Solutions for Ay 124 Problem Set 3

1 - Sersic profile $I(r) = I_0 \exp\{-b_n (r/r_e)^{1/n}\}$

a) Find b_n such that r_e encloses 50% of the total light

$$\frac{2\pi \int_0^{r_e} I(r) r \, dr}{2\pi \int_0^{\infty} I(r) r \, dr} = 0.5$$

$$\Rightarrow 2 e^{b_n} n \pi r_e^2 \left(\Gamma(2n) - \Gamma(2n, b_n)\right) \frac{I_0}{b_n^{-2n}}$$

$$= 2 \cdot 0.5 \, e^{b_n} n \pi r_e^2 \Gamma(2n) \frac{I_0}{b_n^{-2n}}$$

$$\Rightarrow \Gamma(2n) - 2\Gamma(2n, b_n) = 0$$

Find roots using Mathematica

$\Rightarrow$

n	b_n
1	1.678
2	3.672
3	5.670
4	7.669
5	9.669

b) total enclosed light : $2\pi \int_0^{\infty} I(r) r \, dr = 2 I_0 r_e^2 \frac{e^{b_n} n}{b_n^{-2n}} \Gamma(2n)$

n	$I_{tot}/I_0 r_e^2$
1	11.95
2	16.31
3	~~22.67~~ 19.74
4	22.67
5	25.25

converges for all $n > 0$ as Gamma-function defined for all n

2 – Fundamental plane

a) The fundamental plane is a set of relationships between the surface brightness, velocity dispersion and effective radius of elliptical galaxies. Elliptical galaxies occupy a very thin plane with small scatter in this three dimensional parameter space.

b) FP: $R \propto \sigma^{\alpha} I^{\beta}$, Tully Fisher: $L \propto v_{rot}^{4}$

Assume galaxies are virialized and have constant mass-to-light ratio $\langle M/L \rangle$

Virial theorem : $\frac{GM}{\langle R \rangle} \propto \langle v^2 \rangle$, $R \propto \langle R \rangle$, $V^2 \propto \langle v^2 \rangle$

$L \propto I R^2$, $L = M \langle M/L \rangle^{-1}$

From virial theorem : $\langle v^2 \rangle \propto \frac{GM}{\langle R \rangle} \propto \frac{L}{R} \langle M/L \rangle \propto I R \langle M/L \rangle$

$$\Rightarrow R \propto V^2 I^{-1} \langle M/L \rangle^{-1}$$

"virial FP"

Start again from VT: $R \propto \frac{M}{V^2} \propto R^2 I \langle M/L \rangle \frac{1}{V^2}$

$\Rightarrow R \propto V^2 \langle M/L \rangle^{-1} I^{-1} \Rightarrow R^2 \propto V^4 \langle M/L \rangle^{-2} I^{-2}$

$\Rightarrow R^2 I \propto L \propto V^4 \langle M/L \rangle^{-2} I^{-1}$

"virial Tully Fisher relation"

c) see George's slides (lecture 12) for lots of plots and explanations