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THE GALACTIC GLOBULAR CLUSTER SYSTEM

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ABSTRACT

We explore correlations between various properties of Galactic globular clusters, using a database on 143 objects. Our goal is identify correlations and trends which can be used to test and constrain theoretical models of cluster formation and evolution. We use a set of 13 cluster parameters, 9 of which are independently measured. Several arguments suggest that the number of clusters still missing in the obscured regions of the Galaxy is of the order of 10, and thus the selection effects are probably not severe for our sample. Known clusters follow a power-law density distribution with a slope ~ -3.5 to -4, and an apparent core with a core radius ~ 1 kpc. Clusters show a large dynamical range in many of their properties, more so for the core parameters (which are presumably more affected by dynamical evolution) than for the half-light parameters. There are no good correlations with luminosity, although more luminous clusters tend to be more concentrated. When data are binned in luminosity, several trends emerge: more luminous clusters tend to have smaller and denser cores. We interpret this as a differential survival effect, with more massive clusters surviving longer and reaching more evolved dynamical states. Cluster core parameters and concentrations also correlate with the position in the Galaxy, with clusters closer to the Galactic center or plane being more concentrated and having smaller and denser cores. These trends are more pronounced for the fainter (less massive) clusters. This is in an agreement with a picture where tidal shocks form disk or bulge passages accelerate dynamical evolution of clusters. Cluster metallicities do not correlate with any other parameter, including luminosity and velocity dispersion; the only detectable trend is with the position in the Galaxy, probably reflecting Zinn's disk-halo dichotomy. This suggests that globular clusters were not self-enriched systems. Velocity dispersions show excellent correlations with luminosity and surface brightness. Their origin is not well understood, but they may well reflect initial conditions of cluster formation, and perhaps even be used to probe the initial density perturbation spectrum on a $\sim 10^6 \mathscr{M}_\odot$ scale. Core radii and concentrations play a role of a "second parameter" in these correlations. While a global manifold of cluster properties has a high statistical dimensionality ($D>4$), a subset of structural, photometric, and dynamical parameters forms a statistically three-dimensional family, as expected from objects following King models; we propose to call this set of quantities the King Manifold. Some of the observed correlations may be usable as distance indicator relations for globular clusters.

1. INTRODUCTION

The Galactic globular cluster system represents a fossil record of the formation of our Galaxy. It is also a valuable tracer of our Galaxy's structure and dynamics. Its constituents, globular clusters, are among the most interesting astronomical objects, veritable laboratories for studies of stellar dynamics and stellar populations. Analysis of correlations among their properties can yield fundamental clues for our understanding of formation of globular clusters and their evolution in the Galactic tidal field.

In this paper we perform such an analysis of a data set on 143 Galactic globular clusters (hereafter GCs), compiled in the appendices of the volume edited by Djorgovski & Meylan (1993a). Earlier work includes the pioneering studies by Peterson & King (1976) and Brosche & Lentes (1984), and also the works by Chernoff & Djorgovski (1989), Djorgovski (1991, 1993a), and many others. Our preliminary results were presented at conferences (e.g., Djorgovski & Meylan 1993c). Our goal is to identify nontrivial correlations among the structural and dynamical parameters of GCs, their metallicities, and positions within the Galaxy, and then to try to interpret these correlations within the framework of our present understanding of their formation and dynamical evolution. Any such correlations should provide valuable constraints for theoretical models.

It is also important to note what we are *not* trying to cover in this study: properties of color-magnitude diagrams, including horizontal branch and the "second parameter" effects, dynamically induced changes in cluster stellar populations, cluster ages, stellar luminosity, and mass functions, the disk-halo dichotomy, details of chemical abundances and inhomo-

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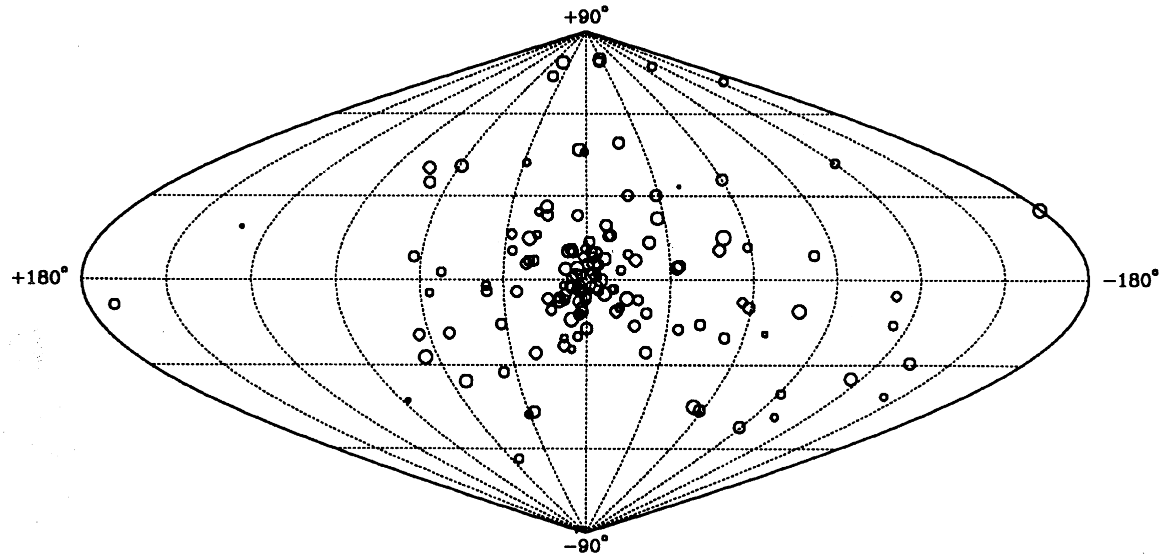

FIG. 1. A projected distribution of the 143 known globulars in Galactic coordinates. The symbol size scales with the logarithm of luminosity. The strong central concentration is obvious.

geneities, internal dynamics of clusters, including binaries, pulsars, etc., cluster shapes and orientations, uses of the GC system as a probe of the Galactic structure and dynamics, and its own kinematics. For more on these fascinating topics, the reader may wish to consult numerous excellent reviews and contributed papers in the recent proceedings edited by Smith & Brodie (1993) and Djorgovski & Meylan (1993a), and references therein.

As noted, the data used for this study are from the compilations by Djorgovski & Meylan (1993b), Peterson (1993), Pryor & Meylan (1993), Trager *et al.* (1993), and Djorgovski (1993b), and we will not repeat them here. The data will be kept and updated in a simple database accessible via Internet, and any interested users wishing to access the data should send an email message to the principal author of this paper. While the quality of the data is still not as high as it can and should be, we believe that this data base represents a substantial improvement over the previous compilations (e.g., by Webbink 1985). Specifically, we concentrate here on the following 13 quantities: the absolute visual magnitude, M_V; the King concentration parameter, c; log of the core radius in parsec, r_c; log of the half-light radius in parsec, r_h; the central surface brightness in the V band, $\mu_V(0)$; the average surface brightness in the V band within the r_h,$\langle\mu_V\rangle_h$; log of the central luminosity density in $L_{\odot V}/\text{pc}^3$, ρ_0; log of the central relaxation time in years, t_{rc}; log of the half-mass relaxation time in years, t_{rh}; the metallicity, [Fe/H]; log of the central velocity dispersion in km/s, σ; log of the distance from the Galactic center in kpc, R_{gc}; and log of the distance from the Galactic plane in kpc, Z_{gp}. The definitions of these quantities, error estimates, and other details can be found in the references quoted. All photometric quantities have been corrected for extinction.

While a few relatively minor corrections have been made since the data were assembled and published, they do not affect this analysis in any way. The following corrections may be worth noting, however: The colors listed as $(U-V)$ in Peterson (1993) are actually $(U-B)$ (Peterson 1993). The metallicity of Rup 106 is [Fe/H]=−1.90, not −1.09 (van den Bergh 1993). The identity of cluster Djorg 3 has been confused: this is really NGC 6540, which is *not* an open cluster, but indeed a previously neglected globular (Skiff 1993).

We treat the clusters with a post-core-collapse (PCC) morphology (Djorgovski & King 1984, 1986) just as if they are a high-concentration end of the King (1966) model (KM) sequence; this is, in most cases, an excellent approximation. We use the observed values of half-width at half-maximum as their core radii, which, strictly speaking is an upper limit. We also assign the constant concentration value of $c=2.5$ to all of them, which is as high a concentration a cluster could reasonably have for any length of time outside brief periods of extreme core collapse. In retrospect, neither approximation seems to affect the observed correlations substantially.

2. DISTRIBUTION OF CLUSTERS, AND SELECTION EFFECTS

Figure 1 shows the distribution of clusters on the Galactic sky. The well known strong central concentration is the most obvious feature of the distribution. Note also the absence of an obvious zone of avoidance near the Galactic plane: this immediately suggests that we cannot be missing too many clusters due to obscuration.

It is very likely that some clusters *are* still missing, lost in the obscured areas near the Galactic plane. Estimating their number is important, as the selection effects could, in principle, affect the correlations we find. We approach this question in several ways.

The radial distribution of clusters is quantified in Fig. 2.

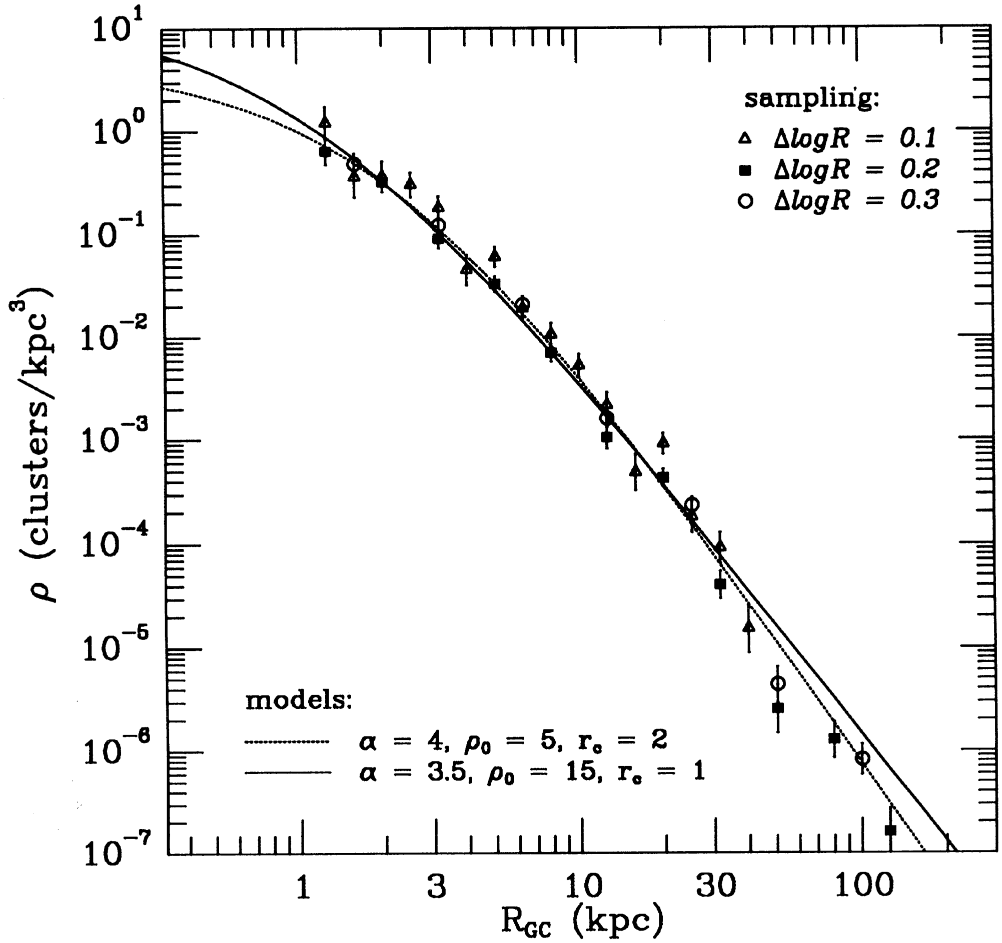

FIG. 2. The derived three-dimensional density distribution for the known clusters. Different symbols correspond to different samplings of the same data. Power-law models with a core [Eq. (1)] are fitted to the data by eye. The two models shown indicate typical best matches, for different values of the slope, core radius, and central density. The flattening near the center is probably partly real, partly due to the smearing by errors in distances, and partly due to the missing clusters, hidden by the dust obscuration. Integrating the best fitting models inwards indicates that at most a few clusters may be missing.

We parametrize it using a simple power law with a core formula

$$\rho(r)=\rho_0\left(1+\frac{r}{r_c}\right)^{-\alpha}. \quad (1)$$

This approach is purely empirical, and is not meant to imply any physical meaning of the distribution given by Eq. (1), reasonable as it might seem. We also neglect the disk-halo dichotomy, and many other fine points. Our purpose here is simply to estimate the number of clusters which we may be missing in the central parts of the Galaxy.

Given the probable incompleteness of the data, and the distance errors, which could be rather substantial for the heavily obscured clusters at small galactocentric radii, we do not perform a rigorous fit to the data, but rather a "chi-by-eye" fitting of model curves with various values of the core radius r_c and the power-law exponent α. Good matches are found for the values of $\alpha \sim 3.5-4$ (a generally quoted value in the literature is 3.5), and the core radii $r_c \sim 0.5-2$ kpc. Integrating the models and subtracting the number of the known clusters, we find that at most a few clusters may be missing near the center, typical numbers being in the range $\sim 1-10$ (more for the models with steeper slopes and smaller cores).

The steep observed slope of this distribution is significantly different from the density law of the dark halo which results in a flat rotation curve, $\rho(r) \sim r^{-2}$. It is hard to imagine an evolutionary process which could convert a r^{-2} distribution into a $r^{-3.5}$ one over the Hubble time for the globular clusters, but not for the dark halo material. This implies a different origin for the GC system (and presumably the visible stellar halo), and the dark halo, whatever its constituents are. For example, if the dark halo was made up of dark clusters of neutron stars or brown dwarfs, or their tidally ground up remnants, such hypothetical clusters must have formed in a process different from that of the known, visible clusters—which sounds rather artificial and perhaps unlikely.

The apparent flattening of the distribution near the center (the core) is probably due to a combination of effects: smearing due to the distance errors, genuine missing clusters due to obscuration, and the real flattening of the distribution. The latter may reflect the initial conditions, but also possible dy-

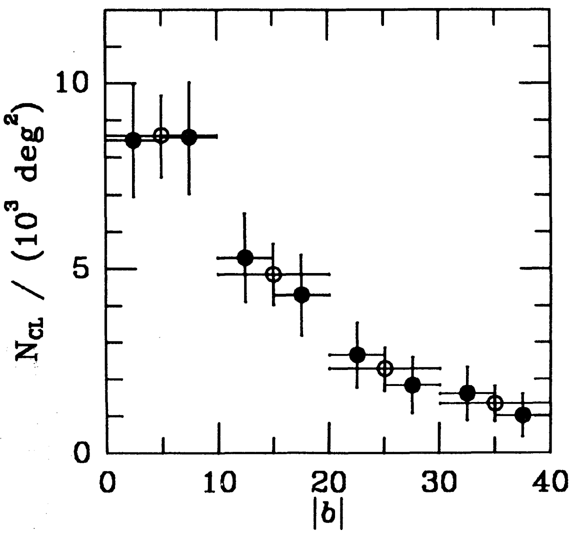

FIG. 3. Projected distribution of clusters in absolute Galactic latitude bins. Different symbols correspond to different samplings of the same data. Here the (often poorly known) distances to clusters are not important. Again, a central flattening is seen. Extrapolating the trend from large values of $|b_{\rm II}|$ inwards, and multiplying by the corresponding solid angle at low latitudes, indicates that at most ~10–20 are still missing in the obscured areas.

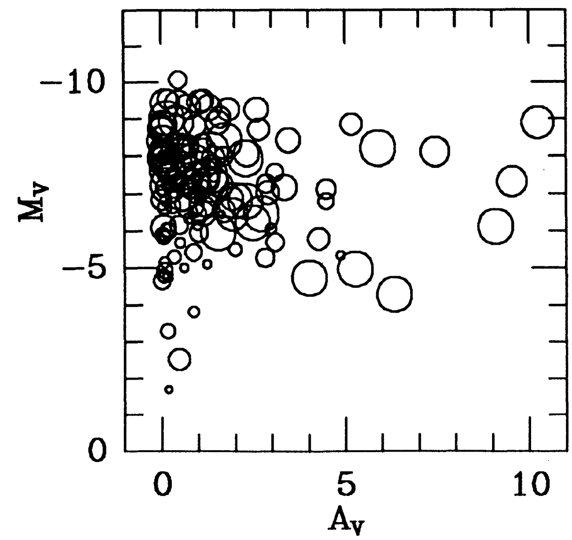

FIG. 4. A plot of the cluster absolute magnitude, M_V, vs the estimated extinction in the V band, A_V, in magnitudes. Symbol size is proportional to the cluster concentration, c. Only very concentrated clusters are seen under a high obscuration. This is clearly a selection effect, although there is a real correlation between c and $R_{\rm gc}$ or $Z_{\rm gp}$, which in turn correlate with A_V. Faint clusters are also missing at high extinction, but this may be largely the luminosity function effect.

namical effects, viz., a more effective tidal destruction of clusters near the center. The apparent core radii we find (~1–2 kpc) are considerably larger than the characteristic radii for the stellar distribution in the bulge: Blanco & Terndrup (1989) give r_c=0.11±0.04 kpc for the bulge light. This is reminiscent of the situation seen in M87, where the core radius of the globular cluster system is some 13 times larger than that of the underlying galaxy's light (Lauer & Kormendy 1986). Ostriker *et al.* (1989) proposed that the efficiency of cluster destruction near the center of M87 is enhanced in a triaxial potential. It is tempting to speculate that the flattening of GC distribution in our Galaxy may be a consequence of the recently discovered central bar (Blitz & Spergel 1991; Long *et al.* 1992).

An alternative approach is to look at the distribution of clusters as a function of Galactic latitude, as shown in Fig. 3. This may be an even more relevant probe of the possible missing clusters, since the main culprit is the foreground obscuration, which directly correlates with the Galactic latitude. The distribution of clusters does appear to flatten at latitudes lower than ~5°–10° (cf. also Woltjer 1975). The missing surface density is at most ~1–2 clusters/(1000 degree^2). Since the solid angle corresponding to this Galactic plane belt is ~(5–10)×10^3 degree^2, the number of missing clusters at low latitudes is at most ~10–20.

Finally, an instructive diagram is shown in Fig. 4, a plot of absolute magnitude versus line-of-sight extinction. The absence of low luminosity clusters at high extinction is almost certainly a selection effect. However, low mass clusters would be less likely to survive tidal destruction at small distances from the Galactic center and plane, where the extinction is higher, so that some of the observed deficiency of clusters may be real. Note also the absence of low concentration clusters of any luminosity (the two are correlated, however; see Sec. *3.1* below) at high extinctions: such clusters would be hard to detect in crowded areas near the plane.

On the whole, we estimate that the number of missing clusters at low latitudes and/or near the Galactic center is perhaps of the order of 10. Similar conclusions have been reached by Racine & Harris (1989), who performed a more detailed analysis, and also by Woltjer (1975) and Oort (1977). We thus conclude that this is not a very serious selection effect (an incompleteness of ~5% of the total number of clusters) for the present analysis.

We note that at any latitude, the selection effects are higher for the faint, low density clusters of the Palomar type, and that very likely more such systems remain to be discovered. For example, we have recently found that four star clusters catalogued by Lauberts (1982) may be previously unrecognized globulars (Djorgovski *et al.* 1994). Since no complete, well defined search for such clusters has ever been done, the selection effects are hard to estimate. It is our expectation that any such objects would largely follow the correlations described below, but the caveat remains.

3. SIMPLE CORRELATIONS AND TRENDS

The first striking thing about the data on globular clusters is the vast dynamical range they span in many of their properties, such as the luminosity, density, etc.—more so than either elliptical or dwarf galaxies (cf. Djorgovski 1993a for comparisons). This is illustrated for selected properties in Fig. 5, which compares parameters measured at or within the two characteristic radial scales, the core radius (r_c), and the half-light radius (r_h). (Here we computed the approximate

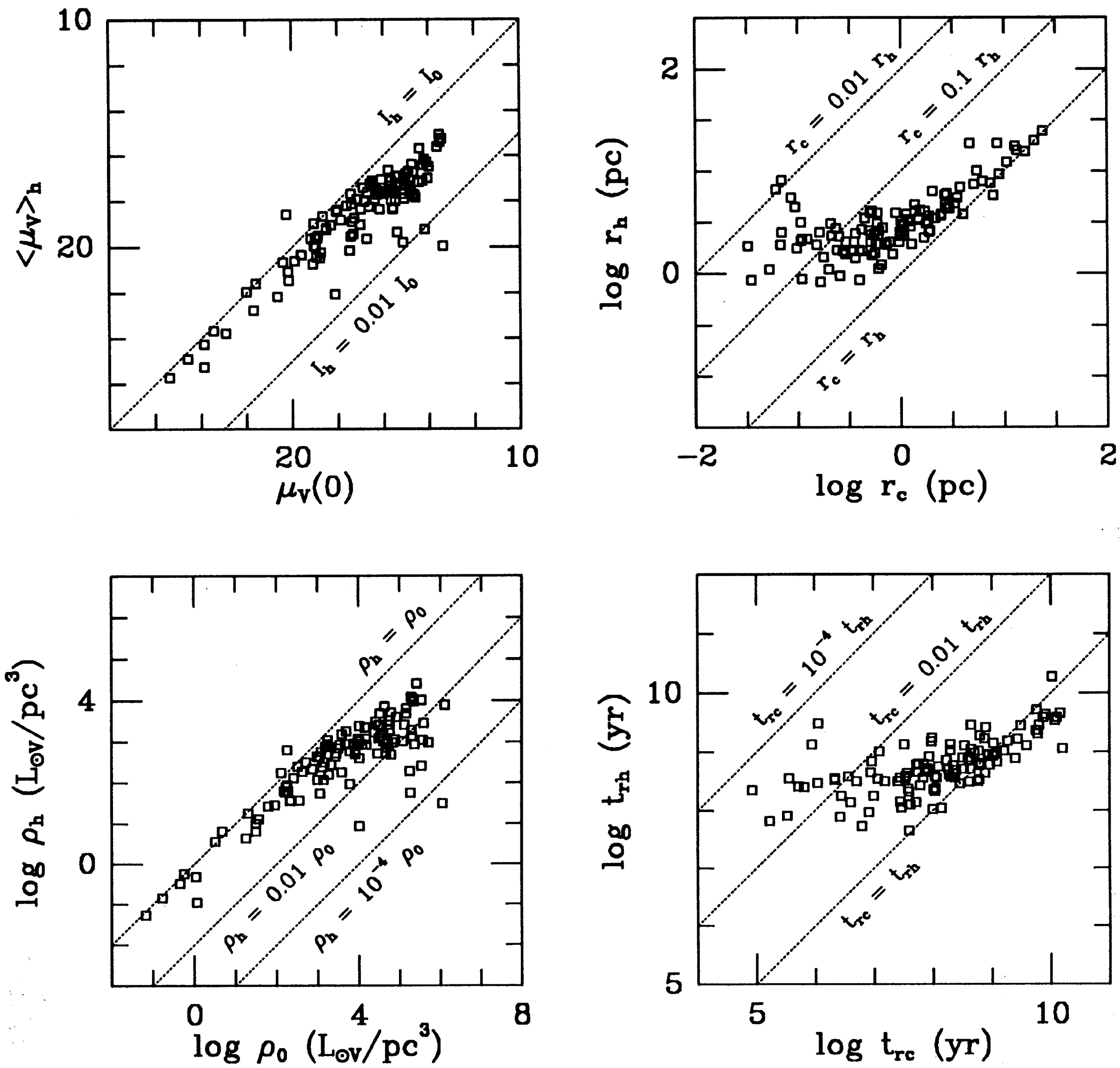

FIG. 5. Large dynamical ranges and relations between selected core (X axis) and half-light (Y axis) cluster parameters are illustrated here. Clusters with largest radii and lowest densities have the core and half-light parameters comparable. At smaller radii, ratio of r_c/r_h decreases, with a corresponding change in all other quantities.

mean luminosity density ρ_h as the one half of the total visual luminosity, averaged over a sphere of a radius r_h.) For the least concentrated, largest, and least dense clusters, $r_c \sim r_h$, and consequently the core and half-mass quantities are comparable, but discrepancies increase as the concentration increases and the core radius gets smaller. The most concentrated clusters have the ratios r_c/r_h of a few percent, in an agreement with the expectations from theory (cf. Vesperini & Chernoff 1994, and references therein).

Most core or central parameters span a larger dynamical range than the corresponding half-light (~ half-mass, or median) quantities. Qualitatively, this may be understood as a consequence of dynamical evolution which operates faster at core scales, where the reference ("relaxation") time scales are shorter, up to a factor of a hundred. It is worth noting that for most part dynamical evolution of clusters tends to *increase* the range of their properties: this is a consequence of the fact that the dominant evolutionary mechanisms for clusters are *instabilities*, such as the gravothermal (core collapse) or mass segregation instabilities. This "stretching of properties" of clusters is inevitable: even if the distributions of cluster properties started as δ functions, some spread would occur over the Hubble time, if for no other reason than because of the different tidal effects for different clusters. Once different dynamical time scales are present, clusters will evolve apart in their properties. The observed fact that the half-mass relaxation times span over two orders of magnitude, and the central relaxation times some five orders of magnitude, practically guarantees that the GC population will contain a range of objects at all stages of dynamical evolution. Evolution of cluster properties, which could generate detectable correlations among them, is thus an essential facet of the GC system.

We first explore the simple, pairwise correlations among

TABLE 1. Matrix of the correlation coefficients.

	M_V	c	$\log r_c$	$\log r_h$	$\mu_V(0)$	$\langle\mu_V\rangle_h$	$\log\rho_0$	$\log t_{rc}$	$\log t_{rh}$	[Fe/H]	$\log\sigma$	$\log R_{gc}$	$\log Z_{gp}$	
M_V	1	−0.435	0.327	0.158	0.634	0.696	−0.486	0.028	−0.290	−0.041	−0.715	0.292	0.198	M_V
c	−0.475	1	−0.889	−0.461	−0.730	−0.498	0.812	−0.853	−0.385	0.106	0.334	−0.469	−0.400	c
$\log r_c$	0.339	−0.890	1	0.709	0.827	0.638	−0.929	0.975	0.678	−0.272	−0.327	0.661	0.622	$\log r_c$
$\log r_h$	0.062	−0.471	0.689	1	0.769	0.819	−0.809	0.657	0.899	−0.226	−0.414	0.580	0.596	$\log r_h$
$\mu_V(0)$	0.537	−0.808	0.853	0.672	1	0.908	−0.976	0.681	0.480	−0.252	−0.803	0.601	0.578	$\mu_V(0)$
$\langle\mu_V\rangle_h$	0.642	−0.542	0.597	0.730	0.829	1	−0.840	0.479	0.485	−0.145	−0.853	0.514	0.503	$\langle\mu_V\rangle_h$
$\log\rho_0$	−0.390	0.854	−0.946	−0.720	−0.969	−0.744	1	−0.823	−0.623	0.271	0.650	−0.631	−0.615	$\log\rho_0$
$\log t_{rc}$	0.059	−0.832	0.979	0.711	0.753	0.501	−0.873	1	0.658	−0.201	−0.141	0.558	0.545	$\log t_{rc}$
$\log t_{rh}$	−0.283	−0.383	0.669	0.905	0.441	0.418	−0.568	0.694	1	−0.258	−0.024	0.511	0.584	$\log t_{rh}$
[Fe/H]	−0.010	0.080	−0.275	−0.193	−0.237	−0.146	0.257	−0.219	−0.236	1	0.216	−0.426	−0.534	[Fe/H]
$\log\sigma$	−0.688	0.327	−0.314	−0.330	−0.774	−0.796	0.555	−0.119	0.029	0.147	1	−0.129	−0.131	$\log\sigma$
$\log R_{gc}$	0.221	−0.447	0.640	0.478	0.486	0.358	−0.546	0.550	0.492	−0.420	−0.097	1	0.788	$\log R_{gc}$
$\log Z_{gp}$	0.092	−0.361	0.584	0.467	0.450	0.329	−0.511	0.524	0.529	−0.506	−0.062	0.756	1	$\log Z_{gp}$
	M_V	c	$\log r_c$	$\log r_h$	$\mu_V(0)$	$\langle\mu_V\rangle_h$	$\log\rho_0$	$\log t_{rc}$	$\log t_{rh}$	[Fe/H]	$\log\sigma$	$\log R_{gc}$	$\log Z_{gp}$	

Upper right: Pearson linear regression correlation coefficients
Lower left: Spearman rank correlation coefficients

the selected GC properties. Table 1 lists the Pearson linear regression correlation coefficients (upper right half of the Table) and the Spearman rank correlation coefficients (lower left), for 13 quantities explored in this study. We have also performed the analysis using cluster ellipticities and orientations (Shawl & White 1987), and found that they do not correlate with any other cluster parameters, and thus provide no useful information for the purposes of the present study; consequently we omitted them from our analysis here. We also used the ratios Z_{gp}/R_{gc}, which are a statistical measure of the orbit inclinations, but they also provided no new insights.

We note that among the 13 quantities studied here, only 9 are measured independently: $\langle\mu_V\rangle_h$ is derived from the M_V and r_h, ρ_0 is derived from the $\mu_V(0)$, c, and r_c, t_{rc} is derived from the M_V, c, and r_c, and t_{rh} is derived from the M_V, and r_h. This will cause spurious correlations. In general correlations of any of the derived quantities with any of their constituent quantities or combinations thereof should not be trusted. Furthermore, R_{gc} and Z_{gp} are derived from the same coordinates and distances to clusters, and are necessarily correlated.

In an earlier study, Djorgovski (1991) also included cluster ellipticities and major axis orientations with respect to the Galactic center. Neither quantity correlated with anything else, perhaps because the dynamical range of cluster ellipticities is comparable to the measurement errors, and position angles of round and "bumpy" isophotes are also liable to be poorly determined (these clusters are called globular for a very good reason). No new data on GC ellipticities are available, even though other data we use are better than what was available previously. We have checked, and there are still no correlations of anything with either ellipticity or position angle. We have thus excluded these quantities from our analysis as presented here.

Inspection of Table 1 immediately suggests some interesting correlations. We will explore them in turn. In some cases, an absence of a correlation may be equally instructing.

3.1 Luminosity Correlations

Luminosity is perhaps the most fundamental observed quantity characterizing a stellar system. For a set of old stellar systems it is a good relative measure of the baryonic mass. Many other properties correlate with luminosity for elliptical and dwarf galaxies; no so for globular clusters (cf. Djorgovski 1993a for comparisons). This is illustrated in Fig. 6.

The only good correlation with luminosity is that with the velocity dispersion, discussed in more detail in Sec. 3.5 below. The only other discernible trends are with the concentration and central surface brightness (or equivalently, central luminosity density); both are shown in Fig. 6. More luminous clusters tend to have higher concentrations and denser cores, but there is a large scatter at every luminosity (cf. also Djorgovski 1991). Interestingly, neither r_c nor r_h correlates with luminosity; this is in a marked contrast to elliptical and dwarf galaxies, where the corresponding correlations are excellent.

The correlation between M_V and c is particularly interesting. The relative scarcity of low luminosity, low concentration clusters could be in part due to an observational selection (such clusters would be hard to find, especially at low Galactic latitudes), but there is no obvious reason for the absence of either luminous, low concentration clusters, or the low luminosity, high concentration clusters: they must be genuinely absent. The absence of the former probably reflects the initial conditions. The absence of the latter may be due to a rapid evaporation of the low mass systems.

While no good correlations are seen, some trends do show up when data are binned in luminosity, and average values are computed in each bin. We have sorted the data in lumi-

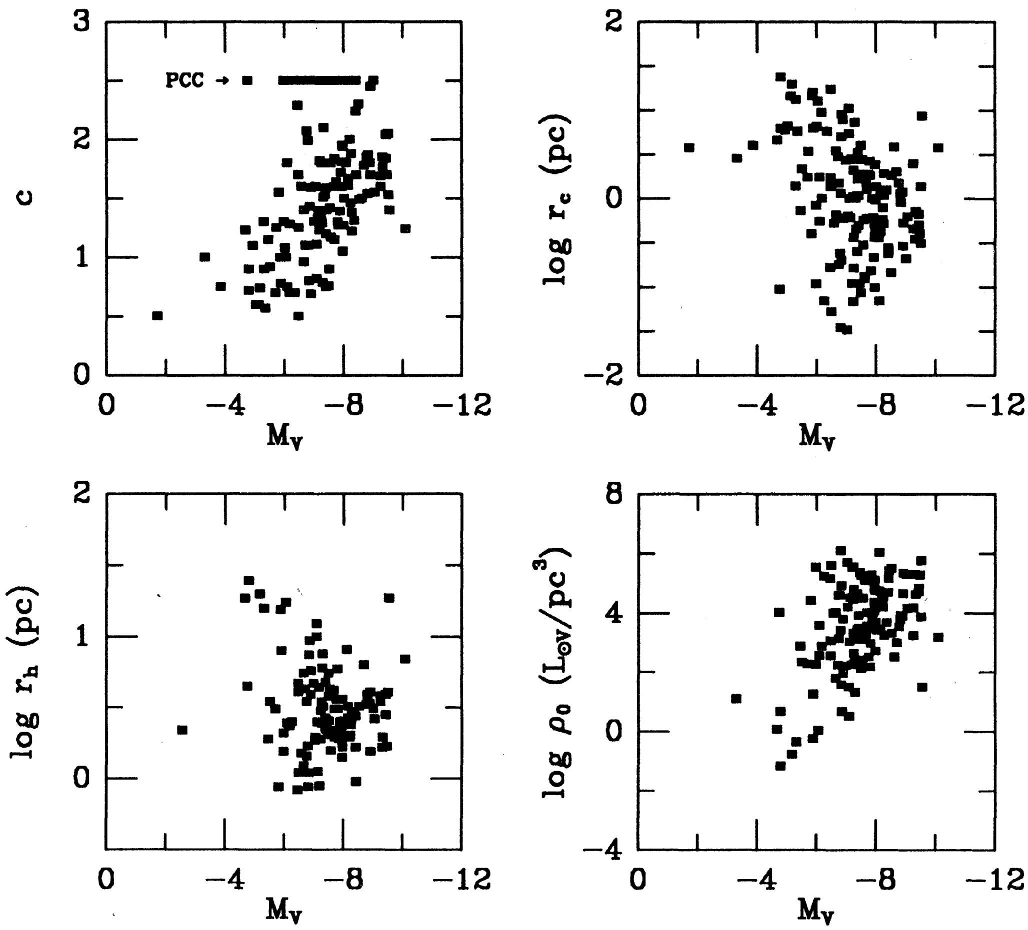

FIG. 6. Correlations, or lacks thereof, of various parameters with absolute magnitude. The only discernible correlation is with the King concentration parameter, c (upper left). The absence of luminous, low-concentration clusters must be genuine: they are unlikely to have been missed due to selection effects. The situation for low-luminosity, high-concentration clusters is less clear: they may have evaporated in the Galactic tidal field, or may have never existed. There may be a corresponding, but much worse, correlation with core radii, r_c (upper right). The rough correlation with central luminosity density, ρ_0, (lower right) is probably just a product of these correlation. Remarkably, there is no correlation at all with the half-light radii, r_h (lower left).

nosity and divided them in bins of 21 points each (17 in the last bin). In each bin, we evaluated median values of other quantities, and estimated the equivalent Gaussian dispersions (denoted $Q\sigma$) from the quartiles of the distribution:

$$Q\sigma = 0.7415(Q_{75} - Q_{25}), \tag{2}$$

where Q_{75} and Q_{25} are the 75th and 25th percentiles of the distribution for the quantity being considered. For a pure Gaussian, $Q\sigma \equiv \sigma$. This use of robust nonparametric statistics guards against possible bad data points and outliers.

Figure 7 shows the trends for the binned data. The trends for the core parameters, $\mu_V(0)$, c, and ρ_0 are now quite apparent. There is also a trend with r_c, which goes in the direction intuitively expected from the other three trends: more luminous clusters tend to have smaller cores. This is the opposite of the corresponding relations for elliptical and dwarf galaxies. The lack of any correlation with t_{rc} is probably due to the countervailing trends of ρ_0 and r_c, from which t_{rc} is derived. The mean trends through the binned data points correspond to the scaling laws:

$$r_c \sim L_V^{-0.5\pm0.25}, \tag{3a}$$

$$\rho_0 \sim L_V^{2\pm1}. \tag{3b}$$

Clusters with higher concentrations and smaller and denser cores are presumably more dynamically evolved. There is no obvious reason why dynamical evolution would produce such correlations with luminosity directly, especially since more massive clusters would have longer relaxation times, everything else being the same. A possible explanation is that we are seeing a differential survival effect: less massive clusters would evaporate faster in the tidal field of the Galaxy. More massive clusters would on the average survive longer, and have better chances of reaching more evolved dynamical states, as apparent in their core properties and concentrations.

Cluster half-light radii, and to a large degree also luminosities, are not expected to have changed much due to dynamical evolution (Murray & Lin 1992). One expects little effect on the half-light radii and the quantities averaged on that radial scale, as is indeed observed: r_h shows no trend

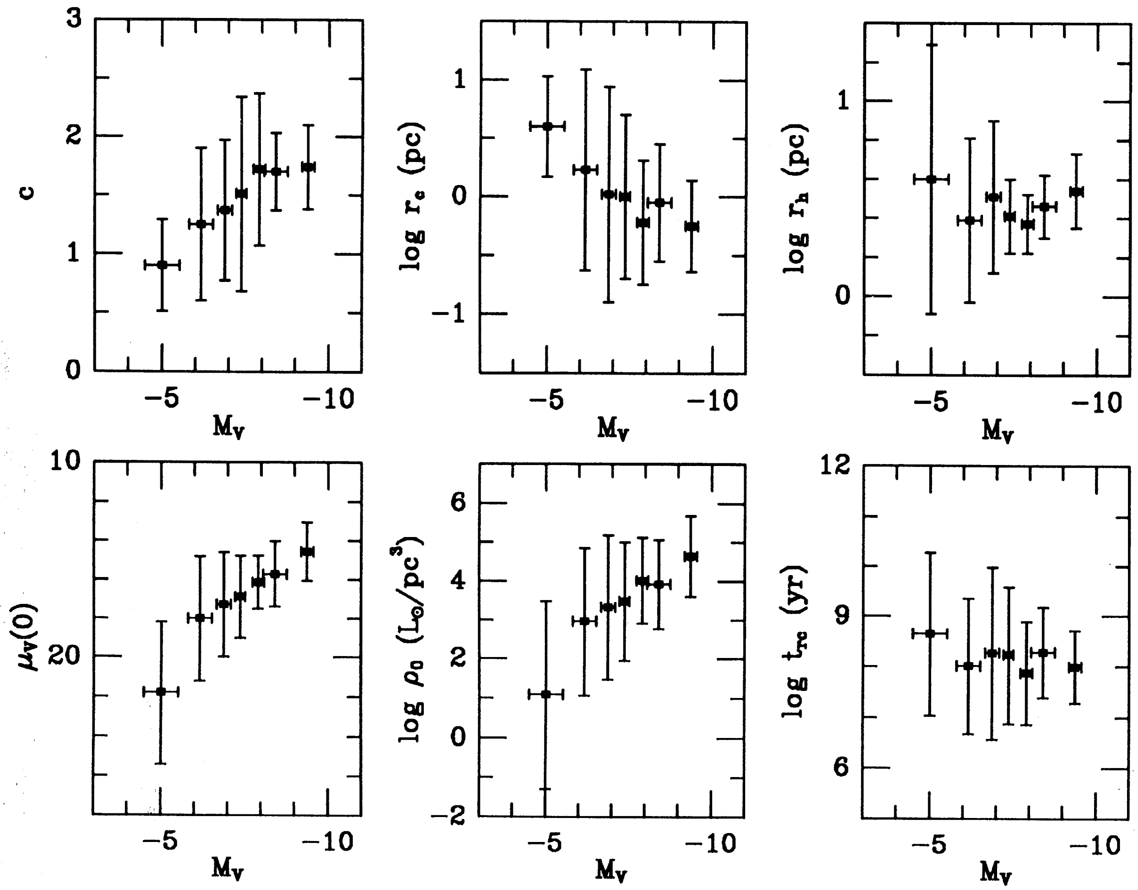

FIG. 7. Trends for the data averaged in luminosity bins. Median values for each quantity are plotted. The error bars correspond to non-parametric estimates of dispersion in each bin, $Q\sigma$ [Eq. (2)]. The trends with the core parameters are now quite evident: more luminous clusters tend to have smaller and denser cores, and higher concentrations. There are still no trends at all for the half-light parameters.

with the luminosity, as already noticed by van den Bergh *et al.* (1991). The lack of a correlation between r_h and M_V thus indicates right away that GCs must be at least a two-parameter family of objects.

3.2 *Trends With the Position in the Galaxy*

Globular clusters live in a tidal field of the Galaxy, and are subject to tidal shocks due both to the bulge and disk passages (Ostriker *et al.* 1972; Chernoff & Shapiro 1987; Aguilar *et al.* 1988; Chernoff & Weinberg 1990, and references therein). Moreover, properties of newly formed clusters may well depend on their position in the proto-Galaxy (Fall & Rees 1977, 1985; Murray & Lin 1992; etc.). It is thus reasonable to expect that some correlations of cluster properties with R_{gc} and/or Z_{gp} will be found. A generic expectation is that clusters closer to the Galactic center will be more dynamically evolved, as tidal shocks accelerate their internal evolution towards the core collapse or dissolution.

Chernoff & Djorgovski (1989) analyzed the frequency of occurrence of PCC clusters as a function of position in the Galaxy, and found them to be highly concentrated towards the Galactic center and plane. This trend continues for KM clusters, in order of decreasing concentration. We confirm and extend their findings by looking at the correlations of core parameters with R_{gc} and Z_{gp}.

Selected correlations are shown in Fig. 8. Similar trends are seen when Z_{gp} is used instead of R_{gc}. Clusters at smaller R_{gc} tend to have smaller and denser cores and higher concentrations, and thus also shorter central relaxation times. These trends are not confined to the core properties, but extend to the half-light radial scales as well, although there the tidal truncation may also play an important role. In all cases, correlations are stronger for the less luminous clusters (we have divided the sample by the median luminosity and recomputed all correlation coefficients). This is exactly as expected from the picture of GC dynamical evolution driven by tidal shocks: less massive clusters should be more susceptible to the tidal effects, and/or be in more advanced stages of evaporation.

Even for the best of these correlations, there is still a large residual scatter at any R_{gc}. We have thus examined trends with binned data sorted in R_{gc}, in the same way as we did with M_V in the previous section.

Figure 9(a) shows the results for the core parameters. The radial trends are now very obvious. These correlations support the picture in which tidal shocks at low R_{gc} can both accelerate the internal evolution of clusters towards the core

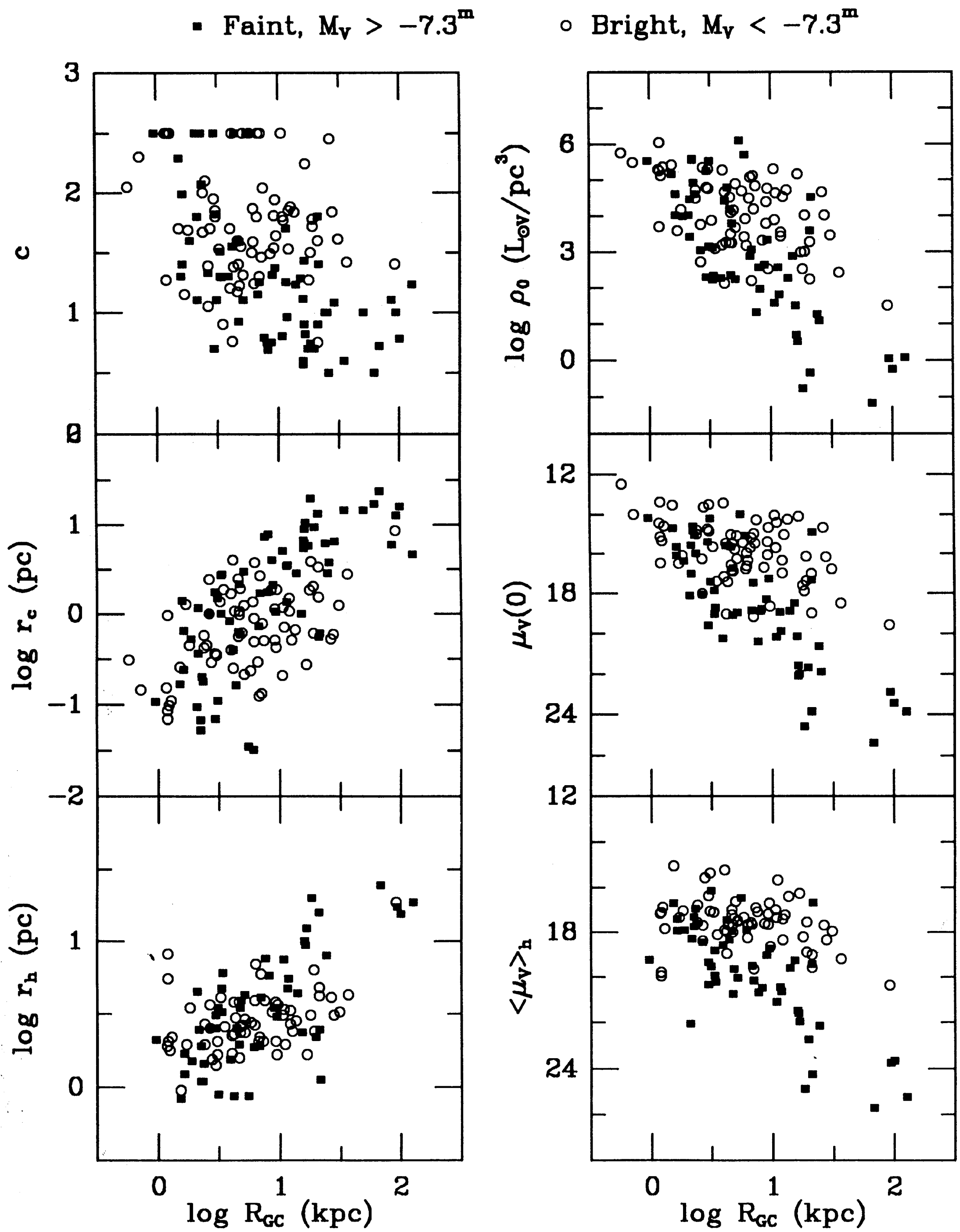

FIG. 8. Dependence of cluster parameters on the distance to Galactic center, R_{gc}. Similar trends are seen with the distance to Galactic plane, Z_{gp}. Clusters were divided by median luminosity ($M_V = -7.3^m$) and plotted with different symbols, as indicated. Clusters closer to the Galactic center or plane tend to have smaller radii (both r_c and r_h), higher surface brightness and luminosity densities, and higher concentrations. These trends are more prominent for the fainter clusters. This is in an agreement with a picture where tidal shocks simultaneously remove stars from clusters, and accelerate their evolution towards the core collapse.

collapse, and remove the low concentration systems by a rapid evaporation. The mean trends through the binned data points correspond to the scaling laws:

$$r_c \sim R_{gc}^{1.1\pm0.5}, \tag{4a}$$

$$\rho_0 \sim R_{gc}^{-2.8\pm1.6}. \tag{4b}$$

Equation (4b) is obviously consistent with constant mass cores shrinking according to Eq. (4a). For clusters moving in a potential corresponding to a flat rotation curve, the frequency of disk or bulge passages would be roughly proportional to R_{gc}^{-1}. Thus, the data suggest shrinkage of cores roughly proportional to the frequency of tidal shocks. Equations (4) can be used as empirical constraints for theoretical models of dynamical evolution of the GC system.

Figure 9(b) shows the the corresponding, weaker trends for the half-light parameters [the correlation between r_h and R_{gc} has been previously noted by van den Bergh *et al.* (1991)]. It is not clear to what extent these trends reflect the importance of tidal shocks in analogy with the core parameters, since the effects of dynamical evolution at the half-light scale are expected to be relatively small (e.g., Murray & Lin 1992, and references therein). It is also possible that these trends reflect in part the initial conditions of cluster formation.

Indeed, the theory by Fall & Rees (1985) predicts a radial trend of the mean cluster densities, bound by the scaling laws given by the thermal instability ($\rho_h \sim R^{-1}$) and by tidal truncation ($\rho_h \sim R^{-2}$, following the density law of the dark halo), although other theoretical explanations are certainly possible. The observed clusters roughly follow such a trend, but an alternative description of the data is that the mean densities are roughly constant up to $R_{gc} \approx 10$ kpc, and decline outward. The outward decline of the mean densities is a consequence of the trend for half-light radii, which increase with R_{gc}. If we ignore the last point, the approximate scaling is $r_h \sim R_{gc}^{0.3}$; including the last point, it is approximately $r_h \sim R_{gc}^{0.5}$. These scaling relations are similar to those proposed respectively by Murray & Lin (1992), and by van den Bergh *et al.* (1991).

Median cluster luminosities also appear to be roughly constant out to $R_{gc} \approx 10$ kpc, and decline thereafter. This effect was discussed by Brown (1993), who proposed that it is due to a change in the Jeans mass for protoclusters. It is also possible that low-mass clusters were effectively destroyed by tidal shocks inside this radius. However, given the very large scatter in M_V at every R_{gc}, and the observational selection against low luminosity clusters closer to the Galactic center, this trend probably should not be taken too seriously.

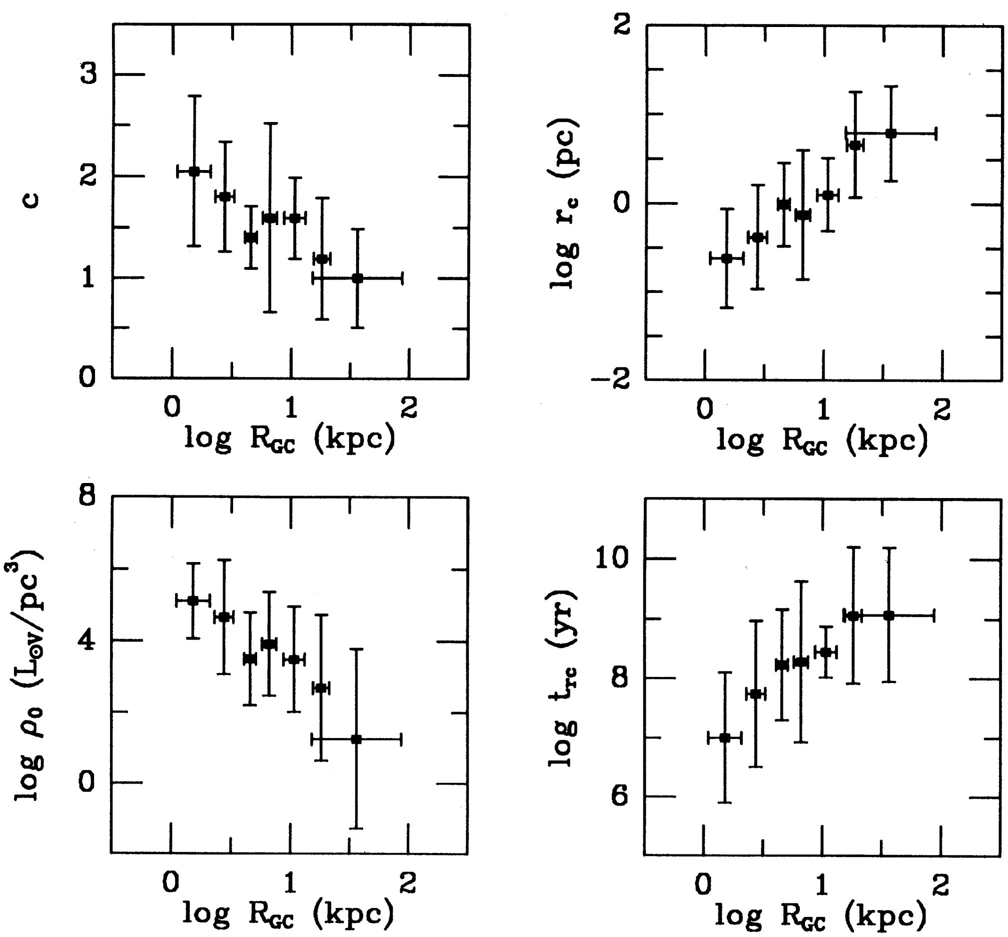

FIG. 9. (a) Trends for the core parameters with the data binned in R_{gc}. Median values for each bin are plotted and $Q\sigma$ estimates used as error bars, for all quantities. (b) Similar to (a), but for the luminosities and half-light parameters. The median cluster luminosities stay roughly constant out to $R_{gc} \sim 10$–15 kpc, and then decline (upper left). This may be a combination of the initial conditions, survival, and selection effects. Cluster mean densities mimic this trend (lower left), and may be bound by the scaling laws corresponding to thermal instability of protocluster clouds ($\rho_h \sim R^{-1}$) and tidal truncation ($\rho_h \sim R^{-2}$, as proposed by Fall & Rees (1985). Cluster half-light radii (upper right), and therefore relaxation times (lower right) decline at smaller R_{gc}, possibly reflecting tidal truncation effects.

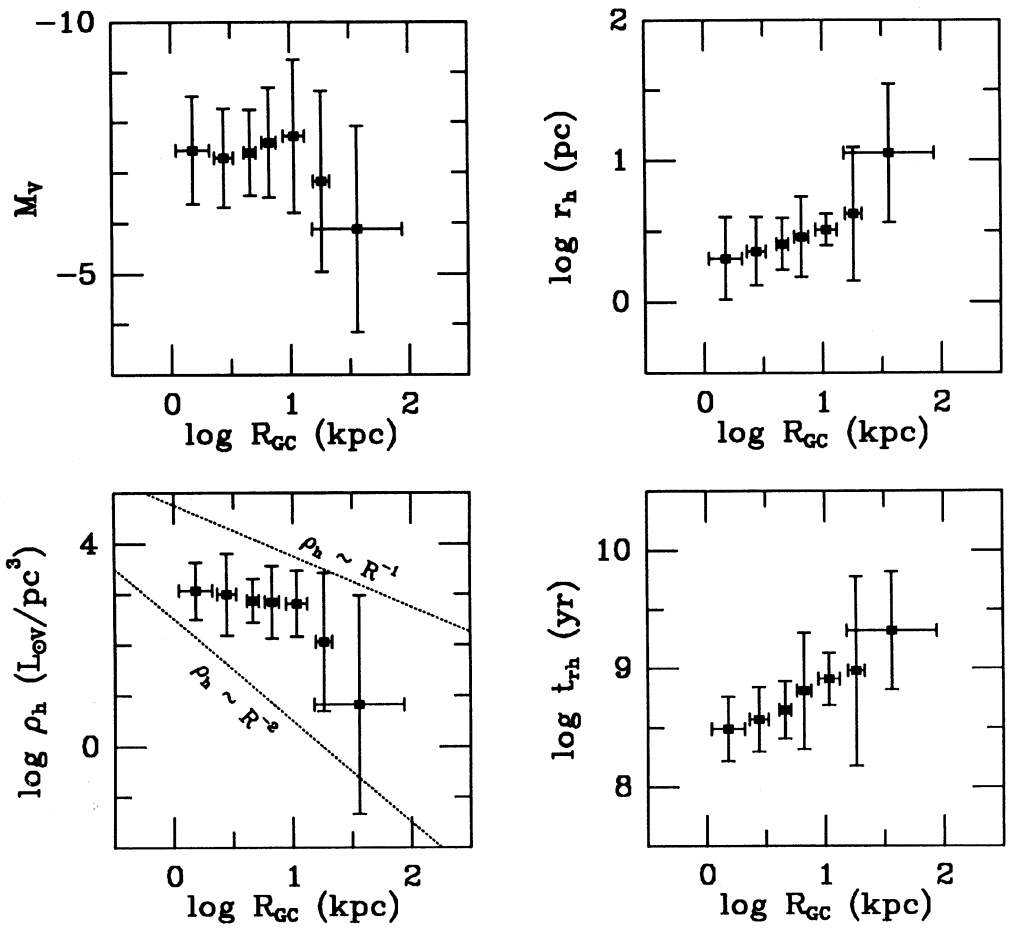

FIG. 9.(b)

3.3 Correlations of Core Properties

Some of the best correlations of GC properties are those between the various core parameters and concentrations (the only comparable ones are correlations involving velocity dispersions; see Sec. *3.5* below). They are illustrated in Fig. 10. We emphasize that all three principal quantities (r_c, $\mu_V(0)$, and c) are measured independently; correlations among them are real. On the other hand, the spectacular apparent correlation between t_{rc} and r_c is entirely artificial: it reflects the derivation of t_{rc}, which depends on $r_c^{3/2}$ (close to the apparent slope of the correlation), and other quantities which also correlate with r_c.

The correlation between the core radius, r_c, and the central surface brightness, $\mu_V(0)$ has been noted by Kormendy (1985). When the surface brightness, I_0, is expressed in linear units (e.g., $L_\odot/pc^2$) rather than in magnitudes, this corresponds to the scaling law:

$$I_0 \sim r_c^{-1.8\pm0.2}. \tag{5a}$$

The equivalent correlation for the central luminosity density corresponds to the scaling law:

$$\rho_0 \sim r_c^{-2.6\pm0.15}. \tag{5b}$$

We note that the volume luminosity density ρ_0 is derived from the projected luminosity density I_0 as: $\rho_0 = I_0/(r_c p)$, where $p = f(c)$ (see Djorgovski 1993b), so that Eqs. (5) are only partly independent. The excellent correlation between ρ_0 and r_c was already noted by Lightman (1982), who used older and less reliable data. Its small scatter corresponds to a relatively small dynamical range in core luminosities.

The correlations between r_c and c or $\mu_V(0)$ are so good, that they may be usable as rough distance indicators for globular clusters, with the 1-σ errors corresponding approximately to a factor of two in distance. While this is not competitive with the more traditional methods involving color–magnitude diagrams and pulsating variables, it may be useful as a rough check for the clusters where little data exist, e.g., many of the obscured systems in the direction of the Galactic bulge.

The obvious implication of the Eqs. (5) is that the core luminosity, or core mass, assuming $(\mathcal{M}/L) \approx$const. is nearly constant, since $L_c \approx \pi I_0 r_c^2$, or $L_c \approx \frac{4}{3}\pi \rho_0 r_c^3$. Taking the mass segregation effects into account, i.e., a higher relative fraction of dark stellar remnants in smaller cores (more concentrated clusters), would make the relations for core mass even

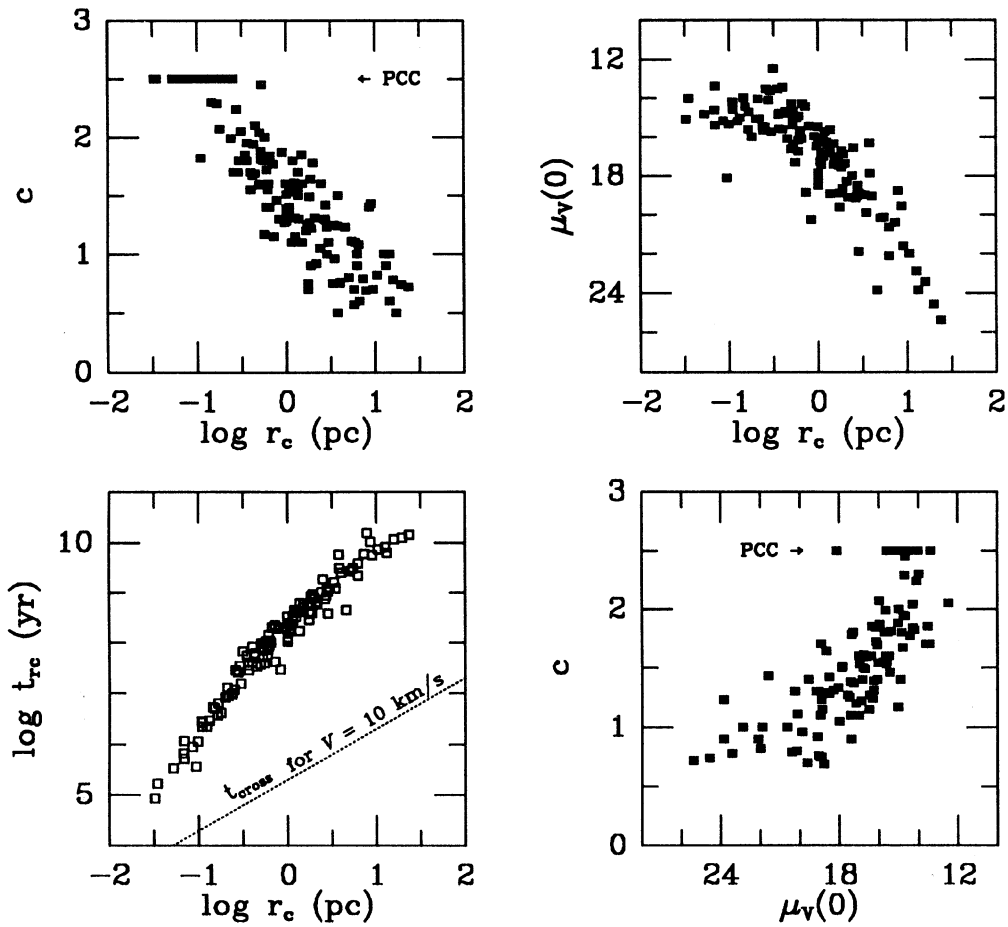

FIG. 10. Correlations between the core parameters. Clusters with smaller cores have higher concentrations, higher central surface brightness and luminosity densities, and therefore also shorter central relaxation times. This is as expected from a family of objects with a roughly constant core mass, evolving towards the core collapse. All three principal parameters, c, r_c, and $\mu_V(0)$, are measured independently, and are thus real. The correlations involving t_{rc} (as in the lower left panel) are entirely artificial, by operational definition of t_{rc}.

closer to $\mathcal{M}_c \approx$const. The observed dynamical range of core luminosities is indeed comparable to, or less than the dynamical range of total cluster luminosities, i.e., at most two orders of magnitude; yet the central densities span over seven orders of magnitude! This remarkable fact has been also pointed out by Lightman (1982).

These relations are exactly what may be expected from dynamical evolution towards the core collapse: while a core shrinks, its mass changes a little, or decreases slightly due to an enhanced evaporation of stars. Simple models of gravothermal catastrophe (core collapse) for isolated clusters predict a scaling law $\rho_0 \sim r_c^{-2.23}$ (Lynden-Bell & Eggleton 1980; Cohn 1980; cf. Goodman 1988 for a review). This is close to the observed scaling [Eq. (5b)], within the measurement errors. The observed relation is steeper, possibly due to the tidal effects: less concentrated clusters with larger cores would be more susceptible to evaporation of stars, which would lower their densities.

The smallest cores have $r_c \sim 0.05$ pc, and $t_{rc} \sim 10 t_{\rm cross} \sim 10^5$ yr. They must be dynamically highly evolved, and can hardly be much smaller, since $t_{rc} \gtrsim t_{\rm cross}$ must hold. The distributions of relaxation times agree with theoretical expectations, and give the rates of core collapse and cluster evaporation in the Galaxy (Hut & Djorgovski 1992). The smallest core radii expected from theory are indeed $r_c \sim 0.1$ pc, due to the presence of primordial binaries (Goodman & Hut 1989; Heggie & Aarseth 1992; Vesperini & Chernoff 1994), or even possible gravothermal core oscillations (Murphy *et al.* 1990). Most post-collapse cluster cores appear to be marginally resolved at just that level from the ground (the data we use here), and are certainly resolved with the *HST* (Lauer *et al.* 1991; Djorgovski *et al.* 1994). Flattenings of surface brightness profiles are observed at small radii, thus operationally defining cores, regardless of whether or not shallow density cusps remain at smaller radii.

This general agreement of observations of GC core properties with theoretical expectations is gratifying. It suggests that we have a reasonably good basic understanding of dynamical evolution of globular clusters, even though many details remain yet to be addressed by careful and extensive modeling.

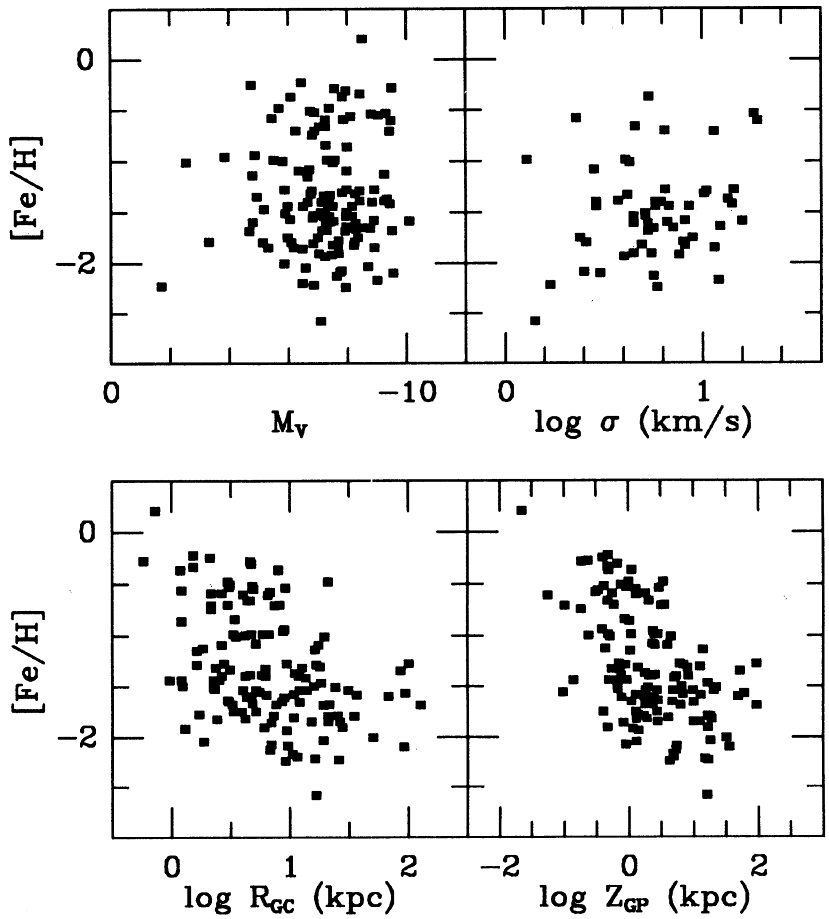

FIG. 11. Cluster metallicities show no correlations whatsoever with luminosities or velocity dispersions (top row), suggesting that globular clusters were not self-enriched systems. They show only a weak dependence on the position within the Galaxy (bottom row), which may be explained in terms of Zinn's disk and halo populations.

3.4 Metallicity Noncorrelations

Unlike elliptical and dwarf galaxies, globular clusters show no correlations between metallicity and luminosity or velocity dispersion (Fig. 11). The standard explanation for these correlations for galaxies is self-enrichment in the presence of galactic winds. It is thus natural to conclude that GCs were not self-enriched systems. This conclusion is also supported by the extreme internal chemical homogeneity of most GCs (cf. Larson 1988). Furthermore, even a single supernova exploding in a still gaseous proto-globular cluster would deposit $\sim 10^{51}$ erg of kinetic energy, which is comparable to the binding energies of GCs today, $E_{\rm bind} \sim 10^{50}-10^{51}$ erg. Thus, with a possible exception of the most massive systems such as ω Cen where some chemical inhomogeneities are seen (Cohen 1981), a still gaseous proto-cluster could be immediately disrupted. Obviously, the exact outcome would depend on many of the as-yet poorly known details of the physics of globular cluster formation. For a more detailed treatment of the problem, see, e.g., Morgan & Lake (1989).

The only correlations of metallicity are with the position in the Galaxy, and even there we see a large scatter in [Fe/H] at every value of $R_{\rm gc}$ and $Z_{\rm gp}$. Whereas these correlations can be interpreted as a noisy radial gradient in mean abundances (cf. Pilachowski 1984), perhaps a better explanation for the apparent gradient is the now well established disk-halo dichotomy (Zinn 1985).

3.5 Velocity Dispersion Correlations

Aside from the correlations of core properties described in Sec. *3.3* above, the best nontrivial correlations of GC properties are between the velocity dispersion and luminosity or surface brightness (Fig. 12). These correlations probably reflect formative processes of GCs more than their subsequent dynamical evolution, and therein lies their significance. The relation between velocity dispersion and luminosity ($L-\sigma$) has been already discussed by Meylan & Mayor (1986), Paturel & Garnier (1992), and Djorgovski (1991, 1993a), and the relation between velocity dispersion and surface brightness ($\sigma-\mu$) by Djorgovski (1993a).

The observed relations correspond to the scaling laws:

$$\sigma \sim L^{0.6\pm0.15}, \tag{6a}$$

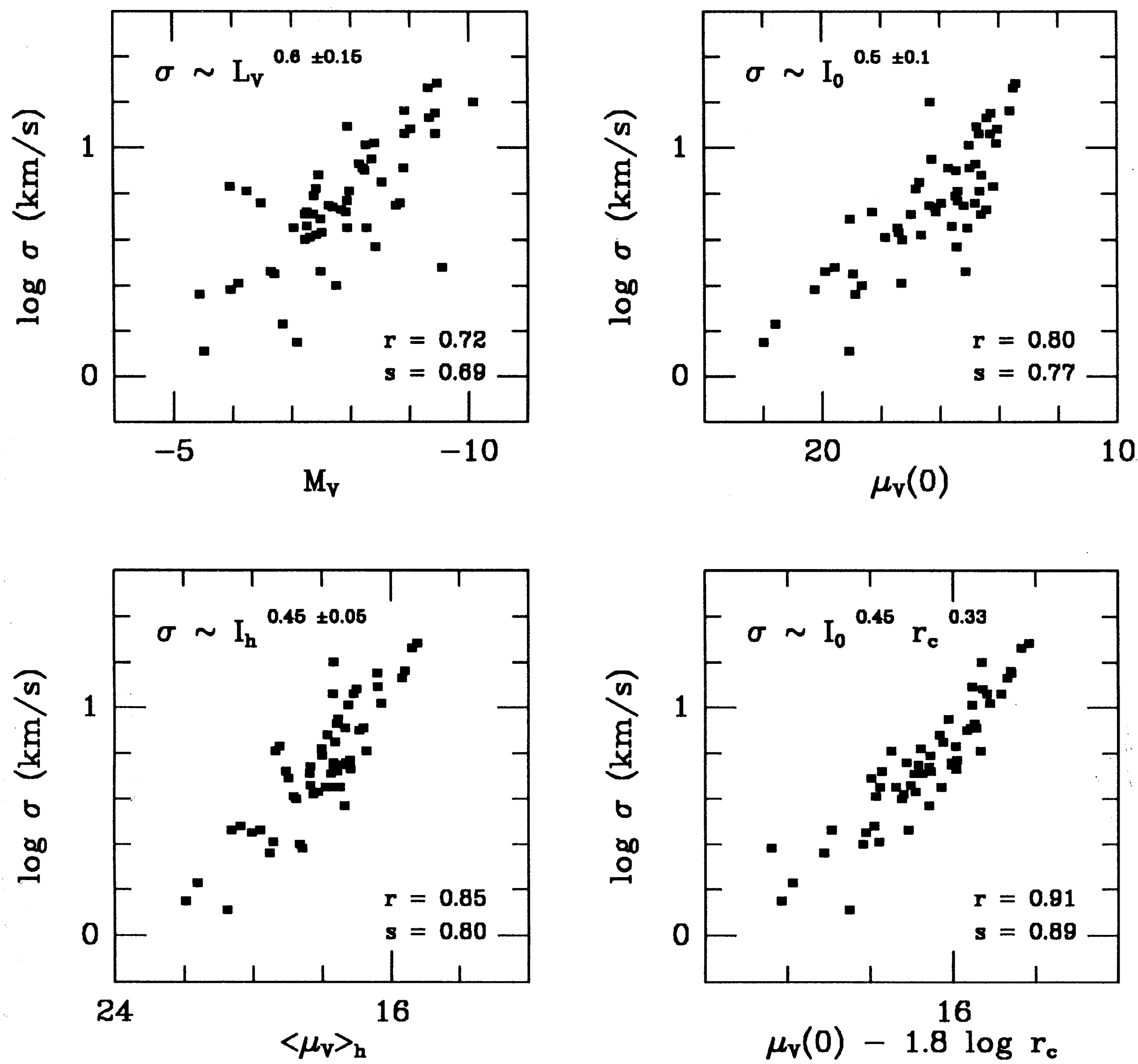

FIG. 12. Velocity dispersion correlations. These are the best nontrivial correlations known for globular clusters, along with the correlations of core properties shown in Fig. 10. The corresponding scaling laws are indicated in the upper left of each panel, and the Pearson (r) and Spearman rank (s) correlation coefficients are listed in the lower right of each panel. The lower right panel shows a bivariate correlation, where core radius was used as a "second parameter" to improve the corresponding correlation shown in the upper right panel.

$$\sigma \sim I_0^{0.5\pm0.1}, \tag{6b}$$

$$\sigma \sim I_h^{0.45\pm0.05}. \tag{6c}$$

Here we again denote the central surface brightness expressed in linear units as I_0, the average surface brightness within r_h as I_h, and the total luminosity as L; all are in the V band. Note that since M_V and r_h are not correlated, the correlation between σ and I_h is *not* simply a consequence of the $L-\sigma$ relation, although they are obviously related.

The slope of the $L-\sigma$ relation for GCs, viz., $L\sim\sigma^{5/3}$, is significantly different from the Faber-Jackson (1976) relation for ellipticals, or its equivalent for dwarf galaxies, viz., $L\sim\sigma^4$. These relations also do not tie-in to one another, and should not be lumped together (cf. Djorgovski 1993a). The slope of the $\sigma-\mu$ relation for GCs has the *opposite sign* from the corresponding relation for ellipticals; furthermore, the GC correlation is much better.

A possible interpretation of these relations, based on an argument originally proposed by Gunn (1978), was suggested by Djorgovski (1991, 1993a), as follows. Dynamical evolution of young globular clusters may have been dominated by adiabatic mass loss due to stellar evolution (cf. Chernoff & Weinberg 1990). If characteristic mass, radius, and velocity scales are denoted as $\mathscr{M}$, R, and σ, the adiabatic invariants are $\mathscr{M}R\simeq$const. and $R\sigma\simeq$const. Thus, $\mathscr{M}\sim\sigma$, and if $(\mathscr{M}/L)\simeq$const. (a reasonable assumption given the data on the present-day globulars), then the primordial scaling relation may be of the form $L\sim\sigma$. Subsequent dynamical evolution of clusters would be dominated by tidal shocks, which would remove mass (and luminosity), but not affect the central velocity dispersions very much. This mass removal would be more efficient for the less massive clusters, and thus the primordial $L-\sigma$ relation should steepen in time, perhaps reaching the observed slope by the present epoch. Since this dynamical evolution process must be operating, the primordial relation must have been shallower, and may well have had the slope of ~ 1. By a similar argument, if $(\mathscr{M}/L) \simeq$const., surface brightness is proportional to $\mathscr{M}R^{-2} \sim \sigma^3$, which is roughly similar to the observed slopes of the $\sigma-\mu_V(0)$ and $\sigma-\langle\mu_V\rangle_h$ relations. Undoubtedly, this is an oversimplified scenario, and a more complete treatment, involving the possible variations in the IMFs, distribution of cluster orbits, etc., is desirable.

A preliminary comparison of $L-\sigma$ relations for Galactic, Fornax dwarf, and Magellanic Clouds globulars has been made by Dubath *et al.* (1994), and Zaggia *et al.* (1994). Dubath *et al.* find that the Fornax and SMC clusters follow within the measurement errors the $L-\sigma$ relation for Galactic GCs, and that LMC clusters tend to have velocity dispersions too high for their luminosity, or that they follow a relation $L \sim \sigma^\gamma$, where $\gamma > 2$. To the extent that GCs in these dwarf galaxies should not suffer from tidal shocks driven evaporation as much as their Galactic counterparts, the observations do not support the simple evolutionary picture described above. On the other hand, it is not clear why should clusters in different galaxies follow the same initial, let alone evolved, $L-\sigma$ scaling relations. The data on extragalactic globulars are still too scarce to tell if this is the case.

Let us consider another intriguing possibility, that at least some GCs formed directly from small-scale density perturbations in the primordial density field (Peebles 1984; Rosenblatt *et al.* 1988; West 1993). At any given mass scale, the power spectrum of the primordial density field can be expressed as $P(k) \sim k^n$, where k is the inverse spatial wavelength corresponding to the enclosed mass. From the Gott & Rees (1975) theory, one can derive the scaling laws between the mass $\mathscr{M}$, characteristic velocity scale σ, and the projected surface density Σ:

$$\sigma \sim \mathscr{M}^{(1-n)/12}, \tag{7a}$$

$$\sigma \sim \Sigma^{(n-1)/(4n+8)}, \tag{7b}$$

where n is the effective power law index of the perturbation spectrum at that mass scale. For GCs, $(\mathscr{M}/L) \simeq$const., and thus we can assume $\mathscr{M} \sim L$ and $\Sigma \sim I_h$. Thus from Eqs. (6a) and (7a) we derive $n = -6.2 \pm 1.8$, and from Eqs. (6c) and (7b) $n = -5.8 \pm 1.9$; the two results are of course not independent. We conclude that *if* GCs formed directly from primordial density fluctuations, their velocity dispersion correlations imply the perturbation spectrum index $n = -6 \pm 2$ on the mass scale corresponding to their progenitors ($\sim 10^6 \mathscr{M}_\odot$?).

For comparison, the CDM spectrum on such mass scales predicts $n \approx -3$, which is within 1.5 σ from our result. In reality, nothing is really known about the primordial density fluctuation spectrum at such small mass scales. While the modern measurements of CMBR fluctuations are beginning to probe the fluctuation spectrum on supercluster or larger mass scales ($>10^{15} \mathscr{M}_\odot$), and indirect inferences about it can be made on the scale of galaxies ($\sim 10^{12} \mathscr{M}_\odot$), properties of globular clusters may be the only handle we can have on the mass scales $\sim 10^6 \mathscr{M}_\odot$—if indeed most globulars formed in this way, which is by no means obvious.

As already found by Djorgovski (1991), core radii and concentrations play the roles of a "second parameter" in these relations, in the sense that clusters with smaller cores and/or higher concentrations have higher central velocity dispersions at a given luminosity or surface brightness. This implies the existence of an important bivariate correlation, which we address in Sec. 4 below.

The $L-\sigma$ relation, with or without a "second parameter," may be usable as a distance indicator for globular clusters in our Galaxy, and perhaps even for other nearby galaxies.

3.6 Subgroups in the GC System?

A question arises of whether GCs can be meaningfully divided by some physical property or a combination of properties into distinct groups, and if so, whether different groups would show different correlations, possibly indicating different formative or evolutionary histories. An obvious example is Zinn's disk-halo dichotomy. We approached this question in several ways.

We divided the sample by the median luminosity (fainter or brighter than $M_V = -7.3^{\mathrm{m}}$), distance to the Galactic center (closer or farther than $R_{\mathrm{gc}} = 6.3$ kpc), and metallicity ([Fe/H]$\geqslant -1.3$ or [Fe/H]< -1.3). We have also tried the traditional disk-halo dividing value of [Fe/H]$= -0.8$, with essentially the same results. The only significant difference in correlation coefficients we find is that the correlations of core properties with position in the Galaxy are stronger for the fainter clusters, as described in Sec. *3.2* above. We thus see no new evidence for distinct subgroups in the GC system.

Recently, van den Bergh (1993) has divided the Galactic GCs into three groups, labeled α, β, and γ. The last group are simply the metal-rich clusters, with [Fe/H]$\geqslant -0.8$. The first two groups are effectively defined by the presence of a "second parameter" in their horizontal branch (HB) morphology, the β group following a well defined relation between the HB type and metallicity, and the α group showing a scatter blueward in the HB morphology at any given metallicity. According to van den Bergh, the β clusters represent an old, coeval population dating from the initial stages of Galaxy's formation, whereas the α clusters were acquired or formed in subsequent merger events, following the Searle & Zinn (1978) scenario. As noted by van den Bergh, the three groups show different radial distributions in the Galaxy, with the α clusters being least centrally concentrated, and γ clusters most centrally concentrated, and possibly also the difference in orbital structures, the α clusters favoring retrograde or radial orbits, and the β clusters favoring prograde and circular orbits.

We were unable to find any other differences in correlations for the three groups, other than the obvious consequences of their differing radial distributions. In particular, we find no obvious differences in the velocity dispersion correlations for the three groups, which is somewhat surprising, if they formed through different mechanisms. The only distinction we find is that half-light parameters (r_h, $\langle\mu_V\rangle_h$, and t_{rh}) correlate better with the luminosity and cluster concentration for the α clusters than the β clusters; r_c also correlates better with M_V for the α clusters. However, these differences may be simply due to a relatively small number of data points, and are dominated by three α clusters with large values of r_h and low values of c. These data thus do not offer any new support for van den Bergh's trichotomy of clusters, and his interpretation; but they certainly do not contradict his picture either.

Éigenson & Yatsuk (1989) made an intriguing attempt to analyze the distribution of GC properties using statistical clustering analysis. While no clear-cut division of GCs into distinct classes emerged in their work, the method is certainly worth exploring further, using more modern data.

4. A MULTIVARIATE APPROACH: THE MANIFOLD OF GLOBULAR CLUSTERS

The GC system is clearly a subject to a number of evolutionary processes and factors, some of which may be connected in complex ways; take, for example, the apparent dependence of internal dynamical evolution towards the core collapse on the cluster position within the Galaxy. Any possible correlations imprinted on GCs in the process of their formation may have been subsequently modified by the dynamical evolutionary processes; perhaps this was the case with the velocity dispersion correlations. And thus, while the simple approach of examining individual monovariate correlations of GC parameters gives us a good first look at the properties of the GC system, the complexity of the situation calls for a more sophisticated approach.

We are dealing with a multidimensional data set, in which sets of several observables may be connected in multivariate correlations. Simple, monovariate correlations are only a very special and rare case. In order to reveal the possible correlations of a more complex nature, we have to apply a multivariate statistical analysis.

4.1 Some Useful Concepts in Multivariate Analysis

Here we describe briefly some of the essential ideas and methods used for such an analysis. Interested readers may find the monograph by Murtagh & Heck (1987) particularly useful.

The data points occupy a volume in an N-dimensional parameter space, where N is the number of input quantities (in our study, $N=13$). If any of the input quantities are derived from the others, the data will occupy a volume of dimension M, where M is the number of *independent* input quantities (in our study, $M=9$). If, in addition, any correlations are present in the data, the dimensionality of the volume occupied by the data points, also called the data manifold, will be reduced further. The effective statistical dimensionality of the data manifold, $D \leqslant M$, gives the number of independent factors which fully describe the data; for example, if all input quantities are perfectly correlated, $D=1$. This minimum number of dimensions needed to account for the spread in the data may reflect the number of physical processes or controlling parameters which determine the observable properties of objects being studied. If there are no correlations, the data points will populate the full M-dimensional volume of the parameter space. The dimensionality of the data manifold is thus a sensitive indicator of whether any correlations are present, regardless of their complexity.

Subsets of input variables, or even all of them, may be mutually connected in a set of K-variate correlations, i.e., each variable can be correlated to within the measurement errors with combinations of K others. Some of the input quantities may be uncorrelated with any of the others, and would thus automatically increase the dimensionality of the data manifold. Finding a subset of the data for which a nontrivial reduction of dimensionality occurs is the process by which the correlations may be inferred.

The procedure by which this can be accomplished is principal component analysis (PCA). In its simplest form, it consists of diagonalizing of the matrix of pairwise correlation coefficients (e.g., those given in Table 1). It is the eigensolution of the data ellipsoid in the N-dimensional parameter space of measured quantities. The data should be properly linearized first; for most astronomical applications, where correlations are power laws, this is usually accomplished by taking logarithms. Ideally, every input quantity should have a Gaussian distribution; if not, the derived correlations may be biased, but the dimensionality will still be right. Typically, the input data are renormalized by subtracting the mean and dividing by the sigma in each of the input variables. The renormalization does not affect the qualitative nature of the PCA solutions, but it may affect the statistical significance of higher data dimensions.

The principal axes (eigenvectors) of the data ellipsoid may be tilted with respect to any of the observables. Their mutual relations are usually represented through correlation vector diagrams (CVDs), which show the projections of input variables' axes to the principal planes given by the eigenvectors. The number of the significant eigenvalues, i.e., those larger than expected from the measurement errors, gives the dimensionality of the data manifold, D. Each of the eigenvectors also accounts for some fraction of the total sample variance.

The exact determination of the number of statistically significant data dimensions, D, is a subject of much debate among the theorists and practitioners of PCA. It is generally agreed that eigenvalues>1 are statistically significant, but somewhat lower ones may be as well, depending on how the data were renormalized. The key problem is how to account for the presence of measurement errors, which will thicken the manifold in all N input dimensions. The additional complications involve missing data, or data of very heterogeneous quality (as it may be the case in our study), etc. Usually, a steep drop in the successive eigenvalues or in the fractional contributions to the sample variance indicates where the number of statistically significant dimensions stops, and where the noise begins, but the case is not always so clearcut.

For example, if $D=2$, the data form a plane embedded in the N-dimensional space, and any of the input variables can be computed from any two others (bivariate correlations). This is the case for elliptical galaxies, and probably other galaxy types as well (Brosche 1973; cf. Djorgovski 1992a, b for reviews and further references). As a rule, correlations of order $D>2$ are seldom very useful, as measurement errors add up in combining too many input quantities. If any correlations are present, their equations can be deduced from the transformations connecting the eigenvector components and input variables, or by several other methods (iterative optimization of correlation coefficients, multivariate least squares, etc.).

At that point, the real task begins: physical interpretation of the correlations found, and their origin.

4.2 The Global Manifold: Dimension>4

This statistical machinery was first applied on GCs by Brosche & Lentes (1984). They used a set of seven input quantities, and derived the dimensionality of the GC manifold of $D=3$. Éigenson & Yatsuk (1986) performed a similar analysis using a data set of 11 input quantities, perhaps only 7 or so of which were really independent. They found that the first two eigenvectors account for ~65% of the total sample variance, from which they decided without providing an adequate justification that $D=2$. Both studies used older data, whose quality was sometimes questionable.

Djorgovski (1991) compiled a more modern data set of 12 input quantities, only 9 of which were independent. He found that the dimensionality of this data set was high, $D\approx5\pm1$. Ellipticities and cluster orientations contributed at least 1 to that dimensionality; in general, quantities which do not correlate with any others are practically guaranteed to increase the statistical dimensionality of the data manifold. Metallicities alone were also largely responsible for 1 dimension in that data set.

Several other recent studies used PCA effectively to address particular problems in the GC research, e.g., the HB morphology (Fusi Pecci *et al.* 1993), stellar mass function slopes (Djorgovski *et al.* 1993), or specific frequencies of GCs in elliptical galaxies (Djorgovski & Santiago 1992; Santiago & Djorgovski 1993).

We performed the PCA on our data set, using all independent input variables, and subsets thereof. We ignore the derived quantities, $\langle\mu_V\rangle_h$, ρ_0, t_{rc}, and t_{rh}, as they by definition do not add to the dimensionality of the data manifold. Since the variable with the least number of available measurements (56) is the velocity dispersion, σ, all PCA solutions involving σ are restricted to that subset of clusters. The next most restricting variables are the r_h and $\langle\mu_V\rangle_h$, which limit the data set to 115 clusters.

We used both Pearson and Spearman correlation coefficients to form input data matrices: the former are less noisy, the later more robust in regard to outlier data points. In all cases the two methods yielded consistent solutions.

As an illustration, we list in Table 2 the eigenvalues (e_i), fractional (V_i), and cumulative (C_i) contributions to the total sample variance, in percent, for two cases: first, the PCA solution for the set of all 9 independent input variables (M_V, c, r_c, r_h, $\mu_V(0)$, [Fe/H], σ, $R_{\rm gc}$, and $Z_{\rm gp}$); second, the set of 7 with [Fe/H] and σ omitted. In both cases, the number of statistically significant dimensions is at least $D=3$. There is, however, no sharp boundary. In the first sample, the successive eigenvalues scale roughly as $e_i/e_{i+1}\approx2$ all the way to the maximum dimension of 9, with a similar scaling for the fractional variances. In the second sample, there may be a drop at the 6th component. Looking at the fractional and cumulative variances, in the first sample we certainly reach the noise level by about $D=6$ or 7, and in the second case by about $D=5$ or 6. We thus conclude that the global statistical dimensionality of this sample is at least $D=3$, and possibly as high as $D=6$. This is still less than the number of input quantities. Thus, some correlations are present, as expected from our simple analysis in Sec. 3.

TABLE 2. PCA Solutions for two subsamples.

	All independent qts.			[Fe/H], η excluded		
i	e_i	V_i	C_i	e_i	V_i	C_i
1	4.12	45.7	45.7	4.31	61.5	61.5
2	2.14	23.8	69.5	1.21	17.2	78.7
3	1.20	13.4	82.9	0.82	11.8	90.5
4	0.60	6.7	89.6	0.40	5.7	96.2
5	0.53	5.9	95.5	0.18	2.6	98.8
6	0.22	2.4	97.9	0.05	0.7	99.5
7	0.11	1.2	99.1	0.03	0.5	100
8	0.05	0.6	99.7	⋯	⋯	⋯
9	0.03	0.3	100	⋯	⋯	⋯

Do the principal components have a clear identity, i.e., can they be easily associated with one or more of the input variables each? The answer is, unfortunately, no. For both of these samples, the first three (i.e., the most significant) eigenvectors are composed of a relatively uniform mix of several input variables—none are "more fundamental" than others. In the first sample, [Fe/H] does dominate the 4th eigenvector, r_h dominates the 5th eigenvector, $R_{\rm gc}$ and $Z_{\rm gp}$ in about equal proportions dominate the 6th eigenvector, and M_V and σ in about equal proportions dominate the 7th eigenvector. In the second sample, we see a similar situation, minus the contribution of [Fe/H]: r_h dominates the 4th eigenvector, and $R_{\rm gc}$ and $Z_{\rm gp}$ in about equal proportions dominate the 5th eigenvector. This does suggest that at least one dimension of the data manifold is added by the metallicity itself, and one by the position in the Galaxy, but not much more can be said.

We see similar behavior in other subsamples which contain [Fe/H] and/or positional variables.

We note that this is a fairly restricted set of GC properties. We know from the previous studies that adding various measures of the HB morphology will increase the dimensionality of the data manifold by 1 or 2 (Fusi Pecci *et al.* 1993), as would the addition of cluster ellipticities and orientations (Djorgovski 1991). Adding clusters colors, properly corrected for the extinction, would probably not add much new information: colors do correlate well with the metallicity (Zinn 1980; Zinn & West 1984), and the stellar populations in GCs are old and roughly coeval, so that age effects cannot play a very significant role. There is probably some spread among the clusters in details of chemical abundances, all of which we simply lumped together in the traditional [Fe/H], and that should also add to the dimensionality of the data manifold (Rose & Tripicco 1986). There are also probably real differences in cluster dynamics, e.g., velocity anisotropy (Meylan & Mayor 1986; Meylan 1987).

In all, we can safely conclude that the data dimensionality found here is a lower limit, and that adding additional observables, such as the HB parameters, cluster shapes, abundance patterns, etc., could only increase it. It is a fairly safe guess that for a global manifold of GCs, with all these variables included, $D>4$, and very likely $D>7$. This large dimensionality can be interpreted as a product of many distinct evolutionary processes shaping the observed properties of GCs today. It could also result from a superposition of many subgroups of globular clusters, possibly formed through different mechanisms and/or in different conditions, with slightly different distributions of their physical properties.

4.3 The King Manifold: Dimension=3

The information richness of the global manifold of GCs, as reflected by its high statistical dimensionality is daunting. Possibly a more practical or useful approach is to restrict the analysis to some heuristically chosen subsets of variables, where a significant reduction of dimensionality may be found. Here we perform just such an analysis.

Consider only the photometric, structural, and dynamical parameters of clusters, M_V, c, r_c, r_h, $\mu_V(0)$, and σ [derived variables, such as $\langle\mu_V\rangle_h$, ρ_0, t_{rc}, and t_{rh}, add no independent new information]. We have such data for 56 clusters; if σ is excluded, we have them for 115 clusters. Thus, we perform the PCA for both samples, with 6 or 5 input quantities. Table 3 lists the eigenvalues and variance fractions for the two solutions.

Regardless of whether σ is included in the analysis, the PCA solutions are unambiguous: there is a sharp drop in the eigenvalues and fractional variances after the third component. The statistical dimensionality of this manifold is clearly $D=3$, even though there are 5 or 6 independently measured input quantities.

This is exactly what can be expected from a family of objects described by King (1966) models. They require 3 input parameters: a scaling of the core radius, a scaling of the surface brightness (the two together determine the scaling of the luminosity), and a shape parameter. The fact that velocity dispersion participates in the manifold suggests that GCs have uniform or even constant global $(\mathscr{M}/L)$ ratios. We also see no systematic deviations for the PCC clusters, indicating that our approximations of a constant $c=2.5$ and r_c=observed HWHM are quite good for most clusters. The King models are a good and robust description of GC structure. After all the advances over the past 30 years, both theoretical and observational, they can still serve as an excellent empirical description of the data. We thus propose to call this manifold of global structural, photometric, and dynamical properties of GCs "the King Manifold."

TABLE 3. PCA solutions for the King manifold samples.

	All six quantities			σ excluded		
i	e_i	V_i	C_i	e_i	V_i	C_i
1	3.50	58.3	58.3	3.38	67.7	67.7
2	1.61	26.9	85.2	0.98	19.7	87.3
3	0.67	11.2	96.4	0.53	10.7	98.0
4	0.11	1.9	98.2	0.07	1.3	99.3
5	0.07	1.2	99.4	0.03	0.7	100
6	0.04	0.6	100	···	···	···

The axes of this data ellipsoid (the first 3 eigenvectors) do not project nicely on the observable axes, and vice versa. This is illustrated in Fig. 13, which shows projections of the axes of observables onto the principal planes of the data manifold. Input observables contribute in a fairly equal manner to all three of the principal eigenvectors: none are apparently more fundamental than others.

The result that $D=3$ for the King Manifold implies that any one input quantity can be expressed as a combination of three others; for example, $\mu_V(0)$ can be derived uniquely from M_V, c, and r_c. While this is not a surprise for a believer in the King models, there is certainly no obvious reason why should real clusters cooperate in this way; but they do.

A natural question arises of whether a further reduction of dimensionality can be obtained for subsets of this data set. The answer is yes. The existence of a $D=2$ manifold of GCs, an equivalent of the so-called Fundamental Plane of elliptical

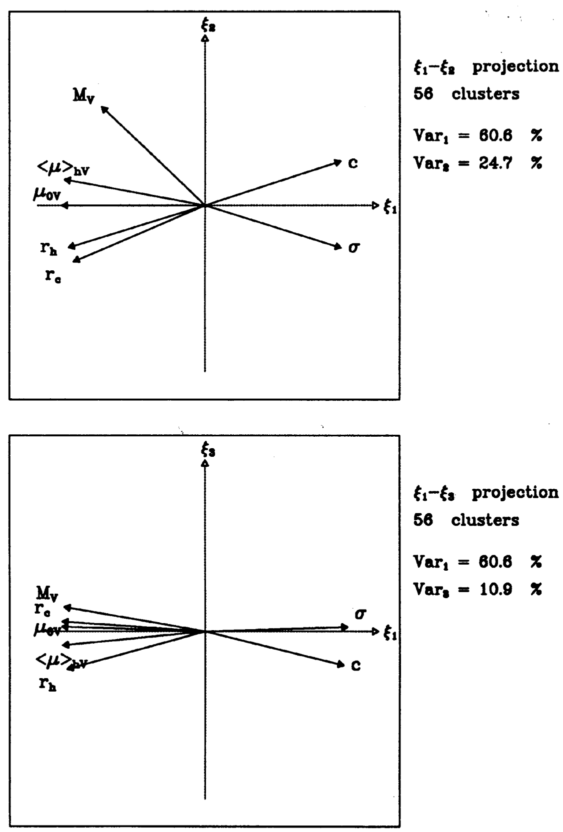

FIG. 13. A correlation vector diagram (CVD) for the King Manifold, showing the projections of the observable axes on the principal planes defined by the first three eigenvectors. The first three eigenvectors account for ~95% of the total sample variance, the rest presumably being due to the measurement errors. Note that the data ellipsoid is fairly flat, with the axial ratios roughly 6:2.5:1.

galaxies, is described by Djorgovski (1994), who discusses both the implications and possible uses of that result.

5. CONCLUDING REMARKS

What emerges is a global picture of a GC system as a rather complex family of objects: they are very diverse in their properties, yet follow a number of nontrivial correlations and trends. Many of these correlations are at least qualitatively as expected from from our current theoretical understanding of GC dynamics and evolution. Others still remain to be explained.

Most of the observed correlations can be expressed as power-laws. They can be used as empirical constraints for theoretical models of GC formation and evolution.

The correlations of core parameters are consistent with our understanding of GC evolution towards the core collapse; median cluster parameters show no such correlations, as expected from the theory. Cluster position in the Galaxy clearly plays a major role in speeding up the rates of internal dynamical evolution, in agreement with the tidal shocks picture. Structural, photometric, and dynamical parameters of clusters, including those with a PCC morphology, are connected in a three-parameter family, as expected from objects following King (1966) models.

There are excellent correlations of cluster velocity dispersions with luminosities and surface brightness. Their origin is not yet understood, and they may contain valuable insights in the formation of globular clusters or their progenitors.

Many other quantities, including cluster metallicities and a number of other parameters with which we did not deal directly in this study (e.g., cluster ellipticities, HB morphology, etc.) show only weak or no correlations with other parameters, increasing the statistical dimensionality of the manifold of GCs. This suggests that they were determined by processes other than those which have produced the observed correlations of GC parameters.

An obvious direction for the future work is to improve the available data, and in particular the distances to clusters, on which so much else depends. Discovery of more clusters, especially sparse, faint systems, would help flesh out many of the correlations. And finally, with the advent of the repaired *HST* and Adaptive Optics for the ground-based telescopes, detailed studies of GC systems in other, nearby galaxies are rapidly becoming a reality (cf. Fusi Pecci *et al.* 1994). Their comparison with the Galactic GC system will probably reveal both similarities and differences, which may help us understand and reconstruct the formation and evolution of galaxies and their GC systems.

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REFERENCES

Aguilar, L., Hut, P., & Ostriker, J. 1988, ApJ, 335, 720
Blanco, V., & Terndrup, D. 1989, AJ, 98, 843
Blitz, L., & Spergel, D. 1991, ApJ, 379, 631
Brosche, P. 1973, A&A, 23, 259
Brosche, P., & Lentes, F.-T. 1984, A&A, 139, 474
Brown, J. H. 1993, in The Globular Cluster—Galaxy Connection, edited by G. Smith and J. Brodie, ASPCS, 48, 766
Chernoff, D., & Shapiro, S. 1987, ApJ, 322, 113
Chernoff, D., & Djorgovski, S. 1989, ApJ, 339, 904
Chernoff, D., & Weinberg, M. 1990, ApJ, 351, 121
Cohen, J. 1981, ApJ, 247, 869
Cohn, H. 1980, ApJ, 242, 765
Djorgovski, S., & King, I. R. 1984, ApJ, 277, L49
Djorgovski, S., & King, I. R. 1986, ApJ, 305, L61
Djorgovski, S. 1991, in Formation and Evolution of Star Clusters, edited by K. Janes, ASPCS, 13, 112
Djorgovski, S. 1992a, in Morphological and Physical Classification of Galaxies, edited by G. Longo, M. Capaccioli, and G. Busarello (Kluwer, Dordrecht), p. 337
Djorgovski, S. 1992b, in Cosmology and Large-Scale Structure in the Universe, edited by R. de Carvalho, ASPCS, 24, 19
Djorgovski, S., & Piotto, G. 1992, AJ, 104, 2112
Djorgovski, S., & Santiago, B. X. 1992, ApJ, 391, L85
Djorgovski, S. 1993a, in The Globular Cluster—Galaxy Connection, edited by G. Smith and J. Brodie, ASPCS, 48, 496
Djorgovski, S. 1993b, in Structure and Dynamics of Globular Clusters, edited by S. Djorgovski and G. Meylan, ASPCS, 50, 373
Djorgovski, S., & Meylan, G. 1993a (ed.), Structure and Dynamics of Globular Clusters, ASPCS, vol. 50 (Astron. Soc. Pacific, Provo)
Djorgovski, S., & Meylan, G. 1993b, in Structure and Dynamics of Globular Clusters, edited by S. Djorgovski and G. Meylan, ASPCS, 50, 325
Djorgovski, S., & Meylan, G. 1993c, BAAS, 25, 885
Djorgovski, S., Piotto, G., & Capaccioli, M. 1993, AJ, 105, 2148
Djorgovski, S. 1994, ApJ (submitted)
Djorgovski, S. *et al.* 1994, in preparation
Dubath, P., Meylan, G., & Mayor, M. 1994, A&A (submitted)
Éigenson, A. M., & Yatsuk, O. S. 1986, Sov. Astron., 30, 390
Éigenson, A. M., & Yatsuk, O. S. 1989, Sov. Astron., 33, 280
Faber, S., & Jackson, R. 1976, ApJ, 204, 668
Fall, S. M., & Rees, M. 1977, MNRAS, 181, 37P
Fall, S. M., & Rees, M. 1985, ApJ, 298, 18
Fusi Pecci, F., Ferraro, F., Bellazzini, M., Djorgovski, S., Piotto, G., & Buonanno, R. 1993, AJ, 105, 1145
Fusi Pecci, F., *et al.* 1994, A&A, 284, 349
Goodman, J. 1988, in Dynamics of Dense Stellar Systems, edited by D. Merritt Cambridge University Press, Cambridge), p. 183
Goodman, J., & Hut, P. 1989, Nature, 339, 40
Gott, J. R., & Rees, M. 1975, A&A, 45, 365
Gunn, J. E. 1978, in Globular Clusters, edited by D. Hanes and B. Madore (Cambridge University Press, Cambridge), p. 301
Heggie, D., & Aarseth, S. 1992, MNRAS, 257, 513
Hut, P., & Djorgovski, S. 1992, Nature, 359, 806
King, I. R. 1966, AJ, 71, 64
Kormendy, J. 1985, ApJ, 295, 73
Larson, R. 1988, in The Harlow Shapley Symposium on Globular Cluster Systems in Galaxies, IAU Symposium No. 126, edited by J. Grindlay and A. G. D. Philip (Kluwer, Dordrecht), p. 311

Lauberts, A. 1982, ESO/Uppsala Survey of the ESO (B) Atlas (ESO, Garching)

Lauer, T., & Kormendy, J. 1986, ApJ, 303, L1

Lauer, T., *et al.* 1991, ApJ, 369, L45

Lightman, A. 1982, ApJ, 263, L19

Long, K., Ostriker, J., & Aguilar, L. 1992, ApJ, 388, 362

Lynden-Bell, D., & Eggleton, P. 1980, MNRAS, 191, 483

Meylan, G., & Mayor, M. 1986, A&A, 166, 122

Meylan, G. 1987, A&A, 184, 144

Morgan, S., & Lake, G. 1989, ApJ, 339, 171

Murphy, B., Cohn, H., & Hut, P. 1990, MNRAS, 245, 335

Murray, S., & Lin, D. 1992, ApJ, 400, 265

Murtagh, F., & Heck, A. 1987, Multivariate Data Analysis (Reidel, Dordrecht)

Oort, J. 1977, ApJ, 218, L97

Ostriker, J. P., Spitzer, L., & Chevalier, R. A., 1972, ApJ, 176, L51

Ostriker, J. P., Binney, J., & Saha, P. 1989, MNRAS, 241, 849

Paturel, G., & Garnier, R. 1992, A&A, 254, 93

Peebles, P. J. E. 1984, ApJ, 277, 470

Peterson, C. 1993, private communication

Peterson, C. 1993, in Structure and Dynamics of Globular Clusters, edited by S. Djorgovski and G. Meylan, ASPCS, 50, 337

Peterson, C., & King, I. R. 1976, AJ, 80, 427

Pilachowski, C. 1984, ApJ, 281, 614

Pryor, C., & Meylan, G. 1993, in Structure and Dynamics of Globular Clusters, edited by S. Djorgovski and G. Meylan, ASPCS, 50, 357

Racine, R., & Harris, W. 1989, AJ, 98, 1609

Rose, J., & Tripicco, M. 1986, AJ, 92, 610

Rosenblatt, E., Faber, S., & Blumenthal, G. 1988, ApJ, 330, 191

Santiago, B. X., & Djorgovski, S. 1993, MNRAS, 261, 753

Searle, L., & Zinn, R. 1978, ApJ, 225, 357

Shawl, S., & White, R. 1987, ApJ, 317, 246

Skiff, B. 1993, private communication

Smith, G., & Brodie, J. 1993 (ed.), The Globular Cluster—Galaxy Connection, ASPCS (Astron. Soc. Pacific, Provo), Vol. 48

Thomas, P. 1989, MNRAS, 238, 1319

Trager, S., Djorgovski, S., & King, I. R. 1993, in Structure and Dynamics of Globular Clusters, edited by S. Djorgovski and G. Meylan, ASPCS, 50, 347

van den Bergh, S., Morbey, C., & Pazder, J. 1991, ApJ, 375, 594

van den Bergh, S. 1993, ApJ, 411, 178

van den Bergh, S. 1993, private communication

Vesperini, E., & Chernoff, D. F. 1994, Cornell preprint # CRSR 1063

Webbink, R. F. 1985, in Dynamics of Star Clusters, IAU Symposium No. 113, edited by J. Goodman and P. Hut (Reidel, Dordrecht), p. 541

West, M. 1993, MNRAS, 265, 755

Woltjer, L. 1975, A&A, 42, 109

Zaggia, S., *et al.*, 1994, in preparation

Zinn, R. 1980, ApJS, 42, 19

Zinn, R., & West, M. 1984, ApJS, 55, 45

Zinn, R. 1985, ApJ, 293, 424