Problem Set #4: Elements of Stellar Dynamics

Structure and Dynamics of Galaxies, Ay 124, Winter 2009

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1. [Binney & Tremaine, Problem 4-3] By analogy with the derivation for cylindrical coordinates, show that in spherical cooordinates the collisionless Boltzmann equation is

$$\begin{aligned} 0 \quad = \quad & \frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r}\frac{\partial f}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial f}{\partial \phi} + \left(\frac{v_\theta^2 + v_\phi^2}{r} - \frac{\partial \Phi}{\partial r}\right)\frac{\partial f}{\partial v_r} \\ & + \frac{1}{r}\left(v_\phi^2 \cot\theta - v_r v_\theta - \frac{\partial \Phi}{\partial \theta}\right)\frac{\partial f}{\partial \theta} - \frac{1}{r}\left[v_\phi(v_r + v_\theta\cot\theta) + \frac{1}{\sin\theta}\frac{\partial \Phi}{\partial \phi}\right]\frac{\partial f}{\partial v_\phi} \qquad (1) \end{aligned}$$

2. [Binney & Tremaine, Problem 4-5] Suppose the principal axes of the velocity ellipsoid near the Sun are always parallel to the unit vectors of spherical coordinates. Then show that for $|z|/R$ small, $\overline{v_R v_z} \approx (\overline{v_R^2} - \overline{v_\theta^2})(z/R)$. (Hint: Write v_R and v_z in terms of v_r and v_θ, and then average $v_R v_z$ using $\overline{v_r v_\theta} = 0$.)

3. [Binney & Tremaine, Problem 4-10] In a spherical stellar system with mass profile $M(r)$, a stellar population with number density $\nu(r)$ has anisotropy parameter $\beta \equiv 1 - (\overline{v_\theta^2}/\overline{v_r^2}) = 1 - (\overline{v_\phi^2}/\overline{v_r^2})$ of the form $\beta(r) = r^2/(r_a^2 + r^2)$, where r_a is a constant. Show that

$$\overline{v_r^2} = \frac{\mathrm{G}\int_r^\infty [(r_a/r')^2 + 1]\nu(r')M(r')\mathrm{d}r'}{(r_a^2 + r^2)\nu(r)}. \qquad (2)$$