Galaxy Evolution: The Milky Way Perspective
ASP Conference Series, Vol. 49, 1993
Steven R. Majewski (ed.)

STARCOUNTS AS A PROBE OF GALACTIC STRUCTURE

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INTRODUCTION

For more than 200 years, astronomers have used starcounts - that is, the variation with position on the sky of the stellar apparent magnitude distribution - to infer the size and shape of the Milky Way system. In reviewing past work in the field, it is clear that starcounting has passed in and out of fashion several times, and that each renewal of interest has been sparked largely by technological developments. Herschel's (1785) first 'star-gages' were the product of visual observations at the telescope; Gill and Kapteyn applied photography to the task, with the plates measured by hand; and the flurry of interest at the beginning of the 1980's reflected, at least in part, the development of automated plate-scanning machines. Each episode also saw the application of new methods of analysis - an assumed uniform stellar luminosity gave way to the (m, log π) diagram (see Bok, 1937) which, in turn, gave way to computer modelling. The most recent advance is the development of large-format, highly sensitive CCD arrays which, used on relatively small telescopes, have the potential to provide considerable insight on what remains a rather murky subject.

In this review I will first present a (selective) review of past work in the field, with particular reference to the more recent studies, before moving on to describe a new model which uses slightly different techniques than other computer models. Finally, I shall apply this model to a preliminary analysis of starcount data towards the North Galactic Pole, including Majewski's (1992) re-analysis of the SA 57 plate material, and will show that even this preliminary work reveals substantial shortcomings in the configurations adopted in other model analyses.

A brief history of starcounting, I - 1750-1970

Probably the first people to indulge in starcounting were the Minoan priest-astronomers who were responsible for naming most of the constellations. However, it is generally recognised that William Herschel was the first astronomer to undertake a systematic survey of the heavens with the intention of mapping the distribution of the stars in space. Starting in the 1780s, Herschel started a programme of 'star-gaging' with his 20-foot (19-inch diameter mirror) telescope, counting the number of stars in 15-arcminute boxes distributed over the sky - a series of selected areas, predating Kapteyn, in

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fact. In doing so he was extending less extensive surveys made with his two previous, smaller telescope. To analyse these 'star-gages', Herschel was forced to assume that all were of equal brightness - the first stellar parallaxes were not measured until the late 1830s - while he also had no knowledge of the extent (or even presence) of interstellar absorption. Nonetheless, it was clear that most stars were distributed in a flattened structure. [1] Herschel conjectured a box-like structure for the Milky Way, with a splitting in the both to correspond to the Great Rift in Cygnus.

From latitude 52 degrees north, Herschel did not have particularly lavish access to the southern celestial hemisphere, and although his son, Sir John Herschel, did make numerous observations from South Africa and Ascension Island (including star-gages of the southern sky which were later analysed by Kapteyn (1907)), the emphasis was on surveys of nebulae and asterisms. It was left to a Scot and a Dutchman to survey the most spectacular regions of the Milky Way. David Gill, the pre-eminent practical astronomer of the late 19th century, was appointed the Astronomer Royal at the Cape in 1879 (Warner, 1979). While at Dun Echt, working for Lord Crawford, he had started experimenting with astrophotography, mainly of the Sun and Moon, but his discovery of the potential of photography for star cataloguing came with his successful photographs of the Great Comet of 1882. Not only was the comet spectacularly visible, but his photographs - 2-3 minute exposures taken with a portrait camera strapped to one of the Cape telescopes - also revealed a multitude of stars. Gill promptly telegraphed a report to the Paris Academy of Sciences and obtained a Royal Society grant to undertake a southern photographic sky survey - the first attempted.

Gill's report and photographs, however, had inspired Admiral Mouchez (director of the Paris Observatory) to put forward (with Gill's co-operation and support) a proposal for what eventually became the Carte du Ciel - an astrographic survey of the whole sky which was to involve most of the major observatories of the European countries (and their overseas dependencies). Despite Gill's enthusiastic involvement, this created problems, since after two years the Royal Society refused the fund the Cape Photographic Durchmusterung, on the official grounds that it was duplicating the Carte du Ciel. In doing so they were going against the wishes of many astronomers - such as Struve at Pulkova (Warner, 1979) - who, with the Carte du Ciel divided amongst so many observatories, realised the possible duration. Even the theoretician, John C. Adams (of Neptune fame) had a better appreciation of the importance of Gill's work than did the Board of the Royal Society (Murray, 1988). However, the main reason for the decision was not astronomical: the then Astronomer Royal, Sir William Christie, had never been able to forgive Gill - with no university degree - for being preferred over himself - a Chief Wrangler at Cambridge - for the Cape job. The result was

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[1] A structure of this type had already been suggested by Immanuel Kant (1755), based partly on a misunderstanding of the writings of Thomas Wright of Durham (1750), and by Heinrich Lambert (1761). Herschel, however, seems to have been unaware of any of these speculative writings, which were not widely circulated, at the time he made his observations (see Hoskins, 1963 and 1983 for a detailed discussion).

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that Gill received no funding, and in fact devoted half of his salary for the next five years towards the completion of the CPD.

Once the plates were taken, there remained the problem of measuring the plates and publishing the results. Enter Jacobus Kapteyn, the newly appointed director of Groningen. He was in the position of running an astronomical establishment with no telescope of its own so, as Seares (1922) later put it, he 'made all the telescopes of the world his'. He undertook the organising of the measurement and reduction of all of the Cape Durchmusterung plates, and the results, covering the southern sky south of declination were published in three volumes between 1896 and 1900. In contrast, the last volume of the Carte du Ciel, the declination zone covered by the Edinborough outstation of the Perth (Australia) Observatory, was not published until 1952.

As discussed by Seares (1922) and by Paul (1983), all of Kapteyn's research work centred around the form and structure of the Universe as described by stars and, as he built up Groningen Observatory and Laboratory, he conceived of his plan of Selected Areas (Kapteyn, 1906), 206 regions distributed over the whole sky, which would be intensively studied with the aim of determining the number, brightnesses, motions, distances and spectral types of all stars - as the best means of elucidating the nature of this structure. Kapteyn had considerable prestige - at his death, Seares compared his influence to that of Sir William Herschel's - and he was able to gain support from many observatories, notably Mt. Wilson, of which he was a research associate from 1908 and which, through Kapteyn's friendship with Hale, started the influx of Dutch astronomers to the U.S.

The bulk of the observations, at least for the starcounts programme, were completed by 1920, allowing Kapteyn (with van Rhijn) to undertake a preliminary analysis of the data before his death in June, 1922 (Kapteyn & van Rhijn, 1921). (The full reductions of the observations were completed and the resulting starcounts published by Seares, van Rhijn, Joyner and Richmond (1925).) Combining the starcount data in zones of galactic latitude, Kapteyn derived a Galactic model (figure 2 in Kapteyn & van Rhijn) similar to that he had deduced originally in 1901 - a nearly-heliocentric model (the sun is offset of centre by $\sim$ 650 parsecs) which led him to conclude that ' in the direction of the galactic poles about 1500 parsecs may be taken as practically the limit of the system, while in a direction in the Plane of the Milky Way the same overall small density is eight times more distant'. It is worth emphasising that this paper was published 3 years after Shapley's Big Galaxy hypothesis had been presented.

The main reason why Kapteyn arrived at this model was, of course, the assumption of the absence of interstellar absorption in the analysis of the counts. Kapteyn did worry (as did others) about the non-Copernican aspect of the model and, in 1909, commented that

'..one of the greatest difficulties.... lies in our uncertainty about the amount of loss suffered by the light of stars on its way to the observer'.

At the Groningen 1983 IAU meeting (Symposium 106), celebrating the centenary of the Kapteyn Institute, there was considerable discussion concerning how such an obvious effect as interstellar absorption could have been ignored for so long - particularly given that Kapteyn had access to Gill's photographs of the southern Milky Way, including some of the most spectacular dark clouds in the sky (see Paul, 1983, and subsequent discussion).

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However, in defence of Kapteyn and others, we should note that even after Barnard and Wolf (1923) convincingly demonstrated the existence of small-scale dark clouds (using star-counts in adjacent comparison fields), there was still no suspicion of extinction *between* these clouds. Indeed, Shapley, on the basis of his globular cluster studies, was one of the strongest advocates of low interstellar absorption, and he held the famous cluster 'zone of avoidance' near the Galactic Plane as due to cluster-disruption rather than extinction (Paul, 1983; Smith, 1983). At least initially, he regarded Kapteyn's Universe as a sub-system within the Big Galaxy. It was left to Trumpler (1930) to demonstrate the existence of extensive general absorption through a comparison of the H-R diagrams of various open clusters.

Even with the inclusion of interstellar absorption, many starcount analyses (Bok, 1931; Seares, 1931; van Rhijn, 1936) still indicated a local density maximum in the vicinity of the Sun. This became known as the local system, and still surfaces in the literature occasionally. However, most of these studies were based on analyses of bright stars (9th magnitude (apparent) or brighter) and, as a result, are biased towards intrinsically luminous stars. As a result, the counts have a disproportionate contribution from young stars - and are influenced heavily by the proximity of the Sun to Gould's Belt. The reality of this 'local system' amongst the older stars of the disk is not clear.

In any event, the picture of the Milky Way presented by Bok (1937) is still of an irregular collection of sub-systems - although, with Hubble's establishment of 'island universes', Seares (1931) had suggested that our Galaxy might resemble M33, with the Sun falling in one of the spiral knots. In fact, the idea that our Galaxy is a spiral had been suggested as early as 1903, by the Dutch amateur Cornelis Easton based on his visual mapping of the Milky Way - although this was partly a reflection of a morphological bias of the times (Smith, 1983). In the 1950 symposium to celebrate the dedication of the Curtis Schmidt, Baade discussed 'our Galaxy as a spiral nebula' - similar to M31, rather than M33, but it was left to Morgan (with the help of Sharpless and Osterbroek) to demonstrate convincingly the presence of spiral arms in the vicinity of the Sun (Gingerich, 1983).

From the 1950s much of the observational emphasis of Galactic structure studies shifted towards studies within the Galactic Plane (such as McCuskey's (1956) extensive starcount analyses towards twelve fields distributed around the Galactic Plane), studies of nearby stars and, of course, the continued work (mainly by Sandage and Arp at Mt. Wilson) on the globular cluster systems. The most important development as far as the interpretation of starcounts is concerned was Baade's (1944) separation of Population I and Population II systems in the Andromeda spiral. This led to the realisation that the metal-poor, high-velocity stars identified by Roman (1954) (and the globular clusters) were representatives of Pop. II within our own Galaxy, and the eventual codification of the properties of these populations at the Vatican conference (Oort, 1958; Baade, 1958) and in volume 5 of *Stars and Stellar Systems.*

II - The 1980s

That observational studies of the general stellar distribution at high latitudes fell into abeyance after the completion of the Selected Area survey

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can be attributed to the limitations of the then-available technology. The Selected Area plates were all measured on iris photometers, either at Mt. Wilson or at Groningen, and endless contortions were required to set all the magnitudes onto a uniform system. Indeed, discussions about the uniformity of the magnitude scale reverberate through the literature for several decades after the publishing of the catalogue. Thus while the construction of the Palomar 48-inch Schmidt (and of the 200-inch telescope) meant that it was now possible to obtain photographic plates both covering larger fields of view and extending to fainter magnitudes than previous surveys, there were no ready means of measuring the plates - nor, once the plates were measured, of assimilating and analysing the data. (The density analysis carried out by Seares, van Rhijn and colleagues basically consisted of trying to match the observed densities using $(m, log\pi)$ tables - taking an estimate of the luminosity function, one calculates the contribution of each absolute magnitude to an onion-ring of spherical shells, deriving suitable scaling factors to match the observed number-counts with apparent magnitude.) The revival of interest in general starcounts from the mid-1970's was sparked by the fact it became possible to undertake surveys with less human intervention. The availability of measuring machines - such as the PDS microdensitometers and the R.O.E. GALAXY machine, the progenitor (directly) of COSMOS and (indirectly) the IOA Kibblewhite machine (APM) - meant that one could, at least semi-automatically, acquire rapid, relatively accurate, objective measurement of plate material, while the continuing development of photomultiplier tubes and of standard photometric systems meant that calibrating the resultant instrumental magnitudes was a more straightforward, if still exacting, process.

The most influential of the new surveys was Kron's survey of three of the Kapteyn selected areas - SA 57, the NGP; SA 68, l=110, b=-45; and SA 51, a lower latitude anticentre field. Other deep surveys (Tyson & Jarvis, 1979; Peterson et al, 1979) were aimed primarily at studying faint galaxy counts and, with data in only one passband, are of less interest for Galactic structure analyses. Kron's survey, based on PDS scans of deep IIIaJ (B_J) and IIIaF (R_F) plates obtained using the Kitt Peak Mayall reflector, was the first to reveal the striking bimodal distribution in colours amongst stars at faint magnitudes. The explanation of this is straightforward (Gilmore, 1981): the relatively steep density law of the disk population leads to most stars being drawn from a relatively narrow range in distance ($\sim 600 - 800$ parsecs for SA 57). Hence as one moves to fainter apparent magnitudes, the sample becomes dominated by intrinsically fainter stars. Add to this the fact that the (B-V) and (J-F) passbands saturate for M-type stars, and it is clear that the red stars are disk dwarfs. The blue peak represents more distant stars drawn from the more extended Galactic halo. Not all colour indices give a neat, bimodal distribution, although at a given apparent magnitude disk stars are always redder than the bulk of the halo population.

At the same time as these surveys were being constructed, and provoked by the (apparently) imminent launch of Space Telescope, Bahcall and Soneira (1980 and see references in Bahcall (1986) for subsequent papers) embarked on the first attempt to apply computers to modeling the expected stellar distribution with apparent magnitude. The methods used in their models - and in the contemporaneous models of Gilmore (1981) and Pritchet (1983) - are straightforward, numerical integrations of von Seeliger's (1898) formula

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$$A(m,S) \quad = \quad \Omega \int \Phi(M,S) D(r) r^2 dr$$

where A(m, S) is the number of stars of given spectral type (or colour index) at given apparent magnitude, m; $\Phi(M,S)$ is the luminosity function and D(r), the density law.

Thus, to predict star counts in a given direction, one requires a luminosity function and density distribution for each population or sub-population in the model. The Bahcall & Soneira (henceforth BS) model is based on two populations - disk and halo (the latter, confusingly, being termed the 'spheroid'). For the disk, an exponential functional form is adopted

$$D(h,z) \quad = \quad \rho_0 e^{\frac{-h}{h_0}} e^{\frac{-z}{z_0}}$$

where h_0 is the scalelength in the Galactic Plane ($\sim$ 3.5$kpc.$) and z_0 the scaleheight perpendicular to the disk (325 pc. in the standard BS model). The use of a radial exponential follows mainly from Freeman's (1970) observations of external spirals - the origin of the single vertical exponential is less clear. Oort (1932), following Kapteyn (1922), showed that, in the plane-parallel approximation (constant (K_z), a population of stars with a Gaussian velocity dispersion will give an exponential distribution. Camm (1950) considers three examples of self-gravitating disks, deriving a $sech^2(z/z_0)$ distribution (applied by van der Kruit & Searle (1982) to the analysis of photometry of external, edge-on spirals) for the plane-parallel approximation. In both cases one expects the plane-parallel approximation to break down for stars which travel far enough from the Plane to "see" the central Bulge of the Galaxy.

The first explicit use of exponential approximations to describe the stellar density perpendicular to the Plane (that I have found) is in Allen's Astrophysical Quantities (1972) - previous *empirical* density determinations (e.g. Elvius, 1965) avoid fitting an analytic form to the observations. Indeed, Schmidt's (1959, 1962) estimates of the 'equivalent width' of the stellar distribution from K and early M dwarfs (where the equivalent width is defined as the ratio of the surface density to the local volume density) have been consistently misinterpreted as exponential scaleheights. Both Allen (1972) and, later, Miller & Scalo (1979) explicitly refer to the use of an exponential *approximation* to the vertical structure of the disk - and it is only with the advent of the models that that important qualifier is dropped.

For the halo, BS adopted the form of a de Vaucouleur's spheroid

$$\rho_s(r) \quad = \quad \frac{C exp[-7.669(\frac{r}{r_e})^{0.25}]}{(\frac{r}{r_e})^{0.875}}$$

although one can also adopt a power-law density distribution

$$\rho_s(r) \quad = \quad \rho_0 (\frac{a_0^n + R_0^n}{a_0^n + r^n})$$

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where R_0 is the Solar Radius, a_0 a core radius , ρ_0 the local density and where n is probably lies in the range $3 < a < 4$ (see, for example, the RR Lyrae surveys by Oort & Plaut (1975) and by Saha (1985)).

Starcount models should not be created in a vacuum without due regard to the constraints imposed by, for example, stellar evolutionary theory. The initial version of the BS model adopted McCuskey's (1966) luminosity function for the disk, although later versions switched to the (better-defined) nearby-star luminosity function derived by Wielen (1974). Compared with McCuskey's data, the latter includes a dip at $M_v = +7$, as well as a stronger peak at $M_v \sim +12$, both of which influence significantly the predicted starcounts.

A more serious problem concerned the luminosity function adopted for halo stars. In the first version of the programme, BS scaled the entire disk luminosity function by a factor of 0.12 %, as determined by Schmidt (1975) from the fraction of high velocity stars included in surveys of high proper motion stars. [1] Adopting this luminosity function (from $-2 \leq M_v \leq +16$) ignores the fact that the halo is an old population - hence everything brighter than the luminosity of the main-sequence turnoff ($M_v \sim +4$) is an evolved star, and the evolutionary lifetimes derived from models are short (e.g. $\sim 4 \times 10^8$ years on the subgiant branch for $M = 0.9 M_\odot$, Y=0.2, Z=0.0001, Vandenberg & Bell, 1985)) compared with the age of the halo (~ 12-18 Gyrs.). Hence, with this luminosity function, the predicted numbers of giants and subgiants are incompatible with the observed density of halo main-sequence stars unless the initial mass function had a *huge* discontinuity just below the turnoff mass. This error has been corrected in the most recent version of the BS model (Bahcall et al., 1985).

Then there is the matter of the 'thick disk' - stars with a density distribution intermediate between that of the disk and the halo. The name originated in the analysis of starcounts towards the South Galactic Cap (Gilmore & Reid, 1983 - GR83). Given multicolour data (in this case BVI photometry), one can use the method of photometric parallaxes to derive distances to individual stars and hence calculate directly the fall-off in density with height above the Plane. Applying this method, we computed density laws which deviated from single-scaleheight exponentials at heights of 1-2 kiloparsecs - well before one expects significant contribution from the halo (see figure 6 of GR83). The steepening of the luminosity function with increasing z (figure 4 of GR83) demonstrated from the outset that these stars belong to a relatively old (several Gyr. at least) population.

As I noted above, none of the earlier starcount studies claimed that single-exponential laws were adequate representations of the density distribution of the disk perpendicular to the Plane. But fitting a second (exponential) density distribution to the more distant stars (and naming those stars as the 'thick disk' - a name previously used by Burstein (1979) to describe a feature of S0 galaxies) drew attention to the extended distribution. Other high-latitude starcount studies (Friel & Cudworth, 1985; Yoshii et al., 1987, and, indeed, Fenkart (1966)) confirm that "extra" stars (over and above a 300-parsec exponential) are required at distances of $z >\sim 1.5 kpc.$, while

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[1] Note that Schmidt derives a halo-to-disk ratio of 0.12 % by *mass*, and since approximately half of the local disk mass is in gaseous form, this correspond to a stellar *number* density ratio of $\sim 0.25\%$ (Hartwick, 1986).

Sandage (1987) has argued that the velocity distribution derived from proper motion stars (Sandage & Fouts, 1987) is consistent with two populations, $\sigma_w = 19 kms^{-1}$ and 42 kms^{-1}, with corresponding density distributions.

However, while the existence of these 'extra' stars is now (almost) universally accepted, there remains some dispute over their nature. Some favour a separate population (the 'thick disk' - Gilmore (1984), Gilmore & Wyse (1987)), others identify them as the high-velocity tail of the disk (the extended disk - Norris & Green, (1987)). From my own viewpoint, it is important to remember that their original identification as an intermediate population hinged on the assumption (based on the σ_w *inferred* for a 1500-parsec scaleheight) that their kinematics corresponded to those of the metal-rich RR Lyraes. It has become clear, however, particularly from studies of proper motion stars (Dawson, 1986 - discussed further below) that the extended disk stars (unlike the metal-rich RR Lyraes) [1] do *not* have a significant rotational lag with respect to the old disk, $\sigma_w \sim 40 kms^{-1}$ rather than $\sigma_w \sim 60 kms^{-1}$ (Ratnatunga & Freeman, 1985; Norris, 1987). Hence they are clearly part of a dissipational disk component. Whether they formed *in situ* or were scattered subsequent formation - the most important question to answer - remains to be proven.

Over the last seven years or so, the emphasis in galactic structure studies has shifted away from starcount analysis toward more detailed observations of individual stars, primarily nearby stars drawn from proper motion surveys. Yet there are still relatively few sets of deep, accurately calibrated (and completed and published) starcounts: Friel & Cudworth (1986) surveyed two fields for K giant stars, but their starcount data only extend to $V \sim 15.5$; the Basle Schmidt surveys (Buser & Kaeser, 1985) extend little fainter than $V \sim 18$, as does the recent Kiso Schmidt NGP survey (Stobie & Ishida, 1987), while the UKST datasets published to date (Reid & Gilmore, 1983; Gilmore, Reid & Hewett, 1985) are becoming incomplete by $V \sim 19 - 19.5$. There has been one new model programme produced - Robin & Crézé's (1987) model, which uses a galactic evolution scheme to predict the stellar distribution, deriving results largely in agreement with Gilmore (1984). Within the last year, however, Majewski (1992 - hereinafter SRM) has complete UJF photometry (and astrometry) within an extended (0.35 square degree) SA 57 field. These data set new constraints which models must match, and the following section discusses some preliminary results that we have obtained by comparing these observations with the predictions of a newly developed starcount model.

Matching the SA 57 data

The first requirements of a model are a set of populations or sub-populations, each with a luminosity function, a colour-magnitude relation and a density law. Most previously constructed star-count models have chosen

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[1] While intermediate rotation values are derived for the metal-rich RR Lyraes in the field (e.g., $< V >= -110 kms^{-1}$ for the 64 $\Delta S \leq 4$ stars analysed by Strugnell et al. (1986)), the parent population of these stars still remains to be identified. It is possible that the observed kinematics arise from including disk and halo stars in the same sample.

to generate what might be referred to as analogue output files - number-magnitude-colour relations, generally expressed as the number of stars per square degree for a given apparent magnitude range. I have chosen to take a digital, rather than analogue, approach, generating a catalogue of stars, each with specified M_v, distance and colours, rather than a smoothed distribution. The advantage of this approach is that one can analyse the model using exactly the same techniques that one would use to analyse observations - whether from COSMOS or PDS scans of photographic plates, or from DoPHOT reductions of CCD data. Visually, the results can be displayed as colour-magnitude diagrams, and compared directly with the real world.

In setting up a model, for each population or sub-population I define the local number density of stars at each absolute magnitude (the luminosity function) and the colours corresponding to that M_v (the colour magnitude diagram). Dividing the conical sampling volume into sections (by distance), I calculate the expected number of stars in each section, n, and then generate (using a random number generator) a distance (within the specified distance limits), an absolute magnitude (within $M_v - \frac{1}{2}\Delta M_v < M_v < M_v - \frac{1}{2}\Delta M_v$, where ΔM_v is the difference is absolute magnitude between adjacent bins) and colours for that absolute magnitude (using linear interpolation between adjacent bins to get the colours corresponding to the M_v for each star). (The distance is selected at random from an r^3 distribution to give the correct distance distribution within the volume segment.) Finally, the colours and the apparent magnitude are perturbed using a gaussian to simulate cosmic dispersion (or observational errors). Since the exact distance of each 'star' is known, this allows one to test the effects of Malmquist-like biases for different magnitude and colour distributions.

The most extensive set of starcount data is that for the galactic poles - BVI observations covering 18 square degree towards the south pole (SGP: Reid & Gilmore, 1982); the Kiso Schmidt data, covering $\sim$ 25 square degrees towards the north pole (NGP: Stobie & Ishida, 1987); BV data (and proper motions) over a similar area in the NGP from Palomar Oschin Schmidt data (Reid, 1990); and, finally, the extensive SA 57 dataset (Kron, 1980; SRM). Note that there is an excess of $\sim$ 10% in the total southern numbercounts over the northern data for $V >\sim 16$ (Reid, 1990, fig. 12). Analysing the colour distributions, Stobie & Ishida found the discrepancy to lie in the number of red dwarfs - disk stars - and suggested the effect arises through the Sun lying $\sim$ 30–50 parsecs north of the Galactic mid-Plane. This appears to be supported by CO observations by Magnani, Blitz & Mundy (1985), and I have adopted an offset of +30 parsecs in the models.

The current analysis concentrates on the Kiso NGP BV data and the SRM SA 57 data. I have transformed the SA57 JF photometry to the BV passbands using the transformations calculated by Majewski giving, taken with the Kiso data, two-colour observations extending from $V \sim 12$ to ~ 22, with the faint sample having been checked thoroughly for contamination by extragalactic objects. The colour-magnitude distribution of the deep sample is plotted in figure 1 - note the striking bimodality in colour.

As an initial 'strawman' model, I have predicted starcounts for the current 'standard' model of the galaxy. The parameters adopted for the various sub-populations used are given in Table 1. The disk (assumed to have a radial scalelength of 3.5 kpc. - although this parameter obviously has no influence

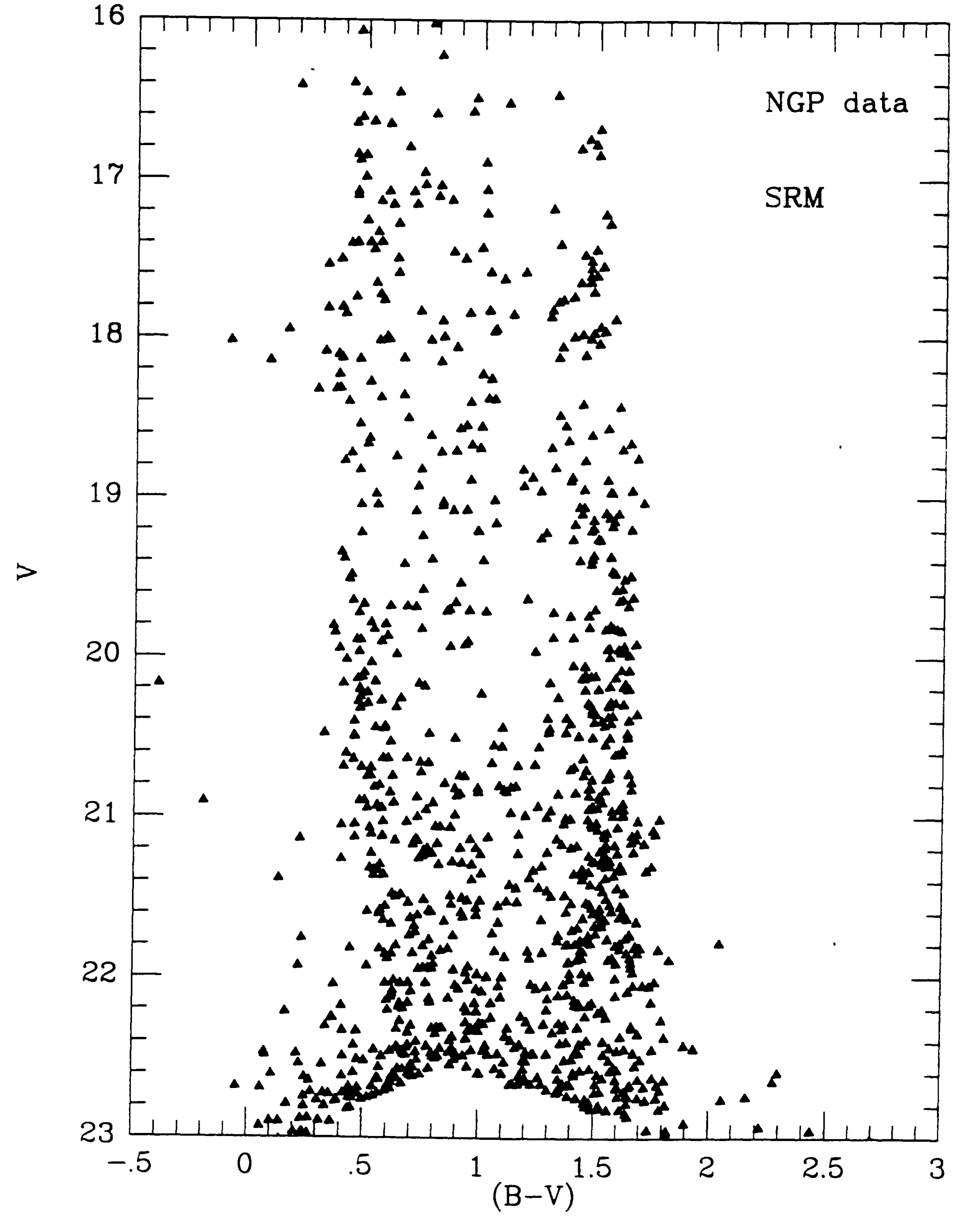

Figure 1. The (V, (B-V)) colour-magnitude diagram for Majewski's (1992) 0.35 square degree SA 57 sample. I have transformed his JF photometry to BV using the relations derived by Majewski.

on the polar counts) is divided into four components (by age), and all of the luminosity functions are based the results from nearby stars (Wielen, Jahreiss & Kruger, 1984 - WJK) and the photometric surveys summarised in Reid (1987 - R87). The young disk component includes all stars brighter

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Table I - Galaxy Model 2

Disk	Density law	exponential
	radial scalelength	3.5 kiloparsecs
	young disk	$\tau \sim 3 \times 10^8$ yrs.
	Luminosity function	
	main-sequence	WJK
	M_v limits	$-6 < M_v < +1$
	main-sequence	30 % of WJK
	M_v limits	+2 to +3
	main-sequence	3 % of WJK + R87
	M_v limits	$M_v \geq +4$
	colours	Gliese
	subgiants & giants	Hyades, Pleiades, NGC 6475, IC 4665
	Densities	
	vertical scaleheight	100 parsecs
	intermediate disk	$\tau \sim 2 \times 10^9$ yrs.
	Luminosity function	
	main-sequence	60 % of WJK
	M_v limits	+2 to +3
	main-sequence	17 % of WJK + R87
	M_v limits	$M_v \geq +4$
	colours	Gliese
	subgiants & giants	NGC 752, NGC 2477
	Densities	
	vertical scaleheight	250 parsecs
	old disk	$\tau > 2 \times 10^9$ yrs.
	Luminosity function	
	main-sequence	80 % of WJK + R87
	M_v limits	$M_v \geq +4$
	colours	Gliese
	subgiants & giants	M 67
	Densities	
	vertical scaleheight	325 parsecs
	extended disk	$\tau \sim 10^{10}$ yrs. (?)
	Luminosity function	
	main-sequence	1.5 % of WJK + R87
	colours	Gliese
	M_v limits	$M_v \geq +4$
	subgiant & giant	NGC 188
	densities	
	vertical scaleheight	1000 parsecs

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Halo	
Density law	radial power-law
exponent (n)	-3.5
core radius	1.0 kiloparsecs
axial ratio	0.9
Luminosity function	
main-sequence	0.2 % of WJK + R87
M_v limits	$M_v \geq +4$
colours	M3

than $M_v = +1$, and proportional segments of fainter stars (based on an assumed age of the disk of 10 Gyrs.). With a scaleheight of only 100 parsecs, these stars make negligible contribution to the polar counts. Similarly, the intermediate disk (age ~ 2 Gyrs.), with an assumed scaleheight of 250 parsecs, and including 17 % of the local stars, has little influence on the counts, which are dominated by the 325 parsec-scaleheight old disk. I have also included an extended disk component, with a local density of 1.5 % of the old disk and a scaleheight of 1000 parsecs - these last values are consistent with the analysis by Friel & Cudworth (1986) and the GR83 analysis, making proper allowance for the contribution from the underlying halo. Each sub-population has an associated giant branch, while for the main-sequence I have used the colour-magnitude relations defined by the nearby stars (figure 2).

The halo population is modelled as a power-law spheroid, axial ratio ($\frac{c}{a}$ = 0.9), with a core-radius of 1 kpc. and a power-law exponent of 3.5. The luminosity function is assumed to be similar in shape to that of the disk for $M_v >\sim +4$, but matching a globular cluster function (e.g. DaCosta, 1982) at brighter magnitudes. The local density zeropoint is determined from proper motion star samples - essentially, one assumes that all stars with tangential motions greater than some limit belong to the halo. Given a kinematic model for the halo (defined by metal-poor RR Lyraes, for example), one can calculate the proper motion selection effects, the fraction of halo stars included in the sample, and hence the local space density. As noted above, Schmidt's (1975) original application of this technique yielded a local halo-to-disk ratio of ~ 0.25% (although this was based on only 17 subdwarfs). More recently, Dawson (1986) has used Monte Carlo methods to model the relative contribution of disk and halo stars to the LHS catalogue (Luyten, 1976). Defining the halo kinematics based on Woolley's (1978) analysis of the metal-poor RR Lyraes ($\sigma_u = 145 kms^{-1}$, $\sigma_v = 125 kms^{-1}$, $\sigma_w = 71 kms^{-1}$), and with $V_c = 220 kms^{-1}$, Dawson derives a halo-to-disk ratio of 1:600 (0.17 %). Clearly, this result depends on the adopted kinematics for the halo, but one should note that most other halo tracers imply lower velocity dispersions, hence a lower mean tangential velocity, a smaller detection fraction (in proper motion surveys) and an increased halo-to-disk ratio.

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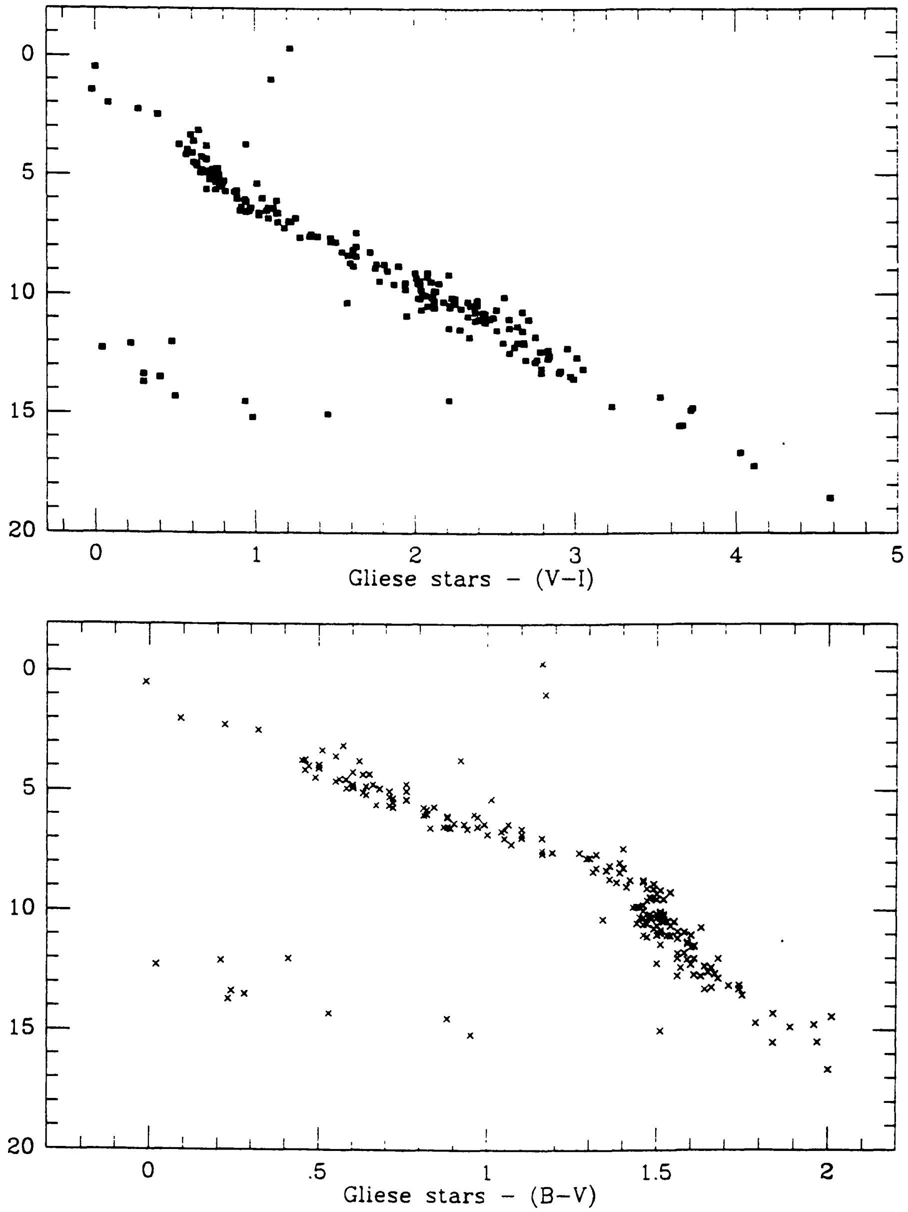

Figure 2. The absolute magnitude-colour relations defined by stars of measured trigonometric parallax - a) $(M_v,(B - V))$; b) $(M_v,(V - I))$. The photometric data are taken from Bessell (1990) and Leggett (1992).

Dawson also uses the LHS data to construct reduced proper motion diagrams, where the reduced proper motion, H, is defined as

$$H \quad = \quad m + 5 + 5log\mu \quad = \quad M + const. + 5logV_t$$

Hence, one can define ridge-lines, $H = M + const. + 5log < V_t >$ in the (H, colour) plane for a population of stars with mean tangential velocity,

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velocity, $< V_t >$. Halo stars have a mean tangential velocity approximately a factor of six higher than disk stars, and Dawson shows that there is a clear separation between the two in the $(H_R, (B-R))$ plane. This implies that any population with kinematics (primarily rotation) intermediate between disk and halo has a local density substantially smaller than 1 % that of the disk.

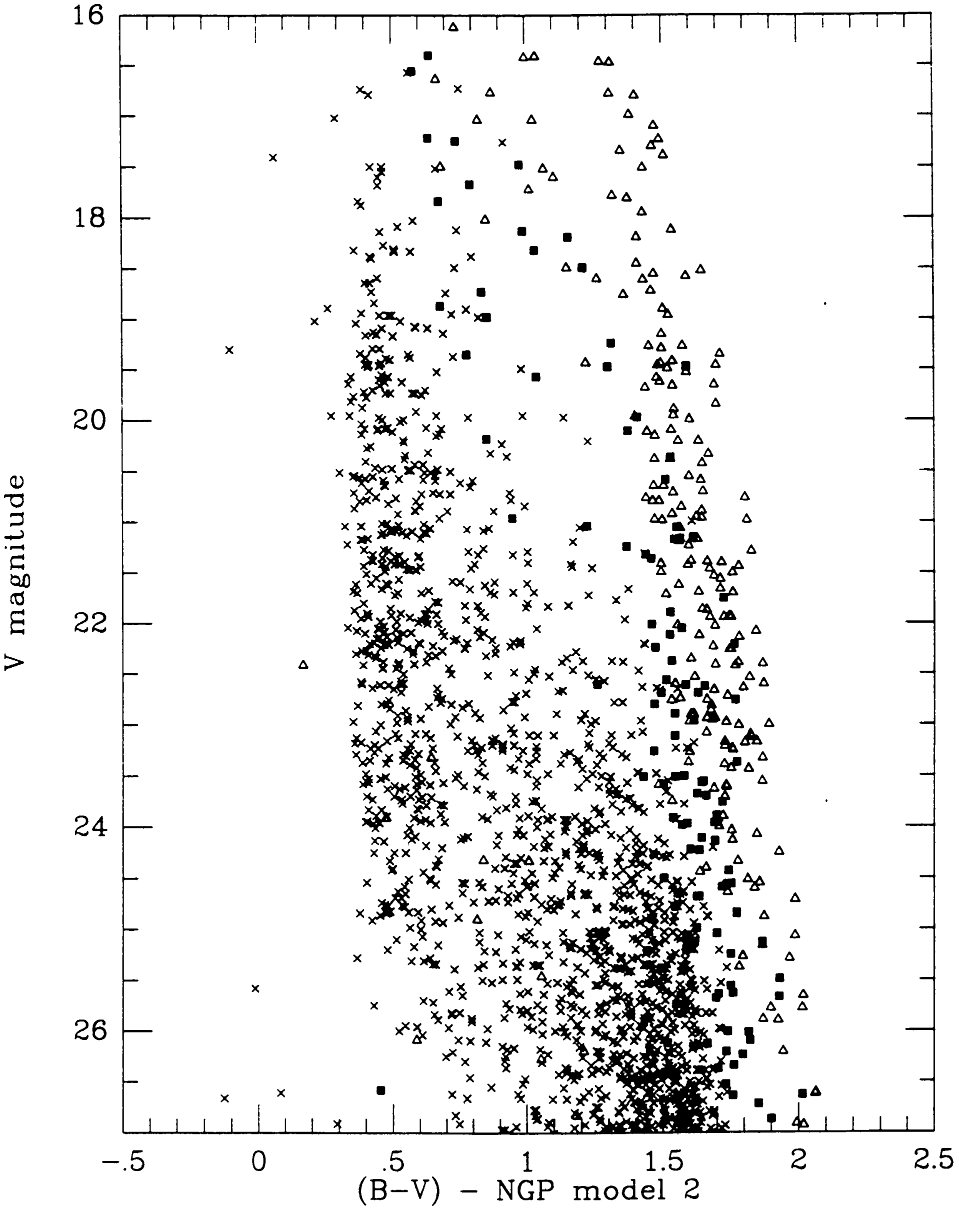

Figure 3. The (V, (B-V)) diagram predicted by model 2, whose parameters are given in Table 1. The open triangles are old disk 'stars', the solid squares represent the extended disk and the crosses are the halo contribution.

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Modelling the NGP

Figure 3 shows the (V, (B-V)) colour-magnitude diagram predicted by the model whose parameters are given in Table 1 (hereafter model 2). In this (and following) diagram, the halo 'stars' are marked as crosses, old disk as open triangles and extended disk as squares. (The contributions of the other disk components are negligible.) One point which can be made immediately is that the model predicts too many blue horizontal branch stars - the relative scarcity of such stars in the halo was one of the first points to emerge from modelling (Bahcall, 1985).

As described above, the bimodal colour distribution stems from two factors - the colour-magnitude relation, and the distance distribution of each sub-population (figure 4). The old disk is drawn from a relatively narrow range in distance, with the median distance modulus $\sim$ 9 ($\sim$ 2 scaleheights). Combining that with the saturation of the (B-V) index at $M_v \sim 8$ evident in figure 2a leads to the strong red peak for $V \geq 18$, while the 'stars' near the halo turnoff start to appear at the same magnitude and, with the broader distance distribution, persist to beyond 23rd magnitude.

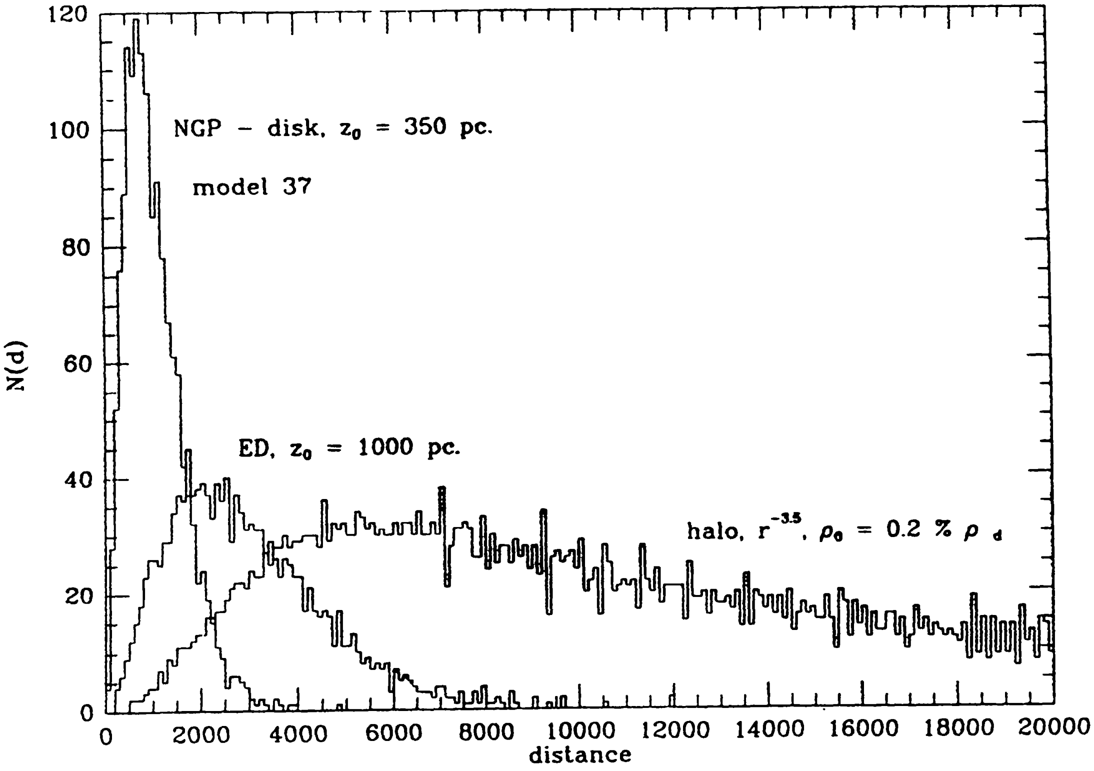

Figure 4. The distance distribution of the three sub-populations identified in figure 3. Note the small range in distance described by the old disk as compared with the much broader halo-star distribution.

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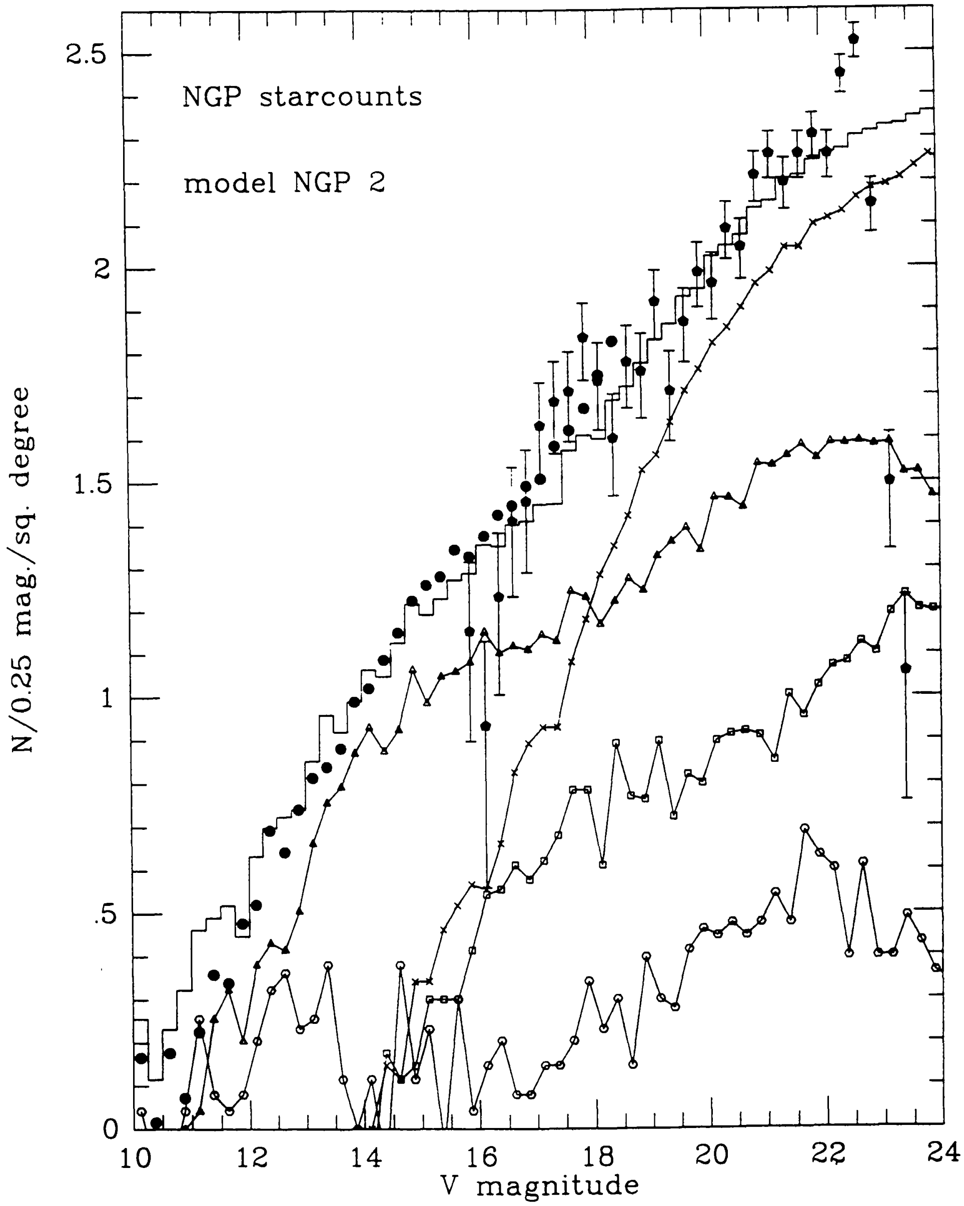

Figure 5. The integral starcounts towards the NGP predicted by model 2. The contributions from four of the sub-populations are identified separately - the open circles represent the intermediate disk. The remaining symbols have the same meaning as in figure 3. The observed counts are from Reid (1990) to $V \sim 18.5$, with the fainter data being drawn from Majewski's SA 57 analysis. The error bars represent the Poissonian uncertainties.

Figure 5 compares the predicted differential starcounts, A(m), from model 2 with observations from Reid (1990) to V=18 and SRM, for $V > 16$. The error bars plotted for the latter points reflect the Poissonian sampling uncertainties. The histogram shows the total counts predicted by the model, while the contributions from the separate components are plotted using the

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same symbols as in figure 3 (the open circles delineate the intermediate disk). The agreement is reasonable - however, this concordance disappears when one considers the colour distributions. Figure 6 plots the observed (histogram) and predicted (line) distributions for 6 magnitude intervals (again, the separate contributions of disk, extended disk and halo are shown). There are two basic problems which start to become evident at $V \sim 17$ - too many halo stars, and too few disk dwarfs.

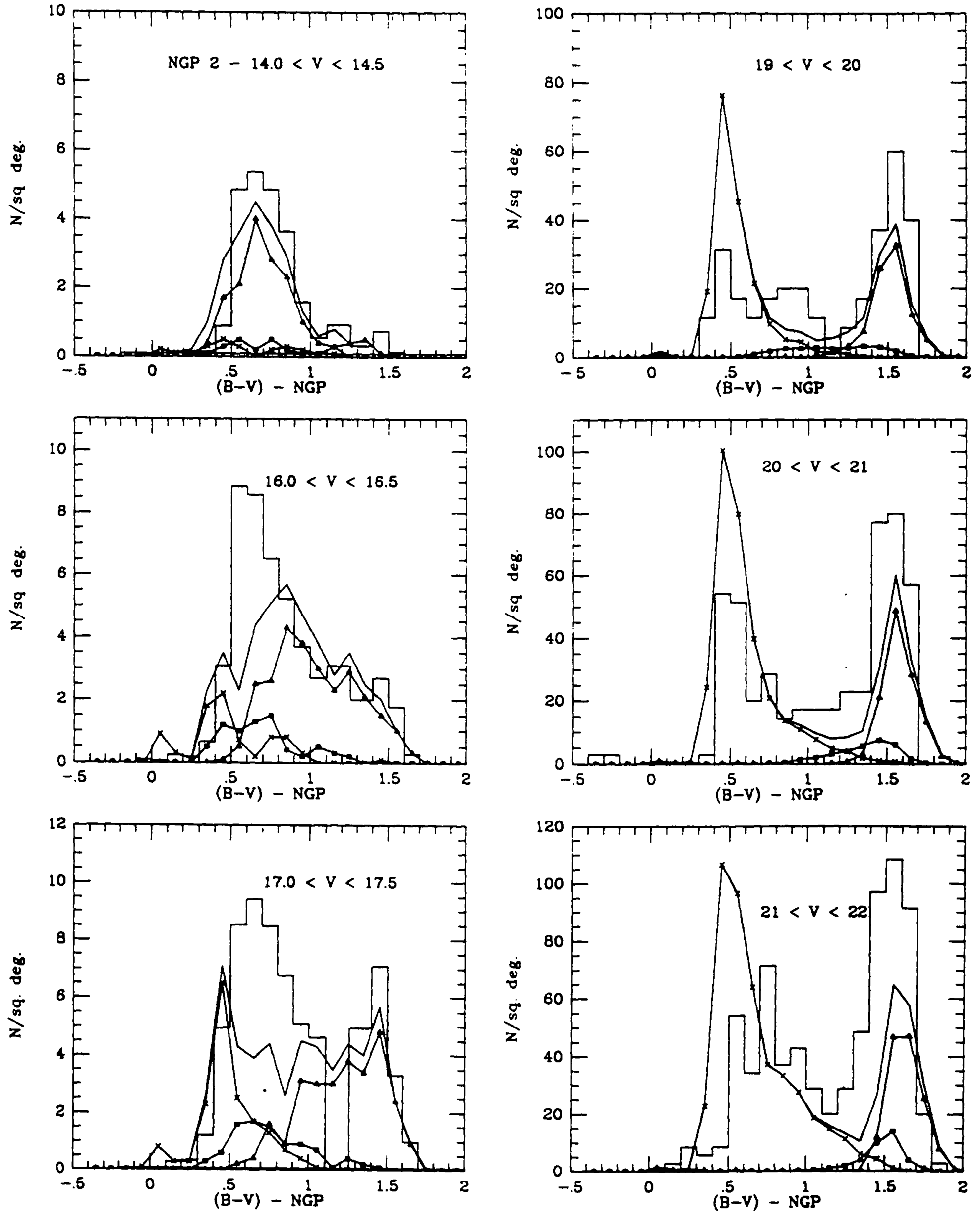

Figure 6. The colour distributions predicted by model 2 for six magnitude intervals. The observed data are from Stobie & Ishida ($V < 17.5$) and Majewski (the three fainter samples). Note the striking failure of the model, which predicts the integral counts with reasonable accuracy, to fit the colour distributions.

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A - the halo problem

There are three possible ways of reducing the contribution of halo stars:

1) Most of the stars in the model lie very close to the turnoff - thus, depressing the luminosity function at this point will decrease the predicted numbers of stars. Figure 7 plots the halo luminosity function used in model 2 together with a globular cluster luminosity function - M3, from Sandage (1957). (Φ (disk) is also plotted as a histogram.) Most models have

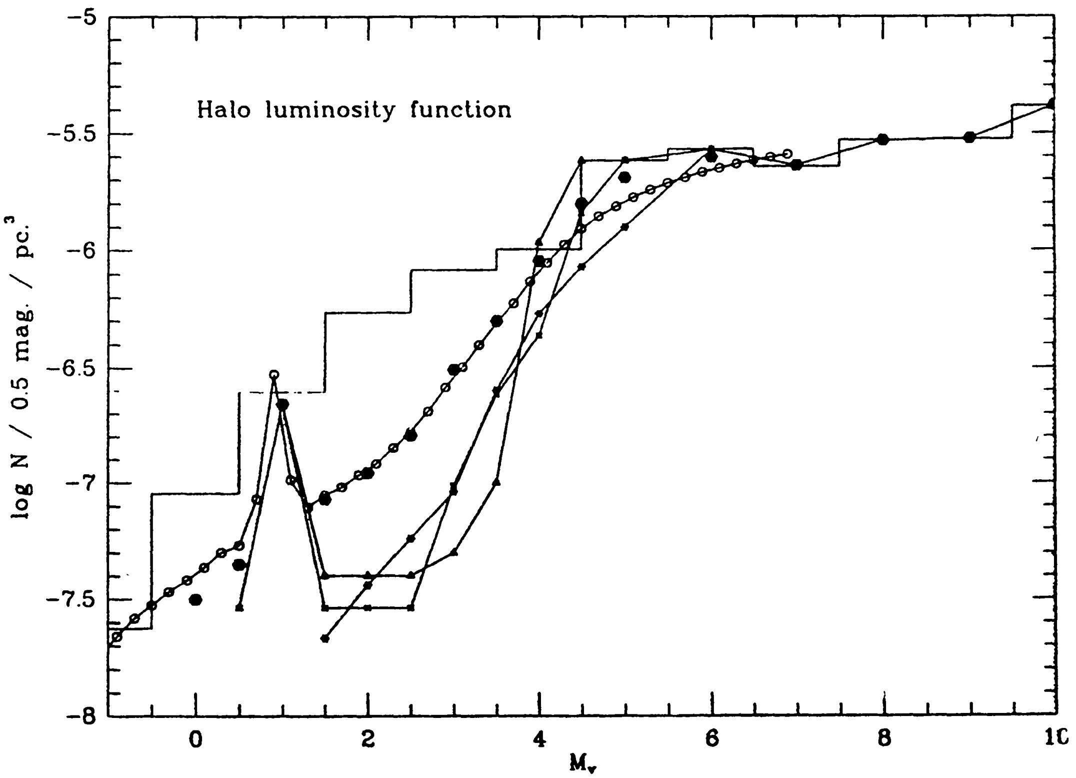

Figure 7. Estimates of the halo-star luminosity function. The open triangles outline the luminosity function adopted in model 2 (scaled to 0.15 % of the disk density for $M_v \geq +4$ - the histogram plots the scaled disk function); the 4-point stars outline the luminosity function derived from the Vandenberg & Bell (1985) theoretical models, the six-point stars delineate the luminosity function derived from the revised Yale tracks (Green et al., 1987) - both of these are for $[Fe/H] \sim -1.5$. The open circles show Sandage's (1957) M3 luminosity function, scaled to match 0.15 % of the disk at $M_v \sim +6$ - note that the observations find a factor of 3-4 more subgiants than are predicted by the theoretical models. The solid points delineate the revised luminosity function we have adopted in model 41.

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chosen to match the halo and disk functions for $M_v > +4$ (Gilmore, 1984, figure 6). Scaling the M3 function to 0.15 % disk at $M_v \sim +6.5$ reduces the predicted number of stars at the main-sequence turnoff by a factor of two. One can also appeal to theoretical models - Vandenberg & Bell's model for a 0.9 $M_\odot$ star with Y=0.20 and Z=0.0003 evolves onto the main sequence at $M_v \sim +5$, reaches $M_v = +4.5$ at age $\sim$ 6.5 Gyrs and +4.0 at $\sim$ 9.5 Gyrs, evolves round the turnoff (to $M_v = +3.2$) in a further 2 Gyrs and to the base of the giant branch by age 12 Gyrs. If we assume an age of 14 Gyrs. for the halo, then scaling densities relative to $\Phi(M_v = +5)$ gives the luminosity function (open squares) plotted in figure 7. We also plot the 16 Gyr., Y=0.30, Z=0.0004 luminosity function from the revised Yale isochrones (Green, Demarque & King, 1987). Note that both theoretical tracks predict substantially fewer subgiants (i.e. shorter lifetimes) than are observed.

We have based the form of our revised halo luminosity function (plotted as stars in figure 7) on the globular cluster data. However, model calculations with this revised function and a near-spherical halo predict just as many halo stars at faint magnitudes, and match the observations as poorly as the model 2 predictions.

2) The local halo-to-disk ratio may be overestimated in the model. However, this would imply that the analyses of proper motion stars were underestimating the halo kinematics and the mean tangential velocity. A reduction of a factor of 2 in ρ_0 requires increasing the mean tangential velocity by $\sim$ 25% to $V_t \sim 310 kms^{-1}$.

3) Model 2 assumes a nearly spherical halo - flattening the halo will, obviously, steepen the density law toward the Pole and reduce the predicted number of stars. Flattened halos have been suggested before - particularly given the anisotropic kinematics of RR Lyrae stars (Strugnell, Reid & Murray, 1986). Oort & Plaut (1975) suggested an axial ratio ($\frac{c}{a}$) of approximately 0.8 from their analysis of the RR Lyrae density distribution, while Wyse & Gilmore (1989) used starcounts from two fields at different galactic latitudes to argue for an axial ratio as low as 0.6.

B - the disk problem

In contrast to the overprediction of the halo, model 2 predicts more than a factor of two too *few* disk stars. I have already alluded to the (over-) simplicity of the single exponential density law - the scaleheights used are based on the GR83 analysis, and one might argue from their figure 5 for an increased scaleheight (perhaps 350 parsecs) for the lower luminosity ($M_v > +7$) stars. To check this, I have re-derived the M-dwarf density distribution, using the relations

$$M_v \quad = \quad 2.89 \quad + \quad 3.37 \times (V - I) \qquad (V - I) > 0.92$$

(Stobie, Ishida & Peacock, 1989; Reid, 1991) and

$$M_v \quad = \quad 1.10 \quad + \quad 5.33 \times (V - I) \qquad 0.5 < (V - I) < 0.92$$

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to derive photometric parallaxes. Figure 8 shows the resulting density laws for $6.5 < M_v < 9.5$. The vertical bars mark the distance at which the sample starts to be come incomplete (V = 19.5). These density laws assume no abundance gradient, but tests show that adding reasonable gradients (up to -0.25 dex [Fe/H] per kiloparsec) leaves the general form of the diagram unaltered.

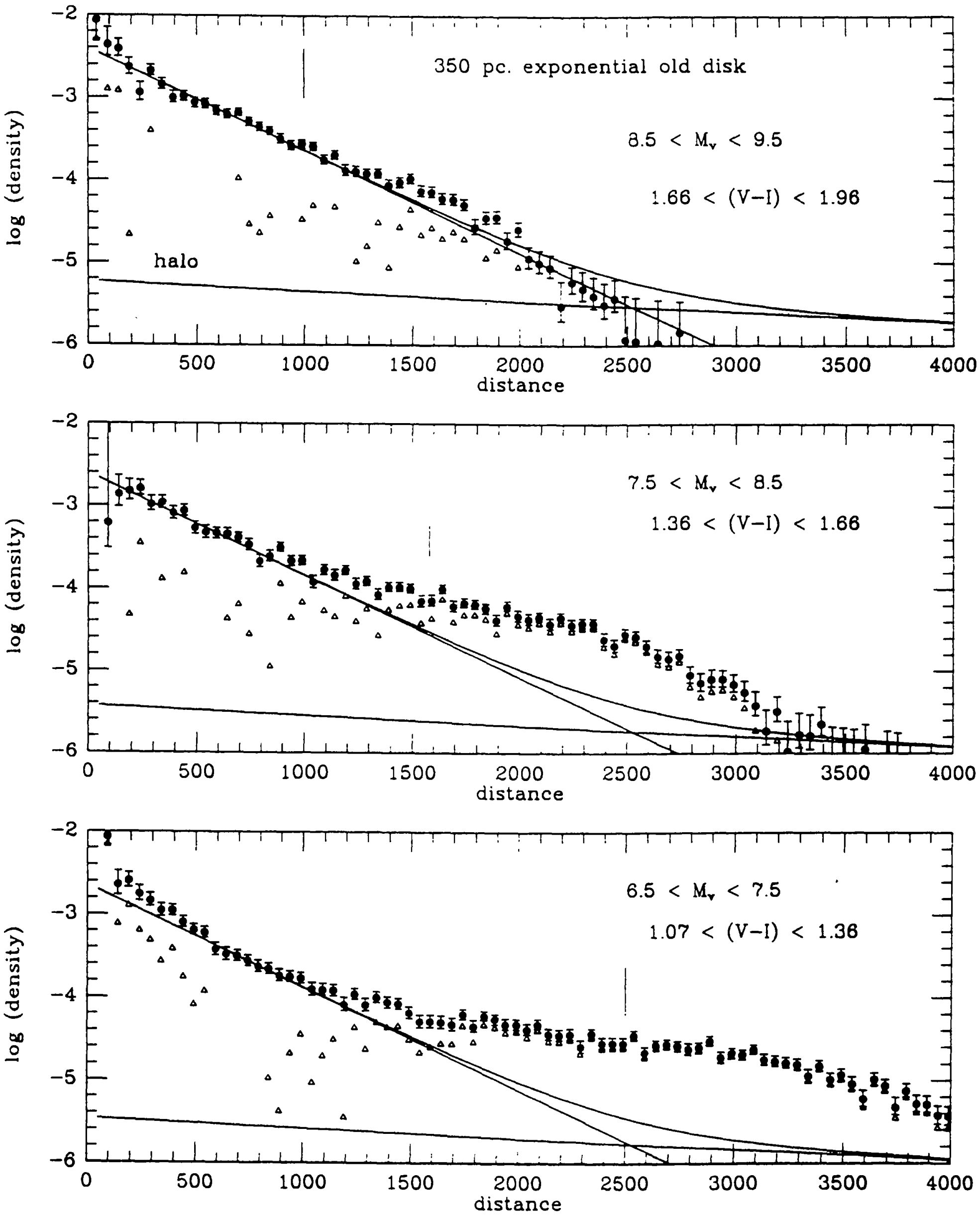

Figure 8. The density distribution derived by applying the photometric parallax method of analysis to GR83s VI SGP photometry. A 350-parsec exponential distribution is shown, together with a halo distribution normalised to 0.2 % of the disk at z=0. The observed counts clearly depart from the former at $z \geq 1.2$ kiloparsecs.

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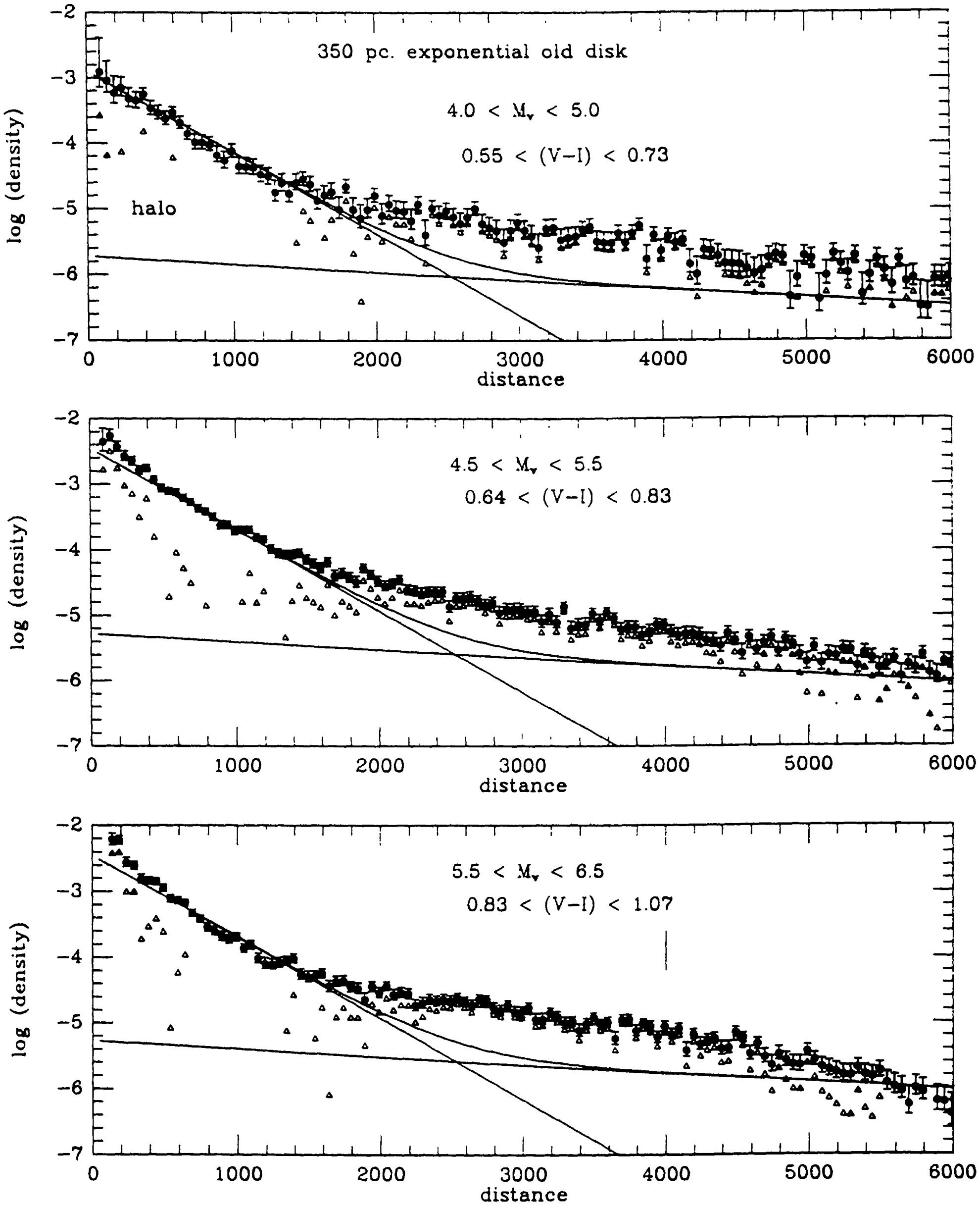

Figure 9. The SGP density distributions derived from stars with $4 < M_v < 6.5$. Note that the colour-magnitude relation steepens for these brighter stars, leading to greater uncertainties in the individual distance determinations.

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Two points emerge from these data - first, the low-z regions ($z <\sim 1$ kpc.) can be represented to a reasonable accuracy by an exponential distribution, scaleheight $\sim$ 350 parsecs. (The extra stars at $z < 600$ pc. (i.e. $V < 16.5$) amongst the brightest (bluest) sub-sample probably reflects the inclusion of halo giant and subgiant stars - the reason for our original decision not to analyse these data. The contamination is, however, negligible at the fainter magnitudes and for the redder stars.)

The second, and more interesting, result lies with the 'extra' stars - the former 'thick disk'. I have subtracted 'thin disk' (350-parsec exponential) and halo from the data, and the residuals are plotted as open triangles. Density laws for the more luminous stars ($4 \leq M_v \leq 6.5$) are shown in figure 9 - note that, although these extend to larger distances, the steepening of the colour-magnitude relation (figure 2b) means that individual distances are correspondingly more uncertain. In both sets of figures, the density law of the 'extra' is poorly determined, but for an exponential scaleheight of $\sim 1 kpc.$, it is clear that the extrapolated local density is closer to 5 % of the 'thin disk' density (cf. Sandage (1987)) than the oft-cited 1.5 to 2 %.

A resolution ?

With the innumerable parameters to hand in a starcount model, it is possible to match just about any set of observations. To do so in a physically self-consistent manner is less easy. For the halo stars, the most plausible option is a flattened halo - although there remain questions about form of the luminosity function adopted. The main-sequence luminosity functions determined for globular clusters (McClure et al., 1988) seldom match the shape of the WJK old disk luminosity function at $M_v \geq +6$. (This may partly reflect the different abundance, hence different bolometric corrections and/or mass-luminosity relation.) While the low-mass ($M_v >\sim +6$) stars themselves make little direct contribution to the predicted starcounts, they determine the normalisation adopted for the brighter stars which *are* observed. [1] In the case of the disk, the evidence for a more substantial extended component appears to be very strong. Again, I would emphasise that while, for modelling purposes, it is convenient to represent these stars as two exponential distributions, there is as yet no reason to attribute any physical significance to these fitting functions.

Bearing in mind these strictures and limitations, I have put together a model which produces a somewhat better representation of the NGP data. This model (no. 41 in an infinite series) differs from model 2 in having a flattened halo, axial ratio ($\frac{c}{a}$=0.6); an old disk with scaleheight 350 parsecs; and an extended disk with local density 4 % of the old disk. The predicted and observed magnitude and colour distributions are plotted in figures 10 and

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[1] In that context, there are very few globular cluster luminosity functions published which extend all the way from the horizontal branch to at least 2 magnitudes below the turnoff. Most of the earlier photographic observations cover the region from the turnoff to the tip of the giant branch, while the more recent CCD data pushes down the main-sequence from the turnoff, and, with separate datasets often covering different areas for the same cluster, grafting old and new together is not straightforward.

11. While still notably imperfect (there are now too many disk stars at bright magnitudes), it at least forms a starting point for future work.

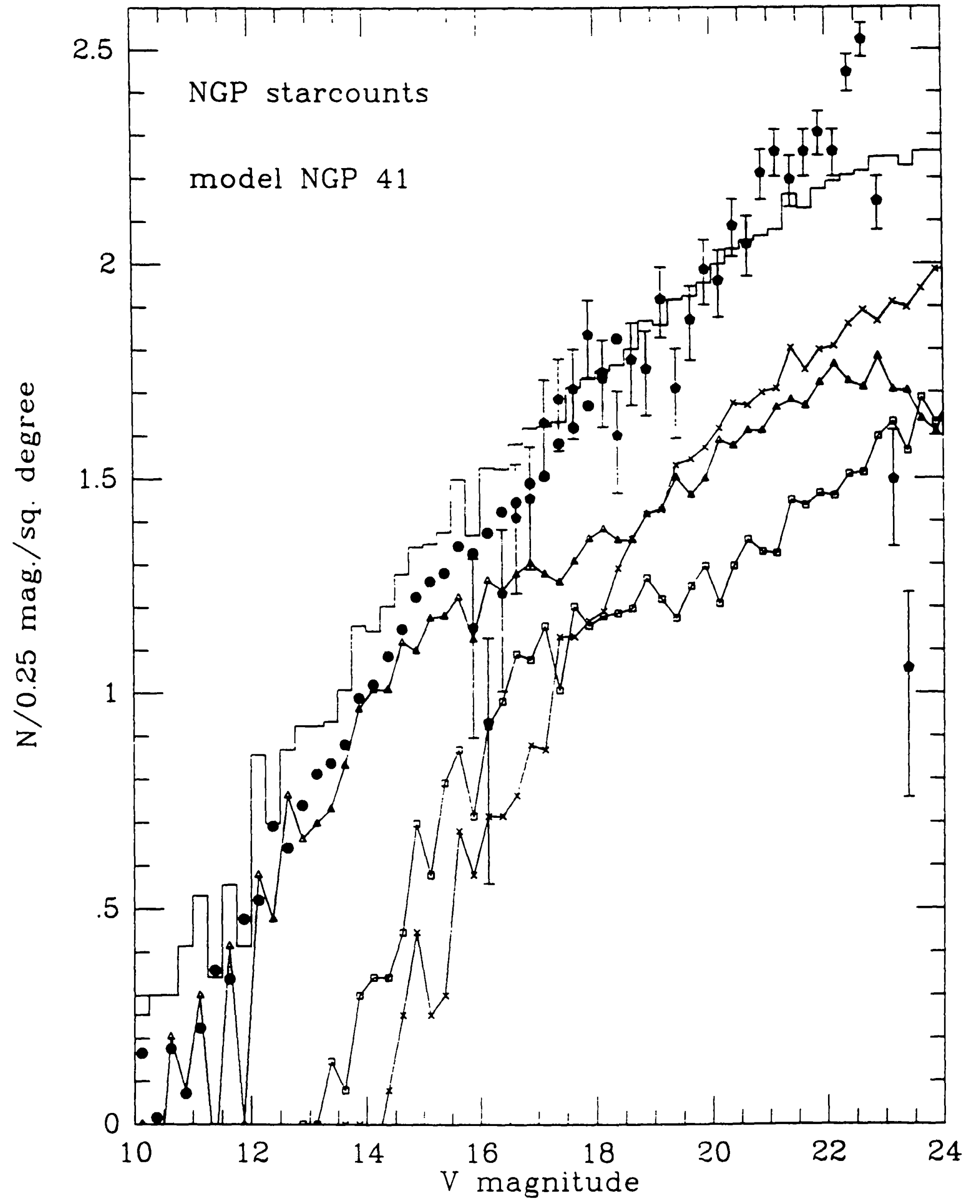

Figure 10. The differential starcounts, A(m), predicted by model 41, which includes the revisions outlined in the text. The model clearly still leaves something to be desired.

Conclusions and future possibilities

The main conclusion is simple and straightforward - after 200 years of work, starcounts still have an important role to play in determining the structure of the Milky Way. It is clear that what has become accepted as the 'standard model' for the Galaxy is a remarkably poor representation of the

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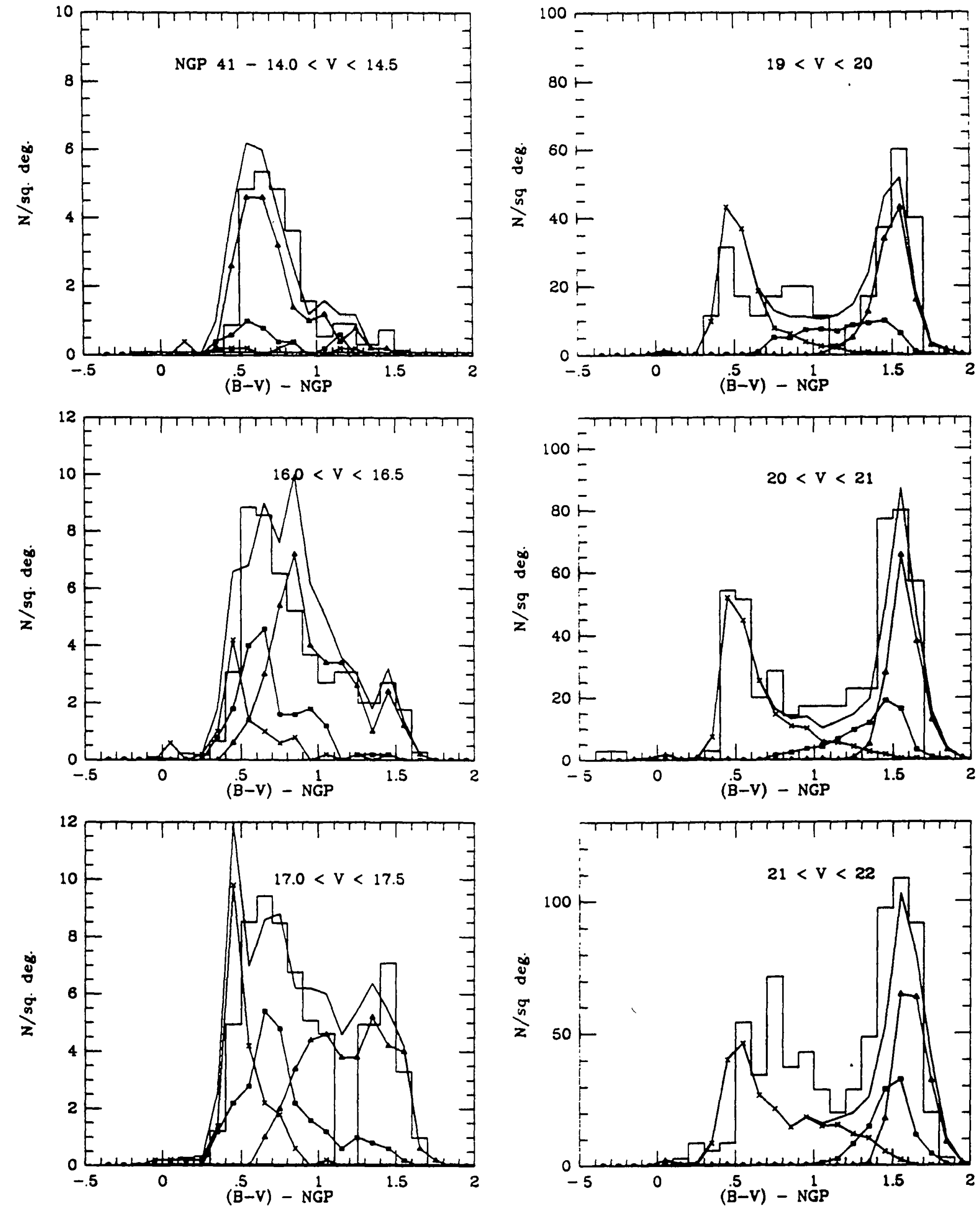

Figure 11. The colour distributions predicted by model 41.

one set of high-quality, deep photometric observations. In most talks delivered by observational astronomers, one of the conclusions is that more data are required. This is emphatically the case in this field - particularly since, while some of the questions raised can be addressed using multicolour, wide-angle, photographic photometry, most of the problems centre on the halo population, and these problems can be addressed using a large format CCD on a relatively small (1-1.5 metre) telescope. Five obvious areas of investigation, which need to be addressed are :

1) The disk density laws - we need a better determination, both at high z (figs. 8 and 9) and as a function of absolute magnitude. It is clear that a 350-parsec exponential overpredicts the disk contribution at bright magnitudes (both apparent and intrinsic), while giving a reasonable fit at fainter magnitudes. It is possible a different form (a $sech^2$ function is an obvious option) of the density law would provide a better representation. Observations of the redder stars ($M_v \sim +7$), which have better-determined magnitudes and are less subject to giant/subgiant contamination questionmarks, are particularly suited to this purpose, requiring accurate (better than 5 %) photometry and star/galaxy separation to at least $V \sim 21$. An estimate of the abundance gradient (if any) in the disk would also be useful.

2) The exact form of the halo luminosity function remains a problem. Globular clusters afford the best means of estimating the relative number of main-sequence and subgiant stars, but this demands data extending from at least the horizontal branch to well below the turnoff. The local normalisation must continue to rest on analysis of proper motion star samples - the acquisition of more observations of those stars should permit both the construction of a purer halo star sample, and a better idea of the appropriate kinematics to use in assessing the incompleteness of that sample.

3) As originally pointed out by Bahcall & Soneira (1980), the flattening of the halo can be determined from the relative number of halo stars in fields at different latitudes on the l=90,270 great circle. A flattened halo can also be detected from deep starcounts towards the Poles, since the halo sequence will move towards redder mean colours with increasing magnitude as one runs out of Galaxy (although this requries accurate data to $V > 24$).

4) The power-law exponent adopted in the models in -3.5, while RR Lyrae studies suggest n=-3 is more appropriate. Adopting the latter value leads to even more halo stars predicted at faint magnitudes, and an even flatter halo.

5) Previous models have essentially treated the abundance of the halo (which defines the blue peak at faint magnitudes - cf. figure 3) as a parameter which can be allowed to vary from one field to the next, with no regard for self-consistency within the model. This makes little physical sense. Again, accurate CCD photometry is required to determine what variations, if any, actually occur as a function of Galactic position.

Other questions (and maybe some answers) will emerge in the course of future work. The most important requirement is that the observations should be used in direct conjunction with the models. The latter can be used to identify specific regions for observation which test specific parameters within the models (exactly the scheme proposed by Bahcall & Soneira, 1980). These new observations allow one to modify the models, leading to new predictions which can be tested against further observations. There is no need to wait until 2080 before rediscovering the efficacy of starcounts (again).

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