Class #16: Kinematics and mass models of the Milky Way

Structure and Dynamics of Galaxies, Ay 124, Winter 2009

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Material in this class is taken mostly from Binney & Merrifield §10.3–10.6.

1 Kinematics in the Solar Neighborhood

Having developed lots of theory of galactic dynamics and structure one of the most obvious things to do is to apply it to the Milky Way and see if it is able to explain observational facts.

1.1 Local Standard of Rest

Throughout this class we've often made use of the Local Standard of Rest, but we've never discussed how it might be determined. We will show below that by measuring the mean motion of different types of stars relative to the Sun (known as the *Solar motion*) we can infer the velocity of the LSR relative to the Sun. We can infer the Solar motion from measurements of either radial velocities or proper motions of stars. Suppose that a star of some type has velocity $\mathbf{v}$ in the frame in which that type of stars is at rest on average. Let $\mathbf{v}_\odot$ be the velocity of the Sun in that frame (the Solar motion). The line of sight velocity of the $k^{\rm th}$ star is just

$$v_{{\rm los}k} = \hat{\mathbf{x}} \cdot \mathbf{v}_k - v_\odot \cos\psi_k, \quad (1)$$

where $\hat{\mathbf{x}}_k$ is a unit vector from the Sun to the star and ψ_k is the angle between $\mathbf{v}_\odot$ and $\hat{\mathbf{x}}_k$. Suppose we average this line of sight velocity over a large number of stars all seen in approximately the same direction. Then $\langle \hat{\mathbf{x}} \cdot \mathbf{v}_k \rangle \approx \hat{\mathbf{x}} \cdot \langle \mathbf{v}_k \rangle \approx 0$ since the mean velocity of the stars must be zero in their rest frame. Therefore

$$\langle v_{\rm los} \rangle \approx -v_\odot \cos\psi. \quad (2)$$

Measuring $\langle v_{\rm los} \rangle$ as a function of position therefore allows us to infer $v_\odot$.

Similarly, suppose we average the proper motions of stars which all lie in approximately the same direction $\hat{\mathbf{x}}$ and approximately at the same distance d:

$$\langle \boldsymbol{\mu} \rangle = \left\langle \frac{((\mathbf{v}_k - \mathbf{v}_\odot) \times \hat{\mathbf{x}}_k) \times \hat{\mathbf{x}}_k}{|\mathbf{x}_k|} \right\rangle$$

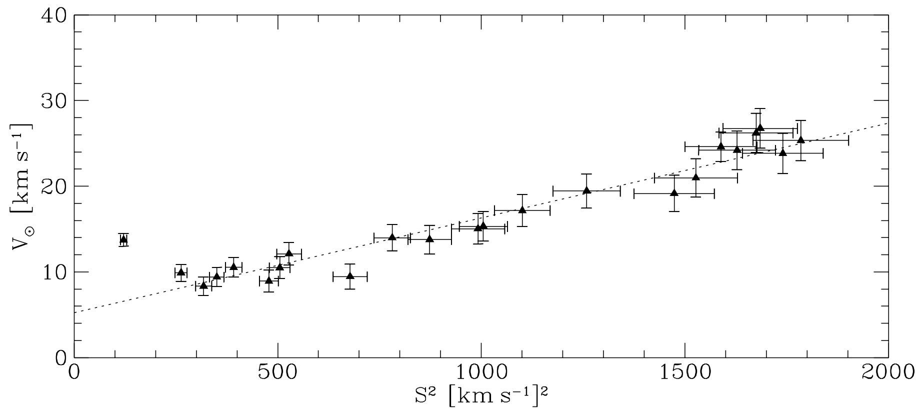

Figure 1: The V-component of Solar motion relative to different stellar types as a function of the random velocity S^2 of each type.

$$
\begin{aligned}
&\approx \frac{((\mathbf{v}_k - \mathbf{v}_\odot) \times \hat{\mathbf{x}}) \times \hat{\mathbf{x}}}{d} \\
&= -\frac{1}{d}(\mathbf{v}_\odot \times \hat{\mathbf{x}}) \times \hat{\mathbf{x}} \\
&= \frac{1}{d}(\mathbf{v}_\odot - v_\odot \cos\phi \hat{\mathbf{x}}). \qquad (3)
\end{aligned}
$$

The direction of the Solar motion is easily found by looking for where $\langle \boldsymbol{\mu} \rangle = 0$ but the magnitude, $v_\odot$, requires an estimate of the distance to the stars.

If we measure $\mathbf{v}_\odot$ in this way for many different types of star we find a strong correlation between V (azimuthal) component and the mean squared random velocity, S^2, of those stars as shown in Fig. 1. This is exactly what we'd expect from our previous study of asymmetric drift—classes of star with greater random velocities rotate more slowly around galactic center (they essentially have a degree of pressure support so don't have to rotate as fast to stay in orbit). The LSR is defined as the mean motion of a population with zero random velocities. We can therefore infer $V_\odot$ relative to such a hypothetical population by extrapolating the relation to $S^2 = 0$. The result of this is that the Sun moves relative to the LSR at a velocity:

$$
\left.
\begin{aligned}
U_\odot &= 10.0 \pm 0.4 \text{km s}^{-1} \\
V_\odot &= 5.2 \pm 0.6 \text{km s}^{-1} \\
W_\odot &= 7.2 \pm 0.4 \text{km s}^{-1}
\end{aligned}
\right\} \Rightarrow |\mathbf{v}_\odot| = 13.4 \text{km s}^{-1}. \qquad (4)
$$

1.2 Vertex Deviation

We previously considered the possibility that the velocity ellipsoid in the galaxy may not be aligned with the principal axes of the coordinate system. If we locally define a Cartesian coordinate system with x pointing towards Galactic center, y in the direction of rotation of the disk and z normal to the plane of the disk then we can test for any such *vertex deviation* by considering correlations such as $\langle v_x(v_y - \langle v_y \rangle \rangle$. (It turns out that correlations involving v_z are zero.) It turns out that

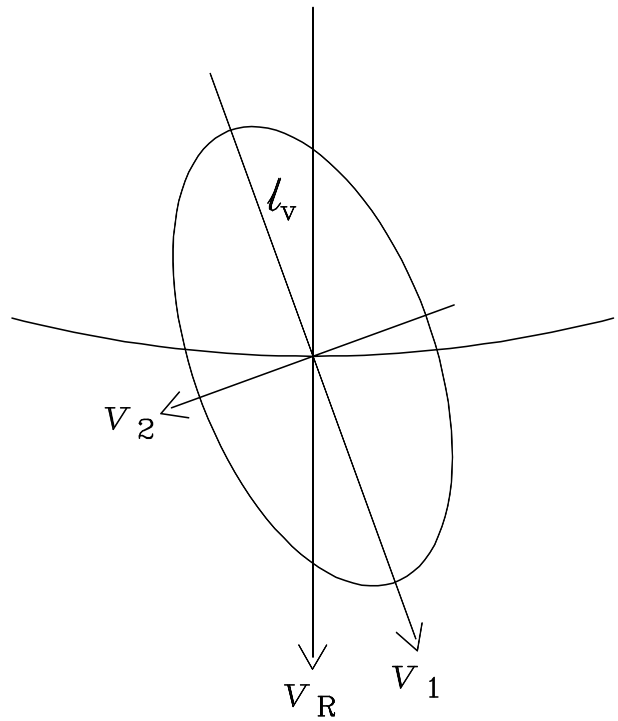

Figure 2: Orientation of the velocity ellipsoid. Galactic center is towards the top of the page. The Sun lies at the center of the ellipse and moves towards the left.

the average $\langle v_x(v_y - \langle v_y\rangle\rangle$ is non-zero for many stellar types. It's therefore useful to find linear combinations of v_x and $(v_y - \langle v_y\rangle$ that are statistically independent (i.e. uncorrelated):

$$\begin{aligned} v_1 &\equiv v_x \cos l_v - (v_y - \langle v_y\rangle) \sin l_v \\ v_2 &\equiv v_x \sin l_v - (v_y - \langle v_y\rangle) \cos l_v \end{aligned} \tag{5}$$

where the angle l_v is the vertex deviation. Taking the average of the product of these quantities gives

$$\langle v_1 v_2\rangle = \frac{1}{2}(\sigma_x^2 - \sigma_y^2) \sin 2l_v + \langle v_x(v_y\langle v_y\rangle)\rangle \cos 2l_v, \tag{6}$$

which will be statistically independent (i.e. $\langle v_1 v_2\rangle = 0$) if

$$l_v = \frac{1}{2} \arctan\left(\frac{2\langle v_x(v_y\langle v_y\rangle)\rangle}{\sigma_x^2 - \sigma_y^2}\right). \tag{7}$$

The linear transformation to v_1 and v_2 is just a rotation of coordinates. The angle l_v therefore measures the angle between the axes of our coordinate system and the principal axes of the velocity ellipsoid (see Fig. 2).

Observationally, vertex deviations of 10–30° are measured (the largest values being for bluest stars). Why do we see vertex deviation? If the Galaxy was axisymmetric, in a steady state with stars distributed randomly along their orbits then we'd expect no vertex deviation purely from the symmetry of the system. That we see l_v differ from zero therefore implies that at least one of these conditions does not hold. One contribution to vertex deviation comes from *moving groups*—groups of stars which share a common motion (due to a common origin). This significantly reduces the number of independent velocities in any determination of velocity correlations and leads to enhanced noise in estimates of the vertex deviation. However, there may also be a more physical origin: the Galaxy isn't axisymmetric as it contains spiral structure. We saw that spiral density waves lead to perturbations in the gravitational potential and, consequently, perturbations in the velocities of stars. These non-axisymmetric perturbations to the velocity can lead to vertex deviation.

1.3 Oort's Constants

We've so far assumed that the Local Standard of Rest is the same throughout the Solar neighborhood. However, when determining the LSR from studies of stars we often have to use a neighborhood that is sufficiently large that we might expect some variation in the SR throughout that volume. We can quantify such effects and, as a bonus, will demonstrate that the motion of stars in the disk is close to circular (rather than being elliptical) and constrain the rotation speed.

We begin by defining a Standard of Rest at every point $\mathbf{x}$ by analogy with our definition of the LSR—it should correspond to the velocity of a star on a closed orbit passing through that point. If the Milky Way disk were everywhere circular the SR would always equal the circular speed and be perpendicular to the radius vector, but our definition is more general. We can consider the variation in the SR around the Sun: $\delta\mathbf{v}(\mathbf{x}) = \mathbf{v}(\mathbf{x}) - \mathbf{v}(\mathbf{x}_\odot)$. Expanding this in a Taylor series we can write the components of this vector as

$$\begin{aligned} \begin{pmatrix} \delta v_x \\ \delta v_y \end{pmatrix} &= \begin{pmatrix} \frac{\partial \delta v_x}{\partial x} & \frac{\partial \delta v_x}{\partial y} \\ \frac{\partial \delta v_y}{\partial x} & \frac{\partial \delta v_y}{\partial y} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \mathcal{O}(x^2+y^2) \\ &= \begin{pmatrix} k+c & a-b \\ a+b & k-c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \mathcal{O}(x^2+y^2), \end{aligned} \tag{8}$$

where the partial derivatives are evaluated at the origin and a, b, c and k are linear combinations of those partial derivatives (we'll see why we want to write them in this way later). The mean line of sight velocity for stars at some position $\mathbf{x} = (x, y)$ is then

$$\begin{aligned} v_{\rm los} &= \frac{1}{d}\mathbf{x}\cdot\delta\mathbf{v} \\ &\approx \frac{1}{d}[(k+c)x^2 + (k-c)y^2 + 2axy], \end{aligned} \tag{9}$$

where $d = (x^2+y^2)^{1/2}$ is the heliocentric distance. In terms of Galactic longitude, $x = d\cos l$ and $y = d\sin l$ so

$$v_{\rm los} = d(k + c\cos 2l + a\sin 2l) + \mathcal{O}(d^2). \tag{10}$$

If we can select a set of stars all at similar distances (e.g. of the same spectral type and apparent magnitude) then by measuring the line of sight velocity as a function of l we could infer a, c and k. Similarly, the constant b can be determined from proper motions since the tangential velocity is

$$\begin{aligned} \delta v_{\rm t} &= \frac{1}{d}(\mathbf{x}\times\delta\mathbf{v})_z = \frac{1}{d}(x\delta v_y - y\delta v_x) \\ &= d(b + a\cos 2l - c\sin 2l) + \mathcal{O}(d^2). \end{aligned} \tag{11}$$

The above makes no assumptions about the form of $\mathbf{v}_{\rm SR}$, only that is varies smoothly. If we assume circular motion with angular frequency $\Omega(R)$ then the results simplify. Expanding our previous exact expression for $v_{\rm los}$ under circular motion and keeping only the first order terms gives

$$\begin{aligned} v_{\rm los}(l,R) &\approx \left.\frac{{\rm d}\Omega}{{\rm d}R}\right|_{R_0}(R-R_0)R_0\sin l \\ &= -2A(R-R_0)\frac{R_0}{R}\sin l, \end{aligned} \tag{12}$$

where *Oort's constant* A is defined by

$$A \equiv -\frac{1}{2}\left(R\frac{{\rm d}\Omega}{{\rm d}R}\right)_{R_0} = \frac{1}{2}\left(\frac{v_{\rm c}}{R}-\frac{{\rm d}v_{\rm c}}{{\rm d}R}\right)_{R_0}. \tag{13}$$

In the Solar neighborhood, $d \ll R$ so

$$(R-R_0)(R+R_0) = R^2 - R_0^2 \approx -2R_0 d\cos l, \tag{14}$$

and since $R+R_0 \approx 2d$ this gives

$$(R-R_0) = -d\cos l. \tag{15}$$

Combining these results

$$v_{\rm los} \approx Ad\sin 2l. \tag{16}$$

Evidently in this special case of circular motion $a=A$, $c=k=0$. For the tangential velocities under circular motion we similarly find

$$\begin{aligned} v_{\rm t} &= \left(\frac{\mathbf{R}-\mathbf{R}_0}{|\mathbf{R}-\mathbf{R}_0|}\times[\boldsymbol{\Omega}(R)\times\mathbf{R}-\boldsymbol{\Omega}(R_0)\times\mathbf{R}_0]\right)_z \\ &= \left(\frac{\mathbf{R}-\mathbf{R}_0}{d}\times\{\boldsymbol{\Omega}(R)\times(\mathbf{R}-\mathbf{R}_0)+[\boldsymbol{\Omega}(R)-\boldsymbol{\Omega}(R_0)]\times\mathbf{R}_0\}\right)_z \\ &\approx \Omega_z(R)d + \left.\frac{{\rm d}\Omega_z}{{\rm d}R}\right|_{R_0}\frac{(\mathbf{R}-\mathbf{R}_0)\cdot\mathbf{R}_0}{d}. \end{aligned} \tag{17}$$

Using the fact that $(\mathbf{R}-\mathbf{R}_0)\cdot\mathbf{R}_0 = -dR_0\cos l$ this simplifies to

$$\begin{aligned} v_{\rm t} &\approx \Omega_z(R_0)d + dR_0\left.\frac{{\rm d}\Omega_z}{{\rm d}R}\right|_{R_0}\cos^2 l \\ &= d\left(\Omega_z(R_0)+\frac{1}{2}R_0\left.\frac{{\rm d}\Omega_z}{{\rm d}R}\right|_{R_0}(1+\cos 2l)\right). \end{aligned} \tag{18}$$

Since the Milky Way rotates clockwise $\Omega_z = -\Omega$ and so we defined Oort's constant B by

$$\begin{aligned} B &\equiv -\left(\Omega+\frac{1}{2}R\frac{{\rm d}\Omega_z}{{\rm d}R}\right)_{R_0} \\ &= -\frac{1}{2}\left(\frac{v_{\rm c}}{R}+\frac{{\rm d}v_{\rm c}}{{\rm d}R}\right)_{R_0}. \end{aligned} \tag{19}$$

Our earlier expression for proper motion can then be written

$$\mu_l = \frac{v_t}{d} = B + A\cos 2l, \tag{20}$$

which implies that $b = B$ for circular motion. A measures the shear in the motion near the Sun—it would be zero if the material were rotating as a solid body. B measures the vorticity of the motion, the tendency for the stars to circulate around a given point. From the above definitions:

$$v_c = R_0(A - B) \text{ and } \left.\frac{\mathrm{d}v_c}{\mathrm{d}R}\right|_{R_0} = -(A + B). \tag{21}$$

Observationally it is found that

$$c = 0.6 \pm 1.1\text{km s}^{-1}\text{ kpc}^{-1}, \qquad k = -0.35 \pm 0.5\text{km s}^{-1}\text{ kpc}^{-1}, \tag{22}$$

which are both consistent with zero. We therefore set them to zero and associate $a = A$ and $b = B$.

A can be determined from studies of radial velocities or proper motions while B can only be determined from proper motion studies. Such studies find

$$A = 14.82 \pm 0.84\text{km s}^{-1}\text{ kpc}^{-1}, \qquad B = -12.37 \pm 0.64\text{km s}^{-1}\text{ kpc}^{-1}. \tag{23}$$

2 Structure of the Stellar Disk

2.1 The Thick Disk

Determination of the variation of the density of a given type of star with distance above the Galactic plane is most easily done via star counts towards the Galactic poles. The number of stars per unit area is counted as a function of their apparent magnitude and their distance (usually determined photometrically from their colors or from parallaxes where available). The relative number density of stars of given absolute magnitude can then be derived as a function of distance from the plane, z.

Observationally, it is found that brighter stars show a more rapid decline in density with z, i.e. they are more confined to the plane than fainter stars. These stars also have lower velocity dispersions than fainter stars (as expected: lower velocity dispersion means that the bright stars don't have the kinetic energy required to rise vary far above the plane and so we should expect them to be confined close to $z = 0$). These facts suggests that stars are born close to the plane, with low velocity dispersion and are gradually "heated", gaining velocity dispersion and rising further above the plane. Bright stars with short lifetimes don't live long enough to be heated significantly and so are only ever seen close to the plane. (This is one explanation of the observed facts, there are others which involve the fainter, older stars being born with a broader distribution in z rather than being heated into this distribution.)

The derived vertical density profile for $4 < M_V < 5$ main sequence stars is shown in Fig. 3. The density profile clearly differs from a single exponential $\nu \propto \exp(-z/z_0)$ (which would be a straight line in this plot). However, it is well fit by the sum of two exponentials with different scale-heights z_0.

We could interpret this in two different ways: 1) The disk is made up of two components, a thin and thick disk, each with exponential profiles or, 2) since we have no a priori reason to expect that disks

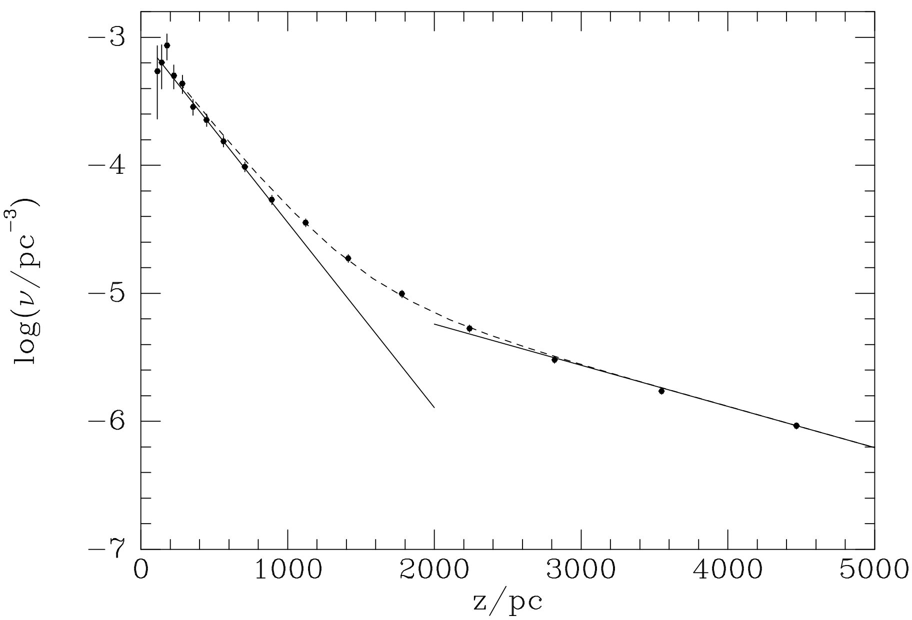

Figure 3: Space density of MS stars above the plane of the disk. Solid lines are exponentials with scale heights of 300pc and 1350pc. The dashed curve shows the sum of these two exponentials.

just have exponential profiles, maybe there is just a single disk which happens to have this particular profile. It's impossible to say anything more about this from the density profile alone. The first possibility would only be physically meaningful if there were some other discernable difference between stars in the two components. In fact, it is found that stars in the thick component (i.e. those at large z) are significantly older and less metal enriched than stars in the thin disk. This suggests that there is a real, physical difference between the thin and thick components, perhaps suggesting different formation mechanisms or, at least, different dynamical histories (e.g. perhaps the thick disk was originally thin, but formed before some significant heating event which thickened it significantly).

2.2 Local Mass Density of the Disk

A key quantity in any understanding of the structure of our Galaxy is the density of the local disk. The only real way to measure the mass of anything astronomically is to measure the strength of its gravitational field, $\mathbf{F}$, and then infer the density through Poisson's equation:

$$\nabla \cdot \mathbf{F} = -4\pi \mathrm{G}\rho. \tag{24}$$

Assuming the Galaxy to be axisymmetric the logical coordinate system to use is a cylindrical coordinate system in which this equation reads

$$\frac{1}{R}\frac{\partial}{\partial R}(RF_R) + \frac{\partial F_z}{\partial z} = -4\pi \mathrm{G}\rho. \tag{25}$$

The circular speed obeys $v_c^2/R = -F_R$ and so

$$\rho = -\frac{1}{4\pi G}\left(\frac{\partial F_z}{\partial z} - \frac{1}{R}\frac{\partial v_c^2}{\partial R}\right). \tag{26}$$

Since the circular speed is approximately constant with R the second term on the right is small compared to the first. Additionally, mass models of the Galaxy (which we'll dicuss soon) suggest that this term depends only weakly on z for $z \ll R$. Therefore, we approximate it as constant and integrate the density over the z direction to get the surface density within some height z above the plane:

$$\Sigma(R, z) \equiv 2\int_0^z dz' \rho(R, z') \approx -\frac{1}{2\pi G}\left(F_z(R, z) - \frac{z}{R}\frac{\partial v_c^2}{\partial R}\right), \tag{27}$$

where we've assumed $F_z(R, 0) = 0$ as appropriate for a disk symmetric about the plane $z = 0$. Close to the plane we expect the thin disk to dominate the mass density, so, crudely, we could take $\Sigma(R, 3z_0)$ to be the mass density of the disk. (A more careful analysis would attempt to subtract off any contibution from other mass components.)

We therefore need to determine $F_z(R, z)$. The basic idea behind how we do this goes as follows: Consider some population of stars lying at a distance z from the plane and with a z velocity dispersion of σ_z. We then identify the same population at $z + \delta z$ and find that the density of such stars is lowe than at z. The argument is that the decline in density is due to the fact that the slower stars at z don't have enough energy to climb up the gravitational potential to $z + \delta z$. Therefore, the greater $F_z(R, z)$ the steeper the potential gradient and the fewer stars will get to $z + \delta z$ leading to a more rapid decline in density with z. Classic studies of this type made use of the Jeans equation to express this mathematically:

$$\nu F_z = \frac{\partial \nu \sigma_z^2}{\partial z} + \frac{1}{R}\frac{\partial}{\partial R}(R\nu\sigma_{Rz}^2), \tag{28}$$

where σ_{Rz}^2 is the average over the population of the product $v_R v_z$. This term actually complicates the analysis significantly as stars can have significant radial velocity dispersion—this cross term arises due to a coupling between radial and vertical motions. Early studies simply neglected this term, but it turns out that it's not negligble and corrections for it must be included.

Current estimates suggest that the surface density within 1.1kpc of the plane is about

$$\Sigma_{1.1}(R_0) = 71 \pm 6 M_\odot \mathrm{pc}^{-2}. \tag{29}$$

Using mass models of the Galaxy to correct for non-disk contributions this number is reduced somewhat to

$$\Sigma_d(R_0) = 48 \pm 9 M_\odot \mathrm{pc}^{-2}. \tag{30}$$

3 Galaxy Models

Having developed some knowledge of the various structural components of the Milky Way and their kinematics it is obviously useful to attempt to put together models which incorporate all of these components and which attempt to match the observational constraints. There are several types of such *galaxy models* which we'll review briefly.

3.1 Mass Models

On obvious approach to do is to specify the mass density distribution of each component of the galaxy using simple, parametric forms, and then adjust the parameters of those forms to fit experimental data (such as the rotation curve, velocity dispersions, disk surface density etc.). This is a well-developed approach. Modern mass models include several components: stellar bulge, thin and thick stellar disks (usually modelled as exponentials in radius and height), a gaseous ISM disk, stellar halo and dark matter halo.

Perhaps not surprisingly, given this large number of components, there is often more than one set of parameters which gives a good fit to the available data. In such situations, one can either wait for better data to be obtained or can attempt to constrain the models further using theoretical expectations. For example, relatively recent work by Klypin et al.[1] impose theoretical expectations for dark matter halo mass and shape and disk mass from the favored Λ-dominated cold dark matter cosmological model to constrain mass models of the Milky Way and M31.

3.2 Star Count Models

A related type of model is the star count model which attempts to predict the number of stars of magnitude M at each position $\mathbf{x}$. It's easy to see that such a model can be constructed by assigning a (possibly position dependent) stellar luminosity function $\Phi(M, \mathbf{x})$ to each component in a mass model. The star counts can then be found by summing the luminosity functions from each component.

3.3 Kinematic Models

Since we can measure the velocity dispersions of stars as a function of position in the Milky Way (from proper motions or radial velocities) it's useful to be able to predict these quantities from a model. A *kinematic model* does just that. Frequently, such models assume that the velocity distribution at each point is given by a Schwarzchild distribution (essentially a triaxial Gaussian) with some additional net streaming motion. These models require specification of the principal axes of this ellipse (directions and magnitudes) at each point together with the streaming motion. This makes them difficult to construct and, in any case, the Schwarzchild distribution is really just an assumption so may not be valid.

3.4 Dynamical Models

As we've seen, dynamics connects the kinematics to the density distribution since stars at some point $\mathbf{x}$ moving with some velocity $\mathbf{v}$ must at a later time find themselves at a new position $\mathbf{x}_1$.

[1] http://adsabs.harvard.edu/abs/2002ApJ...573..597K

If the galaxy is in a steady state then the velocity distributions at these two points cannot be independent.

In principle, Jeans equations describe this connection between structure and kinematics. One could therefore use them to compute a kinematic model given a mass model. Unfortunately, the Jeans equations don't have tractable boundary conditions, so in practice this procedure does not work. A more elaborate approach is to attempt to construct an N-body model which looks just like the Galaxy. This would then predict the density and kinematics at all points. Unfortunately, the very reason why we carry out N-body simulations (to solve the complicated non-linear dynamics of gravitating systems) means that it's not easy to guess a set of initial conditions which will relax to form something which looks like the Milky Way.