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SYSTEMATICS OF GALAXY PROPERTIES: HINTS ABOUT THEIR FORMATION

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ABSTRACT Observed ranges of galaxian global properties and the scaling laws and correlations between them reflect processes of galaxy formation and evolution, and thus contain interesting cosmological information. The basic correlations which tie most of the global properties of galaxies are intrinsically bivariate, both for ellipticals and spirals. The origins of these correlations are not yet fully understood, but some hints about galaxy formation can be deduced from them. Slight environmental variations, possibly caused by differences in the formation histories, may exist in correlations used as distance indicators, and appear as spurious peculiar velocities. Galaxies form two-dimensional sequences in the parameter space whose axes reflect the size (mass, luminosity, or radius), density or surface brightness, and kinetic temperature (velocity dispersion, circular speed for disks). This galaxy parameter space promises to become an equivalent of the H-R diagram for galaxies, a new organizing framework for extragalactic astronomy and cosmology.

1. INTRODUCTION

Understanding of galaxies, their formation and evolution are among the principal goals of modern extragalactic astronomy and cosmology. An observational approach to these problems may involve direct searches for young and rapidly evolving galaxies at large redshifts. However, an alternative strategy is also possible: the history of galactic formation and evolution is encoded in the properties of galaxies today, and in the correlations and scaling laws which organize them. There are both advantages and disadvantages to this paleonto-cosmological approach. On the one hand, we can study relatively nearby galaxies in much more detail and with higher accuracy than their ostensible high-redshift ancestors, and we can have a much better handle on the selection effects. On the other hand, the memory of galactic formation fades with age and an ocassional merger, and we must look carefuly for the best fossil evidence we can find, separating the imprints of the early formative processes or initial conditions from those which may reflect more recent events.

There has been considerable progress in this field over the past several years. This is due to the confluence of two recent developments: the availability of large, homogeneous samples of good-quality data, mainly thanks to the advent of CCDs, and the spread of powerful multivariate statistical analysis techniques. As it will be described below, galaxy families are intrinsically statistically multidimensional in nature, thus defying the simpleminded $X - vs. - Y$ methods of analysis, which unfortunately (and unnecessarly, since more appropriate and more powerful multivariate methods are not hard to master) seem to be the staple of most astronomical data analyses.

Multivariate analysis techniques have been well explored and profitably used in other fields (e.g., economics, biology, etc.) for many years now. To this author's best knowledge, this methodology was first introduced in extragalactic astronomy by Brosche (1973) on the relatively poor data available then, but the significance of his early results and the power of the techniques he pioneered were not widely appreciated. Further studies of galaxy properties using multivariate statistics, in particular the principal component analysis (PCA) followed in the papers by Bujarrabal *et al.* (1981), Brosche & Lentes (1983, 1989), Lentes (1983), Efstathiou & Fall (1984), Whitmore (1984), Okamura *et al.* (1984), Lauer (1985), and others. The one common feature of most of those studies, whether dealing with ellipticals, spirals, or with a mixture or galaxy types, was that galaxies form statistically two-dimensional families, or manifolds: many (but not all) of their observed or derived properties can be expressed as combinations of only two variables, but the physical meaning of these results remained murky. Indeed, many of those authors were primarily interested in using the multivariate analysis to define new objective classification schemes for galaxies; Okamura *et al.* (1989) provide a good review from that viewpoint.

The question of the exact number of statistically significant dimensions in a data set is not always well posed. In practice, one often sees a small number of clearly dominant parameters, and possibly one or two marginally significant ones. Statistical dimensions of data sets (e.g., galaxy manifolds) may well be fractal in nature. It is scientifically useful to adopt a heuristic, even if mathematically non-rigorous approach: uncorrelated observables are often best left ignored (of course, there is some scientific information even in the lack of correlations), while useful correlations can be deduced from those defining the minimal manifolds. Typically, we are looking for subsets of data with a maximum reduction of dimensionality (the smallest number of significant principal components, for a largest number of input observables). This can be viewed as the clustering of data vectors in the parameter space of all observables.

2. ELLIPTICAL GALAXIES

The subject of multivariate analysis of galaxy properties attracted more attention after the discovery of the so-called "fundamental plane" (FP) of elliptical galaxies, independently and simultaneously by the "7 Samurai" collaboration (Burstein *et al.* 1986, Dressler *et al.* 1987*b*), and by Djorgovski & Davis (1986, 1987). The subject of the FP and the manifold of elliptical galaxies has been reviewed by Djorgovski (1987), Djorgovski *et al.* (1988), Kormendy & Djorgovski (1989), Djorgovski & de Carvalho (1990), and Djorgovski (1991). Some applications can be found in the papers by Gorjian *et al.* and Penereiro *et al.* in this volume.

In the parameter space defined by the observed global parameters, such as the luminosity, radius, surface brightness, velocity dispersion, colors, metallicity, etc., elliptical galaxies do not fill all of the available volume, but are confined to a two-dimensional surface, which becomes a plane after the proper mathematical transformations (e.g., logarithms). This is the FP, and it represents a bivariate

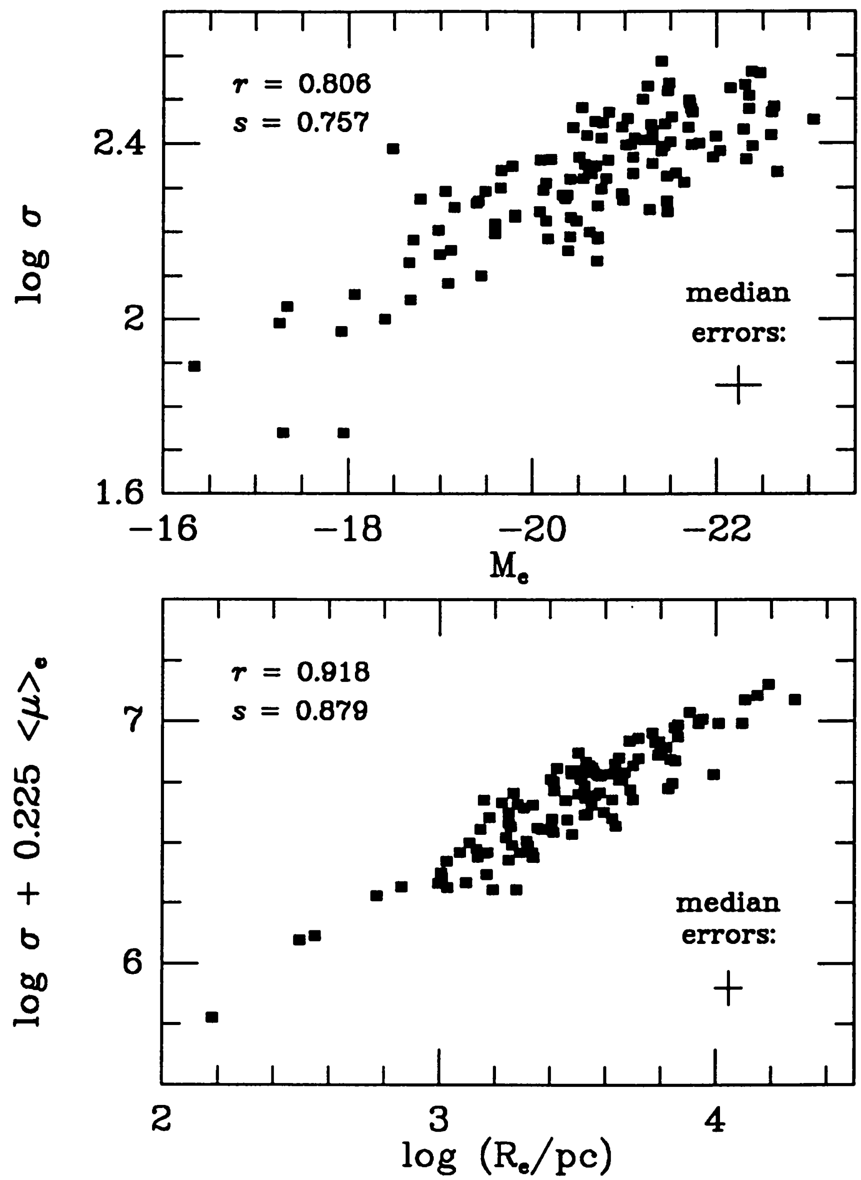

Figure 1. A comparison of an old, monovariate correlation for elliptical galaxies, the Faber-Jackson relation (top), and a bivariate correlation between the radius and a combination of the mean surface brightness and the central velocity dispersion (bottom); the latter is a view of the FP edge-on. Pearson (r) and Spearman rank (s) correlation coefficients are shown in both panels. In addition, the reduced χ^2 per degree of freedom is nearly 6 for the top plot, and less than 1 for the bottom one; the median error bars are indicated. The data used are the sample of ellipticals from Djorgovski (1985) for which velocity dispersions and adequate photometric calibration exist. All photometric quantities are measured within the r_e elliptical isophote, and are in the Johnson R band. The Faber-Burstein (1988) velocity field model with H_0 = 100 km/s/Mpc was assumed to get the distances.

scaling (power) law between various properties of ellipticals. The FP is tilted with respect to all observable axes of the parameter space, and therefore the classical correlations such as the Faber-Jackson (1976) ($L - \sigma$) relation, Kormendy's (1977) radius – surface brightness ($R - \mu$) relation, the correlations between the luminosity and color or metallicity, etc., are simply its oblique projections on the planes defined by the observables (Fig. 1).

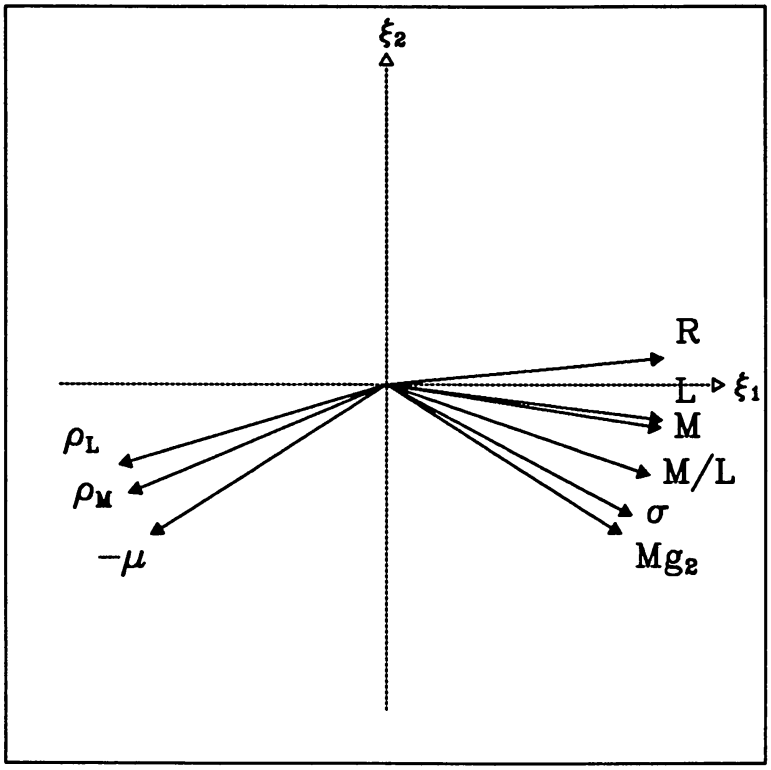

Figure 2. A correlation vector diagram (CVD) for 107 elliptical galaxies from Djorgovski (1985) with good photometry, for which velocity dispersions and metallicity measurements are available. This illustrates the results from PCA: a CVD shows the projections of the axes of observables in the parameter space onto the plane defined by the first two principal components (the eigenvectors of the data manifold, ξ_1 and ξ_2); i.e., the coordinate system of the FP itself. There are 5 independent observed quantities: the effective radius R, enclosed luminosity L, mean surface brightness within that isophote $-\mu$, central velocity dispersion σ, and the metallicity indicator, the Mg_2 index. Also indicated are the projections of the derived quantities, such as the virial mass M, the (M/L) ratio, and mean mass and luminosity densities inside the effective radius ρ_M and ρ_L. The first two eigenvectors account for the 60.7% and 25.4%, respectively, of the total sample variance. A weak third eigenvector is also present, mainly associated with the Mg_2 index. Cosines of angles between the different observable vectors in this coordinate system are directly related to the correlation coefficients: perfectly correlated quantities are parallel, uncorrelated quantities are orthogonal. If there are only 2 significant dimensions, then *all* correlations present in the data are completely described by a CVD like this.

By their definition, many important derived parameters, such as the mass, luminosity or mass density, (M/L), etc., are also contained in the FP (Fig. 2). The simple monovariate correlations for ellipticals thus cannot contain all of the information and must show an intrinsic scatter, unless the variables are carefully constructed to represent the FP seen edge-on (e.g., the relation between the modified isophotal diameter, D_n, and the velocity dispersion). However, the variables which describe the *shape* of the light distribution, be it radial or azimuthal, such as the ellipticity, isophotal twist rate, details of isophotal shapes such as the a_4/a parameter, shapes of the surface brightness profiles, etc., do not correlate with the FP, or even mutually. The *entire* manifold of ellipticals is more accurately described as $(2+N)$-dimensional.

The FP attracted attention for two principal reasons. First, the regularity it implies, its particular slopes, and the remarkably small scatter observed around it (less than a few percent in any given variable) must be telling us something about the formation of elliptical galaxies. Second, it contains (nearly?) optimal distance-indicator relations for early-type galaxies, e.g., the $D_n - \sigma$ relation, and thus, at least in principle, it can be used for measurements of the large-scale peculiar velocity field.

Attempts to interpret the physical meaning of the FP have been made by Faber *et al.* (1987), Djorgovski (1988, 1991), Djorgovski *et al.* (1988), and Djorgovski & de Carvalho (1990). The FP implies a strong regularity among the global properties of elliptical galaxies, which may be a product of the physics of their formation and evolution. On the other hand, the lack of correlations among the "shape" variables (ellipticity, etc.) suggests that the details of dynamical structure are determined by some stochastic process (e.g., mergers), which may operate separately from whatever processes correlate the global properties. The low dimensionality of the FP suggests self-regulation mechanisms in the process of formation of ellipticals: if a single process was dominant, the dimensionality of the manifold could have been 1; if many different processes were equally important, the dimensionality could have been higher. The principal eigenvector of the manifold may be identifiable with the mass, but we still do not understand what physical variable or process causes the spread of properties at a fixed mass.

Probably the best determined expression of the FP is the scaling law between radius (or semimajor axis), R, defined in some consistent way, but not isophotally, e.g., the r_e or r_c, central velocity dispersion, σ, and a surface brightness in linear flux units, I, which could be measured at the radius R, or an average within the R, etc.:

$$R \sim \sigma^A I^B \tag{1a}$$

The typical observed power-law slopes, at least for the ellipticals in major nearby clusters, are $A = 1.35 \pm 0.15$ and $B = -0.9 \pm 0.1$, in the BVR bands, regardless of whether the core radius (e.g., Lauer 1985), or the half-light radius (e.g., Djorgovski & Davis 1987), or some other radial scale (e.g., Djorgovski & Davis 1986) parameters are used. For the field ellipticals, A may be systematically slightly lower, and there also may be possible variations between different clusters. The $D_n - \sigma$ relation of Dressler *et al.* (1987*b*) is equivalent to eq.(1*a*), with the modified isophotal diameter D_n containing both the radius and surface brightness terms, provided that all galaxies have the same surface brightness profiles (which is only approximately true). We have found that, using the same data, the $R-\sigma-I$ formulation is slightly more robust, and shows slightly less scatter than the $D_n-\sigma$ formulation.

Assuming homology, i.e., that all ellipticals have the same luminous and dynamical structure, and that the observed velocity dispersion is always directly proportional to the kinetic energy per unit mass, $\sigma = CE_{kin}/M$, where $C = const.$, one can derive the following scaling relations for the luminosity L, mass M, and (M/L):

$$L \sim \sigma^{2A} I^{2B+1} \tag{1b}$$

$$M \sim \sigma^2 R \sim \sigma^{A+2} I^B \tag{1c}$$

$$(M/L) \sim \sigma^{2-A} I^{-1-B} \tag{1d}$$

The implied scaling for the L is close to the observed relation, which is difficult to evaluate in an unbiased way, because of the strong error coupling between the L and I.

For galaxies bound by Newtonian gravity, it follows from the virial theorem:

$$R = \kappa\sigma^2 I^{-1}(M/L)^{-1} \tag{2}$$

where κ depends on the structure and dynamics of ellipticals, and could be a function of R, L, or other variables (Djorgovski *et al.* 1988). For a purely homologous family of objects, $\kappa = const.$ With that assumption, it follows from eqs. (1a) and (2) that

$$(M/L) \sim M^{\alpha} \tag{3}$$

where $\alpha = 1/(6 \pm 2)$ (Faber *et al.* 1987; Djorgovski 1988). This implies that *if* the assumption of homology is correct, the global properties of ellipticals would be constrained by the virial theorem and the nontrivial relation (3) only. The origin of the homology, and of eq.(3) is in the processes of elliptical galaxy formation, and therefore eqs.(1a) and (3) provide empirical constraints for theories of galaxy formation. Eq.(3) alone implies that formation of ellipticals must have been at least partly dissipative, a conclusion also implied by the high phase-space densities of these systems (Carlberg 1986). Faber *et al.* (1987), Djorgovski (1988, 1991), and Djorgovski & de Carvalho (1990) discuss this topic in more detail.

The trend described by eq.(3) is weak, and on the whole, the spread of the (M/L) ratios is relatively small among the ellipticals, not larger than a factor of 3 (Djorgovski & Davis 1987, Djorgovski 1987). This is in itself an interesting fact: it is not *a priori* obvious why the stellar populations and the mixture of the baryonic (visible) and dark matter should be so uniform among the ellipticals.

Furthermore, measures of the metallicity (colors or the Mg_2 index) are also contained in the FP, and excellent bivariate correlations between the metallicity, surface brightness, and radius can be obtained (de Carvalho & Djorgovski 1989, 1990). The implication is that there is a second parameter in the relation between mass and metallicity, which is identified as the luminosity density. This can be understood, at least qualitatively, in the context of dissipative galaxy formation. If the initial enrichment is regulated by galactic winds (energy input by supernovæ), the final metallicity will depend on the escape velocity, and thus indirectly the mass, and the luminosity. The new term, luminosity density, implies an additional dependence at a fixed mass, similar to the Schmidt law for star formation in protoellipticals, SFR $\sim \rho_{gas}^n$.

This effect can also be seen in the color gradients: Franx & Illingworth (1990) show a strong dependence of a metallicity-sensitive color on the local escape velocity within individual elliptical galaxies. However, there is a substantial spread in the zero-points of their color – velocity dispersion profiles, which is presumably correlated with the surface brightness differences.

Note that the assumption of homology in this discussion ($\kappa = const.$ or a slow function of other parameters) is a highly nontrivial one. The fact that this assumption apparently works also contains some information about the formation of ellipticals. Whereas elliptical galaxies do have rather similar surface brightness profiles, some variation does exist (Djorgovski *et al.* 1985, Djorgovski 1985). Numerical simulations of protogalaxy collapse, with or without dark halos, and with or without dissipation also produce density profiles close to those observed in real ellipticals (Carlberg *et al.* 1984). Hubble-like density profiles are also the maximum entropy solutions if one requires that they have a finite central density, and power-law envelopes (White & Narayan 1987). The origin of this "standard shape" of elliptical galaxies is not yet understood.

We also know that there is some variety in their dynamical properties, which may be roughly correlated with the L, and therefore R (Davies *et al.* 1983). A measure of dynamical anisotropies can be also provided by the boxyness/diskyness parameter a_4/a (Bender 1991, and references therein). Yet, Djorgovski & de Carvalho (1990) obtained only marginally different FP solutions for the boxy and disky samples of ellipticals.

In principle, ellipticals can span a vast diversity of dynamical models, but evidently they do not. The observed small scatter around the FP implies that only a small range of velocity anisotropies is actually being covered, and that perhaps there is some nontrivial coupling between the kinematical and density structure, which keeps ellipticals on a two-dimensional manifold. These restrictions are not yet understood, and probably contain nontrivial information about the formation of elliptical galaxies, or the physics of dense stellar systems in general.

The FP is thus not just a virial theorem in disguise. It implies nontrivial constraints on the dynamical structure of elliptical galaxies, and at the same time on their stellar populations, and their relations with the dynamical structure parameters.

3. SCALING LAWS AND GALAXY FORMATION

A direct attempt to use the observed scaling relations for galaxies to probe their formation can be made using the Gott & Rees (1975) theory, which defines a scaling relation between mass and density for protogalactic clumps which separate from an evolving density field, whose power spectrum is a standard power law, $|\delta_k|^2 \sim k^n$:

$$\rho \sim M^{-3/2-n/2} \tag{4a}$$

Scaling relations for other quantities can be derived by an application of the virial theorem, for example, the proto-Tully-Fisher-Faber-Jackson relation:

$$M \sim \sigma^{12/(1-n)} \tag{4b}$$

where σ is a characteristic velocity. In a pioneering effort, Faber (1982) pointed out that if one identifies the mass with the luminosity, the observed Tully-Fisher or Faber-Jackson slope ~ 4 implies $n \simeq -2$, close to the value predicted by the CDM scenario on the mass scale of galaxies. In that case, we can also try to identify the surface brigtness with the surface density, and derive the following scaling relation:

$$I \sim R^{-2(2+n)/(5+n)} \qquad (4c)$$

The average observed scaling relation for ellipticals, which is better than the Faber-Jackson relation, is: $I \sim R^{-1.2\pm0.1}$ (Kormendy & Djorgovski 1989, and references therein). This implies $n \sim +2.5 \pm 0.8$, which is very different from Faber's CDM-like result, indicating that our approach is too simple. There are several possible weak links in this argument: first, mass is not identical with light, and second, the true scaling relations are bivariate (the FP), while the Gott-Rees hierarchy describes a single-parameter family of objects, parameterized by the mass. (However, one may allow *ad hoc* for a spread of densities at a given mass, thus effectively inventing the second parameter for the hierarchy.) Thus, let us add eq.(3) to the Gott-Rees hierarchy, in order to convert mass to light. From the FP eq.(1a), we derive the condition:

$$2n + 10 = A(1 - n) - B(12\alpha + 4n + 8) \qquad (5)$$

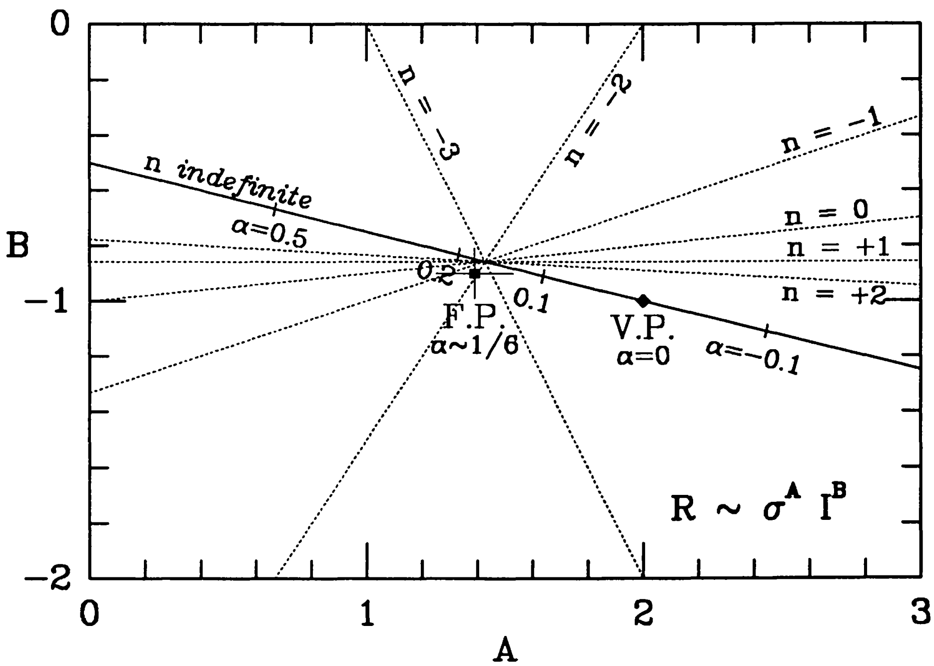

Figure 3. The solution of eq.(5), in the space of the FP power law coefficients (slopes) A and B, as in eq.(1a). There are two parameters: the initial density perturbation spectrum index n, and the power law index of the dependence of the (M/L) on mass, α, as in eq.(3). The solid dot with the error bars indicate the observed values of A and B. A pure virial theorem solution ("virial plane") corresponds to $A = 2$, $B = -1$, and $\alpha = 0$. Since there is only one independent relation in the Gott-Rees hierarchy, eq.(4a), two observed quantities, A and B, and two unknowns, n and α, the solution is degenerate. Another, independent observational or theoretical constraint is needed.

From the observed values of A, B, and α, we find a solution which is degenerate in n (Fig. 3). The observed scaling relations for elliptical galaxies thus contain no memory of the initial density perturbation spectrum, at least in the framework of this analysis. This information may have been erased by the violent relaxation and dissipative formation processes, which convert bound, dark matter dominated protogalactic clumps, which the Gott-Rees theory ostensibly describes, into the luminous ellipticals observed today. However, it is possible that the argument might work at some level for the Tully-Fisher relation for spirals, and that the original argument by Faber (1982) applies to it (eq. 4*b*).

This argument might also work, at least in principle, if we could study the scaling laws and correlations for dark halos, which are presumably the gravitationally dominant components really described by the Gott-Rees theory, and which did not suffer any dissipation (at least if the dark matter is nonbaryonic). Whereas observations of invisible matter present a considerable methodological challenge, indirect methods can be used (e.g., Athanassoula *et al.* 1987), if one dares to make some nontrivial assumptions about the structure and dynamics of dark halos. Kormendy (1988, 1990; and in prep.) obtained for the deduced central densities ρ_0, core radii r_c, and central velocity dispersions σ_0 of dark halos of late-type and dwarf (i.e., dark matter dominated) galaxies the following correlations:

$$\rho_0 \sim r_c^{-0.67} \tag{6a}$$

$$\rho_0 \sim \sigma_0^{-0.64} \tag{6b}$$

$$\sigma_0 \sim r_c^{+0.63} \tag{6c}$$

No error bars are quoted, as the unknown systematics probably dominate them. The corresponding scaling laws derived from the Gott-Rees hierarchy are:

$$\rho \sim R^{-3(3+n)/(5+n)} \tag{7a}$$

$$\rho \sim \sigma^{-6(3+n)/(1-n)} \tag{7b}$$

$$\sigma \sim R^{(1-n)/(10+2n)} \tag{7c}$$

This leads for the solutions $n = -2.43, -2.61$ and -2.35 for the three scaling laws, which, however, are not fully independent. The average value, $n = -2.45$, is tantalizingly close, and probably fully consistent within the errors with the CDM value $n \simeq -2$ on the galactic mass scales. It would still be premature to claim that this analysis gives support to the CDM theory; the data and the possible loopholes in the reasoning used to derive eqs. (6) should be much better understood first.

The observed properties of elliptical galaxies indicate a spread in mass density at any given mass, even though there is some anticorrelation between ρ_M and M. A similar situation holds for the luminosity density ρ_L and luminosity L (Fig. 4). The best fit solutions which take into account the errors in both coordinates correspond to the scaling laws:

$$\rho_L \sim L^{-0.9\pm0.3} \tag{8a}$$

$$\rho_M \sim M^{-0.9\pm0.1} \tag{8b}$$

The mass used here is a virial mass, computed for a de Vaucouleurs galaxy using the measurements of effective radius and velocity dispersion, $M \sim \sigma^2 R$. These fits are for a sample of ellipticals with good measurements taken from the sample of Djorgovski (1985), which is not complete in any way, but probably is representative. Numerous systematic errors and biases can enter into

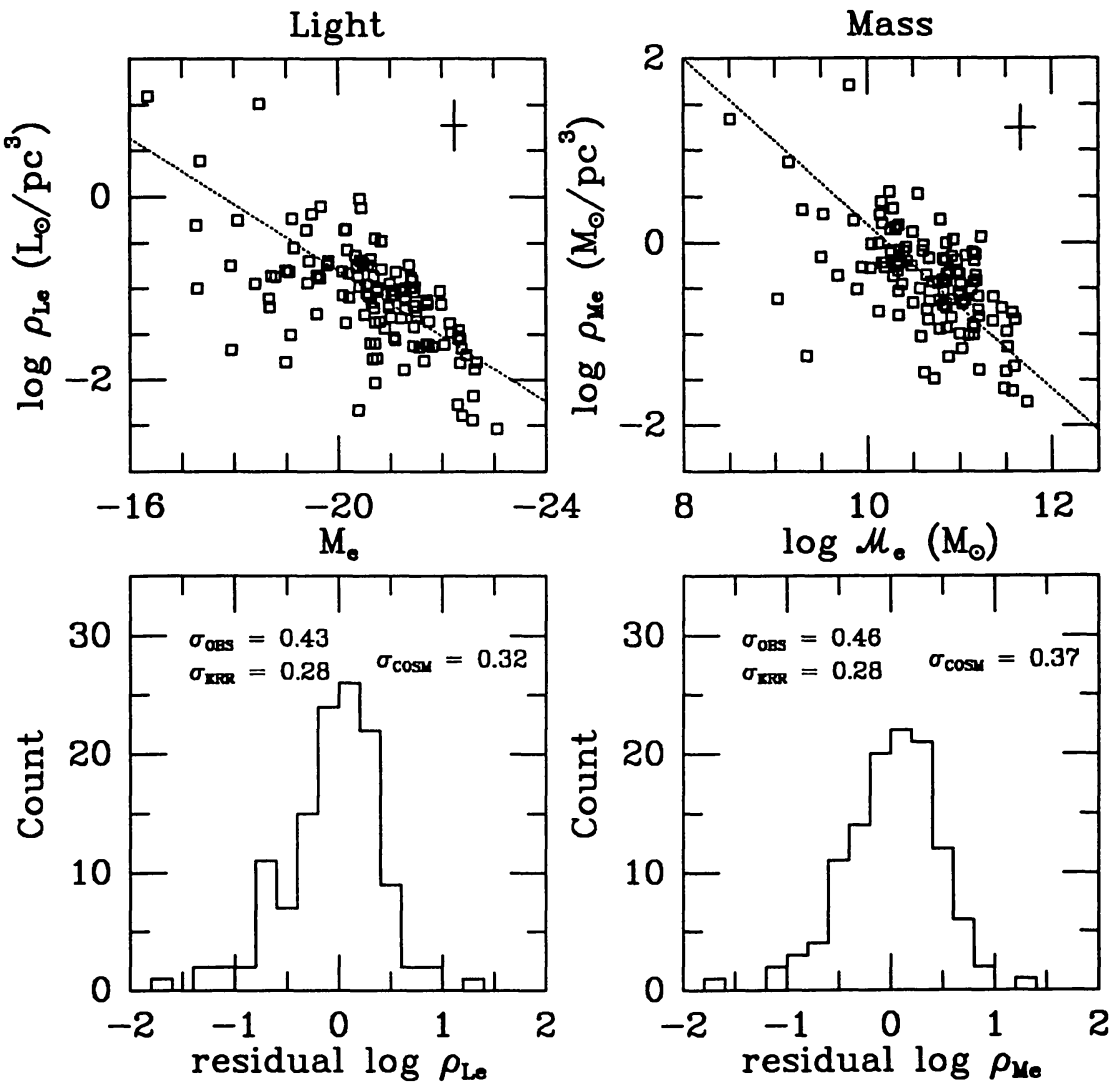

Figure 4. Dependence of the mean luminosity density ρ_L on the absolute magnitude (top left), and mean mass density ρ_M on the virial mass M (top right), for a sample of ellipticals from Djorgovski (1985). All quantities are defined within the r_e (half-light) isophote. The luminosities are in the R band, and the distances assume $H_0 = 100$ km/s/Mpc. Median error bars are shown in the upper right corners. The dotted lines indicate the least-squares fits with error bars in both coordinates, as given in eqs.(8). The bottom panels indicate the distributions of residuals from the fits, which are broader than expected from the error bars alone. The cosmic scatter in $\log \rho$ is derived as: $\sigma^2_{COSM} = \sigma^2_{OBS} - \sigma^2_{ERR}$, where σ_{OBS} is the observed r.m.s. of residuals in $\log \rho$, and σ_{ERR} is the corresponding median error bar.

determinations of these quantities, e.g., variations in shapes and dynamical anisotropies, etc. The errors quoted in eqs.(8) represent the fitting errors only, and thus underestimate the true errors.

Given these caveats, we can compare eq.(8*b*) directly with eq.(4*a*), from which we derive an estimate of the primordial density fluctuation power spectrum index, $n = -1.2 \pm 0.2$ (again, the error is probably underestimated). This should not be taken too seriously, but it is intriguing that the obtained value of n is close to the theoretically expected slope for this mass range in the CDM scenario, $n_{CDM} \simeq -2$. The observed relations must reflect both the initial conditions and the subsequent evolution: for example, dissipationless mergers would on the average increase the mass and decrease the density, thus steepening the observed relation, and increasing the apparent n. Thus, the initial condition may well have corresponded to $n = -2$, but at this point, devolution of the data to the initial relation is a hopeless task.

There is a spread around the best fit lines in these trends, which exceeds the measurement errors. This is because eqs.(8) do not represent an edge-on view of the FP. However, this spread is relatively small. For both relations, the median r.m.s. errors are ~ 0.28 in $\log \rho$, with the observed scatter of ~ 0.45. The quadrature difference implies the intrinsic scatter of residuals of ~ 0.34 in $\log \rho$, or about a factor of 2 at any given M or L; any additional, unknown component of the errors would decrease this cosmic scatter. Even if we do not remove the trends, but measure the spread of densities at all masses or luminosities, the observed r.m.s. is only slightly larger, and the cosmic scatter not more than ~ 0.45 in $\log \rho$. Now, if we identify the turnover epoch, $(1 + z_{turn})$, and the protogalaxy collapse factor R_{init}/R_{final}, as the factors which determine the density of ellipticals today, $\rho^{1/3} \sim (1 + z_{turn}) R_{init}/R_{final}$, the data indicate a very narrow dynamical range of that product, only $\sim 40\%$. This is after accounting for the known measurement errors, and assuming no additional error components or variations in the collapse process. This implies that the mean mass or luminosity densities of ellipticals are remarkably well standardized.

Several types of processes can in principle modify the densities and masses of protoellipticals: dissipative collapse would increase the densities at a fixed mass, disipationless mergers would increase the mass, but decrease the density, dissipative mergers would increase both mass and density, and by supernova-driven galactic winds would decrease both. The relatively narrow range of mean densities of ellipticals is thus somewhat surprizing, and may teach us something nontrivial about the formation of elliptical galaxies: not only their shapes seem to be well standardized, but their densities as well. In this context, the models by Ikeuchi & Norman (1991) are of a considerable interest. One possibility is that self-regulation of star formation and energy input by supernovæ during the initial starburst set the maximum density achievable by a young stellar system; this would be modified only a little by the subsequent dynamical processes, given the long relaxation times in galaxies. This may lead to an establishment of a maximum mass or luminosity density (averaged over a certain region, in this case corresponding to the half-light radius) reached in elliptical galaxies. Inspection of Fig. 4 indicates that a few ellipticals reach mean densities in excess of $\sim 3 M_\odot/\mathrm{pc}^3$ or $\sim 0.5 L_\odot/\mathrm{pc}^3$ (in the R band, and within the r_e isophote). A similar situation may hold for the cores of ellipticals (Lauer 1985), excluding possible luminosity spikes or cusps in the very centers.

The FP may also provide some constraints on biased galaxy formation. Kaiser (1988*a*) proposed a biasing model in which the luminosity of a galaxy scales with its mass and turnover redshift as:

$$L \sim M^{\alpha}(1 + z_{turn})^{\beta} \tag{9}$$

where coefficients α and β are related to the biasing parameter b; $\beta = 0$ is an unbiased model; $\beta = 2$, $\alpha = 4/3$ is the so-called natural or autonomous bias, preferred by n-body simulators. If there is a Tully-Fisher or a Faber-Jackson relation of the form $L \sim \sigma^{\gamma}$, then the following condition applies:

$$\alpha - \beta(n + 3)/6 = \gamma(1 - n)/12 \tag{10}$$

where n is the primordial density spectrum index; for $\beta = \gamma/2$, one gets a scatterless Faber-Jackson relation, unlike the observed one. Now assume that ellipticals form a homologous family, with $\sigma_{stars} \sim \sigma_{halo}$, $M_{baryonic}/M_{total} = const.$, and $R_{init}/R_{final} = const.$ (a reasonable assumption, as shown above). Then the mean halo density at the turnover is a function of the redshift alone, which gives the connection with β. Using the FP eq.(1*a*), after some algebra one obtains:

$$\alpha = (1 - A + 2B)/3B = 0.82 \pm 0.08 \tag{11a}$$

$$\beta = -(2 + A + 4B)/2B = -0.11 \pm 0.3 \tag{11b}$$

Thus, the observed scaling relations for elliptical galaxies, under the stated assumptions, indicate a weak or no bias; the autonomous bias parameters are off by several sigma. A similar discussion and conclusions were presented independently by Peacock (1990).

To summarize, some direct hints about the formation of elliptical galaxies can be obtained directly from observed scaling laws and correlations, e.g., eqs. (1), (3), (8), or (11), from the very existence of the FP and its dimensionality, and from the nonparticipation of the shape parameters in this well-ordered manifold. However, possibly the most important clues, pointing towards some major unsolved puzzles of elliptical galaxy formation, are implied by the overall restrictiveness of the FP, the small scatter around it (a few percent), and the relatively small enviromental dependence of its coefficients (probably less than $\sim 10 - 20\%$). Why are elliptical galaxies such a nearly homologous family? Why are their luminosity density profiles so similar? Why don't they show a larger variety of dynamical structures and anisotropies? Why do their mean densities have such a small dynamical range? What regulation mechanisms produce these restrictions and confine them to the narrow portion of the parameter space spanned by the FP? Why is the statistical dimension of their manifold 2, and not 1, or 3, or greater? These are our clues and challenges.

4. SPIRAL GALAXIES

Let us now turn to spirals, or more generally, galaxies with gaseous, star-forming disks, whether or not they actualy show decorative spiral patterns. Structural properties of disk galaxies were reviewed recently by van der Kruit (1989), and star formation and chemical evolution issues by Kennicutt (1989, 1990) and Dopita (1990). Disks seem to form from the inside out, as suggested by their finite optical radii, which are smaller than the H I radii, and by the radial metallicity gradients. Wyse & Silk (1989) and White (1991) review some theoretical issues of disk formation. Observed rates of star formation and gas masses suggest that the infall continues to the present day, and that formation of disks is an ongoing process. It may therefore be expected that the manifold of their properties is more complex than that of ellipticals.

However, Whitmore (1984) found only two dominant dimensions, the "form", which is a combination of the (B/D) ratio and color, and the "scale", which is a combination of the luminosity and radius (Fig. 5). The former quantifies the Hubble sequence; the later reflects the luminosity spread at a

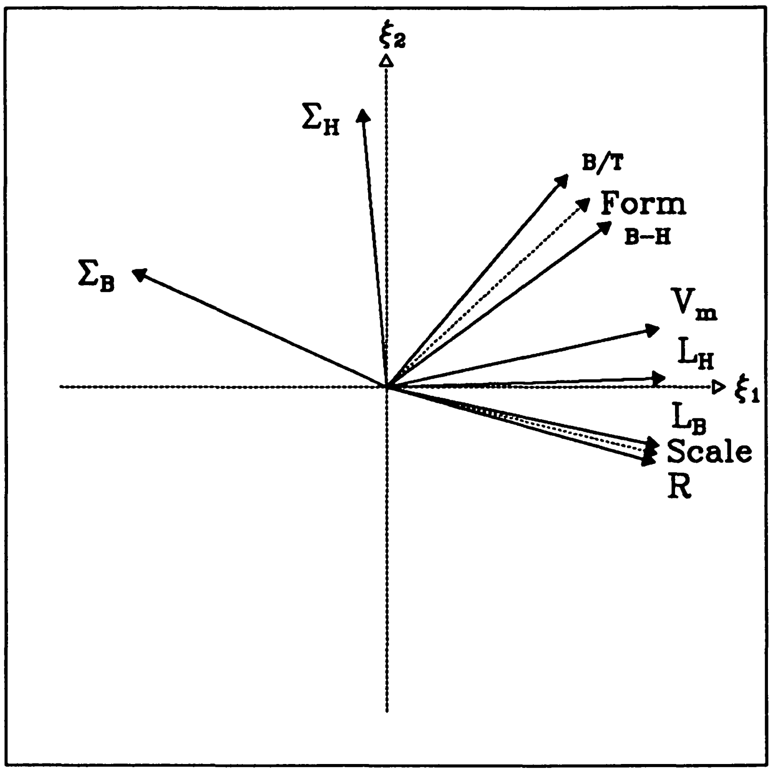

Figure 5. PCA results (CVD; see Fig. 2) for the sample of spirals from Whitmore (1984). The first two eigenvectors carry 43.8% and 27.9% of the total sample variance. The input observed quantities include the luminosities in the B and H bands, L_B and L_H, isophotal radius R, peak rotational velocity V_m, bulge-to-total light ratio B/T, $(B-H)$ color, and the mean surface brightness in the B and H bands, Σ_B and Σ_H; note that there are only 5 independent observables. Whitmore's "scale" and "form" axes are indicated; they are rotated with respect to the true eigenvectors of the manifold, and very weakly correlated. Note the sharpness of the angles between the luminosity and rotation velocity vectors: they represent the Tully-Fisher relations in the B and H. The sharper angle between L_H and V_m reflects the fact that the H band TFR is better than the B band one. The B band TFR can be improved by adding the B/T ratio or the $(B-H)$ color as the second parameter, while such improvements in the H band could be only minor (note that one also increases the errors as more parameters are added, thus diminishing the benefits of the bivariate solutions).

given Hubble type (Rubin 1987). While the Hubble type represents one of the two principal axes of the scale-form plane, it cannot be used to predict galaxy luminosity. The Scale-Form plane (SFP) is roughly an analog of the FP, but for the disk galaxies. Whitmore's results were confirmed and extended by Staveley-Smith & Davies (1988), and Magri (1990), who found that if FIR data are added to the optical and dynamical parameters, the dimensionality of the manifold increases to 3. The most extensive study to date is by Magri (1990).

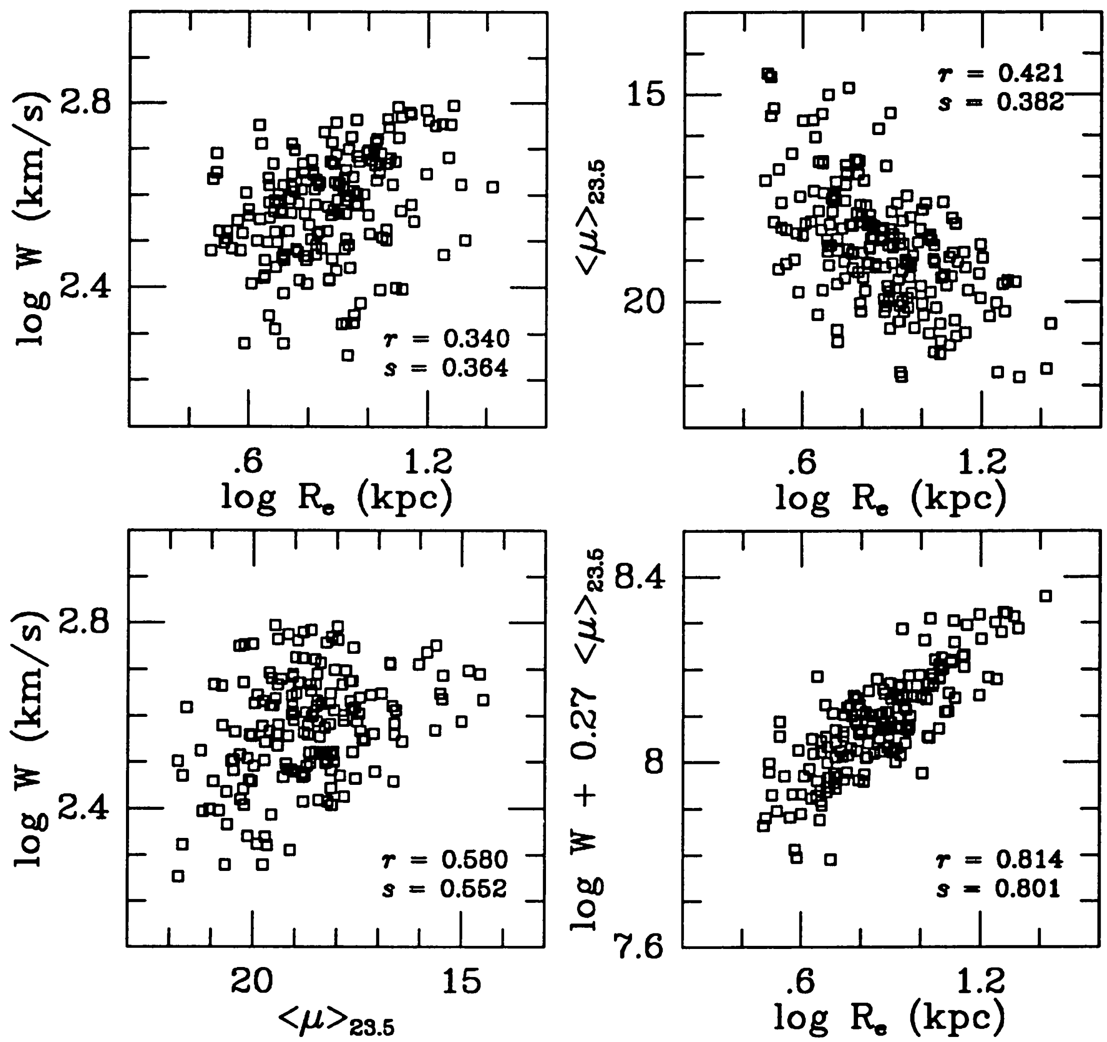

Figure 6. An example of a bivariate solution for spiral galaxies, from the sample by Han (1991), correlating the H I line width (W), e–folding disk scale length (R_e), and the mean surface brightness in the I band inside the 23.5 mag/arcsec^2 isophote ($\langle\mu\rangle_{23.5}$). Pearson (r) and Spearman rank (s) correlation coefficients are indicated in all panels. There is a substantial improvement in the bivariate fit (lower right), relative to the mutual correlations of the input variables. This represents a direct analogue of the FP of ellipticals. The bivariate relation between the R_e and a combination of $\log W$ and $\langle\mu\rangle_{23.5}$ is a new distance indicator relation for spiral galaxies. However, it is not as good as the TFR itself. More details will be presented by Han *et al.* (in prep.)

Bivariate correlations analogous to the FP have now been obtained for the spirals as well (Fig. 6), and it is possible that there is a significant third dimension in the manifold of spirals, even without considering the FIR data. The issue is still not settled, but excellent new data are forthcoming from several groups, and interesting new results are bound to follow.

The most important scaling relation for disk galaxies is the Tully-Fisher (1977) relation (TFR), $L \sim V_m^\gamma$, where $\gamma \simeq 4$, and the residual intrinsic scatter (at least for the near-IR bands) is less than $\sim 0.2^m$ in L, or $\sim 20\%$ per galaxy (expressed as a relative error of distance). Multivariate analysis indicates that a second parameter is present, since the V_m axis is located between the scale and form axes of the manifold; introducing the $(B-H)$ color, surface brightness, (B/D) ratio, or the FIR power as the second parameter improves the scatter of the TFR in the B band; however, the improvement is small in the IR, where the effects of obscuration and recent massive star formation are weaker. In multivariate analysis language, the IR TFR is already a nearly edge-on view of the SFP, which is the fundamental bivariate relation for spirals. This is quite unlike the situation for ellipticals, where the Faber-Jackson relation is very much an oblique projection of the FP. The slope and possibly the intercept of the TFR seem to depend on the Hubble type (Djorgovski *et al.* 1988, and references therein). However, within a narrow range of morphological types and inclination angles, with good CCD photometry in the I band, the intrinsic scatter may be as low as $\sim 10\%$, or even less, which is truly remarkable for a nontrivial astrophysical correlation.

Using the virial theorem, and the tidal torque spinup theory of Fall & Efstathiou (1980), it can be shown that the following scaling relation should apply:

$$L \sim V_m^4 X \tag{12}$$

where

$$X \sim (M/L)^{-1} \Sigma_h^{-1} \tag{13a}$$

or

$$X \sim (M/L)^{-2} I_0^{-1} \lambda^{-2} \tag{13b}$$

where Σ_h is the halo surface mass density, I_0 the disk central surface brightness, and $\lambda = G^{-1}|J|E^{1/2}M^{-5/2}$ is Peebles's spinup parameter, which in turn is related to the collapse factor, by $\lambda = \sqrt{2}R_{disk}/R_{halo}$. The existence of a good TFR implies $X \simeq const.$, or a weak power-law function of L and/or V_m. The mystery of the TFR is in the fact that while each of the quantities which compose the function X is known or suspected to vary by at least a factor of 2; yet, their products are constant to better than 20%, the scatter of the TFR. Thus, not only should the quantities composing X be connected by the appropriate scaling relations, but their scatter should also be correlated. Some coupling mechanism must be at work.

The small scatter of the TFR is quite remarkable, since it connects a halo-driven property (V_m) with a disk property (L, which is roughly proportional to the integrated SFR over the Hubble time). This points to halo-regulated disk formation, as proposed by Silk & Wyse (1989; and Wyse & Silk 1989). In their model, the SFR is regulated both by the gas density (the Schmidt law), and the local velocity shear rate:

$$\mathrm{SFR} = \epsilon\ \mu_{HI}^n\ \Omega \tag{14}$$

where ϵ is the efficiency factor (a few % in the normal disks), μ_{HI} the gas surface density, $n \simeq 1-2$, and $\Omega = V(r)/r$ the local angular frequency, or the velocity shear. Ω reflects the rotation curve and is directly proportional to its amplitude, which in turn is largely determined by the dark halo. This model is in a good agreement with the data and the heuristic model proposed by Kennicutt (1989). He found that the SFR depends on the local gas density, but only above a given threshold, which in turn depends on the local angular frequency, i.e., the velocity shear. These models predict an increasingly good correlation between L and V_m, but do not yet guarantee the right slope and intercept.

Silk & Wyse (1989) suggest that their SFR formula should apply in *all* star forming systems, from protogalaxies on, modulo a different efficiency factor (cf. also Brosche & Lentes 1985). A similar bivariate dependence of SFR was suggested for protoelliptical galaxies on the basis of the mass - metallicity - luminosity density relation (de Carvalho and Djorgovski 1989, 1990; Djorgovski & de Carvalho 1990). It is possible that star formation in galaxies is a universally bivariate process, which along with the virial theorem may be the ultimate causes of the two-dimensionality of galaxy manifolds.

5. DISTANCE-INDICATOR RELATIONS AND PECULIAR VELOCITIES

Both the FP (e.g., in the form of the $D_n - \sigma$ relation) and the SFP (e.g., in the form of the TFR) can be used as (nearly?) optimal distance indicator relations for galaxies of the appropriate types. The origins of these relations are in the processes of galaxy formation and evolution, as discussed above. Therein lies the root of a controversy.

Some authors (e.g., Djorgovski *et al.* 1988, Djorgovski & de Carvalho 1990, or Djorgovski 1991) advocated the view that relatively minor, naturally expected variations in the galaxy formation processes in different large-scale environments would be reflected in small differences in distance-indicator relation slopes and/or intercepts, and that some evidence for such variations exists, typically on a $\sim$ 10% level. In some theoretical models such large-scale environmental variations arise naturally (e.g., Silk 1989). This would cause spurious large-scale peculiar velocities of a few hundred km s^{-1} , in the typical samples studied to date. For example, Lucey *et al.* (1991) measured a peculiar velocity of Abell 2634 of −3,400 km s^{-1} , and concluded that it must be spurious. Others (e.g., Burstein *et al.* 1990) have denied that there is any evidence for such variations.

Whereas some large-scale peculiar motions must exist, the important cosmological questions are their amplitude and coherence. At least some claims for large-scale streaming motions (e.g., Burstein *et al.* 1986, Dressler *et al.* 1987*a*), including the "Great Attractor" model (Lynden-Bell *et al.* 1988; Faber & Burstein 1988) are difficult to reconcile with many theoretical models (cf., e.g., Vittorio *et al.* 1986, Kaiser 1988*b*, etc.) All sources of errors, random or systematic, would add to the observed velocity dispersion at large scales, and could also generate spurious infall signatures (Roth 1990). It is thus important to check on the possible systematics: even a relatively small additional component to the estimated errors could double the inferred peculiar velocities. Recall that for a galaxy or a cluster at a redshift of 5,000 km s^{-1} , a 10% distance error causes a spurious peculiar velocity of 500 km s^{-1} .

Distance-indicator relations are not rigorous laws of physics; they are empirical correlations connecting measured properties of complex and evolving systems, i.e., galaxies. They are tools for measurements of relative distances, and like any other tools, they must have some limits of accuracy. Claims for large-scale streaming motions or for the exitence of the so-called "Great Attractor" are based on the rather poorly tested assumption that the slopes and the intercepts of the FP are universal to better than a couple percent. We need to understand better the origin of the FP and distance-indicator relations in general, and thus the natural limits of their accuracy, before any such claims can be credibly made.

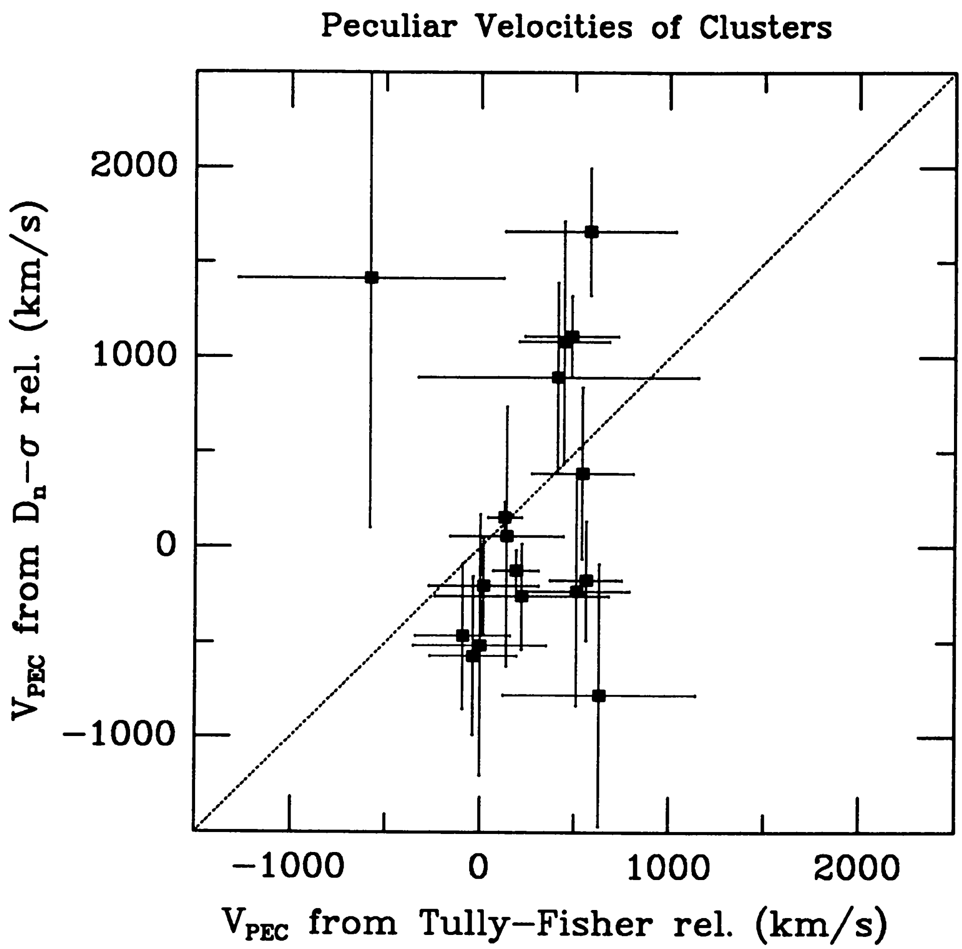

Figure 7. A direct comparison of peculiar velocities for clusters of galaxies, determined using two different distance-indicator relations: the TFR, and the $D_n - \sigma$ relation. The velocities are taken directly from the papers by Aaronson *et al.* (1976), Mould *et al.* (1991), and Faber *et al.* (1989), as summarized by Mould *et al.* No correlation is discernible: the Pearson correlation coefficient for this data set is $r = 0.000$, and the Spearman rank correlation coefficient is $s = 0.116$. It is clear that the error bars are underestimated in at least one axis; the spread is larger in the $V_{pec}(D_n - \sigma)$ axis, suggesting that the errors are larger there. This diagram is a direct measure of the reliablility of measurements of large-scale peculiar velocities (some are *really* peculiar). Measurements for the individual galaxies or galaxy groups can only be worse, given smaller numbers of data points.

A basic test for any physical measurement is a comparison of results obtained using different methods, for example, the peculiar velocities obtained using the $D_n - \sigma$ and Tully-Fisher relations. Such a comparison has been made for rich clusters (where the $1/\sqrt{N_{gal}}$ statistics should help) by Mould *et al.* (1991), and de Carvalho & Djorgovski (in prep.), as shown in Fig. 7. The results are not encouraging: the observed scatter and the lack of any correlation suggest that the claimed errors are underestimated. Comparisons of the peculiar velocities obtained using distance-indicator relations and modeling of the IRAS redshift surveys or optical and IRAS dipoles (e.g., Strauss & Davis 1988, Lynden-Bell & Lahav 1988, Rowan-Robinson *et al.* 1990, Kaiser *et al.* 1991, etc.) give equally dismal results. As a rule, the Tully-Fisher results are in a better agreement with other measurements than the $D_n - \sigma$ results. For example, Tonry (1991) finds that the distances to galaxies measured using the surface brightness fluctuations technique agree very well with those from the TFR, while there are substantial disagreements with the $D_n - \sigma$ distances.

In addition to possible cluster-to-cluster variations, de Carvalho & Djorgovski (1992; and in prep.) find systematic differences in the FP solutions for the field and cluster ellipticals, in the sense of a shallower dependence of R or D_n on σ. Weigelt & Kates (1990) also found that a self-consistent solution for the entire "7 Samurai" sample, which includes many field and loose group ellipticals, has a $D_n - \sigma$ relation slope of 0.98 ± 0.06; this should be contrasted with the cluster-defined slope of 1.33 from Dressler *et al.* (1987*b*). Thus, the cluster-calibrated $D_n - \sigma$ relation may not be applied securely to the ellipticals in the field or in loose groups. Errors in the slope of a distance indicator relation translate to effective errors in intercepts for galaxies at different distances from us, and thus can generate spurious peculiar velocities.

These comparisons suggest that some enviromental variations may exist on a $\sim 10\%$ level. This is comparable to the measurement errors, and thus would represent a very delicate effect. On the other hand, the absence of even larger apparent peculiar velocities suggests that the distance-indicator relations are universal (at least in our ~ 100 Mpc vicinity) at a level of $\sim 20\%$ or better. That is in itself a highly nontrivial constraint on galaxy formation.

A more mundane, but not less important problem is the requirement that the measurements for all-sky samples of galaxies are free of systematic errors at a level of a couple of percent in each quantity, or better. As most of the practitioners of this kind of work know, this is a very difficult task, and it may be even impossible to guarantee this kind of precission with the data available today. This is seldom admited or fully realized in the literature.

6. TOWARDS A UNIFIED PICTURE OF GALAXY POPULATIONS

The manifold of low surface brightness dwarf galaxies is discussed in the paper by de Carvalho & Djorgovski elsewhere in this volume and will not be discussed here; see also Kormendy (1985). The behavior of gas-poor dwarfs is quite different from that of normal ellipticals, pointing to a fundamental difference in their formation histories. At this point, very little is known about the systematics of properties of gas-rich dwarfs.

At least for the normal galaxies, there seems to be a general feature that most global properties are unified in two-dimensional manifolds, the FP, and the SFP. A major technical problem is that we cannot compare very easily the manifolds of spirals and ellipticals, since we measure different things for them, or compare either with the theoretical quantities. The problem is operational, rather than fundamental, and a unified description of all galaxy manifolds in terms of physical quantities (e.g., mass, density, etc.), rather than in terms of raw observables, should be possible. The situation may be analogous to the problem of translating observed H-R diagrams in different photometric systems for young and old star clusters into the same theoretical (L_{bol}, T_{eff}) plane.

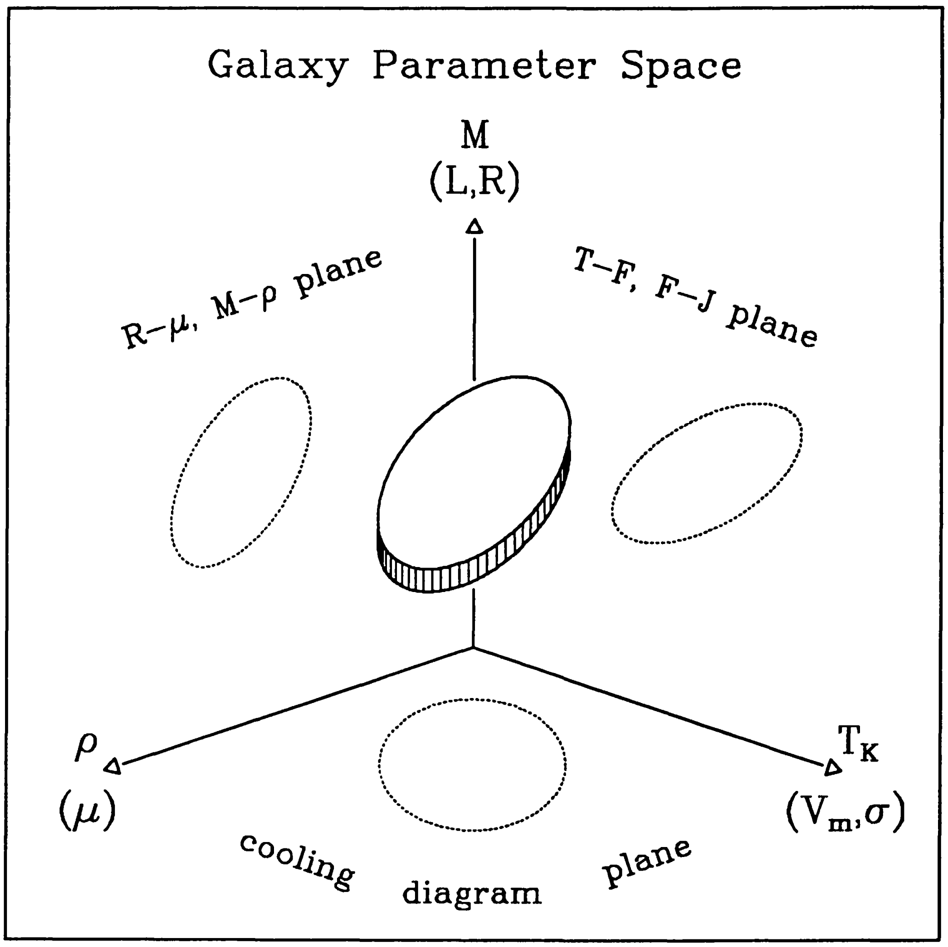

Figure 8. A schematic representation of the galaxy parameter space. Galaxies of a given family (ellipticals or spirals, and probably dwarfs as well) occupy two-dimensional regions (thickened in the third dimension mainly by the measurement errors) in a parameter space whose axes can be called size (mass, luminosity, or radius), density (or surface brightness), and temperature (i.e., kinetic energy per unit mass, typically the maximum rotational velocity for cold disks, or the central velocity dispersion for pressure-supported systems). The particular choice of axes depends on the application and available observables, but the basic picture remains unchanged. The coordinate planes thus defined are some of the well-known diagrams in extragalactic astronomy and cosmology; however, none of them contains all the information, only the oblique projections of the galaxy manifolds. (From Djorgovski 1991).

The observed fact that the fundamental correlations of galaxian properties are very well represented by power laws (even if multivariate), is interesting in itself. It implies that there are no strongly prefered physical scales of global properties for galaxy *families*. This may or may not be strictly true: it is possible that the correlations deviate slightly from the power laws, i.e., show some curvature. Some evidence for such nonlinearity has been seen both for the high-luminosity end of the FP (Djorgovski & de Carvalho 1990), and in the TFR (Aaronson & Mould 1983).

As an approach to this unified view, consider a parameter space whose axes are a measure of galaxy size (e.g., mass, luminosity, or radius), density (or surface density or surface brightness), and kinetic temperature of stars (kinetic energy per unit mass, e.g., rotational V_{max} or velocity dispersion, depending on the dominant support component), as schematically shown in Fig. 8. Individual coordinate planes of this galaxy parameter space (G-space for short) are some of the familiar diagrams in extragalactic astronomy and cosmology: the Tully-Fisher or Faber-Jackson relations (luminosity vs. velocity scale plane), the cooling diagram (projected density - kinetic temperature), or the radius - surface brightness (the Kormendy relation) or mass-density plane (which describes the primordial density fluctuation spectrum). As already discussed, galaxies of all families (e.g., spirals, ellipticals) do not fill the G-space, but sit on two-dimensional surfaces embedded in it, and are in general tilted with respect to all of its axes. Projecting these actual distributions of data points onto any of the coordinate planes smears the intrinsic bivariate correlation, entailing a loss of information; that is why G-space needs three dimensions.

The G-space and galaxy manifolds provide a potentially very useful and powerful new framework for studies of systematic properties of galaxies and their formation. Different formative and evolutionary processes (dissipation, collapse, merging, galactic wind sweeping, etc.) can obviously move galaxies in the G-space, in ways which are only partially and qualitatively understood at this point, and remain yet to be fully explored theoretically. This multivariate approach can also lead to a better understanding of the distance-indicator relations and the limits of their accuracy. At long last, we may have an analog of the H-R diagram for galaxies.

The empirical setup of the H-R diagram provided theories of stellar structure and evolution with some fundamental facts to be explained about the stars, with a testbed and a framework; it continues to play these roles to this day. The G-space may do the same for galaxies. Just as stars represent a one-dimensional sequence of mass embedded in the space defined by the luminosity and temperature (the H-R diagram), galaxies form two-dimensional sequences embedded in the G-space. The low dimensionality of galaxy manifolds suggests that a small number of physical processes were operating in forming galaxies, and/or that they were well-regulated. The resulting bivariate correlations, e.g., the FP or the SFP, may contain some useful information about them.

It may be worthwhile to extend this type of multivariate study to systems of galaxies, such as groups or clusters. An example of such analysis of compact groups is presented in the paper by Djorgovski *et al.* elsewhere in this volume. We are now preparing a similar study of properties of rich clusters of galaxies, including their x-ray parameters as well.

While the observational efforts to secure large, homogeneous data samples on galaxies of all types continue, we probably need more theoretical models with testable predictions in this new framework. The field is wide open. We are not short on fundamental questions posed by the data.

This review is based on the work involving many collaborators, and in particular Reinaldo de Carvalho. The data for our survey of early-type galaxies have been obtained at Palomar, LCO, and CTIO, and I wish to thank their staff for the expert help during our observing runs. Nick Weir made valuable software contributions. This work was supported in part by the Alfred P. Sloan Foundation, the NSF PYI award AST-9157412, and the NASA contract NAS5-31348. I also acknowledge a AAS travel grant.

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