaling Relations and the Standard Galaxian Structures

- In order to achieve the observed small scatter of the scaling relations, it is necessary that galaxies occupy only a narrow range of dynamical structures at any given point in these correlations
- It is also necessary that there is an orderly change of these structure along the galaxian sequences, and the deviations from homology drive the slopes of the scaling relations
- Somehow, this has to be achieved during the processes of galaxy formation an evolution, and this is a real puzzle:
  - Processes of galaxy assembly are messy and diverse
  - It is easy to think of the ways of spoiling the correlations (if they were built in at the start), adding the scatter

… and yet…

Galaxy Formation

- Repeated, random, hierarchical merging is a key process of galaxy assembly
- It involves varying amounts of dissipation
- There are gas inflows into, and outflows from galaxies, driven by star formation
- Supermassive black holes are ubiquitous in normal galaxies; their feedback processes probably play a significant role
- All this is a strong function of the large scale environment

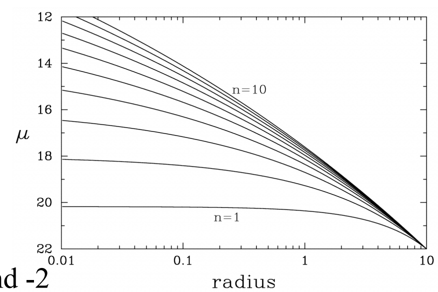
… and yet …

Galaxies come in a fairly narrow range of standard dynamical structures

Standardized Structures of Galaxies

For **ellipticals** and bulges of spirals, projected density or surface brightness is well described by the empirical Sersic formula:

$$I(r) = I_0 \, \mathrm{dex}\left(-r^{1/n}\right)$$



Typical values of the shape index $n \approx 4$

Locally behaves like a power law, typical slopes around -2

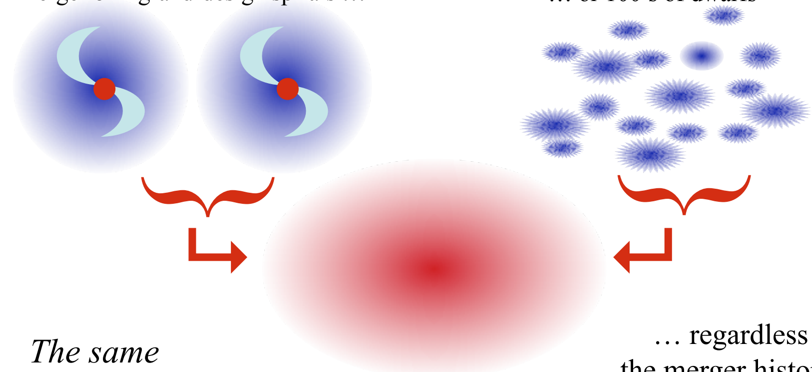
Disks of spirals are well approximated by the n = 1 case, exponential density distributions, both in radius and perpendicularly to the principal plane:

$$I(R) = I(0)\ e^{-R/h_R}$$

Many Paths Towards Building of an Elliptical Galaxy

Merger of 2 grand-design spirals …

… or 100's of dwarfs



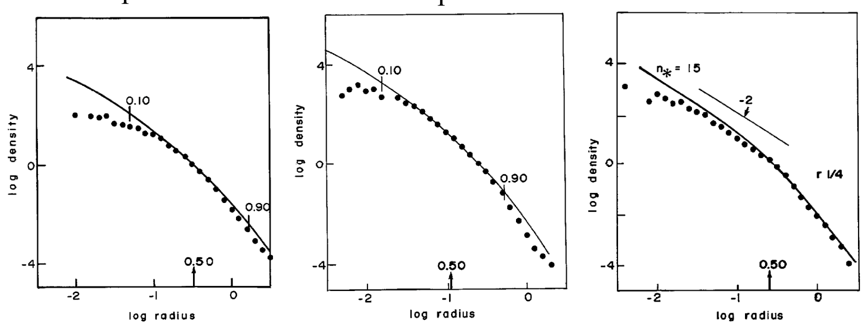
The same outcome is produced …

… regardless of the merger history, initial conditions, the amount of dissipation, etc.

A Remarkable Robustness of Outcomes

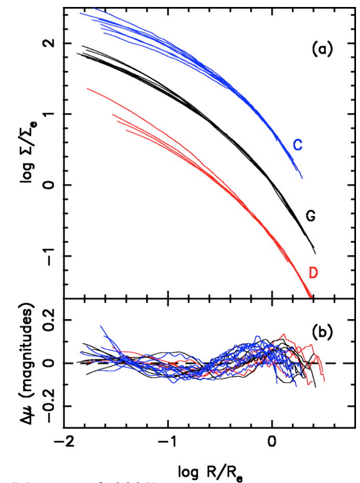
The resulting density profiles from numerical simulations of a formation of an elliptical galaxy

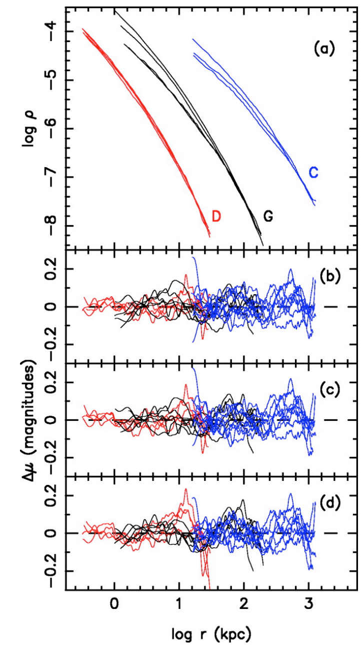
Dissipationless | Dissipative | Dark Matter



(Carlberg, Lake, & Norman 1986)

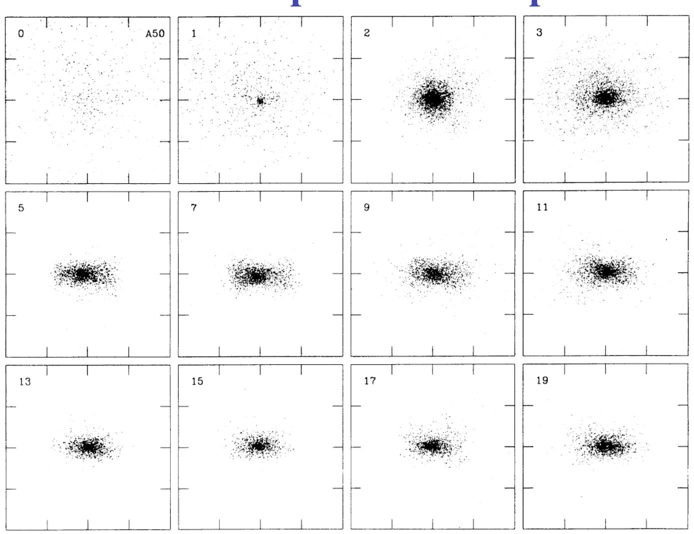
Confirmed by the more modern simulations: a universal Sersic-like profile always results!





(Merrit et al. 2005)

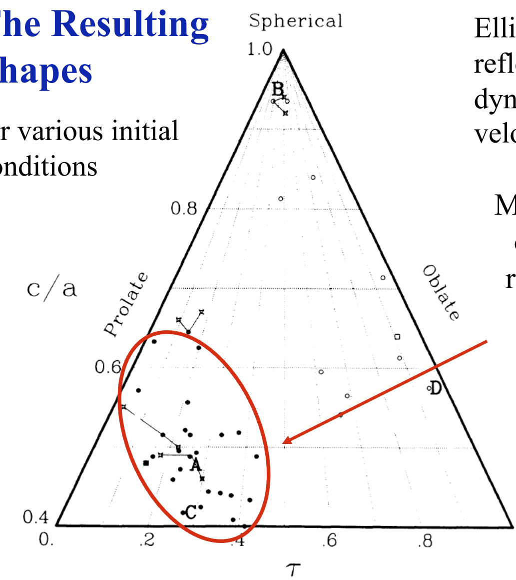
Simulated Formation of an Elliptical Galaxy From a Simple Cold Collapse



(Aguilar & Merritt 1990)

The Resulting Shapes

for various initial conditions



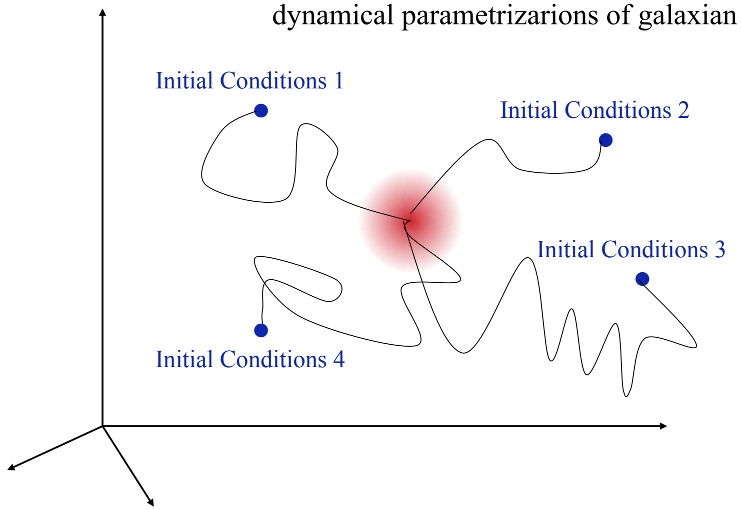
Elliptical galaxy's shape reflects directly its internal dynamical structure: the velocity anisotropy tensor

Most elliptical galaxies occupy only a limited range of all dynamical structures which are in principle available to them!

(Aguilar & Merritt 1990)

Galaxy Structures as Attractors?

Axes represent schematically appropriate structural and dynamical parametrizarions of galaxian forms



Convergent Evolution of Galaxies?

Aside from the major dichotomies (dwarfs/giants, disks/ spheroids), whose origins we think we understand,

Galaxies evolve into a narrow range of dynamical structures, regardless of the details of their formative histories, and are the same everywhere

What drives this natural selection of galaxian forms?

Understanding The Galaxian Philogeny

What is the physical mechanism behind the natural selection of galaxian forms?

- **Dynamical stability of galaxies**
  - Many dynamical structures are possible (follow the conservation laws, etc.), internally consistent, but may not be stable
- **States of maximum entropy**
  - Not yet well defined for stellar dynamics, i.e., collisionless systems dominated by a purely attractive force (gravity)
  - The role of dissipation is hard to fold in
  - Recall that galaxies are open systems

How Stable Are Galaxies?

There is an infinite number of possible, internally consistent dynamical models for stellar systems, but not all of them are stable.

For example, self-gravitating systems have a negative specific heat, and thus are instable to core collapse (gravothermal catastrophe); this is actually known to occur in globular clusters and protostars.

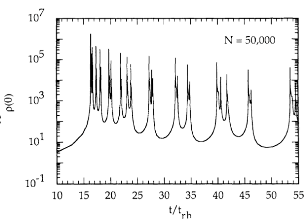
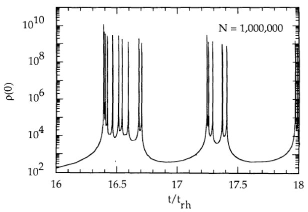


FIG. 3b

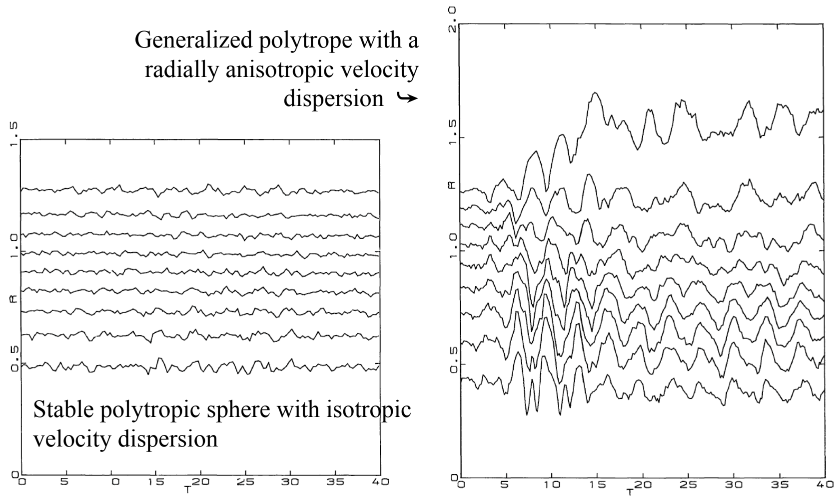
In some situations, quasi-periodic gravothermal oscillations set in:



(Breeden et al. 1994)

An Example: Henon Instability

Radii of shells containing 10%, … 90% of the total mass



(Henon 1973)

Entropy and Galaxian Structures

Could stable galaxian forms be states of ***maximum entropy***? The problem is that the classical Boltzmann entropy,

$$S_B = -\int f \log f \, dx \, dv$$

does not have a maximum for collisionless, self-gravitating stellar systems: it can be made arbitrarily large by making an extreme core-halo system, with a tightly bound core and a weakly bound halo of a large radius.

But for collisionless systems like galaxies, one could define other forms of entropy, e.g., where ***C(f)*** is a convex function.

$$S = -\int C(f) \, dx \, dv$$

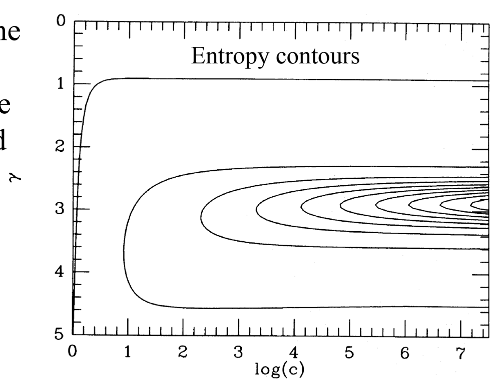
Some additional constraints are needed, but there is no compelling solution yet

A Simple Toy Example

Consider a spherical galaxy with a truncated power-law density profile:

$$\varrho(r) = \begin{cases} \varrho_0; & r < r_0, \\ \varrho_0 (r/r_0)^{-\gamma}; & r_0 < r < c r_0, \\ 0; & c r_0 < r. \end{cases}$$

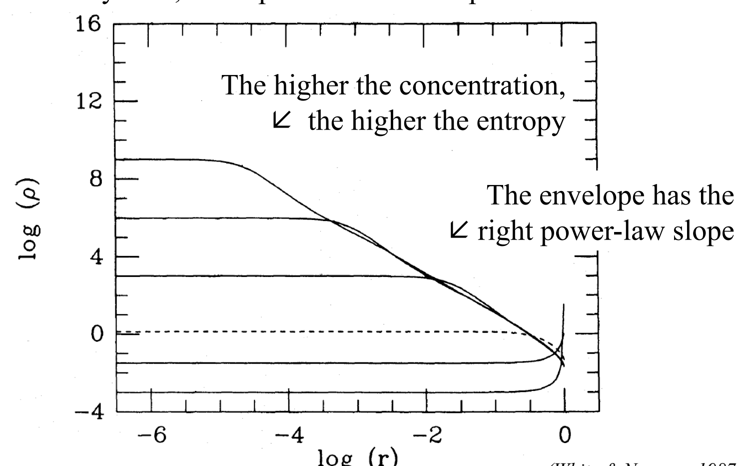
Entropy is maximized for the highest concentrations, but the optimal power-law slope is just right for the observed galaxies:



(White & Narayan 1987)

Constrained Max. Entropy Solution

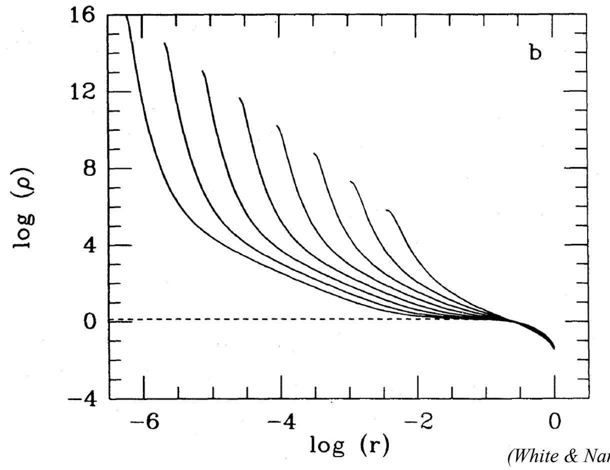
Recall that this is a constrained solution: it requires a finite size and density core, and a power-law envelope



(White & Narayan 1987)

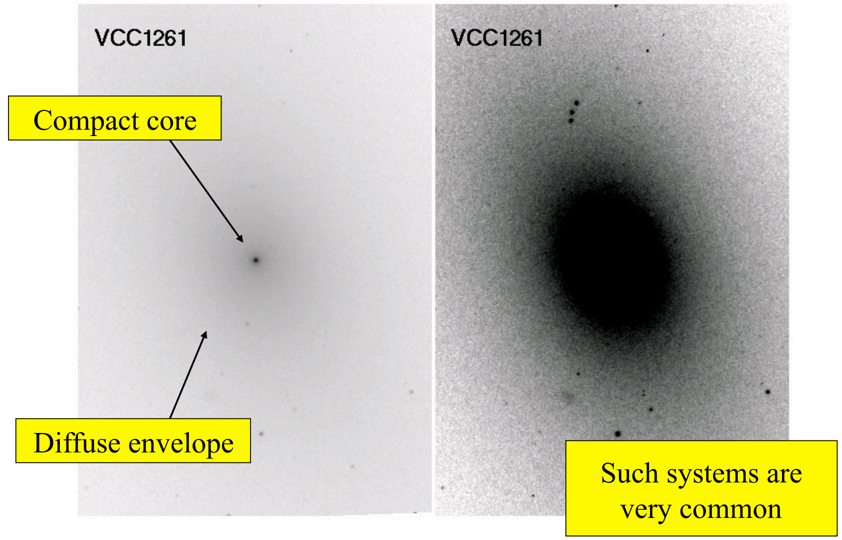
Unconstrained Max. Entropy Solution

Now remove the finite core and power-law requirements:
an ***extreme core-halo structure*** results



(White & Narayan 1987)

Compact Nuclei in Dwarf Ellipticals



Summary

- Galaxies universally show a limited range of structural and dynamical forms, despite a broad range of very messy evolutionary histories and processes
  - A convergent evolution / natural selection
- This is demonstrated dramatically in the existence of non-trivial, small-scatter scaling relations such as FP and TFR
  - A narrow range of dynamical structures is necessary for that
- The origin of these regularities is not yet understood
  - Why these structures, and not others?
- Dissipation, feedback, stability and entropy all play roles, but a complete theory is still missing
- Similar puzzles are posed by the scaling relations for galaxy clusters, star clusters, and even stars themselves (HRD)