Ay124: Homework #1 Solution Set

Problem #1. Apparent Bolometric Magnitudes of

(a) Sun-like star: For the sun: $M = 4.72$, D=50 pc

$$m = M - 5 + 5\log(D/\text{pc}) = 8.21$$

(b) Light Bulb

$$M_{LB} - M_{\odot} = -2.5\log\frac{L_{LB}}{L_{\odot}} = 61.5$$

$$M_{LB} = 66.2$$

$$D = 384400 \text{ km} = 1.3 \times 10^{-8} pc$$

$$m = 66.2 - 5 + 5\log(1.3 \times 10^{-8}) = 21.7$$

(c) Galaxy

$$L_{Gal} = 1.5 \times 10^{10} L_{\odot}$$

$$M_{Gal} - M_{\odot} = -2.5\log\frac{L_{Gal}}{L_{\odot}} = -25.44$$

$$M_{Gal} = -20.72$$

$$m = -20.72 - 5 + 5\log(2 \times 10^{7}) = 10.79$$

(d) Quasar

$$M_Q - M_{\odot} = -2.5\log\frac{10^{46}}{3.8 \times 10^{33}} = -31.05$$

$$M_Q = -26.33$$

$$m = -26.33 - 5 + 5\log 10^{9} = 13.67$$

By the way, (a) is a typical star in one of the major catalogs (HD, BD, SAO, etc.), (c) is a bright galaxy in the Virgo cluster, (d) is similar to 3C273 or 3C48 (the first quasars discovered).

Problem #2. Galaxy flux

Flux from a $V = 22^m$ galaxy

For $V = 0^m$, the flux is:

$$F_{\nu,0} = 3800 \text{ Jy} = 3.8 \times 10^{-20} \text{erg/cm}^2\text{/s/Hz}$$

$$m_{Gal} - m_0 = -2.5\log\frac{F_{\nu,Gal}}{F_{\nu,0}} = 22 - 0 = 22$$

$$\log\frac{F_{\nu,Gal}}{F_{\nu,0}} = -8.8$$

$$F_{\nu,Gal} = 10^{-8.8} \times F_{\nu,0}$$

$$F_{\nu,Gal} = 6.0 \times 10^{-29}\text{erg/cm}^2\text{/s/Hz}$$

We also want F_λ.

$$F_\lambda d\lambda = F_\nu d\nu$$

$$F_\lambda = F_\nu \frac{d\nu}{d\lambda}$$

Since $\nu = c/\lambda$, $\frac{d\nu}{d\lambda} = \frac{c}{\lambda^2}$ We approximate the V band as $\lambda = 5500\mathring{A} = 5.5 \times 10^{-5}$ cm

$$F_\lambda = \frac{3 \times 10^{10}}{(5.5 \times 10^{-5})^2} F_\nu = 9.9 \times 10^{18} F_\nu$$

$$F_\lambda = 6.0 \times 10^{-10}\text{erg/cm}^2\text{/s/cm} = 6.0 \times 10^{-18}\text{erg/cm}^2\text{/s/Å}$$

The energy of a typical photon in the V band is $E = \frac{hc}{\lambda}$. Using our value of lambda above, $h = 6.63 \times 10^{-27}$erg s, and $c = 3 \times 10^{10}$cm/s, we find that $E = 3.62 \times 10^{-12}$ erg.

$$\text{PHOTON RATE} = \frac{F_\lambda \, \Delta\lambda \, \pi R^2}{E}$$

$$\Delta\lambda = 900\mathring{A}$$

$$R = 254 \text{ cm}$$

$$\text{PHOTON RATE} = 300 \text{ photons/sec}$$

Problem #3. Sirius A and B.

First, find the ratio of the masses from the ratio of the distances from the center of mass:

$$\frac{m_A}{m_B} = \frac{r_B}{r_A} = \frac{1}{0.466} = 2.15$$

Next, calculate the total mass from Kepler's third law:

$$P^2 = \frac{4\pi^2 a^3}{G(m_A + m_B)}$$

The distance to the star system is

$$d = \frac{1}{0.377''} = 2.65 \text{ pc}$$

So the semimajor axis is

$$a = d \times 7.62'' \times \frac{1 \text{ rad}}{206265''} = 9.8 \times 10^{-5} pc = 3.03 \times 10^{14} \text{ cm} = 20.2 \text{ AU}$$

$$P = 49.94 \text{ yr} = 1.6 \times 10^{9} \text{ s}$$

Plugging in to Kepler's third law, we find

$$m = m_A + m_B = 6.70 \times 10^{33} \text{ g} = 3.35 \ M_\odot$$

Then compute the individual masses:

$$m_A = \frac{m}{1 + \frac{m_B}{m_A}} = 4.57 \times 10^{33} \text{g} = 2.28 \ M_\odot$$

$$m_B = m - m_A = 2.13 \times 10^{33} \text{ g} = 1.06 \ M_\odot$$

The bolometric luminosities relative to the sun are given by:

$$M_{BOL} - M_{BOL,\odot} = -2.5 \log \frac{L}{L_\odot}$$

$$M_{BOL,\odot} = 4.72, M_A = 1.33, M_B = 8.57$$

So $L_A = 22.7 \ L_\odot$ and $L_B = 2.88 \times 10^{-2} \ L_\odot$

Next, compute the radius of Sirius B ($T = 2.7 \times 10^4$K) from the Stefan-Boltzmann equation:

$$L = 4\pi R^2 \sigma T^4$$

$$R = 5.4 \times 10^{3} \text{ km} = 0.85 \ R_{Earth} = 7.8 \times 10^{-3} \ R_\odot$$

Note that this is the classic estimate of the radius of the first discovered white dwarf, Sirius B. White dwarfs contain the mass of about 1 $M_\odot$ inside a radius the size of the Earth!

Problem #3. Star Cluster

(a) Average stellar mass

$$< m >= \frac{\int m dN}{\int dN}$$

$$< m >= \frac{\int_{m_{min}}^{m_{max}} m \frac{dN}{dM} \quad dm}{\int_{m_{min}}^{m_{max}} \frac{dN}{dM} dm}$$

$$\frac{dN}{dM} = km^{-(1+x)}$$

$$< m >= \frac{\int_{m_{min}}^{m_{max}} \quad m^{-x} \quad dm}{\int_{m_{min}}^{m_{max}} \quad m^{-(1+x)} \quad dm}$$

$$< m >= \frac{-x}{1-x} \quad \frac{m_{max}^{1-x} - m_{min}^{1-x}}{m_{max}^{-x} - m_{min}^{-x}}$$

For our case $x = 1.35$, $m_{min} = 0.08 M_\odot$,$M_{max} = 80 M_\odot$ Plugging in those values to the equation above yields:

$$< m >= 0.28 M_\odot$$

(b) Average stellar luminosity Note that we can easily see the value of the proportionality constant in the mass-luminosity relationship since we know the Sun's values.

$$\frac{L}{L_\odot} = \frac{m^4}{M_\odot^4}$$

¿From now on, I will simply assume that L,m are measured in solar units to avoid writing the constants every line! We evaluate the average in the same way as part (a):

$$< L >= \frac{\int L \frac{dN}{dM} dm}{\int \frac{dN}{dM} dm}$$

$$< L >= \frac{\int_{m_{min}}^{m_{max}} \quad m^{3-x} \quad dm}{\int_{m_{min}}^{m_{max}} \quad m^{-(1+x)} \quad dm}$$

$$< L >= \frac{-x}{4-x} \quad \frac{m_{max}^{4-x} - m_{min}^{4-x}}{m_{max}^{-x} - m_{min}^{-x}}$$

Using our values for this cluster:

$$< L >= 1860 L_\odot$$

(c) Bolometric magnitude

$$M_{TOT} = 10^3 M_\odot$$

$$N_{stars} = \frac{M_{TOT}}{< m >} = \frac{10^3}{0.28} = 3600$$

$$L_{TOT} = N_{stars} \times < L >= 6.6 \times 10^6 L_\odot$$

$$M_{BOL} = M_{BOL,\odot} \ - 2.5 \log \frac{L_{TOT}}{L_\odot} = -12.3$$

(d) Mass distribution

$$M_+ = \frac{\int_{m_\odot}^{m_{max}} m^{-x} dm}{\int_{m_{min}}^{m_{max}} m^{-x} \ dm}$$

$$M_+ = \frac{1-x}{1-x} \ \frac{m_{max}^{1-x} - m_\odot^{1-x}}{m_{max}^{1-x} - m_{min}^{1-x}}$$

For our cluster:

$$M_+ = 0.36$$

Clearly then the fraction of mass due to stars below one solar mass is:

$$M_- = 0.64$$

(e) Luminosity distribution

$$L_+ = \frac{\int_{m_\odot}^{m_{max}} m^4 \ m^{-(1+x)} \ \ dm}{\int_{m_{min}}^{m_{max}} m^4 \ m^{-(1+x)} \ \ dm}$$

$$L_+ = \frac{m_{max}^{4-x} - m_\odot^{4-x}}{m_{max}^{4-x} - m_{min}^{4-x}}$$

$$L_+ = 0.999991$$

$$L_- = 0.000009$$

High mass stars completely dominate the luminosity even though they are less than half of the mass!