Ay 124 – Midterm (Observational Part)

Posted on Tuesday, Feb. 3 – Due by 5 pm on Wednesday, Feb. 11 (directly to the TA)

Note:

You have up to 2 hours to do this part, and it counts for 40% of your midterm grade. Then you have up to 3 hours to do the theoretical part, which counts for 60%. You can do them in any order, with a break in between. The entire exam should be taken in a single, contiguous period of up to 8 hours (including both work parts and any breaks you may wish to take). Please mark your exams with the start and end times.

The Rules:

Closed book, closed notes, etc. No collaboration. You cannot discuss the exam with anyone, until everyone in the class has turned in their test.

Good Luck!

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1. Relaxations and evolution [12 points]

a. Write down or derive the approximate formula for the two-body relaxation time in a stellar system, in terms of the typical size (*R*), typical stellar velocity (*V*), and the number of stars in the system (*N*). [4 pts]

b. Estimate the appropriate values and derive the crossing and the relaxation times for each of the following: (*i*) an open cluster, (*ii*) a globular cluster, (*iii*) an elliptical galaxy. [4 pts]

c. Compare these and discuss what it all means in terms of the dynamical evolution of these systems. [4 pts]

2. [6 points] Consider a red giant in a globular cluster with the parameters you estimated in (1.b.*ii*). The radius of the giant is 1 a.u.

a. What is the chance that it will collide with another star during a single crossing time? (Assume that most other stars have much smaller radii, and approximate the cluster as a sphere of uniform density.) [3 pts]

b. If there are 1,000 red giants in this cluster, what is the mean time between their collisions with other stars? How many have collided over the entire history of the cluster? [3 pts]

3. Binding energies [6 points]

a. Using the virial theorem, estimate the binding energy of a globular cluster with the parameters above. What is the mean velocity and the kinetic energy of its stars? [3 pts]

b. Now consider a binary in the cluster core, where both components have masses of 0.5 $M_\odot$, in a circular orbit with a diameter of 10 a.u. Compute the binding energy of this binary, and compare it with the mean binding energy per star in the cluster as a whole. Is this a soft or a hard binary? [3 pts]

4. Galaxy luminosity function (GLF) [16 points]

a. Define the Schechter luminosity function; draw a diagram; what are the typical values of its parameters? [4 pts]

b. Using these parameters, derive the mean galaxy luminosity. [4 pts]

c. Now derive the mean luminosity density, in $L_\odot Mpc^{-3}$. [4 pts]

d. For what values of α does the GLF diverge? [4 pts]