Ay 124 — Final Exam (Theoretical Part)

Posted on Friday, March 13—Due by 5 pm on Wednesday, March 18 (directly to the TA)

Note:

You have up to 3.5 hours to do this part, and it counts for 65% of your final exam grade. Then you have up to 2 hours to do the observational part, which counts for 35%. You can do them in any order, with a break in between. Please mark your exams with the start and end times.

The Rules:

Closed book, closed notes, etc. No collaboration. You cannot discuss the exam with anyone, until everyone in the class has turned in their test.

Good Luck!

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1. [10 points] Suppose that spherical globular cluster evolves self-similarly as a result of relaxation. In this case its evolution can be described by two functions $M(t)$ and $R(t)$, the mass and characteristic radius as functions of time. Since the evolution is driven by relaxation, we expect that

$$\frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{C_M}{t_{\mathrm{relax}}}; \qquad \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{C_R}{t_{\mathrm{relax}}}, \tag{1}$$

where C_M and C_R are constant of order unity. Neglecting changes in the Coulomb logarithm, the relaxation time is $t_{\mathrm{relax}} \propto R^{3/2}M^{1/2}$.

a) If the evolution is dominated by evaporation, we expect that $M(t)$ declines with time while the cluster energy $E \propto \mathrm{G}M^2/R$ remains constant, since evaporating stars leave with nearly zero energy. Derive how M and R scale with $\tau = t_0 - t$ where t_0 is the time at which the cluster mass reaches zero.

b) Afer core collapse, the evolution of the cluster is dominated by the energy input from binary stars at the cluster center, so the cluster energy E grows but the mass M remains approximately constant. In this case derive the scaling of R with $\tau = t - t_{\mathrm{c}}$ where t_{c} is the time of core collapse.

2. a) [15 points] The velocity at a point in an expanding ring is $\mathbf{\Omega} \times \mathbf{R} + (v_e/R)\mathbf{R}$, where $\mathbf{R}$ is the Galactocentric position vector of the point. For such a ring, show that the line of sight velocity may be written

$$v_{\mathrm{los}}(l) = [\Omega(R) - \Omega(R_0)]R_0 \sin l + \frac{d^2 + R^2 - R_0^2}{2Rd}v_e, \tag{2}$$

where d is the heliocentric distance to the point in the ring that lies along the line of sight at Galactic latitude l. Show that, in the limit $R \to \infty$, the last term tends to v_e.

b) [5 points] If the circular speed of the Milky Way is constant at $v_{\mathrm{c}} = 220$km/s from R_0 outwards, and no 21-cm emission is seen beyond $v_{\mathrm{los}} = 130$km/s at $l = 225°$, derive an upper limit to the radius of the HI disk under the assumption that the disk is perfectly flat.

3. [20 points] By writing Poisson's equation, $\nabla \cdot \mathbf{F} = -4\pi\mathrm{G}\rho$, in spherical polar coordinates, show that the density ρ_{H} of a spherical halo is connected to the contribution v_{H}^2 that it makes to the Milky Way's squared circular speed v_{c}^2 by

$$\rho_{\mathrm{H}} = \frac{1}{4\pi\mathrm{G}r^2}\frac{\partial}{\partial r}(rv_{\mathrm{H}}^2). \tag{3}$$

Hence show from the equation derived for $\Sigma(R,z)$ in class that a good estimate of the surface density of the disk in the solar neighborhood, $\Sigma_{\mathrm{D}}(R_0)$, is given by

$$2\pi\mathrm{G}\Sigma_{\mathrm{D}}(R_0) - \frac{3z_0}{R_0}\left(\frac{\partial(Rv_{\mathrm{D}}^2)}{\partial R}\right)_{R_0} = -F_z(R_0, 3z_0) - \frac{3z_0 v_{\mathrm{c}}^2}{R_0^2}, \tag{4}$$

where $v_{\mathrm{D}}^2 \equiv v_{\mathrm{c}}^2 - v_{\mathrm{H}}^2$. If the disk is assumed to be thin and exponential then the left-hand side of this equation is a known multiple of $\Sigma_{\mathrm{D}}(R_0)$ and the right-hand side can be observationally determined.

4. a) [10 points] Derive the scaling of galaxy radius and mass with circular velocity. You may assume that all dark matter halos have the same spin parameter, λ, the same mean density within their virial radius, have flat rotation curves and that the entire baryonic content of each halo cools and forms a galactic disk. You may ignore the self-gravity of the galaxy and any backreaction on the dark matter.

   b) [5 points] Assume now that supernovae feedback is effective and expels gas from the forming galaxies at a rate $\beta = (V_c/200\text{km/s})^{-2}$ relative to the star formation rate. Assuming again that all gas in each halo cools to form a galactic disk and that feedback can eject the gas at above the escape velocity of the halo (so that it never returns) derive the scaling of galaxy mass with V_C. If the probability for gas to be ejected is independent of location within the disk (and therefore independent of the specific angular momentum of the gas) how is the scaling of the disk radius with V_c altered by this feedback?