

# Galaxy disks

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## Abstract

The disks of disk galaxies contain a substantial fraction of their baryonic matter and angular momentum, and much of the evolutionary activity in these galaxies, such as the formation of stars, spiral arms, bars and rings and the various forms of secular evolution, takes place in their disks. The formation and evolution of galactic disks is therefore particularly important for understanding how galaxies form and evolve, and the cause of the variety in which they appear to us. Ongoing large surveys, made possible by new instrumentation at wavelengths from the ultraviolet (GALEX), via optical (HST and large groundbased telescopes) and infrared (Spitzer) to the radio are providing much new information about disk galaxies over a wide range of redshift. Although progress has been made, the dynamics and structure of stellar disks, including their truncations, are still not well understood. We do now have plausible estimates of disk mass-to-light ratios, and estimates of Toomre's  $Q$  parameter show that they are just locally stable. Disks are mostly very flat and sometimes very thin, and have a range in surface brightness from canonical disks with a central surface brightness of about  $21.5 B\text{-mag arcsec}^{-2}$  down to very low surface brightnesses. It appears that galaxy disks are not maximal, except possibly in the largest systems. Their HI layers display warps whenever HI can be detected beyond the stellar disk, with low-level star formation going on out to large radii. Stellar disks display abundance gradients which flatten at larger radii and sometimes even reverse. The existence of a well-defined baryonic (stellar + HI) Tully-Fisher relation hints at an approximately uniform baryonic to dark matter ratio. Thick disks are common in disk galaxies and their existence appears unrelated to the presence of a bulge component; they are old, but their formation is not yet understood. Disk formation was already advanced at redshifts of  $\sim 2$ , but at that epoch disks were not yet quiescent and in full rotational equilibrium. Downsizing (the gradual reduction with time in the mass of the most actively star-forming galaxies) is now well-established. The formation and history of star formation in S0s is still not fully understood.

The final version of this chapter as it will appear in Annual Reviews of Astronomy & Astrophysics, vol. 49 (2011), will have fewer figures and a somewhat shortened text.

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# 1 INTRODUCTION AND OVERVIEW

## 1.1 Historical Background

Disks are the most prominent parts of late-type spiral galaxies. The disk of our own Milky Way Galaxy stretches as a magnificent band of light from horizon to horizon, particularly from a dark site at southern latitudes, as in the Astronomy Picture of the Day for January 27, 2009 (Pacholka, 2009). Its appearance with the Galactic Center high in the sky is reminiscent of the beautiful edge-on spiral NGC 891 (Hawaiian Starlight, 1999), which can be regarded as a close twin to our Galaxy (van der Kruit, 1984). What might in hindsight be called the first study of the distribution of stars in a galaxy disk is William Herschel’s famous cross-cut of the ‘Sidereal System’ based on his ‘star gauges’ (“*On the Construction of the Heavens*”; Herschel, 1785), from which he concluded that the distribution of stars in space is in the form of a flattened system with the sun near its center. His counts, on which he based this ‘section’ of the system, were performed along a great circle on the sky almost perpendicular to the Galactic equator, crossing it at longitudes  $45^\circ$  and  $225^\circ$  and missing the poles by  $5^\circ$ . Comparison to modern star counts shows that Herschel counted stars consistently down to magnitude  $V \sim 15$  (van der Kruit, 1986). It was the first description of the flattened nature of galactic disks.

The next major step in the study of the distribution of stars in the Milky Way was that of Jacobus Kapteyn (Kapteyn & van Rhijn, 1920; Kapteyn, 1922). In spite of his deduction that interstellar extinction must have the effect of reddening of starlight with increasing distance, Kapteyn (1909a,b, 1914) was unable to establish the existence of interstellar absorption in a convincing way and was led to ignore it. As a result he ended up with a model for what we now know to be the Galactic disk that erroneously had the Sun located near its center. The proceedings ‘*The Legacy of J.C. Kapteyn*’ (van der Kruit & van Berkel, 2000) has a number of interesting studies of Kapteyn and astronomy in his time.

Even before galaxies were established to be ‘Island universes’, spiral structure was discovered in 1845 by William Parsons, the Third Earl of Rosse. His famous drawing of M51 appears in many textbooks, popular literature and books on the history of astronomy. The importance of the disk and the development of spiral structure were the basis for the classification scheme that John Reynolds and Edwin Hubble developed; the background and early development of galaxy classification was described by Sandage (2005). Eventually the concept of Stellar Populations, first proposed by Baade (1944), led to the famous Vatican Symposium of 1957 (O’Connell, 1958), where a consistent picture was defined to interpret the presence of disk and halo populations in the context of the structure and formation of galaxies.

Not long after that, the collapse picture of Eggen, Lynden-Bell & Sandage (1962) provided a working picture for the formation and evolution of the Galaxy and by implication of galaxies in general. It was modified by Searle & Zinn (1978) to include extended evolution, whereby the outer globular clusters originated and underwent chemical evolution in separate fragments that fell into the Galaxy after the collapse of the central halo had been completed. The basic discrete two-component structure of the edge-on galaxy NGC 7814 led van der Kruit & Searle (1982b) to deduce that there are two discrete epochs of star formation, one before and the other after virialization of the spheroid and the formation of the disk.

Two related important developments in understanding the properties of galaxies and their formation were the discovery of dark matter halos, and the appreciation of the role of hierarchical assembly of galaxies. The concepts of hierarchical assembly were already around in the early 1970s, and became widely accepted at a landmark symposium on the *Evolution of Galaxies and Stellar Populations* at Yale University in 1977 (Tinsley & Larson, 1977). The discovery of dark matter halos (see below) led to the two-stage galaxy formation model of White & Rees (1978), in which hierarchical clustering of the dark matter took place under the influence of gravity, followed by collapse and cooling of the gas in the resulting potential wells. The Hubble Space Telescope

made possible the high resolution imaging of galaxies at high redshift, and showed directly that merging and hierarchical assembly are significant in the formation of massive galaxies.

The significance of internal secular evolution for the evolution of disks has become clear in recent years. The presence of oval distortions, bars and spiral structure can have a profound effect on the changing structure of disks, as has been extensively reviewed by Kormendy & Kennicutt (2004) and Kormendy (2007).

The rotation of galaxies was discovered early in the twentieth century. For a historical introduction, see van der Kruit & Allen (1978), van der Kruit (1990, Ch 10) and Sofue & Rubin (2001), documenting the first derivation of rotation velocities as a function of radius using optical absorption lines (Pease, 1914), emission lines (Babcock, 1939) and HI (van de Hulst, Raimond & van Woerden, 1957; Argyle, 1965), all in the Andromeda Nebula M31. The subject developed into the extensive mapping of the velocity fields of disk galaxies in the optical and HI, which eventually led to the discovery of flat rotation curves and the existence of dark matter (see *e.g.* reviews by van der Kruit & Allen, 1978; Faber & Gallagher, 1979; Roberts, 2008).

Quantitative surface photometry of disk galaxies to study their structure and luminosity distributions began with the work of Reynolds (1913) for the bulge of M31: for a review, see van der Kruit (1990), Ch. 5. Photometry of the much fainter disks came later, and revealed the exponential nature of the radial surface brightness distributions. This was first described in a Harvard thesis, using observations of M33 (Patterson, 1940): the data appear as fig. 10 in the review by de Vaucouleurs (1959a). He had undertaken in the late fifties a systematic survey of the light distributions in nearby spirals, particularly in M31 and M33 (de Vaucouleurs, 1958, 1959b) and established the universal ‘exponential disk’ description of the radial light distribution in galactic disks. At about the same time, Holmberg (1958) completed a survey of the diameters of 300 galaxies from micro-photometer tracings of photographic plates in two colors. This heroic effort was the culmination of work started much earlier (Holmberg, 1937).

The exponential nature of the radial surface brightness distributions of disks was discussed in detail by Freeman (1970), who noted that many of the larger spirals had a remarkably small range in the extrapolated central (face-on) surface brightness around  $21.65 B\text{-mag arcsec}^{-2}$ . This result still holds for classical spiral galaxies. The exponential surface brightness distribution of starlight in disks was complemented by the observations of the vertical light distribution in edge-on spirals. The distribution could be approximated very well by an isothermal sheet (Camm, 1950, but for practical purposes an exponential can be used as well) with a scaleheight that - surprisingly - is to an excellent approximation independent of galactocentric radius (van der Kruit & Searle, 1981a,b, 1982a).

Freeman (1970) also noted that for a self-gravitating exponential disk the expected rotation curve peaks at 2.2 scalelengths and then declines. A decline at 2.2 scalelengths was however not observed in the rotation data for NGC 300 and M33 at the time. This 1970 paper appears to be the first indication from rotation curve analysis that the rotation curve is not determined by the mass distribution in the disk alone, but requires a contribution to the amplitude of the rotation curve from an extended distribution of invisible matter. Subsequent observations of rotation curves eventually led to the concept of dark halos in individual galaxies (*e.g.* Faber & Gallagher, 1979; Roberts, 2008).

An important concept in the analysis of rotation curves is that of ‘maximum disk’, introduced by Carignan & Freeman (1985) and van Albada et al. (1985). In this concept, because the  $M/L$  ratio of the disk is unknown, the contribution of the disk mass to the rotation curve is taken to be as large as permitted by the observed rotation curve. This means in practice (see below) that the amplitude of the rotation curve from the disk itself is about 85% of the observed amplitude (Sackett, 1997). In principle an independent measurement of the disk mass distribution can be obtained from hydrostatic considerations, comparing the thickness and velocity dispersion of the stars, as was pioneered for the Galaxy by Kapteyn (1922) and Oort (1932), or the HI gas (van

der Kruit, 1981). Sanders & McGaugh (2002) have reviewed Modified Newtonian Dynamics as an alternative to dark matter.

Within disks, the star formation history was studied first in our Galaxy in the solar neighborhood. The stellar initial mass function IMF (the statistical distribution of stellar masses during star formation) was derived first by Salpeter (1959). Schmidt (1959) defined the ‘Schmidt-law’ for the rate of star formation as a function of the density of the ISM, in which the rate of star formation is proportional to the square of the local gas density; see reviews by Kroupa (2002a) and Kennicutt (1998) for subsequent refinements. Studies of the chemical evolution in the local disk identified the ‘G-dwarf problem’ in the ‘Simple Model’ of chemical evolution (Schmidt, 1963). In this simple model the chemical evolution is followed in a galactic disk starting as pure gas with zero metallicity and without subsequent inflow or outflow; then the result is a much higher fraction of low-metallicity, long-lived stars as G-dwarfs than is observed in the solar neighborhood. This can be rectified by extensions of the model; see e.g. the review by Tinsley (1980). The basic models for chemical evolution were able to represent the radial gradients in metal abundance in the gas of disk galaxies (Searle, 1973) in terms of the extent to which star formation and chemical enrichment has proceeded (*e.g.* Garnett & Shields, 1987, for M81). The mean metal abundance of stars that formed over the lifetime of a disk approaches that of the abundance of the gas at the time of disk formation plus an ‘effective yield’ (the net production of heavy elements, modified by effects of zero-metal inflow or enriched gas outflow). In this ‘simple model with bells and whistles’ (Mould, 1984) it follows that, while gas consumption is still proceeding, the abundance gradients in the stars will be in principle shallower than that in the gas.

Reviews of Stellar Populations include those by King (1971), Sandage (1986), Bahcall (1986), Freeman (1987) and Gilmore, Wyse & Kuijken (1989). An IAU Symposium in 1994 on the subject of Stellar Populations (van der Kruit & Gilmore, 1995) includes a historical session. For a recent review on the structure and evolution of the Galaxy, see Freeman & Bland-Hawthorn (2002).

The integral properties of galaxies and the systematics of their distribution have been used as a tool towards understanding galaxy formation and the origin of the variety among them. Chief among these relations are those between the morphological type and properties of their stellar and gas content, such as HI content and integrated color (Roberts & Hayes, 1994). These latter properties were convincingly interpreted as a measure of the process of depletion of the interstellar gas in star formation and the rate of current star formation relative to that averaged over a galaxy’s lifetime (Searle & Sargent, 1972; Searle, Sargent & Bagnuolo, 1973; Larson & Tinsley, 1978), which then correlate with galaxy type. The Tully-Fisher relation (Tully & Fisher, 1977) provides a tight correlation of rotation velocity and integrated luminosity, although it still is not clear why it is so tight when the rotation velocity is determined not only by the mass in the stars that provide the luminosity but also by the dark matter halo.

We should mention here the discovery of low surface brightness galaxies, which was anticipated by the work of Disney (1976). Disney & Phillipps (1983) showed that the observed range in central surface brightness of galaxy disks (and also of elliptical galaxies) is severely restricted by the necessity for them to stand out against the background sky; the exponential nature of disks naturally restricted samples to a small range in central surface brightness comparable to the value first noted by Freeman (1970). This selection effect had been described earlier in qualitative and more general terms by Arp (1965). Many low surface brightness galaxies are known today, although it appears that the bright limit of the surface brightnesses seen by Freeman (1970) is not an effect of observational selection (Allen & Shu, 1979; Bosma & Freeman, 1993)

## 1.2 Setting the Scene

This brief description of the historical development of our subject already indicates that a comprehensive treatment of all aspects of galaxy disks is beyond the scope of a single chapter in Annual Reviews. Topics that we will not review in detail include radio continuum studies and magnetic fields (van der Kruit & Allen, 1976; Condon, 1992; Beck, 2008), AGNs and black holes in the centers of galaxies (Kormendy & Richstone, 1995; Ferrarini & Ford, 2005; Pastorini et al., 2007), spiral structure (Toomre, 1977), bars (Sellwood, 2010b) and secular evolution (Kormendy & Kennicutt, 2004). Also we will not review issues related to physical or chemical processes in the ISM. We refer the reader to the proceedings of some recent symposia that concentrate on disks in galaxies, including ‘*The dynamics, structure and history of galaxies*’ (Da Costa & Jerjen, 2002), ‘*Island Universes: Structure and evolution of disk galaxies*’ (de Jong, 2007), ‘*Formation and Evolution of Galaxy Disks*’ (Funes & Corsini, 2008), ‘*Unveiling the mass: extracting and interpreting galaxy masses*’ (Courteau & de Jong, 2009)<sup>1</sup> and ‘*Galaxies and their Masks*’ (Block, Freeman & Puerari, 2010).

Despite the correlations between overall properties, there are galaxies with very similar properties but very different morphologies. M33 and the Large Magellanic Cloud provide an example. Both galaxies have very similar central surface brightness ( $\sim 21.2$   $B$ -mag arcsec<sup>-2</sup>), scalelength ( $\sim 1.6$  kpc), integrated magnitude ( $\sim -18.5$  in  $B$ ),  $(B - V)$  color ( $\sim 0.51$ ), IRAS luminosity ( $\sim 1.0 \times 10^8 L_\odot$ ), HI mass ( $\sim 9.5 \times 10^8 M_\odot$ ) and rotation velocity ( $\sim 90 - 100$  km s<sup>-1</sup>) (see van der Kruit, 1990, Ch. 10). Our point is that these two systems differ significantly only in morphological classification and nothing else. The detailed structure of a galaxy, its morphology and spiral structure, may be determined by external properties such as environment or may even be transient, so that during the lifetime of the systems there might have been periods when M33 looked very much like the LMC and *vice versa*.

In the final section, we will discuss the origin of S0 galaxies. Originally introduced by Hubble as a transition class between elliptical and spirals, they were believed to be systems that had quickly used all of their remaining gas. The Sa systems had sufficient gas left after completion of disk formation to support star formation at a low level up to the present, while later types had more gas left and were able to form stars more vigorously up till now (Sandage, Freeman & Stokes, 1970). Alternative theories were suggested, involving the stripping of gas from existing spirals by collisions (Spitzer & Baade, 1951) or intergalactic gas (Gunn & Gott, 1972). Assuming that it is unlikely that we are living at the time just when within a few Gyr all spirals will ‘run out of gas’, Larson, Tinsley & Caldwell (1980) argued that gas must be replenished in normal spirals but not in S0’s.

## 2 SURVEYS

Surveys provide the basis of much of the observational studies of disk galaxies. In the past, major surveys were very time consuming. For example, the *Hubble Atlas of Galaxies* (Sandage, 1961) which provided the basic source list for much of the past work on nearby galaxies, was the culmination of decades of photography of galaxies by Hubble, Sandage and others in order to survey the variety of morphologies among galaxies. For many years, the Humason, Mayall & Sandage (1956) survey was a main source of galaxy redshifts and magnitudes; it was the result of 20 years of observations at Mount Wilson, Palomar and Lick, and was only surpassed decades after its publication. The advent of dedicated, automated survey telescopes, multi-object spectrographs, and high-resolution imaging and spectroscopic space facilities has transformed our ability to make surveys of galaxies. In this section, we will give a brief overview of surveys,

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<sup>1</sup>For this symposium on the occasion of Vera Rubin’s 80th birthday, there will be no printed proceedings; electronic versions of presentations or posters are available through the conference website.

currently or recently undertaken, that are relevant to studies of disks in galaxies as discussed in later sections of this review.

Kinematic surveys aimed at the dynamics of (stellar) disks (see § 3) using integral field spectrographs include DiskMass (Bershady et al., 2010a) (146 nearly face-on galaxies for which H $\alpha$  velocity fields have already been measured, and a subset of 46 galaxies with stellar velocities and velocity dispersions) and PINGS<sup>2</sup> (Rosales-Ortega et al., 2010), which will provide 2-dimensional spectroscopy in 17 nearby galaxies. For these surveys, the data are supplemented by extensive observations at other wavelengths.

Surveys specifically designed to gather detailed information on the properties of disks in galaxies usually involve samples of nearby galaxies that are not statistically complete but are designed to cover the range of morphological types. HI surveys of individual galaxies are often complemented by optical or near-IR surface photometry to aid the analysis of their rotation curves (see § 4). A first such survey of spiral galaxies, combining imaging (three-color photographic surface photometry) at optical wavelengths and mapping of distributions and kinematics of HI, was made by Wevers, van der Kruit & Allen (1986). This *Palomar-Westerbork Survey of northern spiral galaxies* included only 16 galaxies, but required 64 observing periods of 12 hours with the Westerbork Synthesis Radio Telescope (WSRT) and 42 dark nights at the Palomar 48-inch Schmidt. This was extended substantially in the WHISP survey<sup>3</sup> (van der Hulst, 2002; Noordermeer et al., 2005) of a sample of a few hundred galaxies. THINGS<sup>4</sup> (Walter et al., 2008) is the most detailed recent uniform set of high-resolution and high-sensitivity data on 34 nearby disk galaxies available at this time; data were taken with the Very Large Array VLA. A special section, devoted to THINGS, appeared in the December 2008 issue of the *Astronomical Journal*. Another major survey of nearby galaxies is SINGS (*Spitzer Infrared Nearby Galaxies Survey*; sings.stsci.edu Kennicutt et al., 2003). This is a comprehensive imaging and spectroscopic study of 75 nearby galaxies in the infrared.

Other surveys provide large samples of galaxy data of various kinds, in different wavelength regions. Images of galaxies in two UV bands from the *Galaxy Evolution Explorer* GALEX (Martin et al., 2005) survey are particularly useful for estimating the recent star formation history of galaxies. In the optical B-band, the *Millennium Galaxy Catalogue*<sup>5</sup> comes from a 37.5 deg<sup>2</sup> medium-deep imaging survey of galaxies in the range  $13 < B < 24$ , connecting the local and distant universe. The 6dF and 2dF Galaxy Redshift Surveys<sup>6</sup> and the SDSS<sup>7</sup> (York et al., 2000) provide vast samples of optical galaxy redshifts and spectroscopic properties related to their star formation history (see § 5). The 2MASS<sup>8</sup> (Skrutskie et al., 2006) gives integrated near-IR photometry for a very large sample of galaxies, and also relatively shallow near-IR images for the brighter galaxies. High-resolution deep imaging in the near and mid-infrared over a wide redshift range is provided by the *Spitzer Space Telescope* mission : see Soifer, Helou & Werner (2008) for a recent summary of extragalactic studies. Large HI surveys are of interest for studies of the HI mass function in the universe, and also for scaling laws (see § 6). For example, the HIPASS survey<sup>9</sup> gives integrated HI data for galaxies south of declination +25° out to velocities of 12,700 km s<sup>-1</sup>.

Two major surveys are using HST to study resolved stellar populations in nearby galaxies. ANGST<sup>10</sup> (Dalcanton et al., 2009) establishes a legacy of uniform multi-color photometry of

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<sup>2</sup> PPAK IFS Nearby Galaxies Survey; [www.ast.cam.ac.uk/research/pings/html/](http://www.ast.cam.ac.uk/research/pings/html/).

<sup>3</sup> Westerbork observations of neutral Hydrogen in Irregular and SPiral galaxies; [www.astro.rug.nl/~whisp](http://www.astro.rug.nl/~whisp).

<sup>4</sup> The HI Nearby Galaxy Survey; [www.mpia.de/THINGS/](http://www.mpia.de/THINGS/).

<sup>5</sup> see [www.eso.org/~jliske/mgc](http://www.eso.org/~jliske/mgc).

<sup>6</sup> [www.aao.gov.au/local/www/6df](http://www.aao.gov.au/local/www/6df) and [msowww.anu.edu.au/2dFGRS](http://msowww.anu.edu.au/2dFGRS).

<sup>7</sup> Sloan Digital Sky Survey; [www.sdss.org](http://www.sdss.org).

<sup>8</sup> Two Micron All Sky Survey; [www.ipac.caltech.edu/2mass](http://www.ipac.caltech.edu/2mass).

<sup>9</sup> HI Parkes All-Sky Survey; [www.atnf.csiro.au/research/multibeam/release](http://www.atnf.csiro.au/research/multibeam/release)

<sup>10</sup> ACS Nearby Galaxy Survey Treasury; [www.nearbygalaxies.org](http://www.nearbygalaxies.org).

resolved stars for a volume-limited sample of nearby galaxies. GHOSTS<sup>11</sup> (de Jong et al, 2007a) is imaging several edge-on galaxies with a range in masses to study their stellar populations. These population studies are important for understanding the star formation history in galaxies (see § 5 and § 7). For more nearby population studies, SEGUE<sup>12</sup> and RAVE<sup>13</sup> focus on kinematic and chemical surveys of very large samples of stars in the Galactic disk and halo.

For studies of disk galaxies at high redshift, the Hubble (Ultra-)Deep Fields (Williams et al., 1996, 2000; Beckwith et al., 2006) and the GOODS and COSMOS surveys have been very influential. The GOODS<sup>14</sup> (Dickinson, Giavalisco & the GOODS Team, 2003) survey involves two fields centered on the Hubble Deep Field North and the Chandra Deep Field South and combines deep observations from NASA’s Great Observatories, Spitzer, Hubble, and Chandra, ESA’s Herschel and XMM-Newton, and from the most powerful ground-based facilities such as Keck, VLT, Gemini and Subaru. The *Cosmological Evolution Survey* COSMOS (cosmos.astro.caltech.edu) covers a two square degree equatorial field with a similar range of facilities, aimed at probing the formation and evolution of galaxies with cosmic time (see § 8).

Astronomy profits enormously from new facilities, and this is equally true for our subject of disk galaxies. For the future, we look forward to new insights from major facilities. In the submillimeter and radio, Herschel and the Atacama Large (sub-)Millimeter Array ALMA will revolutionize studies of star formation and the interstellar medium in disk galaxies. The LOw Frequency ARray LOFAR, its southern MWA counterpart, the MeerKAT and ASKAP pathfinder arrays and ultimately the SKA itself will have a profound impact on studies of the formation of galaxies and the structure of disk galaxies. Current deep HST surveys (see for example [candels.ucolick.org](http://candels.ucolick.org)) and surveys to come with the James Webb Space Telescope JWST, will bring new insights into the properties of disk galaxies and their assembly. For studies of the structure and evolution of the Milky Way, the Gaia mission will give astrometric data of unparalleled precision. Combined with panoramic surveys like those planned with Pan-Starrs, SkyMapper and LSST, it will help us to understand the structure and genesis of the different components of our disk galaxy.

## 3 STELLAR DISKS

### 3.1 Luminosity Distributions

#### 3.1.1 Exponential disks

The structure and general properties of stellar disks have previously been reviewed by us (*e.g.* van der Kruit, 2002; Freeman, 2007). As mentioned in the introduction, the radial distribution of surface brightness in the disks of face-on or moderately inclined galaxies can be approximated by an exponential:  $I(R) \propto \exp(-R/h)$ . Before we discuss the three-dimensional distribution we first review work on the exponential disks in such galaxies. Fits to actual surface photometry result in two parameters, the radial scalelength  $h$  and the (extrapolated and corrected to face-on) central surface brightness  $\mu_0$ , both as a function of photometric band. The determination of these parameters can in general be done in a reasonably reliable way from component separations (Kormendy, 1977); Schombert & Bothun (1987) and Byun & Freeman (1995) showed from realistic simulations that one-dimensional and two-dimensional bulge-disk separations do return input values for bulge and disk parameters very well. Nevertheless, independent determinations of scalelengths of the same galaxies in the literature give results that differ with a standard deviation of 20% (Knapen & van der Kruit, 1991). In his CCD study of exponential disks in

<sup>11</sup> *Galaxy Halos, Outer disks, Substructure, Thick disks and Star clusters*; [www-int.stsci.edu/~djrs/ghosts](http://www-int.stsci.edu/~djrs/ghosts)

<sup>12</sup> *Sloan Extension for Galactic Understanding and Exploration*; [www.sdss.org/segue/](http://www.sdss.org/segue/).

<sup>13</sup> *RArial Velocity Experiment*; [www.rave-survey.aip.de/rave/](http://www.rave-survey.aip.de/rave/).

<sup>14</sup> *Great Observatories Origin Deep Survey*; [www.stsci.edu/science/goods](http://www.stsci.edu/science/goods).



a sample of bright UGC galaxies, Courteau (1996) also stresses the pitfalls, and cautions that comparison of central surface brightnesses and scalelengths is complicated by the subjective nature of their measurement. We note that older fits adopt the  $R^{1/4}$  law  $I(R) \propto \exp(R^{-1/4})$  for the bulge, whereas most authors now use the Sérsic (1963)-profiles  $I(R) \propto \exp(R^{-1/n})$ , with the Sérsic index  $n = 1$  for the exponential disk and  $n = 4$  for the  $R^{1/4}$ -law. In the context of two-component decompositions of radial surface brightness distributions, we note that the flat pseudo-bulge structures discussed by Kormendy & Kennicutt (2004) have values for  $n$  of about 2.5.

The original publication on exponential disks of Freeman (1970) used observations in the  $B$ -band. In that paper the distribution of the two parameters was discussed, finding an apparent constancy of  $\mu_o$  for about 75% of the sample and that disk galaxies have scalelengths with a wide range of values (predominantly small in later-type galaxies). We already noted in § 1.1 that the apparent constancy of central surface brightness is seriously affected by observational selection (Arp, 1965; Disney, 1976; Disney & Phillipps, 1983), leading to the conclusion that there must be many lower surface brightness galaxies. However, the upper limit is believed to be real (Allen & Shu, 1979; Bosma & Freeman, 1993; de Jong, 1996b).

With the advent of large datasets of surface photometry (such as from the SDSS), it has become possible now to study large samples of galaxies. For example, Gadotti (2009) has collected  $g$ ,  $r$  and  $i$ -band images of a representative sample of nearly 1000 galaxies from the SDSS and decomposed them into bulges, bars and disks. Pohlen & Trujillo (2006) and Pohlen et al. (2007) have used SDSS data to determine radial luminosity distributions and look for radial truncations (see § 3.8). Fathi et al. (2010) determined scalelengths, using an automatic technique, for over 30,000 galaxies in five wavelength bands, together with indices for asymmetry and concentration. Comparison with the overlap with the sample of Gadotti (2009) shows in general terms good agreement (see fig. 1 of Fathi, 2010, which concerns the same sample). Fathi et al. (2010) form sub-samples for which reliable morphological types or central velocity dispersions are available. As before, the average scalelength ( $3.8 \pm 2.0$  kpc) is independent of morphological type and is very similar in the optical bands ( $g$ ,  $r$ ,  $i$  and  $z$ ). In the  $u$ -band, they find a mean scalelength of  $5 \pm 3$  kpc. Galaxies of smaller mass ( $10^9$  to  $10^{10} M_\odot$ ) have smaller scalelengths ( $1.5 \pm 0.7$  kpc) than larger mass ( $10^{11}$  to  $10^{12} M_\odot$ ) galaxies ( $5.7 \pm 1.9$  kpc). The distributions in this study have *not* been corrected for sample selection.

It is possible to study the bi-variate distribution function of the disk parameters. It is most important for such studies that the sample is complete with respect to well-defined selection criteria and that the distribution in the  $(\mu_o, h)$  plane is corrected for the effect of these selection criteria (following the prescriptions of Disney & Phillipps, 1983). This was first done in van der Kruit (1987), later at various optical and near-infrared colors by de Jong & van der Kruit (1994) and de Jong (1996a,b,c) and more recently by Fathi (2010) using the large SDSS Fathi et al. (2010) sample. The study of the distribution of parameters in this plane reveals important results that bear on the formation models of disks.

The distribution in the  $(\mu_o, \log h)$  diagram (Fig. 1) shows a broad band running from bright, large disks to faint, small disks (van der Kruit, 1987; de Jong, 1996b; Graham & de Blok, 2001; Fathi, 2010). Graham & de Blok (2001) find that there is a morphological type dependence in this plane: among low surface brightness galaxies (central surface brightness more than 1 mag fainter than the 21.65  $B$ -mag arcsec $^{-2}$  mean value of Freeman, 1970), the early-type spiral galaxies have large scalelengths (larger than 8-9 kpc), while the late-type spirals have smaller scalelengths. Further, de Jong (1996b) finds that the scale parameters of disks and bulges are correlated at all morphological types, but are not correlated themselves with Hubble type. On the other hand, low surface brightness galaxies are usually of late Hubble type. He also concludes that the bulge-to-disk ratio is not correlated with Hubble type, nor is the disk central surface brightness. The significant parameter that does correlate with morphological type is the effective

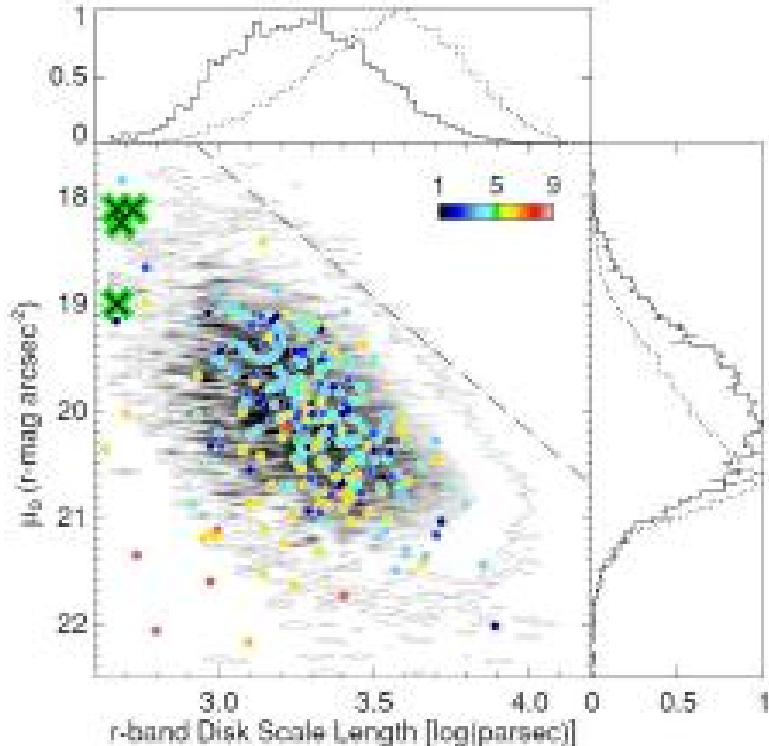


Figure 1: The bivariate distribution function of face-on central surface brightness and scale-length of galaxy disks in a sample of almost 30,000 galaxies taken from SDSS, corrected for selection effects. Superposed are the 282 most reliable data as colored points (coding for revised Hubble type) and a few diskly ellipticals as green crosses. The dashed line shows a slope of 2.5 corresponding to a constant disk luminosity. The distributions on the right and top are as observed (dotted) and after correcting for sample selection (solid). (From Fathi, 2010).

surface brightness of the bulge. Color information shows that, within and among galaxies, low surface brightness corresponds to bluer colors (de Jong, 1996c). This results from the combined effect of mean stellar age and metallicity and not from dust reddening and implies significant mass-to-light ratio variations. Fig. 2 shows the modern version of fig. 5 of Freeman (1970), corrected for volume selection effects. There is still a mean value (but with a large scatter around it), that does not depend on morphological type, except for the later ones.

What properties of galaxies do correlate with  $h$  and  $\mu_0$ ? Courteau et al. (2007) collected surface photometry of 1300 galaxies and determined the photometric parameters from either one-dimensional bulge-disk decompositions of the surface brightness profile or using the so-called ‘marking-the-disk’ method, where the extent of the exponential disk profile is judged by eye. They also find some variation of central surface brightness or scalelength as a function of morphological type, with earlier types having fainter surface brightness and larger scalelengths, but the effects are marginal. In addition, they find well-defined relations between luminosity, scalelength and rotation velocity, but the slopes show a definite dependence on morphological type and a small but significant dependence on the wavelength band. The scalelengths in the  $I$ -band correlate with integrated luminosity and rotation velocity (see eqn. (25) and fig. 20 below). In summary, although  $h$  has no strong dependence on morphological type, it is clearly larger in the mean for more luminous and more massive galaxies.

There is some argument concerning the scalelength of the disk of our own Galaxy. If  $V_{\text{rot}} = 220 \text{ km s}^{-1}$ , then the expected scalelength of the Galactic disk would be 4.4 kpc with a one-

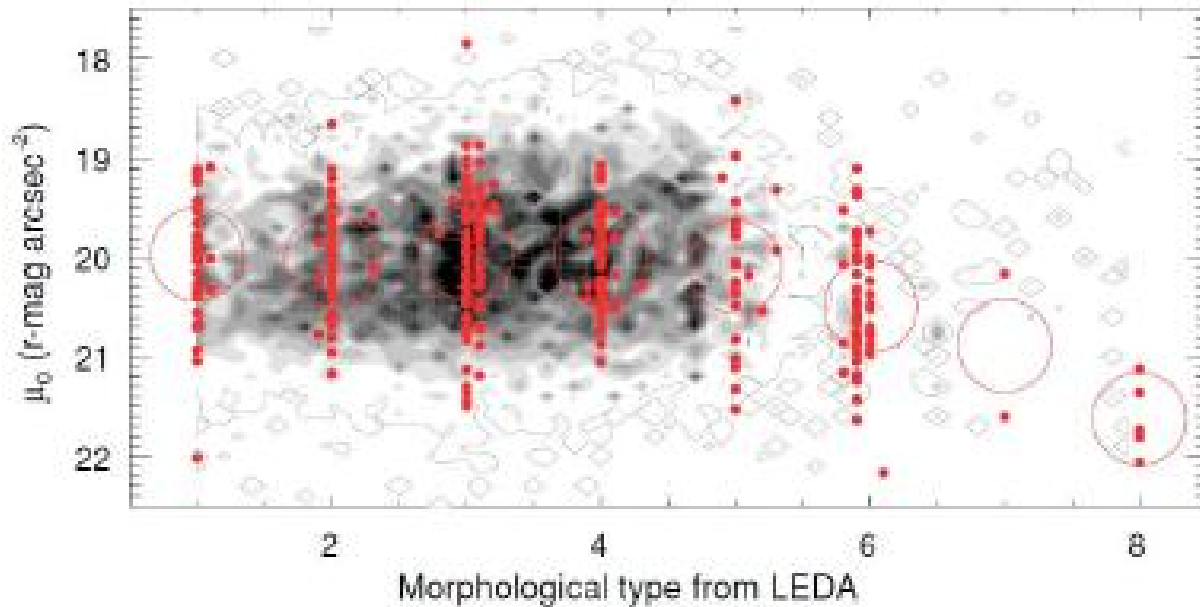


Figure 2: The distribution of face-on central surface brightness for the same sample as in fig. 1 as a function of Hubble type, corrected for selection and again with the 282 most reliable determinations as points. The open circles show the average surface brightness for each type. (From Fathi, 2010).

sigma range between 3.6 and 6.6 kpc (see slide 19 in van der Kruit, 2009). These values would be larger if  $V_{\text{rot}} = 250 \text{ km s}^{-1}$  (Reid et al., 2009). The often quoted values of 2.5 to 3.0 kpc (Sackett, 1997; Freudreich, 1998; Hammer et al., 2007; Yin et al., 2009; Reylé et al., 2009) put the Galaxy outside the one-sigma range of scalelengths for its rotation speed. A value more like 4.5 kpc (van der Kruit, 2008, 2009) (or even the probably too large  $5.5 \pm 1.0$  kpc from the Pioneer 10 photometry alone; van der Kruit, 1986) would be more typical for our Galaxy. On the other hand, Hammer et al. (2007) argue that our Galaxy is exceptional in many aspects: however their adopted low value for the scalelength of the disk is a major contributor to this conclusion.

The origin of the exponential nature of stellar disks is still uncertain. Freeman (1970, 1975) already pointed out that the distribution of angular momentum in a self-gravitating exponential disk resembles that of the uniform, uniformly rotating sphere (Mestel, 1963). This is also true for an exponential density distribution with a flat rotation curve (Gunn, 1982; van der Kruit, 1987). A model in which the disk collapses with detailed conservation of angular momentum (Fall & Efstathiou, 1980) would give a natural explanation for the exponential nature of disks and maybe even their truncations (see below). However, bars or other non-axisymmetric structures may give rise to severe redistribution of angular momentum; nonaxisymmetric instabilities and the secular evolution of disks and their structural parameters may be important (Debattista et al., 2007).

Before leaving the subject of luminosity distributions, we will briefly address the issue of low surface brightness (LSB) disks. Often these have central (face-on) surface brightnesses that are 2 magnitudes or more fainter than the canonical  $21.65 \text{ B-mag arcsec}^{-2}$  of Freeman (1970). Traditionally these are thought to be galaxies with low (gas) surface densities, in which the star formation proceeded slowly. Analysis of available data (HI rotation curves, colors and stellar velocity dispersions) led de Blok & McGaugh (1997) to argue that LSB galaxies are not described well by models with maximum disks (see below). LSB galaxies appear to be slowly evolving,

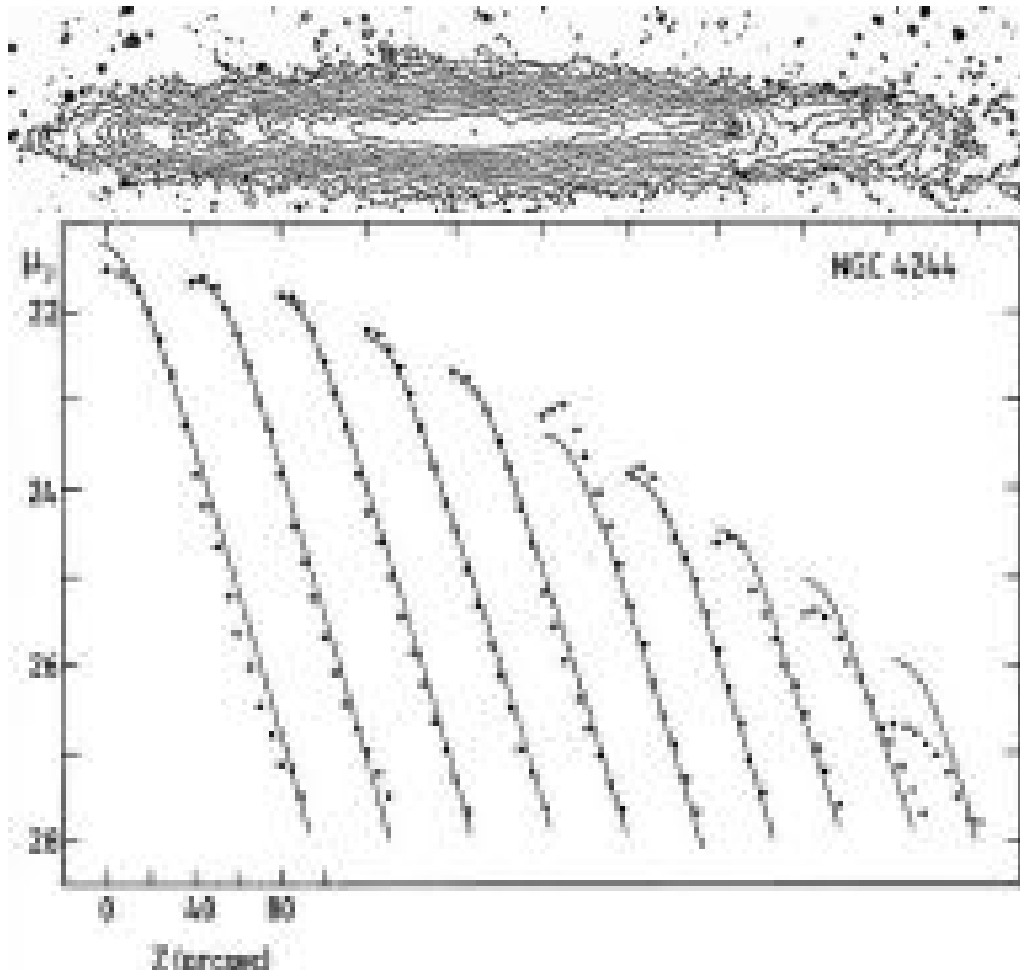


Figure 3: The surface brightness distribution in the edge-on, pure disk galaxy NGC 4244. At the top the isophotes in blue light at  $0.5 \text{ mag arcsec}^{-2}$  intervals. At the bottom vertical  $z$ -profiles at a range of distances from the center after averaging over the four equivalent quadrants. The curves are those for an isothermal sheet with  $n=1$  in eqn. (1). (After van der Kruit & Searle, 1981a)

low density, dark matter dominated systems. The star formation in low surface brightness disks can now be studied with GALEX by directly mapping their near-UV flux. Wyder et al. (2009) combined such data with existing HI observations and optical images from the SDSS for 19 systems. Comparison with far IR-data from Spitzer shows that there is very little extinction in the UV, consistent with the fact that LSB galaxies appear to have little dust and molecular gas (see *e.g.* de Blok & van der Hulst, 1998a,b). The star formation rate in LSB galaxies lies below the extrapolated rate as a function of gas surface density for high surface brightness galaxies, implying a lower mean star formation efficiency in LSB systems. This may be related to the lower density of molecular gas.

### 3.1.2 Three-dimensional distributions

We now turn to the three-dimensional distribution. The vertical distribution of luminosity within a galactic disk can be modelled to a first approximation with an isothermal sheet (Camm, 1950) with a scaleheight that is independent of galactocentric distance (van der Kruit & Searle, 1981a).

This is a surprising observational result. We will discuss its possible origin in § 3.2.3. In a more general form the luminosity density distribution can be written as (van der Kruit, 1988)

$$L(R, z) = L(0, 0) e^{-R/h} \operatorname{sech}^{2/n} \left( \frac{nz}{2h_z} \right), \quad (1)$$

This ranges from the isothermal distribution ( $n = 1$ :  $L(z) \propto \operatorname{sech}^2(z/z_o)$  with  $z_o = 2h_z$ ) to the exponential function ( $n = \infty$ :  $L(z) \propto \exp(-z/h_z)$ ), as was used by Wainscoat, Hyland & Freeman (1989, 1990), and allows for the more realistic case that the stellar distribution is not completely isothermal in the vertical direction. The uncertainty resulting from what the detailed vertical distribution of stellar mass really is in a disk can be estimated by taking a realistic range in the parameter  $n$ , as we have done for example in eqn. (7) below. Fig. 3 shows the fits of this distribution projected in edge-on orientation to the surface brightness distribution in the pure-disk, edge-on galaxy NGC 4244, for the case of the isothermal sheet ( $n=1$ ). The outer profile does not fit, because the truncation (see § 3.8) has not been taken into account. From actual fits in  $I$  and  $K'$  de Grijs, Peletier & van der Kruit (1997) found

$$\frac{2}{n} = 0.54 \pm 0.20 \quad (2)$$

for a sample of edge-on galaxies. A detailed study by de Grijs & Peletier (1997) has shown that the constancy of the vertical scaleheight  $h_z$  is accurate in disks of late-type spirals, but in early type galaxies  $h_z$  may increase outward by as much as 50% per scalelength  $h$ .

The distribution of the scale parameters is most easily studied in edge-on galaxies. Following the work by van der Kruit & Searle (1981a,b, 1982a), an extensive sample of edge-on galaxies was studied by de Grijs (1998) and re-analysed by Kregel, van der Kruit & de Grijs (2002). The results on the distribution of scale parameters can be summarized as follows. Both scalelength  $h$  and scaleheight  $h_z$  correlate well with the rotation velocity of the galaxy: *e.g.* for the scaleheight

$$h_z = (0.45 \pm 0.05) (V_{\text{rot}}/100 \text{ km s}^{-1}) - (0.14 \pm 0.07) \text{ kpc} \quad (3)$$

with a scatter of 0.21 kpc. This relation is important as it can be used to make a statistical estimate of the thickness of disks in galaxies that are not seen edge-on. The correlation between  $h$  and  $V_{\text{rot}}$  is comparable to that found by Courteau et al. (2007). The flattest galaxies (largest ratio of  $h$  and  $h_z$ ) appear to be those with late Hubble type, small rotation velocity and faint (face-on) surface brightness. Among galaxies with large HI content, a large range of flattening is observed, becoming smaller with lower HI mass. The flattest disks occur among galaxies with about  $10^{10} M_{\odot}$  in HI. We will return to this subject in § 3.6 when we discuss ‘super-thin’ galaxies.

## 3.2 Stellar Kinematics, Stability and Mass

### 3.2.1 Vertical dynamics

We will first turn to the dynamics of stellar disks in the vertical ( $z$ ) direction. At the basis of the analysis of the vertical dynamics of a stellar disk we have the Poisson equation for the case of axial symmetry

$$\frac{\partial K_R}{\partial R} + \frac{K_R}{R} + \frac{\partial K_z}{\partial z} = -4\pi G\rho(R, z), \quad (4)$$

where  $K_R$  and  $K_z$  are the gravitational force components. At small  $z$ , the first two terms on the left are equal to  $2(A - B)(A + B)$  (*e.g.* Oort, 1965; Freeman, 1975) and this is zero for a flat rotation curve.<sup>15</sup> So we have

$$\frac{dK_z}{dz} = -4\pi G\rho(z). \quad (5)$$

<sup>15</sup>The Oort constants are  $A = \frac{1}{2}\{V_{\text{rot}}/R - (dV_{\text{rot}}/dR)\}$  and  $B = -\frac{1}{2}\{V_{\text{rot}}/R + (dV_{\text{rot}}/dR)\}$ , so  $A + B = -(dV_{\text{rot}}/dR)$ .

This is the plane-parallel case and flat rotation curves do make this an excellent approximation at low  $z$  (van der Kruit & Freeman, 1986). The Jeans equation then becomes

$$\frac{d}{dz} [\rho(z)\sigma_z^2(z)] = \rho(z)K_z. \quad (6)$$

Combining these gives (*e.g.* van der Kruit, 1988)

$$\sigma_z(R) = \sqrt{c\pi G\Sigma(R)h_z}, \quad (7)$$

where the velocity dispersion  $\sigma_z$  is now the velocity dispersion integrated over all  $z$  (corresponding to the second moment of the distribution observed when the disk is seen face-on) and the constant  $c$  varies between 3/2 for an exponential [ $n = \infty$  in eqn. (1)] to 2 for an isothermal distribution ( $n = 1$ ). Eqn. (7) is the equation for hydrostatic equilibrium that relates the vertical distribution of the stars and their mean vertical velocity dispersion to the distribution of mass; this principle was used already by Kapteyn (1922) and Oort (1932) to derive the mass density in the solar neighborhood. If the mass-to-light ratio  $M/L$  is constant with radius, the exponential radial distribution and the constant scaleheight imply through hydrostatic equilibrium that the vertical velocity dispersion  $\sigma_z(R)$  of the old stars in the disk should be proportional to the square-root of the surface density  $\Sigma$  or as an exponential with galactocentric radius, but with an e-folding of twice the scalelength.

The mass-to-light ratio  $M/L$  is a crucial measure of the contribution of the disk to the rotation curve and the relative importance of disk mass and dark matter halo in a galaxy. An often used hypothesis is that of the ‘maximum disk’ (see also § 4.2), in which the disk contribution to a galaxy’s rotation curve is maximized in the sense that the amplitude of the disk-alone rotation curve is made as large as the observations allow. Using hydrostatic equilibrium, we may estimate  $M/L$  and obtain information on whether or not disks are maximal or sub-maximal. This can in principle be done from eqn. (7) by measuring the velocity dispersion in a face-on galaxy and using a statistical estimate of the scaleheight.

The measurement of stellar velocity dispersions in disks, which has to be done using stellar absorption lines in the optical or near-IR, is seriously hampered by the low surface brightness. In 1984 van der Kruit & Freeman (1984) made the first successful measurements of stellar velocity dispersion in the face-on spirals NGC 628 and 1566.<sup>16</sup> This work was followed by more detailed observations by van der Kruit & Freeman (1986) for NGC 5247 (inclination about 20°), where the prediction was verified: the e-folding length of  $\sigma_z$  was  $2.4 \pm 0.6$  photometric scalelengths, the predicted value of 2.0 being well within the uncertainty. Many studies have since shown that  $\sigma_z$  decreases with galactocentric radius (*e.g.* Bottema, 1993; Kregel, van der Kruit & Freeman, 2004, 2005; Kregel & van der Kruit, 2005, and references therein). Gerssen, Kuijken & Merrifield (1997) found in NGC 488 that the kinematic gradient was comparable to the photometric gradient, which they attributed to the fact that the scalelength should really be measured in  $K$ -band to represent the stellar distribution. The same authors found in NGC 2985, that these scalelengths were indeed as expected from a constant  $M/L$ . There is certainly support from stellar dynamics that in general there are no substantial gradients in mass-to-light ratios in disks. We will come back to this below in the context of photometric models and stellar composition and ages in disks.

Two recent developments are making an impact on this issue. The first is the use of integral field units that enable a more complete sampling of the disks. The DiskMass Project (Verheijen et al., 2007; Westfall et al., 2008; Bershady et al., 2010a,b) aims at mapping the stellar vertical velocity dispersion in 46 face-on or moderately inclined spiral galaxies. This will provide a

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<sup>16</sup>At the same time and independently, Kormendy (1984a,b) succeeded in measuring stellar velocity dispersions in the disks of two S0 galaxies with more or less the same aim; we will discuss this in § 9.

kinematic measurement of the mass surface density of stellar disks. The final results have not yet appeared in the literature, but recent conference presentations show that the ‘kinematics follows the light’, i.e. the velocity dispersions drop off according to the rule described above. Also the actual values indicate relatively low mass-to-light ratios and disk masses that are well below those required for maximum disk fits.

Similarly, the use of planetary nebulae (PNe) as test particles in the disks (Herrmann et al., 2008; Herrmann & Ciardullo, 2009a,b) of five face-on spirals method allows the velocity dispersion of these representative stars of the old disk population to be measured out to much larger radii (see also § 9). In general the findings are similar: except for one system, the  $M/L$  is constant out to about three radial scalelengths of the exponential disks. Outside that radius, the velocity dispersion stops declining and becomes flat with radius. Possible explanations proposed for this behavior include an increase in the disk mass-to-light ratio, an increase in the importance of the thick disk, and heating of the thin disk by halo substructure. They also find that the disks of early type spirals have higher values of  $M/L$  and are closer to maximum disk than later-type spirals.

In summary, the vertical dynamics of stellar disks show that in general the velocity dispersions of the stars falls off with an e-folding length double that of the exponential light distribution, as required for a constant  $M/L$ , while for the majority of disks the inferred mass-to-light ratios are almost certainly lower than required in the maximum disk hypothesis.

### 3.2.2 Stellar velocity dispersions in the plane

The stellar velocity dispersions *in* the plane are more complicated to determine from observations. The radial and tangential components are not independent, but governed by the local Oort constants<sup>17</sup>

$$\frac{\sigma_\theta}{\sigma_R} = \sqrt{\frac{-B}{A-B}}. \quad (8)$$

For a flat rotation curve  $A = -B$  and this ratio is 0.71. In highly inclined or edge-on systems the dispersions can be measured both from the line profiles and the asymmetric drift equation

$$V_{\text{rot}}^2 - V_\theta^2 = \sigma_R^2 \left\{ \frac{R}{h} - R \frac{\partial}{\partial R} \ln(\sigma_R) - \left[ 1 - \frac{B}{B-A} \right] \right\}, \quad (9)$$

where the circular velocity  $V_{\text{rot}}$  can be measured with sufficient accuracy from the gas (optical emission lines or HI observations), which have velocity dispersions of order 10 km s<sup>-1</sup> or less and have therefore very little asymmetric drift.

The stability of a galactic disk to local axisymmetric disturbances depends on the the (stellar) radial velocity dispersion  $\sigma_R$ , the epicyclic frequency  $\kappa$ , and the local mass surface density  $\Sigma$ . Toomre’s (1964) criterion is

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma}. \quad (10)$$

On small scales local stability results from a Jeans-type stability, where tendency to collapse under gravity is balanced by the kinetic energy in random motions, but only up to a certain (Jeans) scale. On scales larger than some minimum radius, shear as a result of galactic differential rotation provides stability. In the Toomre  $Q$ -criterion this smallest scale is just equal to the (maximum) Jeans scale, so that that local stability exists on *all* scales. According to Toomre (1964), local stability requires  $Q > 1$ . Numerical simulations suggest that galaxy disks are on

<sup>17</sup>For small deviations from circular motions around the galactic center, the stellar orbit may be described by a small epicycle superposed on the circular motion around the galactic center. The frequency in the epicycle is  $\kappa = 2\sqrt{-B(A-B)}$  and its axis ratio  $\sqrt{-B/(A-B)}$  (Oort, 1965). The ratio between the two velocity dispersions derives from the shape of the epicycle.

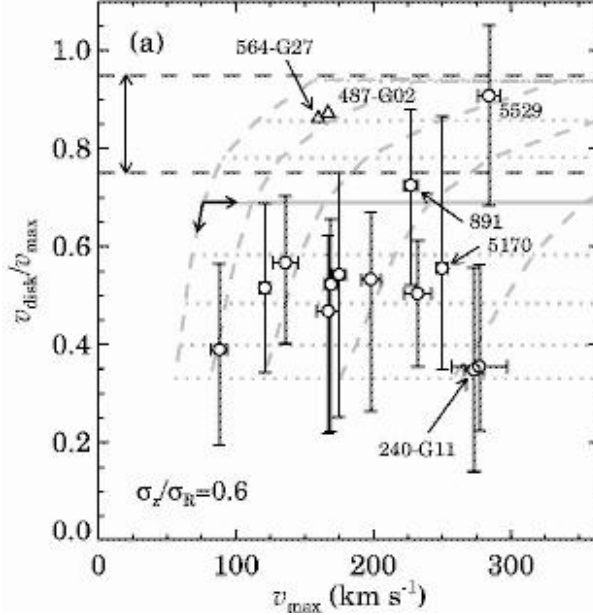


Figure 4: Stellar disk velocity dispersion, measured at one scalelength in edge-on galaxies versus the maximum rotational velocity. The gray lines indicate the relation  $\sigma_R(h) = (0.29 \pm 0.10) V_{\text{rot}}$  (Bottema, 1993). (From Kregel, van der Kruit & Freeman, 2005)

the verge of instability (Hohl, 1971; Sellwood & Carlberg, 1984; Athanassoula & Sellwood, 1986; Mihos, McGaugh & de Blok, 1997; Bottema, 2003), having stellar velocity dispersions that are slightly larger than Toomre’s critical velocity dispersion. The simulations suggest  $Q = 1.5\text{--}2.5$ .

The first attempt to measure these in-plane velocity dispersion components was by van der Kruit & Freeman (1986) for the highly inclined galaxy NGC 7184. They fitted their data using two different assumptions for the radial dependence of the radial velocity dispersion: one that the axis ratio of the velocity ellipsoid (between the vertical and radial dispersion) is the same everywhere, and the other that the Toomre  $Q$  is constant with radius. Both assumptions worked well; over the observed range of one or two scalelengths from the center, the two assumptions correspond to similar variations (see van der Kruit, 1990, page 196).

More extensive observations by Bottema (1993) on a sample of 12 galaxies (including the Milky Way Galaxy from Lewis & Freeman, 1989) resulted in the discovery of a relation between a fiducial value of the velocity dispersion (either the vertical one measured at or extrapolated to the center or the radial velocity dispersion at one scalelength) and the integrated luminosity or the rotation velocity. Luminosity and rotational velocity are equivalent through the Tully-Fisher relation. This has been confirmed by Kregel & van der Kruit (2005) and Kregel, van der Kruit & Freeman (2005), see fig. (4). The relation<sup>18</sup> is

$$\sigma_{z|0} = \sigma_{R|1h} = (0.29 \pm 0.10) V_{\text{rot}}. \quad (11)$$

It extends to very small dwarf galaxies, *e.g.* UGC 4325 with a velocity dispersion of  $19 \text{ km s}^{-1}$  still falls on the relation (Swaters, 1999, chapter 7). The scatter in this relation is not random but appears related to other properties. Galaxies with lower velocity dispersions have higher flattening, lower central surface brightness or dynamical mass ( $4hV_{\text{rot}}^2/G$ ) to disk luminosity ratio.

<sup>18</sup>The formal fit to the sample in Kregel, van der Kruit & Freeman (2005) is  $\sigma_{R|1h} = (-2 \pm 10) + (0.33 \pm 0.05) V_{\text{rot}}$ ; the equation above gives the relation adopted from all available studies.



The linear  $\sigma - V_{\text{rot}}$  relation follows from straightforward arguments, presented in van der Kruit (1990), Ch. 10 (see also Bottema, 1993; van der Kruit & de Grijs, 1999). We evaluate properties at one radial scalelength ( $R = 1h$ ) without using subscripts to indicate this. Using the definition for Toomre  $Q$  for a flat rotation curve, so that  $\kappa = \sqrt{2}V_{\text{rot}}/R$ , and eliminating  $h$  using a Tully-Fisher relation  $L_{\text{disk}} \propto \mu_{\text{o}} h^2 \propto V_{\text{rot}}^4$  results in

$$\sigma_{\text{R}} \propto Q \frac{\mu_{\text{o}}(M/L)_{\text{disk}} h}{V_{\text{rot}}} \propto Q \left( \frac{M}{L} \right)_{\text{disk}} \mu_{\text{o}}^{1/2} V_{\text{rot}}. \quad (12)$$

This shows that, when  $Q$  and  $M/L$  are constant among galaxies, the Bottema relation follows, with indeed the proviso that galaxy disks with lower (face-on) central surface brightness  $\mu_{\text{o}}$  at a given value of  $V_{\text{rot}}$  have lower stellar velocity dispersions than given by the mean  $\sigma - V_{\text{rot}}$  relation.

### 3.2.3 Origin of the constant scaleheight

The origin of the constant scaleheight of stellar disks –or of the fall-off of stellar vertical velocity dispersion such to precisely compensate for the decline in surface density– is not obvious. If the evolution of the stellar velocity dispersions (the ‘heating of the disk’) is similar at all radii and if it evolves to a radial velocity dispersion such that the disk is just stable everywhere, we may expect  $\sigma_{\text{z}}/\sigma_{\text{R}}$  (‘the axis ratio of the velocity ellipsoid’) and Toomre  $Q$  to be independent of galactocentric radius. This would however imply that  $\sigma_{\text{z}} \propto (R/h) \exp(-R/h)$ . Although this is not all that much different from the exponential decline  $\exp(-R/2h)$  that follows from eqn. (7) between say one and three scalelengths<sup>19</sup>, it is significantly different at larger radii. In fact, the simple assumption would result in  $h_{\text{z}} \propto (R/h)^2 \exp(-R/h)$ , which is far from constant over the range  $R = 0$  to  $R = 5h$ .

It is necessary to look first at the evolution of random stellar motions in disk before we can proceed. There are three general classes of models for the origin of the velocity dispersions of stars in galactic disks. The first, going back to Spitzer & Schwarzschild (1951), is scattering by irregularities in the gravitational field, later identified with the effects of Giant Molecular Clouds (GMC’s). The second class of models can be traced back to the work of Barbanis & Woltjer (1967), who suggested transient spiral waves as the scattering agent; this model has been extended by Carlberg & Sellwood (1985). More recently, the possibility of infall of satellite galaxies has been recognized as a third option (*e.g.* Velázquez & White, 1999).

Almost all of the observational information about the evolution of velocity dispersion with age in galactic disks comes from the solar neighborhood, and we must stress that this information remains quite insecure. While much of the earlier work invokes the results of Wielen (1977), which indicates a steady increase of stellar velocity dispersion with age, some of the more recent observational studies indicate that the velocity dispersion increases with age for only 2 to 3 Gyr, and then saturates, remaining constant for disk stars of older age (see *e.g.* Edvardsson et al., 1993; Freeman, 1991; Soubiran et al., 2008). The observational situation regarding disk heating is far from certain, and this in turn must reflect on the various theories of disk heating.

In the solar neighbourhood the ratio of the radial and vertical velocity dispersion of the stars  $\sigma_{\text{z}}/\sigma_{\text{R}}$  is usually taken as roughly 0.5 to 0.6 (Wielen, 1977; Gomez et al., 1990; Dehnen & Binney, 1998; Mignard, 2000), although values on the order of 0.7 are also found in the literature (Woolley et al., 1977; Meusinger, Reimann & Stecklum, 1991). The value of this ratio can be used to test predictions for the secular evolution in disks and perhaps distinguish between the general classes of models. Lacey (1984) and Villumsen (1985) have concluded that the Spitzer-Schwarzschild mechanism is not in agreement with observations; the predicted time dependence

<sup>19</sup>The reason why the two analyses of the measurements of velocity dispersion in van der Kruit & Freeman (1986) and by Bottema et al. (see references in Bottema, 1993) both gave good fits.

of the velocity dispersion of a group of stars as a function of age disagrees with the observed age – velocity dispersion relation (see also Wielen, 1977), while it would not be possible for the axis ratio of the velocity ellipsoid  $\sigma_z/\sigma_R$  to be less than about 0.7 (but see Ida, Kokuba & Makino, 1993).

Jenkins & Binney (1990) argued that it is likely that the dynamical evolution in the directions in the plane and that perpendicular to it could have proceeded with both mechanisms contributing, but in different manners. Scattering by GMC’s would then be responsible for the vertical velocity dispersion, while scattering from spiral irregularities would produce the velocity dispersions in the plane. The latter would be the prime source of the secular evolution with the scattering by molecular clouds being a mechanism by which some of the energy in random motions in the plane is converted into vertical random motions, hence determining the thickness of galactic disks. The effects of a possible slow, but significant accretion of gas onto the disks over their lifetime has been studied by Jenkins (1992), who pointed out strong effects on the time dependence of the vertical velocity dispersions, in particular giving rise to enhanced velocities for the old stars. On the other hand, Hänninen & Flynn (2000, 2002) conclude that observations such as the radial dependence of stellar velocity dispersions in the Milky Way Galaxy by Lewis & Freeman (1989) can be reproduced if scattering occurs by a combination of massive halo objects (black holes) and GMC’s. Dehnen & Binney (1998) conclude that spiral structure is probably a major contributor to disk heating. More recently, Minchev & Quillen (2006) suggested from 2D simulations that multiple patterns of spiral structure could cause strong variations of stellar velocity dispersions with galactocentric radius, which has not been observed. Our conclusion is that there still is much uncertainty about the process of heating of the (thin) disk. Some of this uncertainty is due to uncertainty in the observational relation between stellar ages and velocity dispersions, because stellar ages are so difficult to measure.

Theoretical arguments suggest that a constant axis ratio of the velocity ellipsoid is a fair approximation in the inner parts of galaxy disks (Cuddeford & Amendt, 1992; Famaey, van Caelenberg & DeJonghe, 2002). An observational argument for the approximate constancy of the velocity anisotropy is provided by the ages and kinematics of 182 F and G dwarf stars in the solar neighbourhood (Edvardsson et al., 1993). This indicates that the anisotropy was set after an early heating phase and, although the Galaxy has probably changed much over its lifetime, has remained constant throughout the life of the old disk (Freeman, 1991).

So, where does this leave us with respect to the origin of the constant scaleheight? As long as there is no detailed understanding of the evolution of the velocity dispersions as a function of galactocentric radius, we cannot even begin to address this in a meaningful way. A constant stability parameter  $Q$  and a constant axis ratio of the velocity ellipsoid  $\sigma_z/\sigma_R$  do give an approximate constant thickness over the inner few scalelengths, but this fails at larger radii.

### 3.2.4 Mass distributions from stellar dynamics

The stellar velocity dispersions can still be used to derive information on the disk mass distribution. For a self-gravitating disk which is exponential in both the radial and vertical direction, the vertical velocity dispersion goes as (cf. van der Kruit, 1988):

$$\sigma_z(R, z) = \sqrt{\pi G h_z (2 - e^{-z/h_z}) (M/L) \mu_0} e^{-R/2h}, \quad (13)$$

Assuming a constant (but unknown) axis ratio of the velocity ellipsoid  $\sigma_z/\sigma_R$ , the radial velocity dispersion becomes

$$\sigma_R(R, z) = \sqrt{\pi G h_z (2 - e^{-z/h_z}) (M/L) \mu_0} \left( \frac{\sigma_z}{\sigma_R} \right)^{-1} e^{-R/2h}. \quad (14)$$

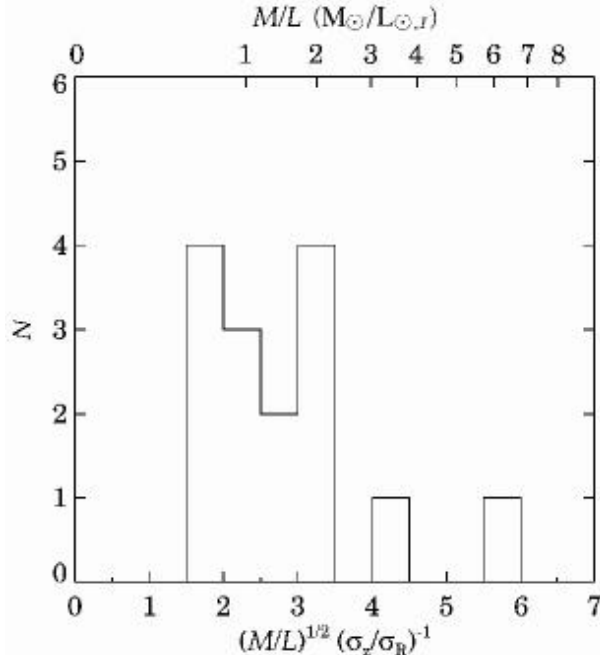


Figure 5: Histogram of the product  $\sqrt{M/L_I} (\sigma_z/\sigma_R)^{-1}$  from stellar kinematics in edge-on galaxies. Except for two outliers the distribution of  $\sqrt{M/L_I} (\sigma_z/\sigma_R)^{-1}$  is rather narrow. The outliers are ESO 487-G02 and 564-G27; data for these galaxies are less complete than for the other ones. Along the top we show the values of  $M/L_I$  implied by  $\sigma_z/\sigma_R = 0.6$ . (From Kregel, van der Kruit & Freeman, 2005)

The distribution of the products  $\sqrt{M/L_I} (\sigma_z/\sigma_R)^{-1}$ , deduced from this equation in the Kregel, van der Kruit & Freeman (2005) sample is shown in fig. 5. This sample of edge-on galaxies has a range of Hubble types from Sb to Scd, absolute  $I$ -magnitudes between -23.5 and -18.5, and a range in rotation velocities from 89 to 274 km s<sup>-1</sup>. Thirteen of the fifteen disks have  $1.8 \lesssim \sqrt{M/L_I} (\sigma_z/\sigma_R)^{-1} \lesssim 3.3$ . The values of the outliers may have been overestimated (see Kregel, van der Kruit & Freeman, 2005). Excluding these, the average is  $\langle \sqrt{M/L_I} (\sigma_z/\sigma_R)^{-1} \rangle = 2.5 \pm 0.2$  with a  $1\sigma$  scatter of 0.6. The near constancy of the product can be used with mass-to-light ratios based on stellar population synthesis models to estimate the axis ratio of the velocity ellipsoid. Conversely, the upper scale of fig. (5) indicates that a typical  $M/L$  in the  $I$ -band of a galactic stellar disk is of order unity and for the majority systems lies between 0.5 and 2.

It is possible to relate the axis ratio of the velocity ellipsoid to the flattening of the stellar disk, i.e. the ratio of the radial exponential scalelength and the vertical exponential scaleheight (van der Kruit & de Grijs, 1999). In the radial direction, the velocity dispersion at one scalelength can be written using the definition of Toome  $Q$  as  $\sigma_{R,h} \propto Q \Sigma(h) h / V_{\text{rot}}$ , where a flat rotation curve has been assumed. At this radius of one scalelength the hydrostatic equation gives  $\sigma_z \propto \sqrt{\Sigma(h) h_z}$ . Eliminating  $\Sigma(h)$  between these two equations then gives

$$\left( \frac{\sigma_z}{\sigma_R} \right)_h^2 \propto \frac{1}{Q} \frac{h_z}{h} \quad (15)$$

If  $Q$  is constant within individual disks, then the disk flattening depends directly on the axis ratio of the velocity ellipsoid.

Eqn. (12) shows that when  $Q$  and  $M/L$  are constant among galaxies, galaxy disks with lower (face-on) central surface brightness  $\mu_o$  have lower stellar velocity dispersions. Combining

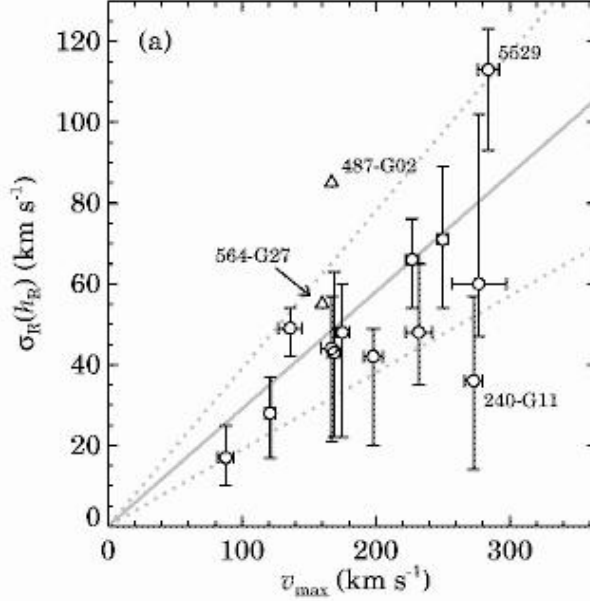


Figure 6: The contribution of the disk to the amplitude of the rotation curve  $V_{\text{disk}}/V_{\text{rot}}$ . for a sample of 15 edge-on galaxies as a function of the rotation velocity itself. The horizontal dashed lines are the limits of  $0.85 \pm 0.10$  from Sackett (1997), which would indicate Fmaximal disks. The axis ratio of the velocity ellipsoid is assumed to be 0.6. The grey lines correspond to collapse models of Dalcanton et al. (1997); dashed lines connect models of the same total mass ( $\log_{10}(M_{\text{tot}}) = 10 - 13$  in steps of 0.5) and dotted lines connect models with the same spin parameter (logarithmically spaced, separated by factors of 0.2 dex, with the solid line at  $\lambda = 0.06$ ). The arrows indicate the direction of increasing  $M_{\text{tot}}$  and  $\lambda$ . The two galaxies without error bars are the same ones as the outliers in fig. 5. (From Kregel, van der Kruit & Freeman, 2005)

eqn. (12) with the hydrostatic equilibrium eqn. (7) and using eqn. (11) gives (Kregel, van der Kruit & Freeman, 2005; van der Kruit & de Grijs, 1999)

$$\frac{h}{h_z} \propto Q \left( \frac{\sigma_R}{\sigma_z} \right) \sigma_z^{-1} V_{\text{rot}} \propto Q \left( \frac{\sigma_R}{\sigma_z} \right). \quad (16)$$

The observed constancy of  $\sqrt{M/L} (\sigma_z/\sigma_R)^{-1}$  implies that the flattening of the disk  $h/h_z$  is proportional to  $Q\sqrt{M/L}$ .

For a self-gravitating exponential disk, the expected rotation curve peaks at 2.2 scalelengths. The ratio of this peak of the rotation velocity of the disk to the maximum rotation velocity of the galaxy ( $V_{\text{disk}}/V_{\text{rot}}$ ) is

$$\frac{V_{\text{disk}}}{V_{\text{rot}}} = \frac{0.880 (\pi G \Sigma_0 h)^{1/2}}{V_{\text{rot}}}. \quad (17)$$

Using eqn. (7) and eqn. (11) this can be rewritten as

$$\frac{V_{\text{disk}}}{V_{\text{rot}}} = (0.21 \pm 0.08) \sqrt{\frac{h}{h_z}}. \quad (18)$$

So we can estimate the disk contribution to the rotation curve from a statistical value for the flattening (see also Bottema, 1993, 1997; van der Kruit, 2002). For the sample of Kregel, van

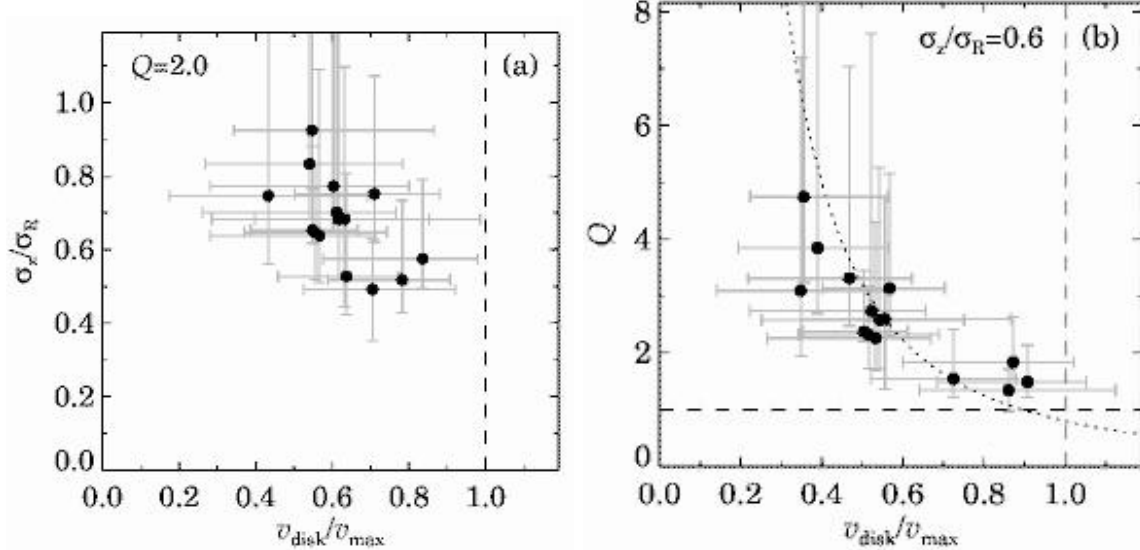


Figure 7: Stellar dynamics parameters for edge-on galaxies. (a) The axis ratio of the velocity ellipsoid as a function of  $V_{\text{disk}}/V_{\text{rot}}$  for  $Q=2.0$ . (b)  $V_{\text{disk}}/V_{\text{rot}}$  as a function of  $Q$  for an assumed axis ratio of the velocity ellipsoid of 0.6. (From Kregel, van der Kruit & Freeman, 2005)

der Kruit & de Grijs (2002) this then results in  $V_{\text{disk}}/V_{\text{rot}} = 0.57 \pm 0.22$  (rms scatter). In the dynamical analysis of Kregel, van der Kruit & Freeman (2005), the ratio  $V_{\text{disk}}/V_{\text{rot}}$  is known up to a factor  $\sigma_z/\sigma_R$  and distance-independent. The derived disk contribution to the observed maximum for the same sample rotation is on average  $V_{\text{disk}}/V_{\text{rot}} = 0.53 \pm 0.04$ , with a  $1\sigma$  scatter of 0.15. Both estimates agree well.

In the maximum disk hypothesis,  $V_{\text{disk}}/V_{\text{rot}}$  will be a bit lower than unity to allow a bulge contribution and let dark matter halos have a low density core. A working definition that has been adopted generally is  $V_{\text{disk}}/V_{\text{rot}} = 0.85 \pm 0.10$  (Sackett, 1997). Thus, at least for this sample, the average spiral has a submaximal disk. Note that eqn. (17) strictly applies to a razor-thin disk. For a disk with a flattening of  $h/h_z \simeq 10$  the radial gravitational force is weaker, leading to decrease of about 5% in  $V_{\text{disk}}/V_{\text{rot}}$  (van der Kruit & Searle, 1982a). Taking the gravity of the gas layer and dark matter halo into account would yield a 10% effect, also in this direction. So, these effects work in the direction of making the disks more sub-maximal.

The values obtained from stellar dynamics are illustrated in Figs. 6 and 7. The measurement of stellar velocity dispersions can be used to derive the disk surface density at some point (e.g. one scalelength) up to a factor  $(\sigma_z/\sigma_R)^2$ , but can be estimated also from the velocity dispersion for an assumed value of  $Q$ . Comparing the two gives then an estimate of the axis ratio of the velocity ellipsoid. In fig. 7 on the left  $Q$  is assumed 2.0 and on the left the velocity anisotropy is assumed to be 0.6 and then a value for  $Q$  results. Most galaxies are not ‘maximum-disk’. The ones that may be maximum disk have a high surface density according to fig. 5. From the panels we also note that disks that are maximal appear to have more anisotropic velocity distributions or are less stable according to Toomre  $Q$ . We will return to the maximum disk hypothesis below (§ 4.2).

### 3.3 Age Gradients and Photometric $M/L$ Ratios

Colors contain information on the history of star formation, as can be studied in the context of integrated colors of galaxies, pioneered by Searle, Sargent & Bagnuolo (1973) and Larson &

Tinsley (1978), and described in much detail by Tinsley (1980), and also as a function of radius in a galaxy disk. Observing and interpreting color gradients in galactic disks is not straightforward. Obviously one needs accurate photometry for unambiguous interpretation in terms of stellar synthesis and star formation histories. However, the effects of age<sup>20</sup> and metallicity are difficult to separate. Dust absorption is also a major factor, often making degenerate the effects of stellar age and metallicity on the one hand and extinction and reddening by dust on the other. Fig. 7 of Larson & Tinsley (1978) is instructive. It shows a sequence of population synthesis models in the two-color ( $U - B$ ) vs. ( $B - V$ ) diagram, with ages of  $10^{10}$  years and star formation histories ranging from initial burst to constant with time. The effects of age, metallicity and absorption, and even changes in the IMF, shift the models in very similar directions!

Wevers, van der Kruit & Allen (1986) were the first to undertake a systematic survey of the luminosity, color and HI distributions in a well-defined set of spiral galaxies. The surface photometry was based on photographic plates and, although the data did show color gradients, Wevers (1984) was not sufficiently confident to conclude that these were significant. In hindsight this was not justified: a detailed comparison by Begeman (1987) with later CCD-photometry of Kent (1987) for three systems showed deviations of at most 0.2 magnitudes in the radial profiles down to 26  $r$ -magnitudes arcsec<sup>-2</sup>. Although common wisdom holds that old photographic surface photometry is not reliable, at least some of it certainly is.

A comprehensive study of the broadband optical and near-infrared colors in a sample of 86 disk galaxies was performed by de Jong & van der Kruit (1994) and de Jong (1996a,b,c). These studies established the existence of color gradients both within and among galaxy disks, fainter surface brightness systematically corresponding to bluer colors. It was also found that the degeneracies between dust absorption, stellar age and metallicity can be broken to a large extent by use of a set of photometric bands from the blue ( $B$ -band) to the near infrared ( $K$ ) and it was concluded from 3D radiative transfer models that dust extinction cannot be the major cause of the observed gradients. The color gradients must be the result of significant differences in star formation history, whereby the outer regions are younger and of lower metallicity than the central parts. The lack of suitable stellar population models made it impossible to quantify the trends, although the extreme variations predicted by the models of Larson (1976) seemed outside the range of possibilities offered by the observed color gradients.

Peletier & de Grijs (1998) used ( $I - K$ ) colors in edge-on galaxies away from the central planes to derive a dust-free near-IR color-magnitude relation for spiral galaxies. The slope of this relation is steeper for spirals than for elliptical galaxies. This is most likely not a result of vertical abundance gradients, but of average age with height. The surprising thing is that the scatter in this relation is small, possibly even only due to observational uncertainty. Average stellar age must be an important contributor to variations in broadband colors.

Bell & de Jong (2000) made an important step forward by using maximum-likelihood methods to match observed colors with stellar population synthesis models, using simple star formation histories. These showed that spiral galaxies almost all have significant gradients in the sense that the inner regions are older and more metal rich than the outer regions. The amplitude of these gradients is larger in high-surface brightness galaxies than in low-surface brightness ones, and the progress of star formation (as evidenced by decreasing age and increasing metallicity) depends primarily on the surface brightness (most clearly in the  $K$ -band) or surface density. The local surface density seems to shape the star formation history in a disk more strongly than the overall mass of the galaxy.

These models can also be used to derive values and gradients of the mass-to-light ratio  $M/L$  in and among disks. This was done by Bell & de Jong (2001) under the assumption of a

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<sup>20</sup>It should be noted that in discussions of these subjects the property ‘stellar age’ is usually the mean age of all stars derived as a luminosity-weighted average, further weighted by the star formation rate over the lifetime of the disk, and should not be confused with the age of the oldest stars.

universal initial mass function (IMF). They conclude that their relative trends in  $M/L$  with color are robust to uncertainties in the stellar populations and galaxy evolution models. Corrections for dust extinction are not critical in the final determination of the stellar masses. They also find that limits on the  $M/L$  ratios derived from maximum disk fits to rotation curves (for galaxies in the Ursa Major cluster by Verheijen, 2001; Verheijen & Sancisi, 2001) match their  $M/L$ 's well, providing support for the universality of the IMF and the notion that at least some high surface brightness galaxies are close to maximum disk. The variations in  $M/L$  span a factor between 3 and 7 in the optical and about 2 in the near-infrared.

The IMF provides the normalisation of the  $M/L$  through the numbers of low-mass stars, but the slope of the relation between color and  $M/L$  is largely independent of what models are used or what IMF is adopted (de Jong & Bell, 2009). The Salpeter IMF gives too massive a normalisation (Bell & de Jong, 2001), which can be remedied by using a ‘diet’ Salpeter IMF (i.e. deficient in low-mass stars such that it has only 70% of the mass for the same color; Bell & de Jong, 2001), or adopting an IMF that is itself more deficient in low-mass stars (Kennicutt, 1983; Kroupa, 2001; Chabrier, 2003). Kroupa (2002a) has rather convincingly argued that the IMF is universal to the extent that its variations are smaller than would follow from the expected varying conditions on the basis of elementary considerations. Bastian, Covey & Meyer (2010) have recently concluded that “there is no clear evidence that the IMF varies strongly and systematically as a function of initial conditions after the first few generations of stars”.

Default models, produced by adopting a declining star formation rate, the population synthesis models of Bruzual & Charlot (2003) and the IMFs listed above, give consistent estimates of  $M/L$  (de Jong & Bell, 2009). In fact, the  $M/L_I$  values implied in fig. (5) on the top-axis (derived for an axis ratio of the velocity ellipsoid of 0.6) are 0.2 dex lower than from Bell & de Jong (2001) but, as de Jong & Bell (2009) point out, the axis ratio of the velocity ellipsoid scales with the *square* of  $M/L$ . The conclusion is that the determination of mass-to-light ratios from broadband colors is reliable and robust in a relative sense, but that there are still some uncertainties in the normalisation resulting from imprecise knowledge of the faint part of the IMF.

### 3.4 Global Stability, Bars and Spiral Structure

Local stability of stellar disks has already been discussed in relation to local stellar velocity dispersions, Toomre’s  $Q$  and the secular evolution (‘heating’) of disks. We will say a few words here about global stability, bars in galaxies and spiral structure. Much of these subjects has been covered recently in the reviews, such as that on dynamics of galactic disk by Sellwood (2010a) and for the case of bars in relation to pseudo-bulges by Kormendy & Kennicutt (2004).

Global stability of disks has been a subject ever since numerical simulations became possible, starting about 1970 (*e.g.* Miller et al., 1970; Hohl, 1971). Criteria for stability were formulated empirically by Ostriker & Peebles (1973) and Efstathiou et al. (1982). In the latter criterion the halo stabilizes the disk; the criterion is in terms of ‘observables’

$$Y = V_{\text{rot}} \left( \frac{h}{GM_{\text{disk}}} \right)^{1/2} \gtrsim 1.1, \quad (19)$$

where the disk is assumed exponential with scalelength  $h$  and total mass  $M_{\text{disk}}$ . Since the rotation velocity  $V_{\text{rot}}$  is related to the total mass, it is a criterion that relates to the relative mass in disk and halo. It can be rewritten to say that within the radial distance from the center corresponding to the edge of the disk, the dark matter halo contains up to 60–70% of the total mass (van der Kruit & Freeman, 1986). Such galaxies are in fact sub-maximal. Sellwood (2010a) concludes that these criteria are only necessary for disks that have no dense centers, since central concentrations of mass in disks themselves could also provide global stability. It was

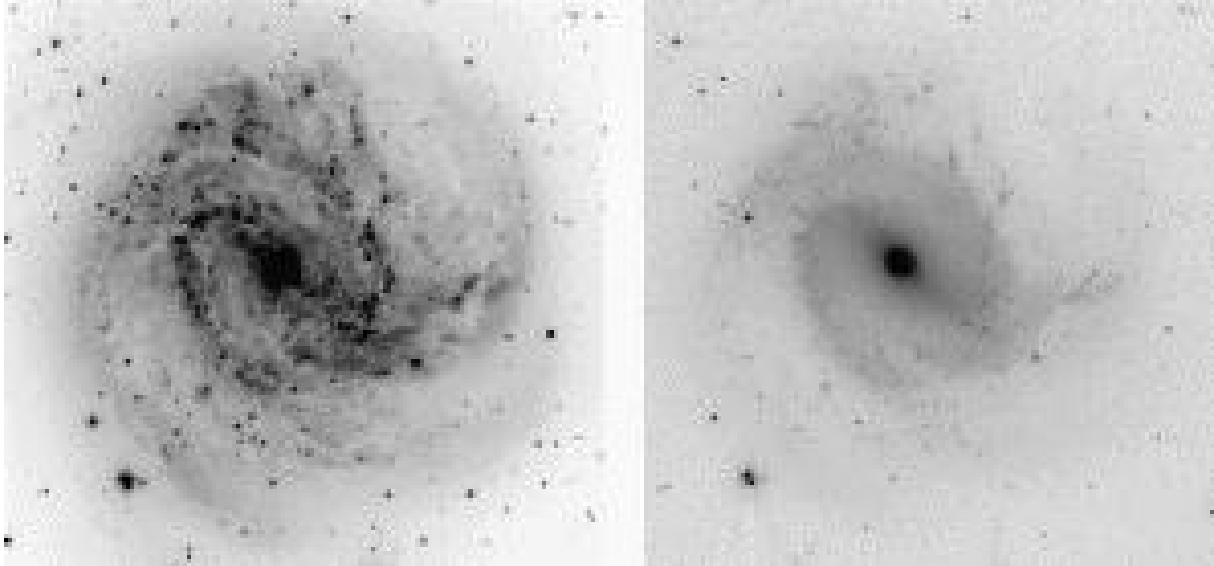


Figure 8: M83 in blue light at the left and in the  $K$ -band on the right. The bar is much more obvious in the near-IR. (Unpublished images by Park & Freeman).

shown already some decades ago (Kalnajs, 1987) that halos are not very efficient in stabilising disks compared to bulges.

We will not discuss the formation of bars in galaxies, as this subject has been covered in detail by Kormendy & Kennicutt (2004) in relation to pseudo-bulges, and by Sellwood (2010a). We do want to stress the fundamental point that the incidence of bars is much larger than traditionally thought; a typical fraction that figured in previous decades –although admittedly for strongly barred galaxies as in Sandage (1961)– was of the order of a quarter to a third. Current estimates are much higher; Sheth et al. (2008) found in the COSMOS field that in the local Universe about 65% of luminous spiral galaxies are barred. This fraction is a strong function of redshift, dropping to 20% at a redshift of 0.8. The *Spitzer Survey of Stellar Structure in Galaxies* S<sup>4</sup>G (Sheth et al., 2010) aims among others at studying this in the near IR. As an example, we show in fig. 8 a blue and near-IR image of the large spiral M83. Although it appears mildly barred in the optical, it is clear that in  $K$ -band the bar is very prominent and extended.

Throughout the previous century much attention has been paid to the matter of the formation and maintenance of spiral structure. It was extensively reviewed by Toomre (1977, 1981). Spiral structure in itself is unquestionably an important issue (see the quote to Richard Feynman in the introduction in Toomre’s review), as it is so obvious in galaxy disks and appears to play a determining role in the evolution of disks through the regulation of star formation and therefore the dynamical, photometric and chemical evolution. We will not discuss theories of spiral structure itself as progress in this area has recently been somewhat slow. We refer the reader to the contributions of Kormendy & Norman (1979), Sellwood & Carlberg (1984), Elmegreen, Elmegreen & Leitner (2003) and Sellwood (2008, 2010a,b). Spiral structure is often related to gravitational interaction between galaxies; for interactions and subsequent merging see the work of Toomre & Toomre (1972), Schweizer (1986) and Barnes (1988).

### 3.5 The Flatness of Disks

The inner disks of galaxies are often remarkably flat. For the stellar disks this can be studied in edge-on systems by determining the centroid in the direction perpendicular to the major axis



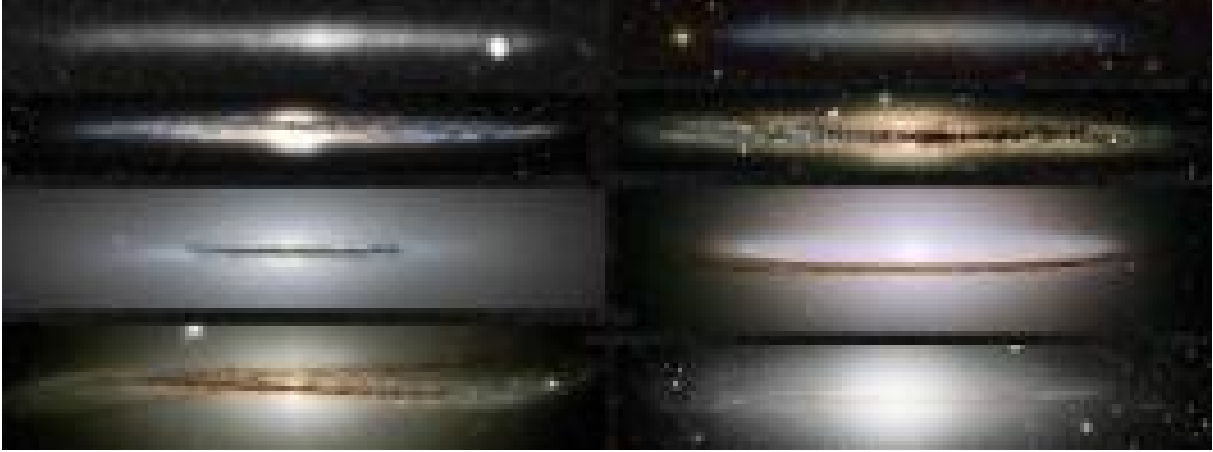


Figure 9: Selected images of edge-on disks and dustlanes from various public Web-galleries. Top: ‘Superthin’ galaxies IC 5249 (from the *Sloan Digital Sky Survey*, van der Kruit et al., 2001) and UGC 7321 ([cosmo.nyu.edu/hogg/rc3/UGC\\_732\\_irc\\_hard.jpg](http://cosmo.nyu.edu/hogg/rc3/UGC_732_irc_hard.jpg)); second row: NGC 4565 ([www.cfht.hawaii.edu/HawaiianStarlight/AIOM/English/2004/Images/Nov-Image2003-CFHT-Coelum.jpg](http://www.cfht.hawaii.edu/HawaiianStarlight/AIOM/English/2004/Images/Nov-Image2003-CFHT-Coelum.jpg)) and NGC 891 ([www.cfht.hawaii.edu/HawaiianStarlight/Posters/NGC891-CFHT-Cuillandre-Coelum-1999.jpg](http://www.cfht.hawaii.edu/HawaiianStarlight/Posters/NGC891-CFHT-Cuillandre-Coelum-1999.jpg)); third row: NGC 5866 ([heritage.stsci.edu/2006/24/big.html](http://heritage.stsci.edu/2006/24/big.html)) and M104 ([heritage.stsci.edu/2003/28/big.html](http://heritage.stsci.edu/2003/28/big.html)); bottom row: ESO 510-G013 ([heritage.stsci.edu/2001/23/big.html](http://heritage.stsci.edu/2001/23/big.html)) and NGC 7814 ([www.cfht.hawaii.edu/HawaiianStarlight/English/Poster50x70-NGC7814.html](http://www.cfht.hawaii.edu/HawaiianStarlight/English/Poster50x70-NGC7814.html)).

at various galactocentric distances (*e.g.* Sanchez-Saavedra, Battaner & Florido, 1990; Florido et al., 1991; de Grijs, 1997, chapter 5). These studies were aimed at looking for warps in the outer parts of the stellar disks (see below), but it is obvious from the distributions that in the inner parts the systematic deviations are very small.

The evidence for the flatness of stellar disks is more compelling when we look at the flatness of the layers of the ISM. First look at the dustlanes. In fig. 9 we collect some images of edge-on disk galaxies. At the top are two ‘super-thin’ galaxies (which we discuss further in § 3.6); the disks are straight lines to within a few percent. The same holds for the dustlanes in NGC 4565 (allow for the curvature due to its imperfectly edge-on nature) and NGC 891. Again the dustlanes indicate flat layers to a few percent. In the third row the peculiar structure of NGC 5866 has no measurable deviation from a straight line, while for the Sombrero Nebula the outline of the dustlane fits very accurately to an ellipse. In the bottom row, NGC 7814 (right) is straight again to within a few percent, but NGC 5866 is an example of a galaxy with a large warp in the dust layer.

The HI kinematics provide probably the strongest indications for flatness. In three almost completely face-on spirals (NGC 3938, 628 and 1058), van der Kruit & Shostak (1982, 1984) and Shostak & van der Kruit (1984) found that the residual velocity field after subtraction of that of the systematic rotation shows no systematic pattern and had an r.m.s. value of only 3 - 4 km s<sup>-1</sup>. So, vertical motions must be restricted to only a few km s<sup>-1</sup> (a few pc per Myr). Then, even a vertical oscillation with a period equal to the typical time of vertical oscillation of a star in the Solar Neighborhood (10<sup>7</sup> years) or of the rotation of the Sun around the Galactic Center (10<sup>8</sup> years) would have an amplitude of only ten to a hundred pc. Again, this is of order a few percent or less of the diameter of a galaxy like our own. The absence of such residual patterns shows that the HI layers and, since by implication they are more massive, the stellar disks must be extraordinarily flat, except maybe in their outer regions. This obviously does not

hold for galaxies that are or have recently been in interaction.

Recently, Matthews & Uson (2008a,b) found evidence for a pattern of corrugation in the disk of the edge-on galaxy IC 2233. The excursion of the plane is most pronounced in younger tracer populations, such as HI or Young stars. The amplitude is up to 250 pc (compared to a radius of order 10 kpc). The older disk shows much less of an effect. IC 2233 is a relatively small galaxy (rotation velocity about  $100 \text{ km s}^{-1}$ ) and has extensive star formation; it appears that the effect is related to the process of star formation.

### 3.6 ‘Superthin’ Galaxies

We have indicated above that the flattening of the stellar disk  $h_z/h$  is smallest for systems that are of late Hubble type, small rotation velocity and faint (face-on) surface brightness. It is of interest then to look more closely at systems at this extreme end of the range of flattening; such systems are referred to as ‘superthin’. Of course, the ones we can identify are seen edge-on. A prime example is the galaxy UGC 7321, studied extensively by Matthews, Gallagher & van Driel (1999), Matthews (2000), Matthews & Wood (2003), Uson & Matthews (2003) and Banerjee, Matthews & Jog (2010). This is a very low surface brightness galaxy (its face-on  $B$ -band central surface brightness would be  $\sim 23.4 \text{ mag arcsec}^{-2}$ ) with a scalelength of about 2 kpc, but a projected vertical scaleheight of only 150 pc. There is evidence for vertical structure: it has a color gradient (bluer near the central plane) and appears to consist of two components. Rotation curve analysis (Banerjee, Matthews & Jog, 2010; O’Brien, Freeman & van der Kruit, 2010c) indicates that it has a large amount of mass in its dark matter halo compared to the luminous component. Its HI is warped in the outer parts, starting at the edge of the light distribution. Extended HI emission is visible at relative high  $z$  (more than 2 kpc out of the plane). Pohlen et al. (2003) argue that the deviation in the light profile in the central regions and the shape of the isophotes point at a presence of a large bar.

Another good example of a superthin galaxy is IC 5249 (Byun, 1998; Abe et al., 1999; van der Kruit et al., 2001). This also is a low surface brightness galaxy with presumably a small fraction of the mass in the luminous disk. However, the disk scaleheight is not small (0.65 kpc). It has a very long radial scalelength (17 kpc); its faint surface brightness  $\mu_o$  then causes only the parts close to the plane to be easily visible against the background sky, while the long radial scalelength assures this to happen over a large range of  $R$ . Therefore it appears thin on the sky. The flattening  $h_z/h$  is 0.09 (versus 0.07 for UGC 7321). The stellar velocity dispersions are similar to those in the Solar Neighborhood; disk heating must have proceeded at a pace comparable to that in the Galaxy.

The flattest galaxies appear not only very flat on the sky, but have indeed very small values of  $h_z/h$ . However, these two examples show that the detailed structure may be different. Superthin galaxies do share the property of late type, faint face-on surface brightness and small amounts of luminous disk mass compared to that in the dark matter halo.

Kautsch (2009) has reviewed the observations of ‘flat and superthin’ galaxies, especially in view of the fact that these late-type, bulgeless systems present challenges to models of disk galaxy formation within the hierarchical growth context of  $\Lambda$ CDM. These pure disk systems have low surface brightness blue structures with low angular momenta, that may have formed with a lower frequency of merging events than disk galaxies with bulges and thick disks. In large and giant galaxies the question of the frequency of the presence of a ‘classical’ bulge has been addressed by Kormendy et al. (2010). They find that giant, pure disk galaxies are far from rare and their existence presents a major challenge to formation pictures with histories of merging in an hierarchical clustering scenario.

### 3.7 Warps in Stellar Disks

In their outer parts, stellar disks have deviations from both the plane of the inner parts (warps) as well as deviations from the extrapolated exponential surface brightness distributions (truncations). We will discuss these phenomena in turn.

First we turn to warps in the outer parts of stellar disks. Studies referred to above (Sanchez-Saavedra, Battaner & Florido, 1990; Florido et al., 1991; de Grijs, 1997; Reshetnikov et al., 2002) have indicated that most, if not all, disks display warps in their very outer parts, often up to  $0.5h_z$  or more. Recently, Saha, de Jong & Holwerda (2009) have studied edge-on galaxies observed with the *Spitzer Space Telescope* in the  $4.5\mu$  band.<sup>21</sup> Out of 24 galaxies they found evidence for warps in 10. The radius of the onset of the warp indicates that there must also be a moderate amount of flaring, in order to match the response to the indicated mass distribution from the light distribution and rotation curve. The warp onset is asymmetric and the more so in small scalelength systems. The reason for this is not clear, but could point to asymmetries in the dark matter distribution. The warp profiles shown in their figures reinforces the point made above about the flatness of disks; in the inner parts the deviations from a straight line are exceedingly small (only a percent or less of the radial extent). Theoretical work related to warps and dynamics in stellar disk has recently been reviewed by Sellwood (2010a), in the context of collective global instabilities, bending waves, bars and spiral structure.

Sometimes optical warps are very pronounced, such as in the so-called ‘Integral Sign’ galaxy UGC 3697 (Burbidge et al., 1967; Ann, 2007). There have recently been a number of statistical studies (*e.g.* Schwarzkopf & Dettmar, 2001; Ann & Park, 2006) from large samples, that contain more and less isolated systems. The conclusions are that strong warps are probably all a result of interactions, while at least a fraction may arise from accretion of gaseous material. An important point to note is that even isolated galaxies show signs of accretion. Beautiful examples have recently been presented in much detail, including NGC 5907 (Morrison, Boroson & Harding, 94; Martínez-Delgado et al., 2008), NGC 4736 (Trujillo et al., 2009), NGC 4013 and NGC 5055 (Martínez-Delgado et al., 2009). In NGC 5055 the brightest part of the faint loops have been registered also in the photographic surface photometry of van der Kruit (1979) in two colors; it appeared definitely red and presumably dominated by older stars. These relatively isolated systems appear to show signs for recent accretion events, which therefore must be common. Of course, much is known now about substructure in the halo of our Galaxy (Helmi, 2008) and M31 (Ferguson, 2007) and the evidence for continuing accretion that this provides, but that is beyond the scope of this review.

### 3.8 Truncations

Truncations in stellar disks were first found in edge-on galaxies, where the remarkable feature was noted that the radial extent did not grow with deeper photographic exposures (van der Kruit, 1979). Especially, when a bulge was present the minor axis did grow considerably on IIIa-J images compared to the Palomar Sky Survey IIa-D exposures in contrast to the major axes that did not grow at all. Detailed surface photometry (van der Kruit & Searle, 1981a,b) confirmed the presence of these truncations in the four brightest, edge-on, disk-dominated galaxies in the northern sky, NGC 891, 4244, 4565 and 5907. For the last two we illustrate this phenomenon of truncation in fig. 10. The truncations appear very sharp, although of course not infinitely.<sup>22</sup> Sharp outer profiles are actually obtained after deprojecting near-IR observations of edge-on galaxies (*e.g.* Florido et al., 2006). Fry et al. (1999), using CCD surface photometry, and

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<sup>21</sup>This paper contains also a rather complete inventory of publications concerning warps; optical, near-IR as well as HI

<sup>22</sup>In fact, the statement in van der Kruit & Searle (1981a) reads: “*This cut-off is very sharp with an e-folding of less than about 1 kpc*”, based on the spacing of the outer isophotes.

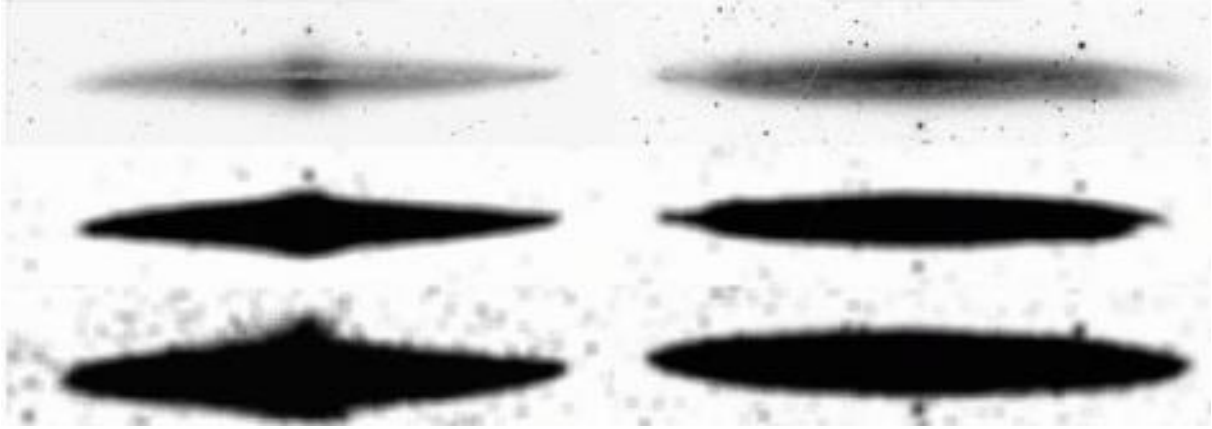


Figure 10: NGC 4565 and NGC 5907 at various light levels. These have been produced from images of the *Sloan Digital Sky Survey*, which were clipped at three different levels (top to bottom) and turned into two-bit images and subsequently smoothed (see van der Kruit, 2007, for an explanation of the details). Note that the disks grow significantly along the minor axes but not in radial extent.

de Jong et al. (2007b), from HST star counts, show that the disk of NGC 4244 has a sharp truncation, occurring over only about 1 kpc.

Various models have been proposed for the origin of truncations. In the model by Larson (1976), the truncations are the current extent of the disks while they are growing from the inside out from accretion of external material. This predicts larger age gradients across disks than are observed (de Jong, 1996b). Another possibility is that star formation is inhibited when the gas surface (or space?) density falls below a certain threshold for local stability (Fall & Efstathiou, 1980; Kennicutt, 1989; Schaye, 2004). The Goldreich–Lynden-Bell criterion for stability of gas layers gives a poor prediction for the truncation radii (van der Kruit & Searle, 1982a). Another problem is that the rotation curves of some galaxies, *e.g.* NGC 5907 and NGC 4013 (Casertano, 1983; Bottema, 1996), show features near the truncations that indicate that the *mass* distributions are also truncated. Schaye (2004) predicts an anti-correlation between  $R_{\text{max}}/h$  and  $h$ , which is not observed.

Obviously, the truncation corresponds to the maximum in the specific angular momentum distribution of the present disk, which would correspond to that in the protogalaxy (van der Kruit, 1987) if the collapse occurs with detailed conservation of specific angular momentum (Fall & Efstathiou, 1980). As noted above, if the protogalaxy starts out as a Mestel (1963) sphere with uniform density and angular rotation in the force field of a dark matter halo with a flat rotation curve, a roughly exponential disk results. This disk has then a truncation at about 4.5 scalelengths, so this hypothesis provides at the same time an explanation for the exponential nature of disk as well as for the occurrence of the truncations. On the other hand it is possible that substantial redistribution of angular momentum takes place, so that its distribution now is unrelated to the initial distribution in the material that formed the disks. Bars may play an important role in such redistribution, as suggested by Debattista et al. (2007) and Erwin et al. (2007). In fact a range of possible agents in addition to bars, such as density waves, heating and stripping of stars by bombardment of dark matter sub-halos, has been invoked (de Jong et al., 2007b). Roškar et al. (2008a,b) have studied the origin of truncations or breaks in the radial distributions in stellar disks as related to a rapid drop in star formation and include the effects of radial migration of stars. Observations of stellar populations and their ages in the regions near the truncation (Yoachim, Roškar & Debattista, 2010) have been used to provide

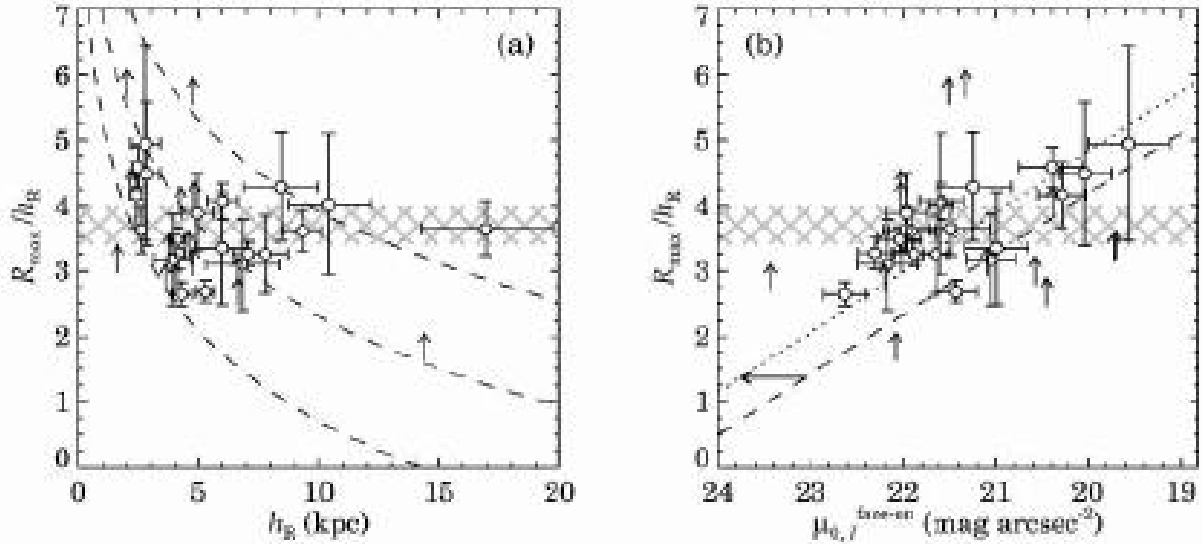


Figure 11: Correlations of  $R_{\max}/h$  with scalelength  $h$  and face-on central surface brightness  $\mu_{o,fo}$  for a sample of edge-on galaxies. The cross-hatched regions show the prediction from a collapse model as in van der Kruit (1987) and Dalcanton et al. (1997); the dotted and dashed lines show predictions from the star formation threshold model of Schaye (2004) for three different values of the disk mass. (from Kregel & van der Kruit, 2004b, ; see there for details of the models.)

evidence that migration of stars is a significant phenomenon in the formation and evolution of stellar disks. Finally, there are models (Battener, Florido & Jiménez-Vicente, 2002; Florido et al., 2006) in which a magnetic force breaks down as a result of star formation so that stars escape. The evidence for sufficiently strong magnetic fields needs strengthening.

Kregel & van der Kruit (2004b) derive correlations of the ratio of the cut-off radius  $R_{\max}$  to the radial scalelength  $h$  with  $h$  itself and with the face-on central surface brightness  $\mu_{o,fo}$  (fig. 11).  $R_{\max}/h$  does not depend strongly on  $h$ , but is somewhat less than the 4.5 predicted from the collapse from a simple Mestel-sphere. There is some correlation between  $R_{\max}/h$  and  $\mu_{o,fo}$ , indicating approximately constant disk surface density at the truncations, as possibly expected in the star-formation threshold model. But this model predicts an anti-correlation between  $R_{\max}/h$  and  $h$  (Schaye, 2004), which is not observed. The maximum angular momentum hypothesis predicts that  $R_{\max}/h$  should not depend on  $h$  or  $\mu_{o,fo}$  and such a model therefore requires some redistribution of angular momentum in the collapse or somewhat different initial conditions.

The truncation in the stellar disk in our Galaxy has been identified using star counts in a number of surveys (Robin, Crézé & Mohan, 1992; Ruphy et al., 1996) at a galactocentric radius of 14 to 15 kpc. Typical values for the ratio between the truncation radius and the radial scalelength  $R_{\max}/h$  are 3.5 to 4 (see fig. 11), so that the Galaxy’s scalelength is expected on this basis to be 3.5 to 4 kpc.

Due to line-of-sight integration, truncations should more difficult to detect in face-on galaxies than in edge-on ones. The expected surface brightness at 4 scalelengths is about 26  $B$ -mag arcsec $^{-2}$ , only a few percent of the dark sky.<sup>23</sup> In face-on galaxies like NGC 628 (Shostak &

<sup>23</sup> This surface brightness is close to that often associated with the ‘Holmberg diameters’ (Holmberg, 1958), which are often assumed to be diameters at 26.5  $B$ -mag arcsec $^{-2}$  and corrected for inclination. For a discussion of the history of Holmberg radii, see the appendix in van der Kruit (2007). Contrary to common belief, they are defined in terms of photographic density (rather than a well-defined surface brightness), in two bands (photographic and photovisual rather than the  $B$ -band) and not corrected for inclination.

van der Kruit, 1984; van der Kruit, 1988) an isophote map shows that the outer contours have a much smaller spacing than the inner ones. The usual analysis uses an inclination and major axis determined from kinematics if available, (otherwise these are estimated from the average shape of isophotes) and then determines an azimuthally averaged radial surface brightness profile. But this will smooth out any truncation if its radius is not exactly constant with azimuthal angle. At this level spiral disks are indeed often lopsided, as is seen from the  $m = 1$  Fourier component of surface brightness maps (Rix & Zaritsky, 1995; Zaritsky & Rix, 1997), presumably as a result of interactions and merging. The effects are nicely illustrated in the study of NGC 5923 by Pohlen et al. (2002) (their fig. 9), which has isophotes in polar coordinates. The irregular outline shows that some smoothing will occur contrary to observations in edge-on systems. Unless special care is taken we will always find a (much) less sharp truncation in face-on than in edge-on systems.

Although we will not discuss oval distortions of disks here because they were reviewed by Kormendy & Kennicutt (2004), we emphasize that they are potentially important for studies of truncations and are also of intrinsic interest. Oval distortions in unbarred galaxies can have significant kinematical and dynamical effects.

Pohlen & Trujillo (2006) studied a sample of moderately inclined systems through ellipse-fitting of isophotes in SDSS data. They distinguish three types of profiles: *Type I*: no break; *Type II*: downbending break; *Type III*: upbending break. Pohlen et al. (2007) have reported that the same profiles occur in edge-on systems; however, of their 11 systems there was only one for each of the types I and III.

Various correlations have been reviewed by van der Kruit (2009). In general, the edge-on and face-on samples agree in the distribution of  $R_{\max}/h$ ; however the fits in moderately inclined systems result in small values of the scalelength. A prime example of a Type III profile in Pohlen & Trujillo (2006) is NGC 3310, which is a well-known case of an merging galaxy (van der Kruit, 1976; Kregel & Sancisi, 2001). In fact, on close examination of their images at faint levels, many of the Type III systems show signs of outer distortions, presumably due to interactions.

There is a good correlation between the truncation radius  $R_{\max}$  and the rotation velocity (van der Kruit, 2008). On average a galaxy like our own would have an  $R_{\max}$  of 15 - 25 kpc (and a scalelength of 4 - 5 kpc). It is of interest to compare this correlation to the case of NGC 300 (Bland-Hawthorn, Vlajić & Freeman, 2005), which has no truncation even at 10 scalelengths from the center ( $R_{\max} > 14.4$  kpc), and therefore is an example of a type I disk in the terminology of Pohlen & Trujillo (2006). Despite showing no sign of truncation down to a very faint surface brightness level, we note that its lower limit to  $R_{\max}$  is still consistent with the observed correlation between  $R_{\max}$  and  $V_{\text{rot}}$  in edge-on systems (NGC 300 has a rotational velocity of  $\sim 105$  km s $^{-1}$ , which would give an  $R_{\max}$  of 8 - 15 kpc and an  $h$  of 2 - 4 kpc). NGC 300 could be interpreted as having an unusually small  $h$  for its  $V_{\text{rot}}$  rather than an unusually large  $R_{\max}$  for its scalelength.

These examples show that at least some of the type III galaxies could arise from the effects of interactions and merging, and type I systems could at least partly be disks with normal truncation radii, but large  $R_{\max}/h$  and small scalelengths, so that their truncations occur at much lower surface brightness.

### 3.9 Nuclei of Pure-Disk Galaxies

Late type pure disk galaxies are commonly nucleated, with central nuclear star clusters (e.g. M33: Kormendy & McClure, 1993). For a sample studied by Walcher et al. (2005), the dynamical masses of the nuclear clusters are in the range  $8 \times 10^5$  to  $6 \times 10^7 M_{\odot}$ . These star clusters usually lie within a few arcsec of the isophotal centers of the galaxies (e.g. Böker et al., 2002). How are the nuclei able to locate accurately the centers of the apparently shallow central potential wells of their exponential disks? The reason may be that the center of the gravitational

field of an exponential disk is actually well-defined: the radial gradient of its potential does not vanish at its center, so the force field defines the center of the disk to within a fraction of the scaleheight of the interstellar medium of the disk, which is of order 100 pc.

Structurally the nuclear star clusters are much like Galactic globular clusters (Böker et al., 2004). Their stellar content is however very different. The light of the nuclear star clusters is typically dominated by a relatively young star population (Rossa et al., 2006; Kormendy et al., 2010), but the young population provides only a few percent of the stellar mass. They have an underlying older population with an extended history of episodic star formation (Walcher et al., 2006). This episodic star formation may come from gas funnelled in to the center of the galaxy by local torques.

AGNs are rare or absent from these nuclei of pure disk galaxies (Satyapal et al., 2009). For the nucleus of the nearby system M33, with a total nuclear mass of about  $2 \times 10^6 M_{\odot}$ , Gebhardt et al. (2001) were able to derive an upper limit of  $1500 M_{\odot}$  for a supermassive black hole within the nuclear star cluster.

These nuclear star clusters, with their episodic and extended star formation history, are interesting in their possible relation to some of the Galactic globular clusters, such as the massive cluster  $\omega$  Centauri which also shows evidence of an extended history of episodic star formation (*e.g.* Bellini et al., 2010) and an inhomogeneous distribution of heavy element abundances. This is unusual in globular clusters. Based on chemical evolution arguments, Searle & Zinn (1978) proposed that the Galactic globular clusters originated in small satellite galaxies which were accreted long ago by the Milky Way. The small galaxies are tidally disrupted but the globular clusters survive. Freeman (1993) and Böker (2008) argued that at least some of the globular clusters may have been the nuclei of such satellite systems.

## 4 HI DISKS

### 4.1 HI Distributions, Kinematics and Dynamics

The study of the distribution of HI in samples of more than a few disks in galaxies has been possible only since the advent of aperture synthesis measurements of the 21-cm line. Early observations with single disk instruments could be made only for the very nearest systems, notably the Andromeda Nebula (in particular Roberts & Whitehurst, 1975). Observations with the necessary angular resolution started with the Owens Valley Two-Element Interferometer (*e.g.* Rogstad & Shostak, 1971), the Half-Mile Telescope at Cambridge (*e.g.* Baldwin, Field & Wright, 1971) and the Westerbork Synthesis Radio Telescope (*e.g.* Allen, Goss & van Woerden, 1973). This early work up to about 1977 has been reviewed in van der Kruit & Allen (1978), although mostly in the context of kinematics.

These studies revealed distributions of the HI in most cases to be much more extended than the stellar disks and often warped away from the plane of the stellar disk beyond the boundaries of the light distribution, both in edge-on (Sancisi, 1976) and moderately inclined systems (Rogstad & Shostak, 1971; Bosma, 1981a,b). The most important finding was that the rotation curves at these radii remained flat (see § 1.1).

Since then many observations of disk galaxies have been taken. The first extensive survey, including comparison with optical surface photometry, was done by Wevers, van der Kruit & Allen (1986). More recently the extensive *Westerbork HI Survey of Spiral and irregular galaxies* WHISP (Kamphuis, Sijbring & van Albada, 1996; van der Hulst, van Albada & Sancisi, 2001; Noordermeer et al., 2005; Swaters et al., 2002; Swaters & Balcells, 2002; García-Ruiz, Sancisi & Kuijken, 2002; Noordermeer, van der Hulst, Sancisi, 2007) was made. The most advanced survey at this stage is *The HI Nearby Galaxies Survey* THINGS (Walter et al., 2008; de Blok et al., 2008), which provides very high-resolution HI maps and rotation curves and has been analysed by

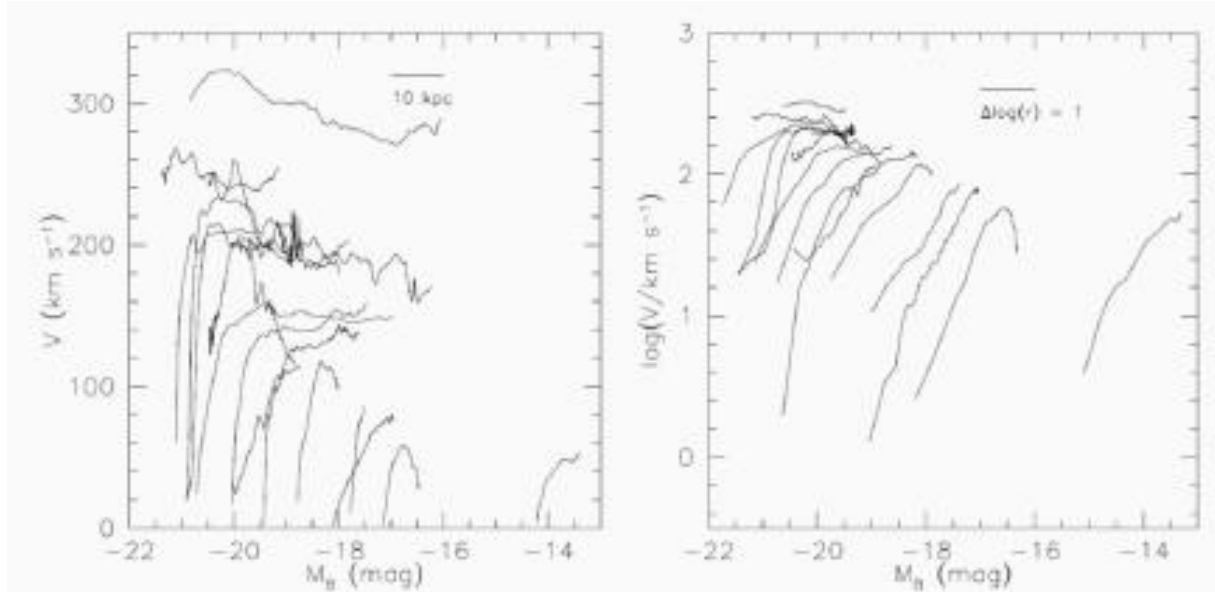


Figure 12: The rotation curves from *The HI Nearby Galaxies Survey* THINGS, plotted in linear units in the left panel and in logarithmic units in the right panel. The origin of the rotation curves has been shifted according to their absolute luminosity as indicated on the horizontal axis such that that the innermost point of each rotation curve is plotted at the  $M_B$  value of the galaxy. The bar in the respective panels indicates the radial scale. (From de Blok et al., 2008).

comparison with  $3.6 \mu\text{m}$  data from the *Spitzer Infrared Nearby Galaxies Survey* SINGS. Fig. 12 shows the rotation curves from THINGS. The details of the rotation curves correlate with the absolute magnitude of the galaxies, as was first described by Broeils (1992). Luminous galaxies have rotation curves that rise steeply, followed by a decline and an asymptotic approach to the flat outer part of the curve; low-luminosity galaxies show a more gradual increase, never quite reaching the flat part of the curve over the extent of their HI disks. Although in some curves the rotation velocity does decrease at large radii, none of the galaxies shows an decline in their rotation curves that can unambiguously be associated with a cut-off in the mass distribution so that in no case has the rotation curve been traced to the limit of the dark matter distribution (de Blok et al., 2008).

We comment on a number of properties of the surface density distribution of the neutral hydrogen first. The total content, especially relative to the amount of starlight  $M_{\text{HI}}/L$  (a property that is distance independent) is well-known to correlate with morphological type (*e.g.* Roberts & Hayes, 1994), while in later types the more luminous galaxies contain relatively less HI (Verheijen & Sancisi, 2001). The HI diameter compared to the optical diameter (at  $25 B\text{-mag arcsec}^{-2}$ ) is about 1.7 with a large scatter, but does not depend on morphological type or luminosity, according to Broeils & Rhee (1997). However, there are very good correlations between HI-mass and - diameter,  $\log M_{\text{HI}}$  and  $\log D_{\text{HI}}$ , both with slopes of about 2. This implies that the HI surface density averaged over the whole HI disk is constant from galaxy to galaxy, independent of luminosity or type. Also note that there is a relatively well-defined maximum surface density in disks of galaxies observed (at least with resolutions of the current synthesis telescopes), which amounts to about  $10 M_{\odot}\text{pc}^{-2}$  (Wevers, van der Kruit & Allen, 1986).

Many systems have asymmetries in their HI-morphologies or -kinematics. Often this is in the form of lopsidedness in the surface density distributions. This can to some extent already be seen in the asymmetries in integrated profiles; studies have claimed that up to 75% of galaxies



are asymmetric or lopsided (Matthews, van Driel & Gallagher, 1998; Hayes et al., 1998; Swaters et al., 1999; Noordermeer et al., 2005) at some detectable level. Lopsidedness appears to be independent of whether or not the galaxy is isolated, so interactions or mergings cannot always be invoked as its origin. It is suggested that it is either an intrinsic property of the disks or is induced by asymmetries in the dark matter distribution (Noordermeer, Sparke & Levine, 2001).

The extraction of kinematic data from the raw observations of moderately inclined systems has often been discussed. The basics are summarized in van der Kruit & Allen (1978), Bosma (1981a), Wevers, van der Kruit & Allen (1986) and Begeman (1987). The results are a radial distribution of HI surface brightness, a rotation curve and a radial distribution of velocity dispersion; from these one can in principle make maps of residuals compared to these azimuthal averages. The derivation of the HI velocity dispersions is easiest in face-on spirals, where there is no gradient in the systematic motions across a telescope beam (van der Kruit & Shostak, 1982, 1984; Shostak & van der Kruit, 1984). In moderately inclined systems it is possible to measure the velocity dispersion, provided that the angular resolution and signal-to-noise are sufficiently high and that the effects of HI layer thickness can be separated from that of random motions in the observed profiles (see Sicking, 1997).

For edge-on systems the procedure is more complicated as a result of the line-of-sight integration. Various methods have been devised, initially only to derive the rotation curve and the radial distribution of HI surface brightness, later in some cases also the flaring (the increasing thickness as a function of galactocentric radius) of the HI layer (*e.g.* Sancisi & Allen, 1979; van der Kruit, 1981; Sofue, 1986; Rubin, Burstein & Ford, 1985; Mathewson, Ford & Buchhorn, 1992; Takamiya & Sofue, 2002; García-Ruiz, Sancisi & Kuijken, 2002; Uson & Matthews, 2003; Kregel, van der Kruit & de Blok, 2004; Kregel & van der Kruit, 2004a). Recently Olling (1996a,b) and O’Brien et al. (2010); O’Brien, Freeman & van der Kruit (2010a,b) have also fit for the HI velocity dispersion. The paper by O’Brien, Freeman & van der Kruit (2010a) provides a detailed description of the various methods and a discussion on the relative merits and pitfalls.

In general, for larger disk galaxies, the radial distributions show (sometimes in addition to a central depression) a radial surface density that falls off more slowly than that of the starlight, a rotation curve that rises to a maximum and stays at that level, and a velocity dispersion of 7 – 10 km s<sup>-1</sup>, often near the higher end of this range in the inner regions and in spiral arms and in the lower range in the outer parts and inter-arm regions (Shostak & van der Kruit, 1984; van der Kruit & Shostak, 1984; Sicking, 1997).

The velocity dispersion of the HI can be measured only in cases where there is a negligible gradient in the overall radial velocity over the beam of the radio telescope. Early determinations have been made in our Galaxy; van Woerden (*e.g.* 1967) found 7 km s<sup>-1</sup> for the Solar Neighborhood and Emerson (1976) found  $12 \pm 1$  km s<sup>-1</sup> from aperture synthesis observations using the Half-Mile Telescope at Cambridge. For larger angular size galaxies, the use of face-on galaxies (as judged from the narrowness of their integrated HI profiles) is required to fulfill this condition. It was done in much detail on three galaxies that are only a few degrees from face-on (NGC 3938, 628 and 1058; van der Kruit & Shostak, 1982, 1984; Shostak & van der Kruit, 1984). They found that the dispersions over the optical extent of the disks were rather constant, with no significant decline with radius at 7 – 10 km s<sup>-1</sup>. Only in NGC 628 was a systematic difference seen between the inter-arm region (at the lower end of the range quoted) and the spiral arms themselves (the higher end).

The most recent study of galaxies that are not very close to face-on, is *The HI Nearby Galaxy Survey* (THINGS) by Tamburro et al. (2009), where references to other earlier work can be found. They do find significant declines of velocity dispersion with radius in their sample, but there appears a characteristic value of  $10 \pm 2$  km s<sup>-1</sup> at the outer extent of the star-forming part of the disk. Inward of this, the dispersion correlates with indicators of star formation, suggesting that the supernovae associated with this star formation activity are driving the turbulence, although

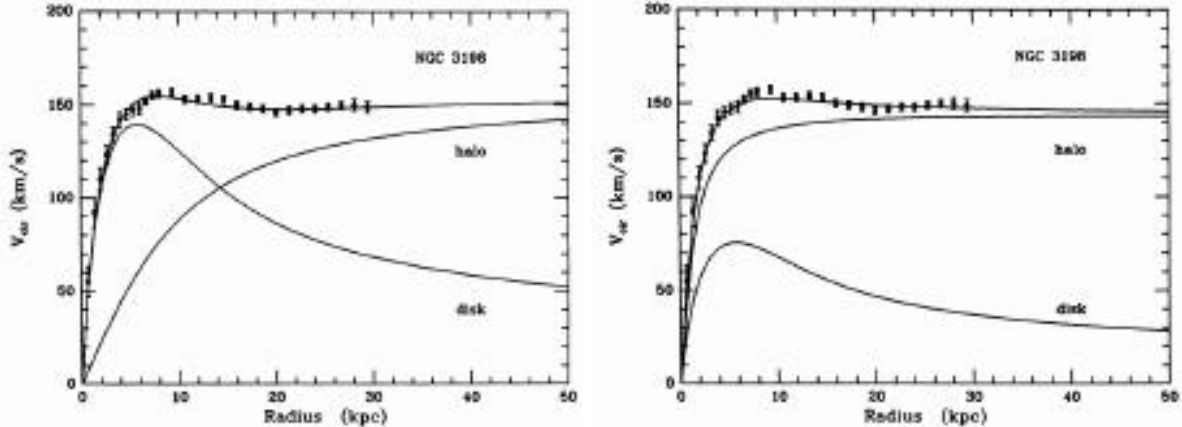


Figure 13: The rotation curve decomposition for NGC 3198. The shape of the curve for the dark matter halo is set by a core radius that determines the initial rise and tends asymptotically to that of a mass density distribution that falls off  $\propto R^{-\gamma}$  with  $\gamma$  close to 2. The shape of the disk curve is that of the exponential disk with a scalelength from the light distribution. On the left the ‘maximum disk’ fit that maximizes the amplitude of the disk rotation curve and on the right an arbitrary fit with a disk mass 0.3 times that of the maximum disk. (From van Albada et al., 1985)

magnetorotational instabilities may add significantly to the random motions as well. In edge-on galaxies, O’Brien et al. (2010) and O’Brien, Freeman & van der Kruit (2010a,b) made detailed analyses to retrieve the radial distributions of HI surface density, rotation velocity and velocity dispersion, finding that there is significant structure, but not much systematic radial decline. This study involved HI rich, dwarf systems, quite dissimilar from the SINGS sample. Typical dispersions are  $6.5$  to  $7.5$   $\text{km s}^{-1}$ , increasing with the amplitude of the rotation curve. In the solar neighborhood the velocity dispersion of OB stars is comparable to that in the HI, so stars are born with that same amount of random motion which then increases as a result of secular evolution. The same will happen in other spirals, but in dwarf galaxies the stellar velocity dispersions are similar to the ones reported in HI. This may suggest that no significant secular evolution of random motions occurs in dwarf systems.

## 4.2 Dark Matter Halos

The discovery of dark matter halos has been described very briefly in § 1.1, where also the concept of the ‘maximum-disk hypothesis’ (see fig. 13) was introduced, in which the contribution of the stellar disk to the rotation curve is taken to be as large as possible, consistent with the observations (Carignan & Freeman, 1985; van Albada et al., 1985; van Albada & Sancisi, 1986). As mentioned above, this contribution has in practice an amplitude at its maximum within the range  $0.85 \pm 0.10$  of the observed maximum rotation, following Sackett (1997). The evidence from stellar dynamics (see § 3.2) is that the majority of galaxies have disk masses significantly below maximum disk, except for some galaxies with the highest surface brightness.

Recent reviews of disk masses in galaxies and implications for decompositions of rotation curves into contributions from dark and baryonic matter have been presented by van der Kruit (2009) and McGaugh (2009) at the Kingston symposium. These reviewers take the view that most galaxy disks are sub-maximal, except possibly those with the highest surface brightness and surface density. In contrast to these studies that involve mostly non-barred galaxies, we note that Weiner, Sellwood & Williams (2001) and Pérez et al. (2004) find from detailed fluid

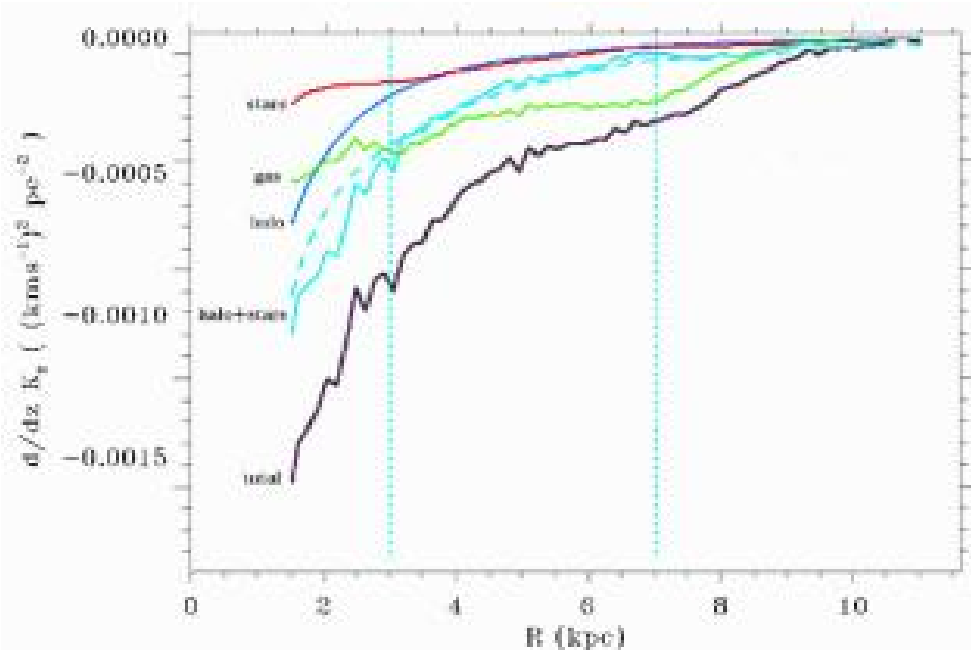


Figure 14: The vertical gradients of the force field for various components in the edge-on galaxy UGC 7321. For an explanation see the text. The measured halo flattening was  $q = 1.0 \pm 0.1$ . The vertical dotted (cyan) lines show the radial regime used for the fit. (From O’Brien, Freeman & van der Kruit, 2010c)

dynamical gas flows in some barred galaxies that their disks are close to maximal. Debattista & Sellwood (2000) argue from the observed rapid rotation of the bars of barred galaxies that the stars are dominating the gravitational field in the inner regions of these galaxies.

The thickness of the gas layer in a disk galaxy can be used to measure the surface density of the disk. It has been known for a long time that this layer in our Galaxy is ‘flaring’ (for a recent discussion see Kalberla & Kerp, 2009). Assume that the vertical density distribution of the exponential stellar disk is locally isothermal ( $n = 1$  in eqn. (1)). If the HI velocity dispersion is  $\langle V_z^2 \rangle_{\text{HI}}^{1/2}$  and isotropic, and if the stars dominate the gravitational field, then the HI layer has a full width at half maximum (to  $\lesssim 3\%$ ) of

$$W_{\text{HI}} = 1.7 \langle V_z^2 \rangle_{\text{HI}}^{1/2} \left[ \frac{z_0}{\pi G(M/L)\mu_0} \right]^{1.2} e^{R/2h}. \quad (20)$$

So, if the HI velocity dispersion is independent of radius, the HI layer increases exponentially in thickness with an e-folding length  $2h$ . This was first derived and applied to HI observations of NGC 891 by van der Kruit (1981) to demonstrate that the dark matter indicated by the rotation curve did indeed not reside in the disk but in a more spherical volume. Using the photometry in van der Kruit & Searle (1981b) and the observed HI flaring (Sancisi & Allen, 1979) and taking a gaseous velocity dispersion of  $10 \text{ km s}^{-1}$  resulted in a rotation curve of the disk alone with a maximum of  $\sim 140 \text{ km s}^{-1}$ . A smaller value for  $\langle V_z^2 \rangle_{\text{HI}}^{1/2}$  would, according to eqn. (20), result in a smaller value for the inferred  $M/L$  for the same HI thickness and therefore to a lower value for the maximum disk-alone rotation. The observed maximum rotation velocity is  $225 \pm 10 \text{ km s}^{-1}$ , which indicates that NGC 891 is not maximum disk. In a similar analysis, Olling (1996b) inferred for NGC 4244 that the maximum disk-alone rotation velocity is between 40 and 80% of the observed rotational velocity.

Another important property of dark matter halos for understanding galaxy formation is their three-dimensional shape. There are various ways to address this issue, of which the use of the flaring of the HI layer is the most prominent. It was first developed by Olling (1995) and subsequently applied to the nearby edge-on dwarf galaxy NGC 4244 Olling (1996a,b). He found the dark halo of this galaxy to be highly flattened. Observations of the kinematics in polar ring galaxies provide estimates of the potential gradients in the two orthogonal planes of the galaxies. These galaxies can potentially give useful information about the shapes of their dark halos (Sackett, 1999), although it is possible that special halos are needed to host these rare polar ring systems. The resulting halo flattenings derived for polar ring galaxies and also for streams in the halo of the Galaxy (Helmi, 2004) range from a few tenths to unity.

A more extensive survey of a sample of eight late-type, HI-rich dwarfs in which the dark halo appears to dominate the gravitational field even in the disk, was undertaken by O’Brien et al. (2010). The basic premise of the approach is that the radial gradient of the dark matter halo force  $\partial K_R/\partial R$  can be measured from the rotation curve after correction for the contribution of the stellar disk and its ISM, while the flaring of the HI layer together with a measurement of the velocity dispersion of the HI provides a measure of the vertical gradient  $\partial K_z/\partial z$ . The ratio of the two force gradients is related to the flattening of the dark halo (assumed spheroidal), as measured by the axis ratio  $c/a$ . The derivation of the necessary properties from the HI observations have been presented in O’Brien, Freeman & van der Kruit (2010a,b). The method relies on two assumptions concerning the HI velocity dispersion, that it is isotropic (since we measure in an edge-on galaxy the line-of-sight dispersion component parallel to the plane and use in the analysis the vertical component) and isothermal with  $z$ . High signal-to-noise observations in edge-on and moderately inclined galaxies are needed.

The first galaxy for which this analysis has been completed is UGC 7321 (O’Brien, Freeman & van der Kruit, 2010c). The halo density distribution was modelled as a pseudo-isothermal spheroid (Sackett et al., 1994) and the disk as a double exponential determined from  $R$ -band surface photometry. After allowing for the gravitational field of the gas layer, the hydrostatic equations gives the  $M/L$  of the disk and a value for the flattening of the dark matter halo. For UGC 7321, the  $M/L$  in the disk is very low: the contribution of the stellar disk to the radial force is small and is far below ‘maximum disk’ in this low surface brightness galaxy (Banerjee, Matthews & Jog, 2010, reached the same conclusion). The best fits of the force gradients  $\partial K_z/\partial z$  from the various components are shown in fig. 14. The vertical gradient of the total vertical force field derived from the hydrostatics of the flaring gas layer (for stars + gas + halo) is shown in black as a function of radius. After taking off the known contribution from the gas layer, the blue curve labelled ‘halo + stars’ shows the remaining contribution from the halo + stars. This is well modelled for  $R > 3$  kpc by the adopted contributions from the stars and the dark halo model (dashed curve). Note that the shape of the observed and fitted curves are remarkably similar for  $R > 3$  kpc. For this best fit, the dark halo model is spherical. More analysis is required for the rest of the sample to draw firm conclusions.

In the Galaxy the flaring of the disk can be studied in much more detail. An extensive study has been performed recently by Kalberla et al. (2007) based on the Leiden/Argentina/Bonn all-sky survey of Galactic HI (see also Kalberla & Kerp, 2009). In this study they map the Galactic HI layer and its flaring and warp in much detail and fit the observations with models containing a dark matter disk as well as a dark matter halo. Their best fitting model has a local disk surface density of  $52.5 M_\odot \text{ pc}^{-2}$  with a scalelength of 2.5 to 4.5 kpc. This corresponds to a maximum ‘disk-alone’ rotation velocity of respectively 200 to 130  $\text{km sec}^{-1}$ . So again, if the Galaxy has a normal scalelength ( $\sim 4$  kpc) for its rotation velocity, it is far below maximum disk, but if the scalelength is 2.5 kpc it is close to maximum disk. A complete analysis of the flaring HI disk in terms of Galactic dark matter distribution yields evidence for a significant dark matter disk with a large scalelength of order 8 kpc and in addition a dark matter ring at

13 to 18 kpc. This ring could be the remnant of a merged dwarf galaxy.

The distribution of the density in the inner parts of dark halos has been a subject of much attention, as a consequence of the Cold Dark Matter paradigm (Blumenthal et al., 1984, 1986), in which structure grows hierarchically with small objects collapsing first and then merging into massive objects. Cosmological  $n$ -body simulations based on  $\Lambda$ CDM (Dubinski & Carlberg, 1991; Navarro, Frenk & White, 1996, 1997) have long predicted that the inner density profile of the dark matter halo ( $\rho \propto r^\alpha$ ) would have an exponential slope  $\alpha$  of about  $-1$  (a ‘*cusp*’), while observations seemed to suggest a slope near zero (a ‘*core*’). The high spatial resolution of the THINGS data (de Blok et al., 2008) are well suited to investigate this matter. They find that, for massive disk-dominated galaxies, all halo models appear to fit equally well, while for low-mass galaxies a core-dominated halo is clearly preferred over a cusp-like halo. This cusp-core controversy (with  $\alpha$  assuming somewhat different values) is a long-lasting hot item in the study of HI-rich, dwarf galaxies and low surface brightness systems, where the contribution of the dark matter to the rotation curve is large even in the inner regions. Recently, de Blok (2010) has summarised the situation in such systems, concluding that the problem is still unsolved, even with the use of current high resolution rotation curves. This issue has potentially important implications for galaxy formation models in  $\Lambda$ CDM.

### 4.3 Outer HI and Warps

The warping of the outer parts of the neutral hydrogen layer of our Galaxy has been known for a long time. It was discovered independently in early surveys of the Galactic HI in the north by Burke (1957) and in the south by Kerr and Hindman (see Kerr, 1957). In external spiral galaxies the first indication came from the work of Rogstad, Lockart & Wright (1974), when they obtained aperture synthesis observations of the HI in M83. The distribution and the velocity field of the HI both showed features that could be interpreted as a warping of the gaseous disk in circular motions in inclined rings. This later became known as the ‘tilted-ring’ description for ‘kinematic warps’ and many more galaxies have been shown to have such deviations using this method (*e.g.* Bosma, 1981a,b). The case that this warping occurs in many spiral galaxies was strengthened by the early observations of edge-on systems. Sancisi (1976) was the first to perform such observations and showed that the HI in four out of five observed edge-ons (including NGC 4565 and NGC 5907) displayed strong deviations from a single plane. Sancisi (1983) discussed these warps in somewhat more detail and in particular noted that in the radial direction the HI surface densities often display steep drop-offs followed by a ‘shoulder’ or ‘tail’ at larger radii.

The most extreme (‘prodigious’) warp in an edge-on system was observed by Bottema, Shostak & van der Kruit (1987) (see also Bottema, 1995, 1996) in NGC 4013. García-Ruiz (2001); García-Ruiz, Sancisi & Kuijken (2002) presented 21-cm observations of a sample of 26 edge-on galaxies in the northern hemisphere. This showed that HI-warps are ubiquitous; the authors state that “*all galaxies that have an extended HI-disk with respect to the optical are warped*”. Studies of possible warps in the stellar disks have also been made; for recent results see § 3.7; although there is evidence for such stellar warps in most edge-on galaxies, the amplitude is very small compared to what is observed in the HI.

The origin of warps has been the subject of extensive study and has been reviewed for example by García-Ruiz, Sancisi & Kuijken (2002), Shen & Sellwood (2006), Binney (2007) and Sellwood (2010a). Although none of the models is completely satisfactory, most workers seem to agree that it has something to do with a constant accretion of material with an angular momentum vector misaligned to that of the main disk. In models by Jiang & Binney (1999) and Shen & Sellwood (2006), this results in an inclined outer torus in the dark matter halo that distorts the existing disk and causes it to become warped. The possibility of a misalignment in the angular

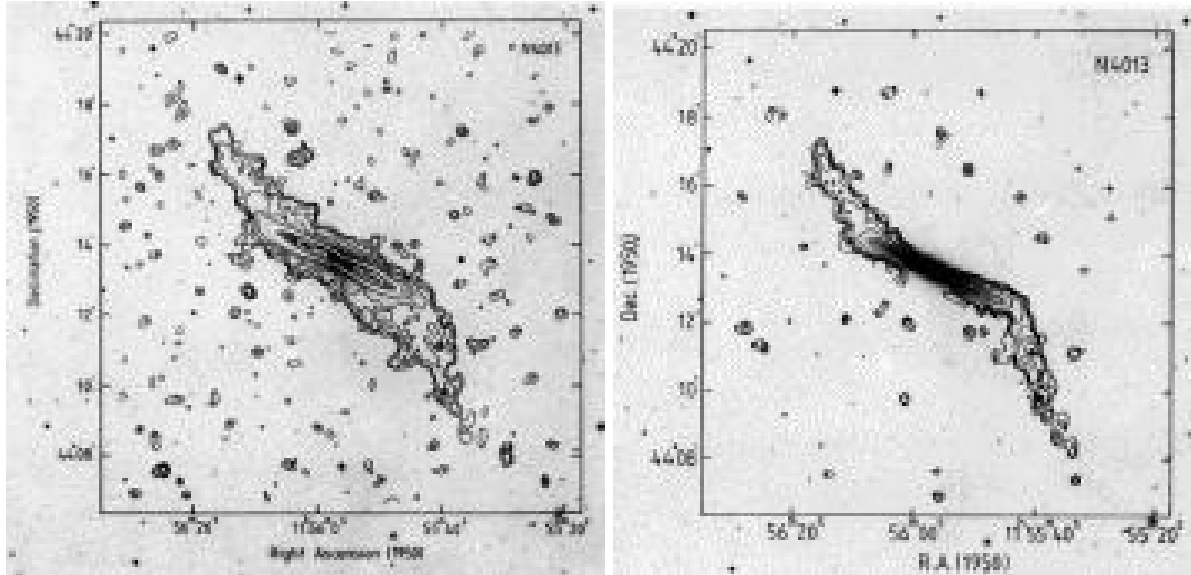


Figure 15: The HI distribution in the edge-on spiral galaxy NGC 4013 (Bottema, Shostak & van der Kruit, 1987). On the left the integrated HI map and on the right only the ‘high’ velocity channel maps. The latter selects the HI along a line perpendicular to the line-of-sight to the galaxy. The warp is less well-defined in the view of the left, since the azimuthal angle in the plane of the galaxy, along which the deviation of the warp is largest, is not precisely perpendicular to the line-of-sight. The warp starts very abruptly at almost exactly the truncation radius of the stellar disk. (From Bottema, 1995)

momenta and therefore of the principal planes of the stellar disk and the dark halo (Debattista & Sellwood, 1999) has recently received some observational support from the observations of Battaglia et al. (2006) of NGC 5055. The HI data suggest that the inner flat disk and the outer warped part of the HI have different kinematic centers and systemic velocities, suggesting a dark matter halo with not only a different orientation, but also an offset with respect to the disk. The detailed study of the HI in NGC 3718 by Sparke et al. (2009) shows an extensive warping, for which the observed twist can be explained as a result of differential precession in a fairly round dark halo with the same orientation as the disk. Briggs (1990) used existing observations and tilted-ring models in moderately inclined galaxies to define a set of “rules of behavior for galactic warps”. One was that “warps change character at a transition radius near  $R_{Ho}$ ”. The latter radius is the Holmberg-radius (see footnote 23) listed for 300 bright galaxies in Holmberg (1958).

The onset of HI warps seems to occur very close to the radius of the truncation in the stellar disk (van der Kruit, 2001). This can be illustrated with two archetypal examples. In an edge-on galaxy, the most pronounced warp known is in NGC 4013 (Bottema, Shostak & van der Kruit, 1987; Bottema, 1995, 1996). The HI warp starts very abruptly at the truncation of the stellar disk (see fig. 15) and is accompanied by a significant drop in rotational velocity. The latter suggests a truncation in the disk mass distribution as well as in the light. A face-on spiral with an HI-warp is NGC 628 (Shostak & van der Kruit, 1984; Kamphuis & Briggs, 1996). The velocity field suggest a warp in the HI, starting at the edge of the optical disk.

In an extended study, van der Kruit (2008) listed the following general characteristics of HI warps. Whenever a galaxy has an HI-disk extended with respect to the optical disk, it has an HI-warp (García-Ruiz, Sancisi & Kuijken, 2002). Many galaxies, but not all, in this sample have relatively sharp truncations in their stellar disks. When an edge-on galaxy has an HI warp, the

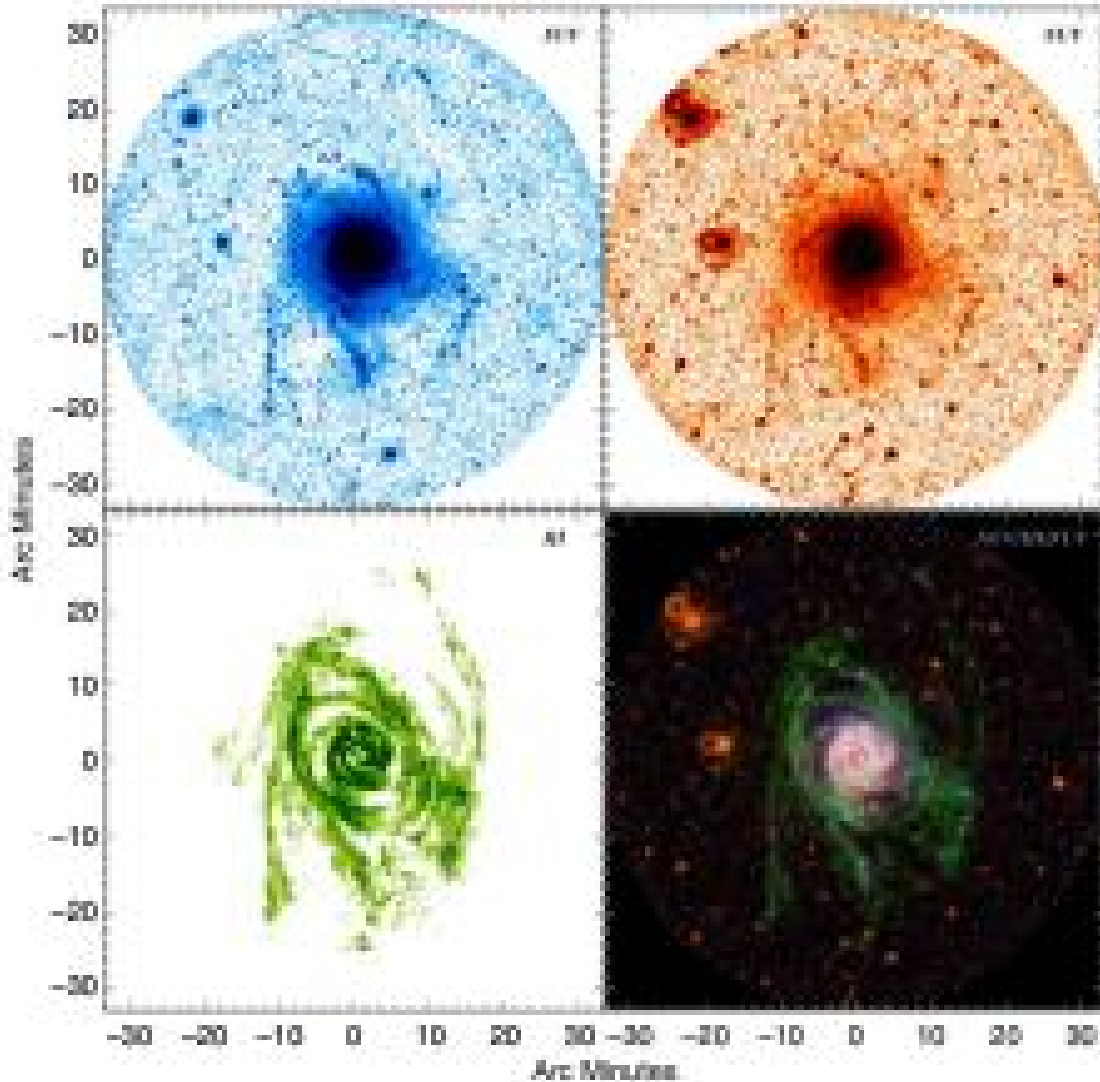


Figure 16: Images of M83, Top: GALEX far-UV and near-UV; Bottom: HI map from THINGS and a combination of the three with the HI smoothed. (From Bigiel et al., 2010a)

onset occurs just beyond the truncation radius. Similarly, in less inclined galaxies, the warp is seen at the boundaries of the observable optical disk (Briggs, 1990). In many cases the rotation curve shows a feature at about the truncation radius (Casertano, 1983; Bottema, 1996), which indicates that the truncation occurs also in the mass. The onset of the warp is abrupt and discontinuous and coincides in the large majority of cases with a steep drop in the radial HI surface density distribution, after which this distribution flattens off considerably. The inner disks are extremely flat (both stellar disks as well as in gas and dust) and the onset of the warp is *abrupt*; beyond that, according to Briggs (1990), the warp defines a “new reference frame”.

These findings suggest that the inner flat disk and the outer warped disk are distinct components with different formation histories, probably involving different epochs. The inner disk forms initially (either in a monolithic process in a short period or hierarchically on a somewhat more protracted timescale) and the warped outer disk forms as a result of much later infall of gas with a higher angular momentum in a different orientation. This is also consistent with an origin of the disk truncations that is related to the maximum specific angular momentum

available during its formation, since then the truncation is also in the disk mass, giving rise to the abruptness of the onset of the warps.

The misalignment of the material in the inner disk and the outer disk and warp has been modelled by Roškar et al. (2010) as a result of the interaction between infalling cold gas and a misaligned hot gaseous halo. The gas that forms the warp is torqued and aligned with the hot gaseous halo rather than the inner disk. In this model the outer accreted gas thus responds differently to the halo than the gas that has formed the inner disk. Although this may be consistent with the observed correlation of the onset of the warp and the truncation or break in the stellar disk, the abruptness of the onset of the warp may not follow naturally. Furthermore, this mechanism requires the existence of a hot gaseous halo and the question is whether these are present in galaxies with smaller rotation amplitudes; the presence of warps in M33-size galaxies would then be another argument against this hypothesis.

To what extent has star formation and chemical enrichment taken place in these gaseous warps? The early work of Ferguson et al. (1998a,b) established the presence of star formation in regions beyond two  $R_{25}$  in a few galaxies through the detection of faint HII-regions, while emission line ratios indicated very low abundances of oxygen and nitrogen in these regions. Since then much work has been done and more is in progress, in particular in the *ACS Nearby Galaxy Survey Treasury* (ANGST, Dalcanton et al., 2009). Early results of this project have been published for M81 (Williams et al., 2009; Gogarten et al., 2009) and NGC 300 (Gogarten et al., 2010), but those do not yet refer to distances beyond the visible spiral structure. However, the disks do contain mostly old stars at these radii, in agreement with the view that the disk inside the truncation region and the radial onset of the HI-warp forms at an early epoch in the galaxy’s evolution. In NGC 2976 (Williams et al., 2010), populations of old ages are found at all radii, also beyond the break in the luminosity profile, but star formation does not appear now to extend into this outer zone. This galaxy may have been in interaction recently with the M81-group.

Shostak & van der Kruit (1984; see their fig. 3) pointed out that in NGC 628 (M74) the extended, warped HI displays spiral structure that shows a smooth continuation of the prominent spiral structure in the main disk, right through the onset of the warp. Particularly interesting for studying star formation and its history in disks beyond the truncation is the comparison of outer HI with UV imaging from GALEX. Bigiel et al. (2010a) compare the HI from the THINGS (HI) survey with the far UV data from GALEX and conclude that, although star formation does clearly take place in the outer HI, its efficiency is extremely low compared to that within the optical radius (truncation). A detailed comparison of images for M83 (Bigiel et al., 2010b, see fig. 16) shows a clear correlation between star formation and HI surface density. The conclusion that star formation is proceeding in this galaxy in the very outer parts also follows from the work of Davidge (2010), comparing GALEX imaging to deep optical (Gemini) images of the outer regions of M83.

#### 4.4 Dustlanes in Disks

We will not review in detail the distributions and properties of dust in disks of galaxies. However, we noted above that dustlanes are very straight, such as illustrated in fig. 9, at least in disks of massive galaxies and indicate that galaxy disks are extraordinarily flat. It has been known for a long time that dustlanes sometimes are less well-defined in galaxies of later type and dwarfs. A good example is NGC 4244, which is a late-type, relatively small, pure-disk galaxy (van der Kruit & Searle, 1981a), where the dustlane is not sharply outlined against the stellar disk. Dalcanton, Yoachim & Bernstein (2004) studied a sample of edge-on galaxies and found that systems with maximum rotation velocities larger than  $120 \text{ km s}^{-1}$  have well-defined dustlanes, while in those with smaller rotations the scaleheight of the dust is systematically larger and the



distribution much more diffuse. Indeed, NGC 4244 has a maximum rotation velocity of about  $115 \text{ km s}^{-1}$ . This finding may have important implications for our understanding of the star formation history and evolution of disks.

Dalcanton, Yoachim & Bernstein (2004) suggest that the transition at  $120 \text{ km s}^{-1}$  marks the rotation speed above which the disks become gravitationally unstable, whereby instabilities in the disk lead to fragmentation of the gas component with high density during a collapse which then gives rise to thin dustlanes. In a study of the vertical distributions in a number of low-mass edge-on galaxies, Seth, Dalcanton & de Jong (2005) find that not only does the dynamical heating of the stellar population appear to occur at a much reduced rate compared to the Galactic disk in the Solar Neighborhood, but in these low-mass systems the vertical distribution of the young stellar population and of the dust layer is thicker than those in the Milky Way. This is consistent with a cold interstellar medium in slowly rotating galaxies that has a larger scaleheight and therefore with an absence of well-defined dustlanes in such systems. Seth, Dalcanton & de Jong (2005) fit the distributions with an isothermal sheet ( $n = 1$  in eqn. (1) with  $z_o = 2h_z$ ). In NGC 4144 (rotation velocity  $67 \text{ km s}^{-1}$ ), the dust has a scaleheight  $z_o$  of about 0.5 kpc or an exponential scaleheight of half that. For comparison, in the Milky Way the three-dimensional distribution of dust has been modelled by Drimmel & Spergel (2001): their flaring dust disk has a similar exponential scaleheight at the solar radius of about 0.2 kpc.

We may examine more massive edge-on galaxies with rotation velocities well over  $200 \text{ km s}^{-1}$  to compare with the low-rotation systems. For example, three such galaxies with obvious dustlanes are NGC 4565, 891 and 7814 for which minor axis profiles (in blue light) are available in the papers of van der Kruit & Searle (1981a,b, 1982b). When we determine the height above the symmetry plane at which the effect of the dust extinction is about one magnitude compared to the extrapolated minor axis profile of the stellar light, we get values of respectively 0.9, 0.7 and 0.6 kpc. These are undoubtedly overestimates when used as indications for the dust scaleheights; Kylafis & Bahcall (1987) for example find that the dust scaleheight in NGC 891 is 0.22 kpc, compared to the stellar scaleheight of 0.5 kpc (van der Kruit & Searle, 1981b). Wainscoat, Hyland & Freeman (1989) determined in IC 2531 (an NGC 891-like edge-on galaxy) that the scaleheight of the old disk stars is about 0.5 kpc, while that of the dust is a quarter of that. A similar determination of the height at which the minor-axis profile indicates an absorption of one magnitude compared to the extrapolated profile yields about 0.5 kpc. On this basis the exponential scaleheights of the dust layers in the three systems just mentioned are of the order of 3 or 4 times less than those of the old stellar disks. The important inference is that the diffuse dust layers in slow rotators have, in absolute measures, thicknesses that are not very different from those in massive galaxies with high rotation velocities.

In early-type edge-on galaxies such as NGC 7814 and NGC 7123, where the spheroids dominate the light (and presumably stellar mass) distributions, the situation appears different. The dust distributions in these systems have scaleheights comparable to those of the stellar disks (Wainscoat, Hyland & Freeman, 1989), which are of the order of 0.5 to 1 kpc. This may be the result of much longer dissipation times due to the lower gas content, so that the scaleheights of the stars (from which the dust comes) and the dust are similar.

In the sample of late-type, gas-rich dwarf edge-on galaxies studied by O'Brien, Freeman & van der Kruit (2010b), the mean HI velocity dispersion increases as a function of the maximum rotational velocity of the HI disk from about 5 to  $8 \text{ km s}^{-1}$  for rotation velocities of 70 to  $120 \text{ km s}^{-1}$ . The scaleheight  $h$  of a Gaussian dust layer is related to the ISM velocity dispersion  $\sigma$  by

$$\frac{\sigma^2}{h^2} = -\frac{\partial K_z}{\partial z} = 4\pi G\rho_o, \quad (21)$$

where  $\rho_o$  is the mid-plane total density. The density  $\rho_o = \Sigma/2h_z$  where  $\Sigma$  is the typical surface density of a disk.  $\Sigma$  and the stellar scaleheight are both approximately proportional to the

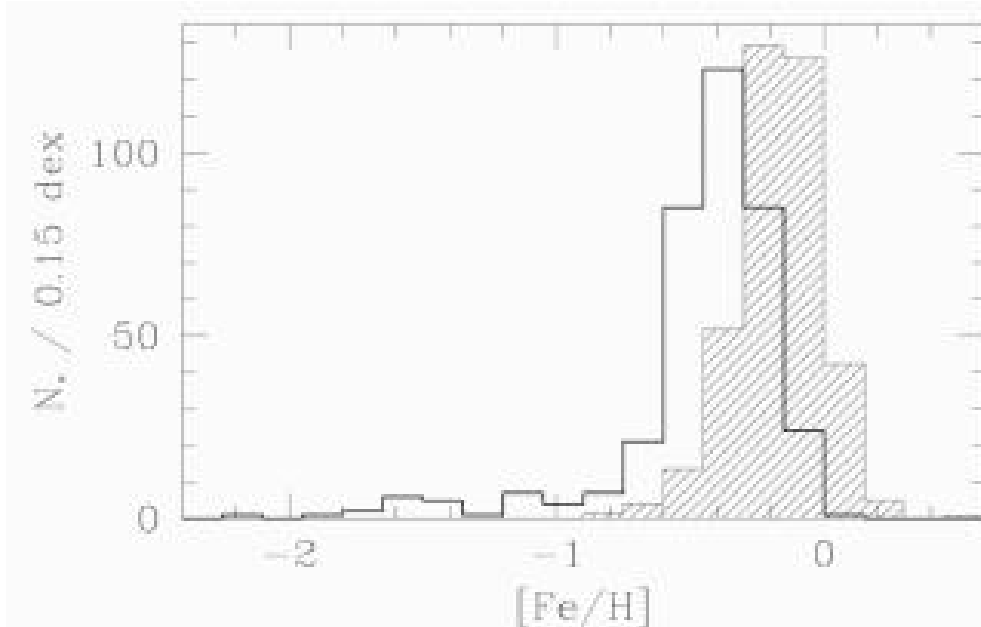


Figure 17: Metallicity distribution function for stars in the bar of the LMC, compared to the solar neighborhood MDF from McWilliam (1990) (hatched). The solar neighborhood MDF is about 0.2 dex more metal-rich. (From Cole et al., 2005).

maximum circular velocity  $V_c$  (Gurovich et al., 2010; Kregel, van der Kruit & de Grijs, 2002; Kregel, van der Kruit & Freeman, 2005), so the typical value of  $\rho_o$  is independent of  $V_c$  and we expect the scaleheight of the dust layer to be directly proportional to the ISM velocity dispersion  $\sigma$ . In the O’Brien sample of galaxies, the galaxy with the largest  $V_c$  (IC2531) shows a clean well-defined dustlane as expected from the Dalcanton et al. observations. However there is no evidence for a decrease in  $\sigma$  as  $V_c$  increases in the O’Brien et al. sample, so those data do therefore not support the variable turbulence explanation for the change in dust lane morphology with rotation velocity.

## 5 CHEMICAL EVOLUTION AND ABUNDANCE GRADIENTS

The stars and gas in galactic disks have a mean metallicity which depends on the luminosity of the galaxy (*e.g.* Tremonti et al., 2004) and often shows a radial gradient. In a large galaxy like the Milky Way, the typical disk metallicity  $[\text{Fe}/\text{H}]$  is near that of the sun. For example, in the solar neighborhood, the metallicity of the disk stars has a mean of about -0.2 and ranges from about +0.3 to -1.0 (*e.g.* McWilliam, 1990). Fig. 17 compares the stellar metallicity distribution functions (MDFs) of the solar neighborhood and the bar of the LMC. As expected from the lower luminosity of the LMC, the LMC MDF is displaced towards lower metallicities, but the shapes of the two MDFs are similar.

The metallicity distribution function (MDF) is believed to come from the local chemical evolution of the stars and ISM, including the effect of infall of gas from outside the Galaxy. The presence of a fairly tightly defined radial abundance gradient in the stars and in the gas in many disk galaxies suggests that the chemical evolution of the disk is determined mainly by local chemical evolution with limited radial exchange of evolution products. Much of the theory of chemical evolution of disk galaxies is based on this assumption (*e.g.* Chiappini, Matteucci &

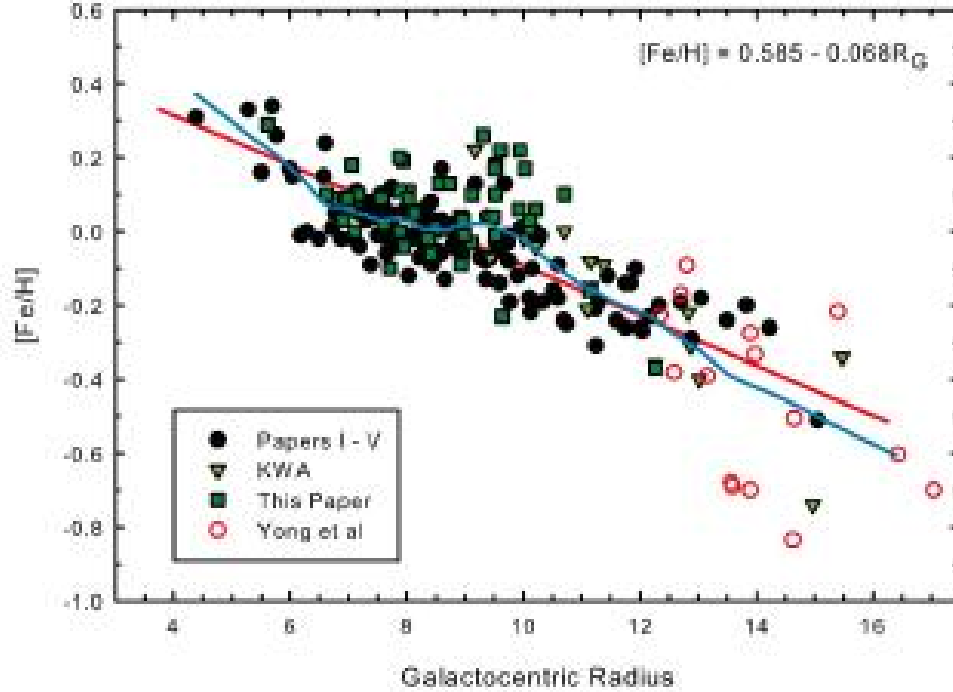


Figure 18: The radial abundance distribution for Cepheids in the Galactic disk. (From Luck, Kovtyukh & Andrievsky, 2006).

Gratton, 1997); this view is however currently under challenge.

From the basic theory of chemical evolution via the continued formation and evolution of stars and enrichment of the interstellar medium, one might expect that the metallicity of disk stars would gradually increase with time. McWilliam (1997) pointed out in his review that it was not clear observationally at the time that any age-metallicity relation (AMR) exists among the stars of the solar neighborhood. This is still the situation today. Edvardsson et al. (1993) found evidence for a weak decrease of  $[Fe/H]$  with age among the disk stars, but later work (*e.g.* Nordström et al., 2004) is less conclusive. From the white dwarf luminosity function and the directly measured ages of disk stars (*e.g.* Knox, Hawkins & Hambly, 1999; Edvardsson et al., 1993; Sandage, Lubin & Vandenberg, 2003), the age of the Galactic thin disk is about 8–10 Gyr. The open cluster NGC 6791 has an age of about 8–10 Gyr and an abundance  $[Fe/H]= +0.2$ , indicating that enrichment to solar level occurred very quickly in the Galactic disk, on a Gyr timescale. An early AMR figure by Sandage & Eggen (1969), based partly on open cluster ages and metallicities, shows a very rapid early evolution of the galactic abundances up to near-solar abundances and then little further change; this is still a fair representation of the current state of knowledge.

The rapidly rising and then flat AMR in the disk of the Milky Way contrasts with the situation in the LMC. Dolphin (2000) derived the star formation history and AMR for two fields in the LMC disk. The star formation rate shows early and late phases of star formation, with a very slow period between about 3 and 7 Gyr ago, and the AMR shows a smooth rise from below  $-1.5$  at 10 Gyr ago to the present metallicity of  $[Fe/H]= -0.4$ . A smoothly rising AMR is seen also in the outer regions of M33 (Barker et al., 2007b). This difference in the morphology of the AMR between the Milky Way and the smaller galaxies is likely to come from the different star formation histories of larger and smaller disk systems (the downsizing phenomenon).

## 5.1 Gas-phase Abundance Gradients

Many galaxies show a clear radial gradient in their gas-phase abundances. Zaritsky, Kennicutt & Huchra (1994) assembled data on oxygen abundance gradients in 39 disk galaxies covering a range of luminosities. They found the now-familiar correlations between oxygen abundance and luminosity, circular velocity and morphological type. The size of the abundance gradients (in dex/isophotal radius) did not correlate with luminosity or type. The presence of a bar appears to flatten or even erase the abundance gradient (see also Alloin et al., 1981), probably due to the non-circular motions which the bar induces in the gas of the disk.

Magrini et al. (2007) modelled the chemical evolution of M33, assuming that the galaxy is accreting gas from an external reservoir. A model with an infall rate of about  $1 M_{\odot} \text{ yr}^{-1}$  reproduces the observational constraints, including the relatively high star formation rate and the shallow abundance gradient. The model indicates that the metallicity in the disk has increases with time at all radii, and the abundance gradient has continuously flattened over the past 8 Gyr (see also Gogarten et al., 2010, for a comparison with the evolution of the somewhat similar disk galaxy NGC 300).

For the Milky Way, Shaver et al. (1983) combined radio and optical spectroscopy to measure abundances for HII-regions between about 3 and 14 kpc from the galactic center. Fich & Silkey (1991) extended the observations to a radius of about 18 kpc. A well defined gradient in the oxygen and nitrogen abundances of  $-0.07$  to  $-0.08 \text{ dex kpc}^{-1}$  is found. Dennefeld & Kunth (1981) see a comparable gradient in nitrogen in the HII-regions of the disk of M31.

## 5.2 Stellar Abundance Gradients

The abundance gradient for relatively young stars in the disk of the Milky Way is nicely delineated by the Cepheids (Luck, Kovtyukh & Andrievsky, 2006), shown in fig. 18. The gradient is about  $-0.06 \text{ dex kpc}^{-1}$ , in good agreement with the gas-phase gradient derived by Shaver et al. (1983). The two-dimensional distribution of the Cepheid abundances over the Galactic plane shows some localized departures from axisymmetry at the level of about 0.2 dex in abundance. These departures may come from radial gas flows associated with the spiral structure.

For the older stars in the outer disk (open clusters and red giants), the abundance gradient appears to be somewhat steeper, as seen in the study by Carney, Yong & Teixeira de Almeida (2005). The abundances fall to about  $-0.5$  at 11 kpc but then stay approximately constant at this level out to radii beyond 20 kpc (fig. 19). The figure also compares the  $\alpha/\text{Fe}$  ratio for the older objects and the Cepheids. It indicates that the abundance gradient in the outer disk is flatter now than it was a few Gyr ago, and also that the  $[\alpha/\text{Fe}]$  ratio is nearer the solar value now than it was at the time of formation of the outer old clusters. These observations suggest that the chemical evolution of the disk gradually flattens the abundance gradient and reduces the  $[\alpha/\text{Fe}]$  ratio. In the outer regions, episodic accretion of gas may trigger bursts of star formation which could erase the abundance gradient and produce the  $\alpha$ -enriched abundances seen in the older objects. A similar bottoming out of the abundance gradient beyond about 15 kpc is seen by Worthey et al. (2005) in the outer disk of M31. We note that the environment of the outer disk in M31 has had a complex star formation history, with much evidence for an extended period of accretion of smaller galaxies (*e.g.* McConnachie et al., 2009) which complicates the interpretation of radial gradients in this system.

Cioni (2009) used the radial change of the ratio of C and M-type AGB stars in the LMC and M33 to evaluate their stellar abundance gradients. Both show a radial abundance gradient. This is particularly clearly seen in M33: the gradient in the disk persists to a radius of about 8 kpc, which is the radius at which the radial truncation of the disk occurs. At larger radii, the abundance gradient becomes much flatter. Barker et al. (2007a) also found evidence that the radial abundance gradient of M33 flattens in the outer regions.

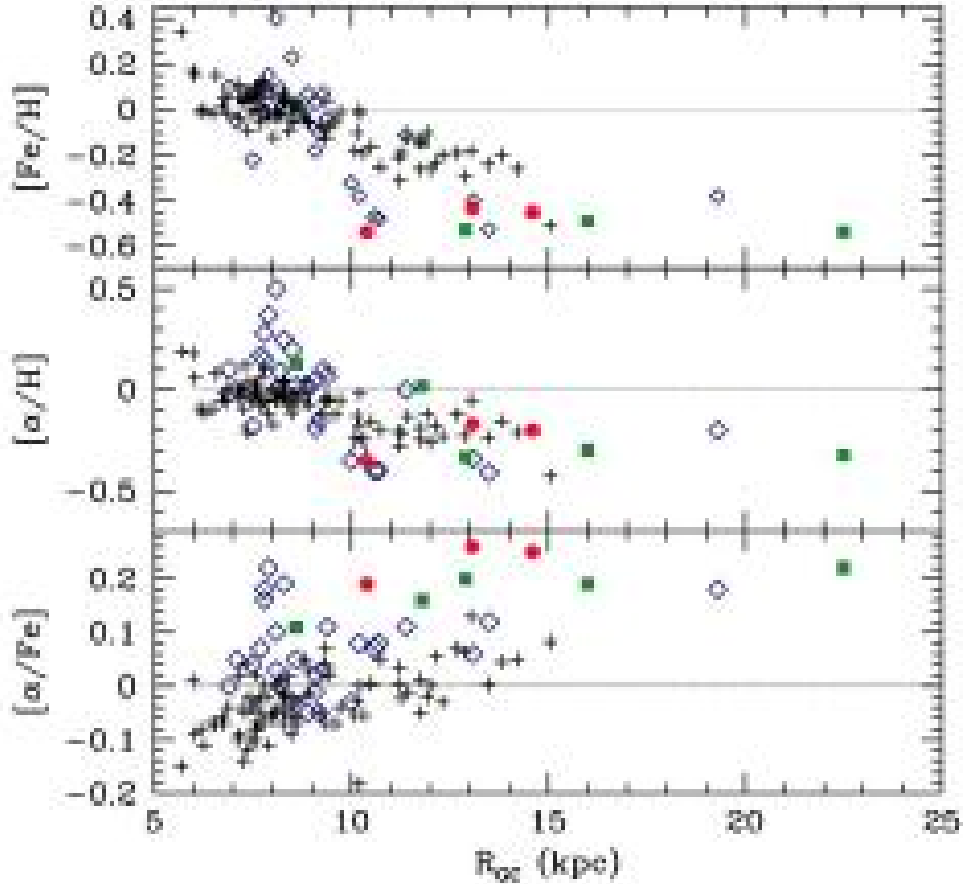


Figure 19: *Upper panel:* The radial abundance distribution for Cepheids (crosses) and older clusters (blue diamonds and green squares) and red giants (red circles) in the Galactic disk. *Lower panels:* the radial  $[\alpha/\text{H}]$  and  $[\alpha/\text{Fe}]$  distributions for the same objects. (From Carney, Yong & Teixeira de Almeida, 2005).

The abundance gradients of disk galaxies may not only flatten at large radii but can also reverse. The galaxy NGC 300 is similar to M33 in appearance and absolute magnitude, and it also has a negligible bulge component. The two galaxies do however differ in the details of the structure of their disks. M33 shows a very well defined truncation of its disk, at a radius of a few scalelengths (Ferguson et al., 2007), while the disk of NGC 300 has an unbroken exponential surface density distribution extending to a radius of at least 10 scalelengths (Bland-Hawthorn, Vlajić & Freeman, 2005). The negative abundance gradient in NGC 300 persists to a radius of about 10 kpc and then appears to reverse, with the metal abundance increasing with radius. Vlajić, Bland-Hawthorn & Freeman (2009) present two possible scenarios for the reversal of the abundance gradient. One is associated with accretion and local chemical evolution. The other scenario involves radial mixing driven by resonant scattering of stars *via* transient spiral disturbances, while preserving their near-circular orbits, as discussed by Sellwood & Binney (2002) and Roškar et al. (2008a). In this picture, the stars in the outermost disk did not form *in situ* but were scattered from the inner galaxy into the outer disk. The scattering occurs from near the co-rotation radius of the individual spiral disturbance. The radial extent of the scattering depends on the strength of the transient spiral wave, and the radius from which the scattering occurs depends on its pattern speed. If the inner disk of the galaxy has the usual negative

abundance gradient, then the abundance distribution in the outer disk (which is populated by stars scattered from the inner disk) would depend on the distribution of pattern speeds and pattern strengths in the inner disk. A reversal in the current abundance gradient could come from an epoch of spiral disturbances whose strength increases with their pattern speed: more metal-rich stars from smaller radii are then more strongly scattered radially. Such an epoch of spiral disturbances could also explain the difference in structure between the truncated disk of M33 and the exponential disk of NGC 300. In this scenario, the outer disk of NGC 300 is more strongly populated by radial mixing, building up the continued exponential. For an alternative view, see Gogarten et al. (2010).

The chemical evolution of disks is usually calculated assuming that each annulus in the disk evolves independently, with infall of gas from intergalactic space but with no radial exchange of processed gas or stars (*e.g.* Chiappini, Matteucci & Gratton, 1997). In this picture, one might expect a relationship between the age of stars and their metallicity. The apparent lack of a well-defined AMR in the solar neighborhood has motivated alternative views of the chemical evolution of disks, involving radial flows of gas and radial mixing of stars. Schönrich & Binney (2009a) have constructed such a theory which seems to fit very well the observed stellar distribution of thin disk stars in the  $[\alpha/\text{Fe}]$  -  $[\text{Fe}/\text{H}]$  plane, and also the existing AMR data. We caution however that the AMR data in the solar neighborhood is still quite uncertain.

## 6 SCALING LAWS FOR DISK GALAXIES

Disk galaxies demonstrate several scaling laws, *i.e.* relations of observable parameters to the luminosity or stellar mass of the galaxies. Some scaling laws involve parameters associated with the stellar component of the galaxies: *e.g.* the stellar mass - metallicity relation and the luminosity - radius relation. Other relate stellar properties to dark matter properties: *e.g.* the Tully-Fisher relation between the stellar luminosity  $L$  (or stellar mass or baryonic mass) and the rotation speed of the galaxy, and the scaling relations between the the luminosity of the stellar component and the central density of the dark matter halos. These scaling laws provide insight into the formation and evolution of disk galaxies.

When examining the scaling laws and other relations between observables for disk galaxies, it is useful to ask how many independent parameters there are in the ensemble of information. Principal component analysis (PCA) can provide insight, although it may be difficult to identify which of the parameters are fundamental (Brosche, 1973). Recently, Disney et al. (2008) combined data from the HIPASS survey of HI in galaxies (Meyer et al., 2004) with SDSS data and derived six parameters: two optical radii (containing 50 and 90% of the light), luminosity, HI mass, dynamical mass and  $(g - r)$  color. PCA shows that the correlations are dominated by only one significant principal component. This indicates a somewhat unexpected organizational uniformity in galaxy properties.

### 6.1 The Tully-Fisher Law

This law, in its original form of absolute blue magnitude  $M_B$  *vs* the velocity width of the integrated HI profile, was discovered by Tully & Fisher (1977). For their profile width, they used  $W_{20}$ , the line width at 20% of the peak intensity, which has become widely but not universally used in Tully-Fisher relation (TFR) studies. The TFR quickly became an important tool for measuring the absolute magnitudes and distances of galaxies from their HI profile width. The magnitudes need to be corrected for Galactic and internal absorption, and the velocity widths corrected for the inclination of the disk and turbulence in the galaxy's ISM. The slope of the TFR depends on wavelength; see for example Sakai et al. (2000). The slope of the  $\log L - \log W_{20}$

relation goes from about 3.2 at  $B$  to about 4.4 at  $H$ , and the scatter about the TFR is smaller at longer wavelengths.

The TFR is an important and complex constraint on galaxy formation theory. Each of the variables in the TFR (luminosity and profile width) is itself the product of many complex and interacting processes involved in the formation and evolution of disk galaxies, and these processes contribute to the slope, zero-point and scatter of the TFR. The luminosity measures the integrated star formation history and evolution of the baryons. The HI profile width is a measure of the maximum value of  $R\partial\Phi/\partial R$  in the plane of the HI disk, where  $R$  is the radius and  $\Phi$  the potential. This potential comes partly from the stellar component and partly from the dark matter halo. It therefore depends on the lengthscale of the stellar component (*i.e.* on how much the baryons have contracted during the formation of the galaxy) and hence on the angular momentum of the stellar component.

The dimensionless spin parameter (Peebles, 1971)

$$\lambda = \frac{J|E|^{1/2}}{GM^{5/2}}, \quad (22)$$

(where  $J$  is the system's spin angular momentum,  $E$  its binding energy and  $M$  its mass) is relevant here.  $\lambda$  is a measure of how far a system is from centrifugal equilibrium. Cosmological simulations predict the distribution of  $\lambda$  for dark halos which form in a CDM universe (*e.g.* Efstathiou & Jones (1979)). The distribution of  $\lambda$  is typically lognormal, with a mean value of about 0.06. In systems with higher values of  $\lambda$ , the baryons settle into disks which are more extended, more slowly rotating and of lower surface brightness in the mean. See *e.g.* Dalcanton et al. (1997) for more details. Furthermore, the parameters involved in the TFR are likely to evolve as the galaxy grows: the stellar mass will increase according to the star formation history, the baryonic mass will be affected by feedback and accretion, and the rotational velocity is likely to change as the stellar mass increases and the dark matter halo is gradually built up. We can expect all of these changes to continue to the present time.

Although the brighter disk galaxies lie on a well-defined luminosity-velocity relation, the fainter galaxies with circular velocities less than about 100 km s<sup>-1</sup> are observed to have luminosities that lie below the relation defined by the brighter galaxies. Many of these fainter disk galaxies are gas (HI) rich, so the stellar mass is only a fraction of their baryonic content. If the baryonic (stellar + HI) mass is used instead of the stellar luminosity or mass, the fainter disk systems move up to lie on the same baryonic TFR as the brighter galaxies (Freeman, 1999; McGaugh et al., 2000), with a relatively small scatter of 0.33 mag. This suggests that the TFR is really a relation between the rotational velocity and the total baryonic mass  $M_{bar}$  (see also McGaugh, 2005). The change in the slope of the  $L - W_{20}$  relation with wavelength is partly due to the way in which the  $L/M_{bar}$  ratio changes with  $M_{bar}$  at different wavelengths, which in turn probably reflects the different star formation histories of disk galaxies of different masses. In the mean, the less massive galaxies are now more affected by current star formation (downsizing), and we can expect more scatter in their  $L/M_{bar}$  ratio.

There is still some disagreement about the slope of the baryonic Tully-Fisher relation (BTFR). Various authors find slopes between  $M_{bar} \propto V^3$  and  $M_{bar} \propto V^4$  where  $V$  is the rotational velocity (*e.g.* Gurovich et al., 2010; Tracternach et al., 2009; Stark et al., 2009). CDM theory predicts a slope closer to 3, so it is important to settle this question observationally. The studies quoted estimate the stellar masses from  $M/L$  ratios based on various models. In a more direct approach, Kregel, van der Kruit & Freeman (2005) use stellar disk masses derived from stellar dynamical analysis; after adding the HI content, they find a slope for the BTFR of  $3.33 \pm 0.37$  over 2 dex, with a scatter of 0.21 dex.

One potentially important difference in approach comes from the way in which different authors measure the rotational velocity (see Cantinella et al., 2007, for more discussion). Some

authors use the profile width  $W_{20}$  as their velocity measure; the relationship between this quantity and the asymptotic (flat) rotational velocity of the galaxy becomes less well defined for lower-mass galaxies, in which the HI often does not extend to the flat part of the rotation curve. Other authors restrict their samples to galaxies in which the rotation curve is observed to reach the flat region, and use this flat level of the rotation curve as their velocity measure. Although the flat level of the rotation curve provides a consistent estimate of the rotation, this is itself a selection effect, restricting the sample of galaxies to those in which the HI extends to the flat rotation curve region. The selected galaxies are therefore biased towards systems in which either the HI is intrinsically more extended, or the dark matter halos are more centrally concentrated with relatively smaller scalelengths and larger concentrations. As one might expect from the above discussion, analyses based on  $W_{20}$  appear to give lower (flatter) slopes for the BTFR than those based on the flat region of the rotation curve. Ideally, to relate the baryonic content of disk galaxies to the properties of the dark halos, we would like to use the circular velocity of the dark halo at the virial radius, but for most galaxies this quantity cannot be measured at present (see Courteau et al., 2007, for a useful discussion).

The interpretation of the BTFR is not straightforward. For normal high surface brightness disk galaxies, the rotation speed depends on the contributions to the gravitational potential from both the stellar distribution and the dark matter, including any effects of the stellar distribution on the dark matter (baryonic contraction). This in turn involves the evolution of the stellar disk itself. In gas-rich and low surface brightness galaxies, the contribution of the luminous matter to the potential field is relatively small but, even in this simpler situation, the BTFR involves the ratio of baryon to dark matter mass, the structure of the dark matter halo, and the radial extent or maximum angular momentum of the HI gas. Zwaan et al. (1995) and Sprayberry et al. (1995) constructed a TFR law for low surface brightness (LSB) galaxies: these galaxies have surface brightnesses that are typically at least a magnitude fainter than the normal galaxies. They found that the TFR for LSB and normal disk galaxies were very similar in slope and zero-point. Although this result may seem surprising, it makes sense if interpreted in the context of the BTFR. The stellar luminosity is used as a proxy for the baryonic mass, with some scatter and bias depending on the gas fraction (and adopted stellar  $M/L$  ratio). The velocity for these galaxies reflects the circular velocity of the dark matter potential, because the contribution to the gravitational field from the stellar component is relatively insignificant.

The following clever argument of Courteau & Rix (1999) makes use of the scatter in the Tully-Fisher relation. The amplitude of the rotation curve of the self-gravitating exponential disk is

$$V_{\text{disk}} \propto \sqrt{h\Sigma_0} \propto \sqrt{\frac{M_{\text{disk}}}{h}}. \quad (23)$$

For fixed disk-mass  $M_{\text{disk}}$  we then get by differentiation

$$\frac{\partial \log V_{\text{max}}}{\partial \log h} = -0.5. \quad (24)$$

So at a given absolute magnitude (or mass) a lower scalelength disk should have a higher rotation velocity. If all galaxies were maximum disk, then this anticorrelation should be visible in the scatter of the Tully-Fisher relation. It is however not observed and the inference is that on average  $V_{\text{disk}} \sim 0.6V_{\text{total}}$  and galaxies in general do not have maximal disks. We note that this argument ignores the contribution of the gas to the total baryonic mass, which can be significant even for relatively bright galaxies (*e.g.* Gurovich et al., 2010).

## 6.2 Scaling Laws involving the Galaxy Diameter

In their pioneering paper, Tully & Fisher (1977) also considered the relationship between galaxy optical diameter and rotational velocity. Courteau et al. (2007) made an extensive study of



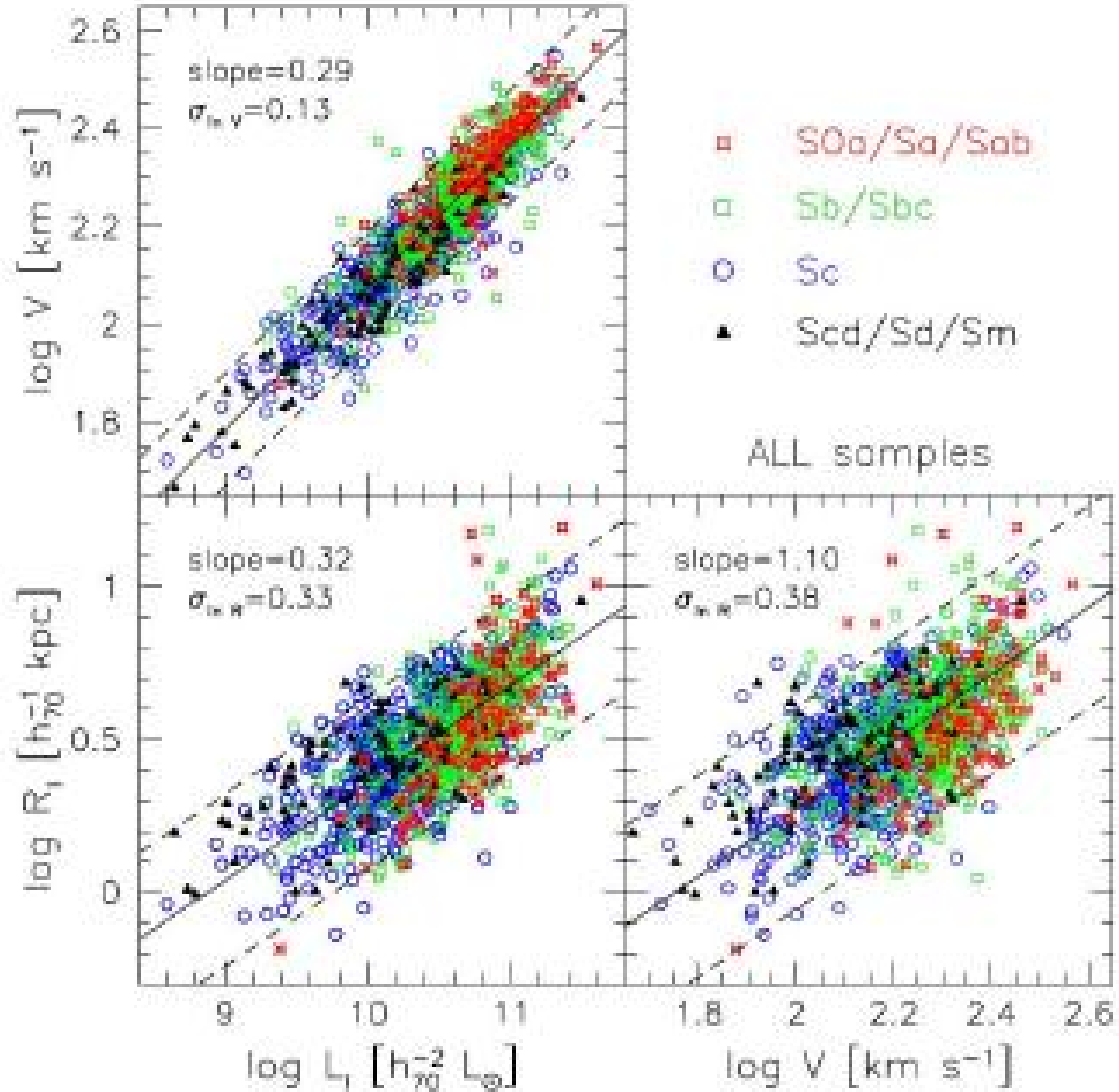


Figure 20: Scaling relations between the rotation velocity  $V$ , the scalelength  $R$  and the luminosity  $L$ , the latter two in the  $I$ -band. (From Courteau et al., 2007).

the distributions of  $L$  and stellar scalelength  $h$  (in the  $I$  and  $K$  bands) and  $V$  for a sample of 1300 galaxies. The scalelength reflects the angular momentum distribution of the star-forming baryons but probably includes further complications of the baryonic dissipation and settling to equilibrium.

The scatter in the relations involving  $h$  is larger than for the  $L - V$  relation (see fig. 20), as one might expect from the additional complexity of the underlying physics and also the difficulty of measuring disk scalelengths accurately. The mean relations found by Courteau et al. (2007) are

$$V \propto L^{0.29}, \quad h \propto L^{0.32} \quad \text{and} \quad h \propto V^{1.10} \quad (25)$$

in the  $I$ -band. The relations involving  $h$  show some morphological dependence, but no significant morphological dependence is seen in the  $V - L$  relation.

### 6.3 The Mass-Metallicity Relation

The mass-metallicity relation pertains to the baryonic component of the galaxies. The relationship between stellar mass or luminosity and metallicity in galaxies goes back at least to Lequeux et al. (1979). A study of a large sample of SDSS galaxies by Tremonti et al. (2004) gives references and demonstrates a tight correlation between the stellar mass and the gas-phase oxygen abundance extending over 3 orders of magnitude in stellar mass and a factor of 10 in oxygen abundance. Kirby et al. (2008) show that the relation between luminosity and metallicity continues down to the faintest known dwarf spheroidal galaxies with absolute V magnitudes fainter than  $-4$ . The deeper potential wells of the more massive galaxies are believed to retain more effectively the metal-enriched ejecta of supernovae (Larson, 1974; Dekel & Silk, 1986), which can be lost via galactic winds. Tassis et al. (2008) have argued however that this kind of supernova feedback may not be essential for generating the mass-metallicity relation; the increasingly inefficient conversion of gas into stars in the lower mass galaxies may be responsible.

### 6.4 The Surface Density-Mass Relation

This relation again pertains to the baryonic component of the galaxies; as for the mass-metallicity relation, it provides some constraints on the theory of the evolution of the baryonic component. In the mean, the surface density of galaxies appears to increase with increasing stellar mass or luminosity, as shown first by Kormendy (1985) and followed up by Dekel & Silk (1986) for a sample of nearby ellipticals and irregular galaxies with stellar masses less than about  $10^{10.5} M_{\odot}$ . For disk galaxies, Gurovich et al. (2010) found a similar relation: the baryonic (stellar + HI) surface density is observed to increase approximately linearly with  $W_{20}$ . Again, this scaling law can be interpreted in terms of supernova-driven loss of gas or as due to increasingly inefficient star formation for systems of lower masses.

### 6.5 Scaling Laws for Dark Matter Halos

The properties of the dark matter halos of disk galaxies appear to scale with the luminosity of their stellar component. Kormendy & Freeman (2004) analysed the rotation curves of spirals and dwarf irregular galaxies to estimate parameters for their dark matter halos. They modelled the dark halo density distributions as isothermal spheres with a central core of density  $\rho_{\circ}$  and a core radius  $r_c$ . Estimates for the dark halos of dwarf spheroidal galaxies were also included, using the velocity dispersion profiles of these systems. Kormendy & Freeman found that the core density  $\rho_{\circ}$  decreases with luminosity, as  $\rho_{\circ} \propto L^{-0.28}$ : the core densities increase from about  $10^{-2.5} M_{\odot} \text{ pc}^{-3}$  for the brighter spirals to a rather high value of about  $10^{-0.5} M_{\odot} \text{ pc}^{-3}$  for the fainter dwarf irregular and spheroidal galaxies. The core radius  $r_c$  increases with luminosity, as  $r_c \propto L^{+0.32}$ , so the surface density  $\rho_{\circ} r_c$  of the dark matter halos is approximately independent of the luminosity of the stellar component. This remarkable observational result was confirmed by Donato et al. (2009) with more recent data for the dark halos of the dwarf galaxies. The surface density  $\rho_{\circ} r_c$  is constant at about  $140 M_{\odot} \text{ pc}^{-2}$  over about 15 mag in stellar luminosity.

The dark halo scaling laws reflect the changing density of the universe as halos of different masses are formed, with the less massive halos forming earlier. The difference in halo densities indicates that the smallest dwarfs formed about 7 units of redshift earlier than the largest spirals. Djorgovsky (1992) showed that protogalactic clumps which separate from an evolving density field with power spectrum  $|\delta_k|^2 \sim k^n$  have a scaling law between density and radius  $\rho \sim r^{-3(3+n)/(5+n)}$ . The observed scaling laws for dark matter halos correspond to  $n \approx -2$ , close to what is expected for  $\Lambda$ CDM on galactic scales.

## 7 THICK DISKS

Most spirals, including our Galaxy, have a second thicker disk component surrounding the thin disk. Thick disks were discovered in other galaxies via surface photometry (Tsikoudi, 1980; Burstein, 1979) and then in the Milky Way through star counts (Gilmore & Reid, 1983). It appears that thick disks are very common in disk galaxies and that they are mostly very old. The thick disk is therefore an important component in understanding the assembly of disk galaxies.

### 7.1 Statistics of Incidence

The photographic surface photometry of van der Kruit & Searle (1981a,b, 1982a) showed that thick disks were common in disk galaxies but perhaps not ubiquitous (see also Fry et al., 1999). At the time, evidence indicated that the thick disk was associated with the presence of a central bulge. A more recent extensive study of a large sample of edge-on galaxies by Yoachim & Dalcanton (2006) showed that thick disks are probably present in all or almost all disk galaxies. They found that the ratio of thick disk stars to thin disk stars depends on the luminosity or circular velocity of the galaxy: it is about 10% for large spirals like the Milky Way, and rises to about 50% for the smallest disk systems.

Our Galaxy has a thick disk. Star counts by Gilmore & Reid (1983) at high Galactic latitude showed two vertically exponential components: the thin disk and the more extended thick disk. Its scaleheight is about 1000 pc, compared to about 300 pc for the old thin disk, and its surface brightness is about 10% of the surface brightness of the thin disk, but there is still some disagreement about these parameters

### 7.2 Structure of Thick Disks

From within our Galaxy, it is difficult to estimate reliably the scaleheight and scalelength of the thick disk. Values for the scaleheight between about 0.5 and 1.2 kpc have been reported. A recent analysis of the SEGUE photometry gives a relatively short exponential scaleheight of  $0.75 \pm 0.07$  kpc and an exponential scalelength of  $4.1 \pm 0.4$  kpc (de Jong et al., 2010). For comparison, the scaleheight of the thin disk is about 250 to 300 pc; the scalelength of the thin disk is poorly determined but is probably between 2 and 4 kpc. Their model gives the stellar density of the thick disk near the sun to be about  $0.0050 M_{\odot} \text{ pc}^{-3}$ , about 7% of the stellar density of the thin disk near the sun (about  $0.07 M_{\odot} \text{ pc}^{-3}$ ).

Thick disks are not easy to see in galaxy images. Fig. 21 shows that in NGC 4762 (that has a bright thick disk Tsikoudi, 1980) the outer extent of the faint starlight has an approximate diamond shape, indicating the double exponential light distribution of a thick disk. In fig. 10, we see the same thing in the outer outline in the bottom picture for NGC 4565 at the deepest stretch. The luminosity distribution of the edge-on galaxy NGC 891, which is often regarded as an analog for the Milky Way (van der Kruit, 1984), shows after subtraction of the disk a light distribution that becomes progressively more flattened at fainter levels (see fig. 7 in van der Kruit & Searle, 1981b); in van der Kruit (1984) it is shown that this distribution can be interpreted as a superposition of a thin and thick disk plus a small, central bulge. Recent studies show that the scaleheight of its thick disk is  $1.44 \pm 0.03$  kpc and its radial scalelength is  $4.8 \pm 0.1$  kpc, only slightly longer than that of the thin disk (Ibata, Mouhcine & Rejkuba, 2009). The relationship between the scalelengths of the thin and thick disk is an important constraint on the various formation mechanisms of thick disks, as discussed below.

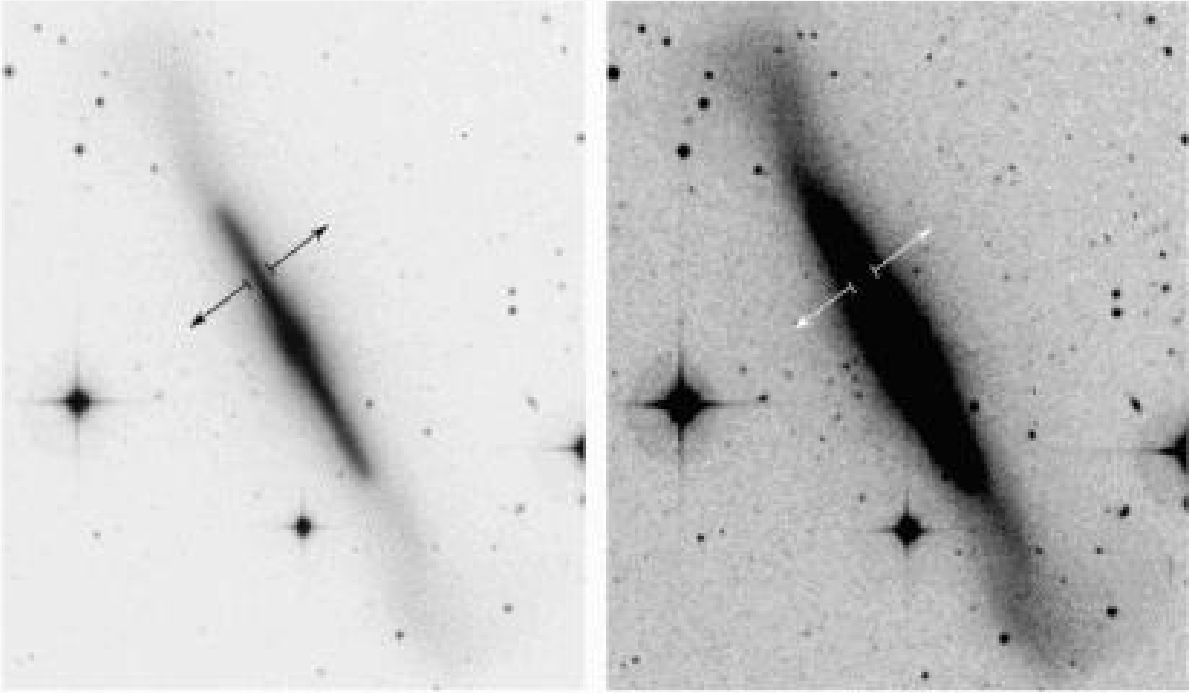


Figure 21: The S0-galaxy NGC 4762, which has a very bright thick disk, as was first described by Tsikoudi (1980). The  $z$ -extent indicated by the arrows is where the thin disk dominates. On the right the outer extent of the thick disk is slanted w.r.t. the symmetry plane (producing an approximately diamond shape), indicative of a double exponential light distribution. These images were produced with the use of the *Digital Sky Survey*.

### 7.3 Kinematics and Chemical Properties

Little information is available on the kinematics and chemical properties of thick disks in galaxies other than the Milky Way. The larger scaleheight of the Galactic thick disk means that its velocity dispersion is higher than for the thin disk (about  $40 \text{ km s}^{-1}$  in the vertical direction near the sun, compared to about  $20 \text{ km s}^{-1}$  for the thin disk (*e.g.* Quillen & Garnett, 2000). The stars of the thick disk are usually identified by their larger motions relative to the Local Standard of Rest, but kinematic selection is inevitably prone to contamination by the more abundant thin disk stars. Recently it has become clear that the Galactic thick disk is a discrete component, kinematically and chemically distinct from the thin disk. It now appears that thick disk stars can be more reliably selected by their chemical properties.

Near the Galactic plane, the rotational lag of the thick disk relative to the LSR is only about  $30 \text{ km s}^{-1}$  (Chiba & Beers, 2000; Dambis, 2009), but its rotational velocity appears to decrease with height above the plane. The stars of the thick disk are old ( $> 10 \text{ Gyr}$ ) and more metal-poor than the thin disk. The metallicity distribution of the thick disk has most of the stars with  $[\text{Fe}/\text{H}]$  between about  $-0.5$  and  $-1.0$ , with a tail of metal-poor stars extending to about  $-2.2$ . The thick disk stars are enhanced in  $\alpha$ -elements relative to thin disk stars of the same  $[\text{Fe}/\text{H}]$  (*e.g.* fig. 22 from Fuhrmann, 2008), indicating a more rapid history of chemical evolution. The thick disk does not show a significant vertical abundance gradient (Gilmore, Wyse & Jones, 1995). It appears to be chemically and kinematically distinct from the thin disk.

A decomposition of the kinematics and distribution of stars near the Galactic poles, by Veltz et al. (2008), nicely illustrates the three main kinematically discrete stellar components of the

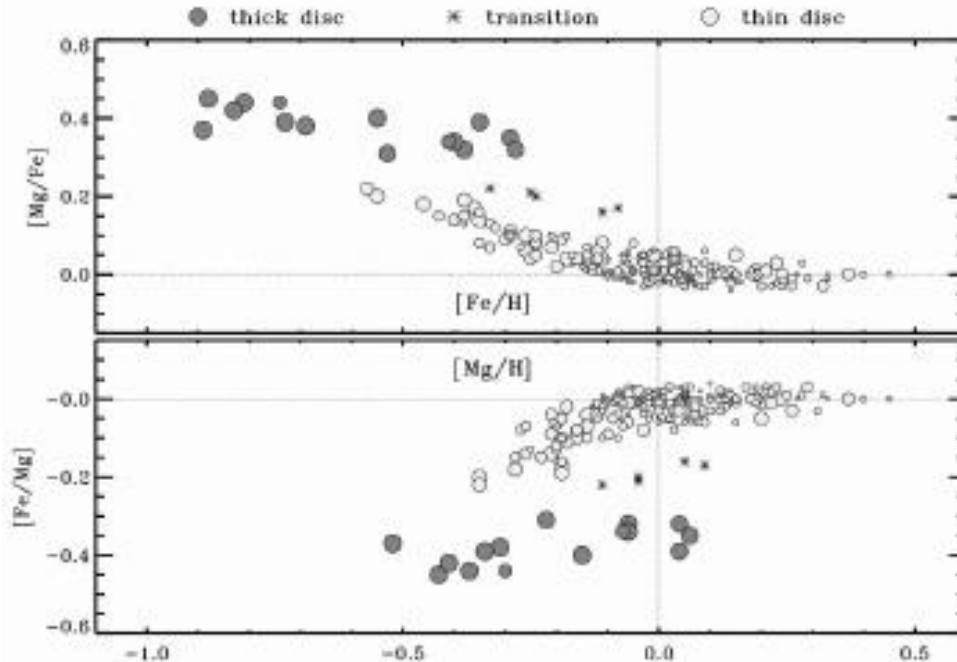


Figure 22: Mg-enrichment of the thick disk relative to the thin disk, indicating that the thick disk is a chemically distinct population. (From Fuhrmann, 2008)

Galaxy: a thin disk with a scaleheight of 225 pc and a mean vertical velocity dispersion of about  $18 \text{ km s}^{-1}$ , a thick disk with a scaleheight of 1048 pc and mean velocity dispersion of  $40 \text{ km s}^{-1}$ , and a halo component with a velocity dispersion of about  $65 \text{ km s}^{-1}$ .

The old thick disk presents a kinematically recognizable relic of the early Galaxy and is therefore a very significant component for studying galaxy formation. Because its stars spend most of their time away from the Galactic plane, the thick disk is unlikely to have suffered much secular heating since the time of its formation. Its dynamical evolution was probably dominated by the changing potential field of the Galaxy associated with the continuing growth of the Galaxy since the time at which the thick disk was formed.

For the thick disks of other galaxies, broadband colors suggest that the thick disks are again old and metal-poor relative to their thin disks. Yoachim & Dalcanton (2008) studied the rotation of the thick disks of a small sample of extragalactic thick disks. In one galaxy (FGC 227)<sup>24</sup>, they found that the thick disk appeared to be counter-rotating relative to its thin disk. If confirmed, this difficult observation would have important implications for the origin of thick disks.

## 7.4 Relation of Thick Disk to the other Galactic Components

At this time, there is no convincing evidence that the thick disk is in any way related to the halo or the bulge of its parent galaxy. The almost ubiquitous nature of thick disks, independent of the bulge to disk ratio, argues against any causal relation with the bulge. In the Milky Way, some chemical similarities of bulge and thick disk stars are observed. For example, the  $[\alpha/\text{Fe}]$  ratio of the thin disk stars near the sun are somewhat like the enhanced  $[\alpha/\text{Fe}]$  ratios seen in bulge stars (Meléndez et al., 2008) but this is probably more a reflection of the rapid star formation history of both components, rather than any deeper relation. The metal-poor tail of the metallicity distribution of the thick disk reaches down to metallicities usually associated with halo stars

<sup>24</sup> FGC is the Flat Galaxy Catalogue of Karachentsev, Karachentseva & Parnovskij (1993).

( $[\text{Fe}/\text{H}] < -1$ ), but the kinematics are different (the metal-poor thick disk stars are rotating more rapidly than the halo stars (*e.g.* Carollo et al., 2010) and do not hint at any cosmogonic relation of thick disk and halo. Gilmore (1995) found that in the Galaxy the cumulative angular momentum distribution function for stars in the thick disk is rather similar to that of the thin disk, but distinctly different from that of the stellar halo and the bulge.

Thin disks and thick disks do appear to be causally linked: the survey of Yoachim & Dalcanton (2006) shows that all or almost all galaxies selected as having a thin disk also have a thick disk. Thick disk formation seems to be a normal event in the early formation of thin disks, but the details of how disks form are not yet understood.

## 7.5 Thick Disk Formation Scenarios

Thick disks appear to be old and very common, puffed up relative to their parent thin disks, and (at least in the case of the Milky Way) to have suffered rapid chemical evolution. Scenarios for their formation include:

- thick disks come from energetic early star burst events, maybe associated with gas-rich mergers (Samland & Gerhard, 2003; Brook et al., 2004)
- thick disks are the debris of accreted galaxies which were dragged down by dynamical friction into the plane of the parent galaxy and then disrupted (Abadi et al., 2003; Walker, Mihos & Hernquist, 1996). To provide the observed metallicity of the Galactic thick disk ( $[\text{Fe}/\text{H}] \sim -0.7$ ), the accreted galaxies that built up the Galactic thick disk would have been more massive than the SMC and would have had to be chemically evolved at the time of their accretion. The possible discovery of a counter-rotating thick disk (Yoachim & Dalcanton, 2008) (FGC 227; see above) would favor this mechanism.
- the thick disk’s energy comes from the disruption of massive clusters or star-forming aggregates (Kroupa, 2002b), possibly like the massive clumps seen in the high redshift clump cluster galaxies. Other authors have discussed the formation of thick disks through the merging of clumps and heating by clumps in clump cluster galaxies (*e.g.* Bournaud, Elmegreen & Martig, 2009).
- the thick disk may be associated with the effects of radial mixing of stars and gas in the evolving Galaxy (Schönrich & Binney, 2009a,b).
- the thick disk represent the remnant early thin disk, heated by accretion events. In this picture, the thin disk begins to form at a redshift of 2 or 3, and is partly disrupted and puffed up during the active merger epoch. Subsequently the rest of the gas gradually settles to form the present thin disk (*e.g.* Quinn & Goodman, 1986; Freeman, 1987).

Sales et al. (2009) have shown that the predicted distribution of orbital eccentricities for nearby thick disk stars is different for several of these formation scenarios. As large samples of accurate orbital eccentricities become available for thick disk stars, they can be used to exclude some of the proposed formation mechanisms.

# 8 FORMATION OF DISKS

## 8.1 Disk Formation Scenarios

Stellar disks are close to centrifugal equilibrium, suggesting dissipation of baryons to a near-equilibrium structure before the gas became dense enough to initiate the onset of the main epoch of star formation. This picture goes back at least to Eggen, Lynden-Bell & Sandage

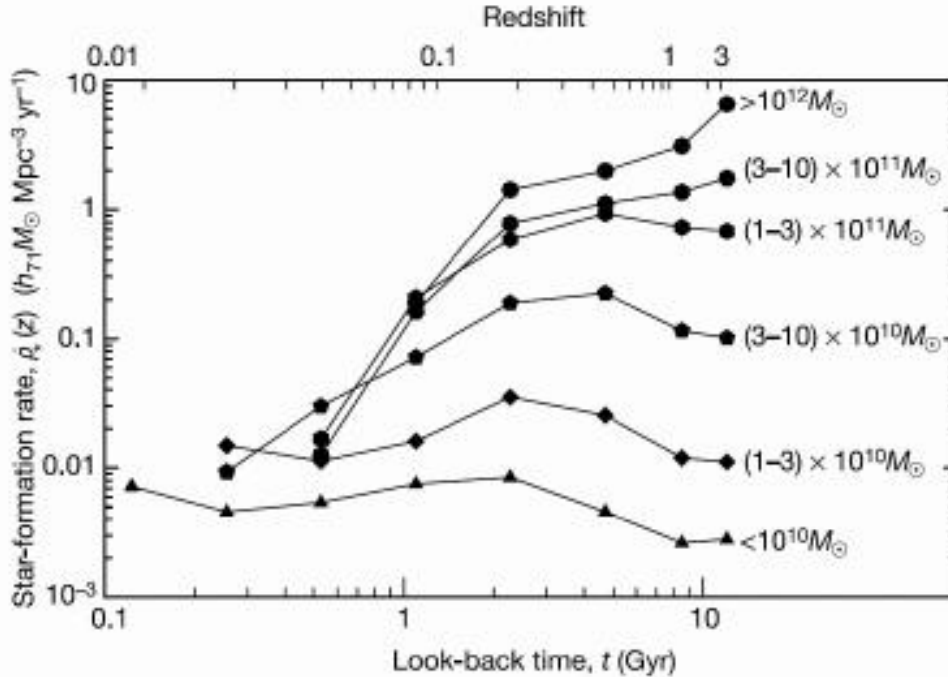


Figure 23: The history of star formation as a function of stellar mass of the galaxy. The curves have been offset by 0.5 in the log, except for the most massive one, where it has been an additional 1.0. The phenomenon of star formation occurring in the early universe mainly in large systems while later shifting to smaller systems is known as ‘downsizing’. (From Heavens et al., 2004)

(1962) and Sandage, Freeman & Stokes (1970) in the pre-dark-matter era, and was followed by many innovative landmark papers in the late 1970s and early 1980s on the cooling of gas and dissipational formation of disks in the potential of the pre-formed dark matter halo. Rees & Ostriker (1977) discussed the cooling of gas and the thermalising of infalling gas. White & Rees (1978) described the two-stage theory of galaxy formation, in which the dark matter clustered under the influence of gravity and the gas cooled into the dark matter halos. Fall & Efstathiou (1980) considered the dissipational collapse of a disk within a dark halo, conserving its detailed  $M(h)$  distribution as suggested by Mestel (1963). Assuming that the specific angular momenta of the dark matter and gas were similar, they showed that the gas would collapse by at least a factor of 10. Gunn (1982) showed that in this hypothesis approximately exponential stellar disks arise naturally and van der Kruit (1987) showed that these would then have radial truncations at about 4.5 scalelengths. Blumenthal et al. (1984) discussed the various possible kinds of dark matter (cold, warm, hot) and their implications for the formation of galaxies by cooling. Later versions of this basic picture were discussed by Dalcanton et al. (1997) and Mo, Mao & White (1998). The so-called ‘rotation curve conspiracy’, that the properties of disk and halo as precisely those necessary to produce essentially flat, featureless rotation curves and to provide the Tully-Fisher law was heuristically explained in the context discussed here by Gunn (1987) and Ryden & Gunn (1987).

In summary, this paradigm involves the dark matter halo forming gravitationally and relaxing to virial equilibrium; then infalling gas is shock-heated to the halo virial temperature and then cools radiatively from inside out, gradually building up the disk and forming stars quiescently.

The subsequent star formation history of disks is very different from galaxy to galaxy. In

the S0 galaxies, the star formation has already come to a halt through gas exhaustion or gas stripping. In the Galactic thin disk, star formation began about 10 Gyr ago (Edvardsson et al., 1993; Knox, Hawkins & Hambly, 1999) and a wide range of stellar age is seen, indicating that disk formation was an extended process starting about 10 Gyr ago and continuing to the present time. The star formation history of the disk is not very well constrained, but is consistent with a roughly constant star formation rate (*e.g.* Rocha-Pinto et al., 2000) with fluctuations of about a factor 2 on Gyr timescales. In the lower luminosity systems, the main epoch of star formation did not begin right away, and much of the current star formation in the universe is now taking place in these smaller disk galaxies. This is the phenomenon of downsizing: star formation in the early universe occurred mainly in the larger systems and has gradually proceeded to progressively smaller galaxies (see fig. 23).

This is nicely shown by Noeske et al. (2007a,b). They analysed the star formation rate as a function of stellar mass  $M_*$  and redshift. In galaxies which are forming stars, the star formation rate depends on the stellar mass of the galaxies. The observed range of star formation rate remains approximately constant with redshift, but the sequence moves to higher star formation rate with increasing redshift. The dominant mode of evolution since  $z = 2$  is a gradual decline of the average star formation rate. At a given mass, the star formation rate at  $z = 2$  was larger by a factor  $\sim 4$  and  $\sim 30$  than that in star forming galaxies at  $z = 1$  and 0 (Daddi et al., 2007). The star formation drives the growth of disks. Trujillo et al. (2006) studied the evolution of the luminosity-size and stellar mass-size relation in galaxies since  $z = 3$  and found that, at a given luminosity, galaxies were 3 times smaller at  $z = 2.5$  than now; at a given  $M_*$  they are 2 times smaller. In the Local Group, Williams et al. (2009) directly measured the evolution of the scale length of M33, which has increased from 1 kpc 10 Gyr ago to 1.8 kpc at more recent times.

The present high specific star formation rate of many less massive galaxies reflects the late onset of their dominant star formation episode, after which the star formation rate gradually declines. The less massive galaxies appear to have a longer e-folding time and a later onset of their star formation history.

The color distributions of galaxies reflect this mass dependence of their star formation history. At low redshifts, the color distribution is bimodal, showing the blue cloud of star forming galaxies at lower stellar masses and the overlapping red sequence of more luminous systems in which star formation has now stopped or is at a very low rate (Kauffmann et al., 2003a; Blanton et al., 2003). The restframe color distribution is now known to be bimodal at all redshifts  $z < 2.5$  (*e.g.* Brammer et al., 2009). The red sequence persists to beyond  $z = 2$ , indicating that many galaxies had completed their star-forming lives by this time. Galaxies in the “green valley” between the red sequence and the blue cloud may be in transition between the two sequences, as blue cloud galaxies whose star formation is running down, but also as red sequence systems in which the star formation has been revitalised (*e.g.* Beaulieu et al., 2010). However it seems more likely now that most of these green valley galaxies are reddened star forming systems.

The bimodality manifests itself in several ways. Blue galaxies, as defined by their color and their star formation rates, dominate the stellar mass function below a transition mass of about  $3 \times 10^{10} M_\odot$ . It persists to redshifts of at least 2.5. The bulge-to-disk ratio shows a related transition from disk-dominated blue galaxies below the transition mass to spheroid-dominated red sequence galaxies above the transition mass (*e.g.* Kauffmann et al., 2003b). The surface brightness  $\mu$  - stellar mass  $M_*$  relation changes from  $\mu \propto M_*^{0.6}$  at lower masses to  $\mu \sim \text{constant}$  at higher masses. The mean mass - metallicity relation also shows a break at the transition mass (*e.g.* Tremonti et al., 2004). Dalcanton, Yoachim & Bernstein (2004) observed a break in the morphology of dustlanes of edge-on disk galaxies at a similar transition mass, changing from diffuse for lower-mass disks to sharply-defined for higher-mass disks. This could be related to infalling cold gas streams enhancing star formation and turbulence in the less massive galaxies (see §§ 4.4 and 8.3).



The role of mergers in the evolution of disk galaxies remains uncertain. The thin disks of disk galaxies are relatively fragile and are easily puffed up by even minor mergers (Quinn & Goodman, 1986; Toth & Ostriker, 1992). CDM simulations show a high level of merger activity on all scales. These mergers tend to excite star formation, puff up the disks and build up spheroidal components in the simulations. This makes it difficult for CDM simulations to generate large spirals like the Milky Way, with relatively small spheroidal components. While there has been discussion by simulators about the unusually quiescent merger history of the Milky Way, such systems are not unusual. Kormendy et al. (2010) have shown that at least 11 of 19 nearby disk galaxies with circular velocities  $> 150 \text{ km s}^{-1}$  show no evidence for a classical bulge. They argue that pure-disk galaxies are far from rare. Lotz et al. (2008) studied the evolution of the merger rate since redshift  $z = 1.2$ . They find that the fraction of galaxies involved in mergers is about 10%, and conclude that the decrease in star formation rate density since  $z = 1$  is a result of the declining star formation rate in disk galaxies rather than a decrease in major mergers. From their GOODS galaxy sample, Bundy et al. (2009) show that mergers contribute little to galaxy growth since  $z = 1.2$  for galaxies with  $M_* < 3 \times 10^{10} M_\odot$ . For the more massive galaxies, which are mostly spheroidal systems, mergers are more important; they estimate that about 30% have undergone (mostly dry or gas-free) mergers. The Milky Way will be an important part of assessing the significance of mergers in building up large spiral galaxies. The unique possibility to make very detailed chemical studies of stars in the Milky Way provides an independent opportunity to evaluate the merger history of our large disk galaxy via chemical tagging techniques (Freeman & Bland-Hawthorn, 2002).

## 8.2 Disks at High Redshift

One of the most fundamental observations that Hubble Space Telescope made possible is the imaging of galaxies when they were very young. Studies of the Hubble Deep Fields (Williams et al., 1996, 2000; Beckwith et al., 2006) showed observationally that few galaxies are present at redshifts  $> 4$  which resemble present-day spirals or ellipticals. Simulations indicate that disks should be present at redshifts around 2 (*e.g.* Sommer-Larsen et al., 2003): is this consistent with observations? If disks are present at these redshifts, what are they like: how do their baryonic mass distributions compare with those of disk galaxies at low redshifts?

Labbé et al. (2003) observed the Hubble Deep Field-South in the near-infrared (from the ground) and found six galaxies at redshifts  $z = 1.4$  to 3.0 that have disklike morphologies. The galaxies are regular and large in the near-infrared (corresponding to rest-frame optical), with face-on effective radii of 5.0 – 7.5 kpc, which is comparable to the Milky Way. The surface brightness profiles are consistent with an exponential law over 2 to 3 effective radii. The HST morphologies (rest-frame UV) are irregular and show large complex aggregates of star-forming regions ( $\sim 15$  kpc across), symmetrically distributed around the centers.

Genzel et al. (2006) described the rapid formation of a luminous star-forming disk galaxy at a redshift  $z = 2.38$ . Their IFU observations indicate that a large protodisk is channelling gas towards a growing central bulge. The high surface density of gas and the high rate of star formation show a system in rapid assembly, with no obvious evidence for a major merger. Integral-field spectroscopy by Förster Schreiber et al. (2009) of several UV-selected galaxies with stellar masses  $\sim 3 \times 10^{10} M_\odot$ , star formation rates of  $\sim 70 M_\odot \text{ yr}^{-1}$  and redshifts between 1.3 and 2.6 provides rotation curves and indicators of dynamical evolution. The morphology is typically clumpy. About 1/3 of the galaxies are turbulent and rotation-dominated, another 1/3 are compact and dominated by velocity dispersion, while the rest are interacting or merging systems. The rotation-dominated fraction is higher for higher masses.

Elmegreen et al. (2009a,b) investigated the relation to modern spirals of clumpy high redshift galaxies in the GOODS, GEMS and Hubble UDF. The clump properties indicate the gradual

dispersal of clumps to form disks and bulges, with little indication of merger activity. The morphological similarity of these systems to modern dwarf irregulars suggests that the clumpy morphology comes from gravitational instability in the turbulent gas. They note that about 50% of these clump cluster galaxies have massive red clumps which could be interpreted as young bulges. We have already discussed the clump cluster galaxies in the context of the formation of the thick disk component (§ 6.5)

Kriek et al. (2009) studied 19 massive galaxies at  $z \sim 2.3$ : nine of them are compact quiescent systems and 10 are emission line systems (6 star-forming galaxies and 4 AGNs). The star forming galaxies again have clumpy morphologies. In the rest-frame  $(U - B) - M_*$  plane, the galaxies appear bimodal: the large star-forming galaxies lie in a blue cloud and the compact quiescent galaxies in a red sequence. A bimodal distribution similar to that at lower redshifts is already in place at redshifts  $> 2$ .

In summary, it appears that the formation of disk galaxies is already well advanced at redshifts  $> 2$ , but the systems have mostly not yet settled to a quiescent disk in rotational equilibrium. The clumpy structure of the massive star forming systems is likely to be an important factor in the subsequent evolution of these systems and in the formation of their thick disks and bulges. The role of mergers in building up these disk galaxies may not be as important as it appears to be from CDM simulations.

### 8.3 Baryon Acquisition by Disk Galaxies

In the scenarios for disk formation that emerged soon after the discovery of dark matter in disk galaxies, the gas was shock-thermalised to the virial temperature and then gradually cooled to form the disk. Simulations (*e.g.* Sommer-Larsen et al., 1999) suggest that this hot halo is further populated by gas blown out from the disk via feedback into the hot halo. This feedback provides a way to reduce the problem of angular momentum loss which led to unrealistically low angular momenta for disks seen in earlier simulations. These simulations are successful in reproducing the peak in the star formation rate seen within individual galaxies at  $z \sim 2$ .

More recent simulations point to the likelihood of cold gas accretion into disk galaxies. SPH simulations by Kereš et al. (2005) showed that typically about half of the gas shock-heats to the virial temperature of the potential well ( $\sim 10^6\text{K}$  in a Milky-Way-like galaxy), while the other half radiates its gravitational energy at  $T < 10^5\text{K}$ . A cold mode of infall is seen for stellar masses  $< 2 \times 10^{10} M_\odot$ . This cold gas often falls in via the cosmic filaments, allowing galaxies to draw their gas from a large volume. Kereš et al. (2009) found that most of the baryonic mass is acquired through the filamentary cold accretion of gas that was never shock-heated to its virial temperature. This cold accretion is the main driver of the cosmic star formation history.

Hot halos are seen only for dark halo masses  $> 2 - 3 \times 10^{11} M_\odot$ . Dekel & Birnboim (2006) ascribed the bimodality of galaxy properties to the nature of the gas acquisition. Galaxies with stellar masses  $< 3 \times 10^{10} M_\odot$  are mostly ungrouped star-forming disk systems, while the more massive galaxies are mostly grouped old red spheroids. They argue that the bimodality is driven by the thermal properties of the inflowing gas. In halos with masses  $< 10^{12} M_\odot$ , the disks are built by cold streams, giving efficient early star formation regulated by supernova feedback. In the more massive halos, the infalling gas is shock-heated and is further vulnerable to AGN feedback, shutting off the gas supply and leading to red and dead spheroids at redshift  $z \sim 1$ . Simulations by Dekel, Sari & Ceverino (2009) showed that the evolution of massive disk systems is governed by interplay between smooth and clumpy cold streams, disk instability and bulge formation. The streams maintain an unstable gas-rich disk, generating giant clumps which can migrate into the bulge in a few dynamical times. The streams prolong this clumpy phase for several Gyr. The clumps form stars in dense subclumps and each clump converts to stars in  $\sim 0.5$  Gyr. The star forming disk is extended because the incoming streams keep the outer

disk dense and unstable, and also because of angular momentum transport by secular processes within the disk (*e.g.* Kormendy & Kennicutt, 2004). Observationally, the large chemical tagging surveys which will soon begin (HERMES, APOGEE) will be able to evaluate the role of giant clumps in the formation of the thin and thick disks of the Milky Way (*e.g.* Bland-Hawthorn et al., 2010). The debris of the dispersed giant clumps should be very apparent from the chemical tagging analysis.

The Milky Way is surrounded by a system of infalling high velocity HI clouds (HVCs) whose nature is not yet fully understood. The associated infall rate is estimated at about  $0.2 M_{\odot} \text{ yr}^{-1}$  (*e.g.* Peek, Putman & Sommer-Larsen, 2008), an order of magnitude smaller than the current star formation rate of the Milky Way. Maller & Bullock (2004) proposed that the cooling of the Galactic hot corona is thermally unstable and generates pressure-confined HVCs with masses  $\sim 5 \times 10^6 M_{\odot}$ , which contribute to fueling the continued star formation of the disk. Binney et al. (2009) argued however that thermal instability of the hot halo is unlikely to be the source of the Galactic HVC system.

Some disk galaxies (*e.g.* NGC 891: Oosterloo, Fraternali & Sancisi, 2007) show thick HI layers surrounding their galactic disks, which is lagging in rotation relative to the gas in the disk. This gas is accompanied by ionized gas (observed in H $\alpha$ ) that shares the lag in rotation velocity (Heald et al., 2007; Kamphuis et al., 2007a) and by dust (Kamphuis et al., 2007b). In more face-on systems this HI appears associated with regions of star formation (*e.g.* Kamphuis, Sancisi & van der Hulst, 1991), indicating that at least a part of it may have originated in the disk. Fraternali & Binney (2008) and Marinacci et al. (2010) suggest that these layers are associated with gas that has been swept up from the hot corona by galactic fountain clouds ejected from the disk by star formation. In this way, the star formation in the disk is self-fueling, through the gas brought down from the hot corona.

Deep HI images of other disk systems show a very extended rotating HI distribution. M83 (see § 4.3 and fig. 16, and [www.atnf.csiro.au/people/bkoribal/m83/m83.html](http://www.atnf.csiro.au/people/bkoribal/m83/m83.html)) is an example, with HI extending far beyond the optical extent of the system. Its outermost HI shows spectacular HI arms and filaments, some of which are forming stars at a low level (Bigiel et al., 2010a). It seems unlikely that this structure should be interpreted as spiral structure in an extended HI disk, because the density of the HI is so low. It may represent a slow filamentary infall of HI into the disk. The observed star formation in this outer HI disk indicates that the outer disk is still in the process of construction. NGC 6946 is another more orderly example of such a very extended HI disk (Boomsma et al., 2008).

## 9 S0 GALAXIES

Hubble (1936) introduced S0 galaxies as the transition type between elliptical galaxies and spirals. Spitzer & Baade (1951) described them as spiral galaxies without arms and suggested they were spirals stripped of their dust, gas and arms as a result of collisions in clusters of galaxies. Sandage, Freeman & Stokes (1970) argued that the morphological type of a galaxy is defined at the time of the formation of the old disk stars, and S0 galaxies are those in which at the time of the completion of the formation of the disk there was little gas left for star formation. Subsequently, the observation of a high proportion of S0 galaxies in clusters (Oemler, 1974), the evolution of blue galaxy populations in clusters with redshift (Butcher & Oemler, 1978) and the increasing ratio of S0s compared to spiral in regions of higher galaxy density (Dressler, 1980) led to the general acceptance that S0 galaxies in clusters are stripped spirals. Larson, Tinsley & Caldwell (1980) reconsidered the issue. They remarked that the apparent rate of consumption of gas by star formation leads to the paradoxical situation that spirals will exhaust their gas supplies on a timescale ‘considerably less than the Hubble time’. The solution they suggested to this was that spirals constantly replenish their gas content from a reservoir in a gaseous envelop

remaining from their formation and therefore can sustain star formation over a much longer timescale, while S0 galaxies would have lost these envelopes early on and therefore do run out of gas on a relatively short timescale.

One way to address the issue of whether S0s are stripped spirals further is to investigate the structure, and in particular the kinematics of disks in S0s in order to investigate whether or not there are differences in the stellar dynamics. We will not fully review the work on S0 galaxies, but concentrate on this aspect and refer for comprehensive reviews of S0 galaxies to those presented by Quilis, Moore & Bower (2000) and Fritze-von Alvensleben (2004) and also point out that the importance of ram-pressure stripping in environments like Virgo has convincingly been demonstrated by Chung et al. (2009).

Although S0s have a relatively bright surface brightness, measurements of their kinematics remained difficult and this prevented for a long time a detailed understanding of their dynamics. It is illustrative in this context to consider the historical development of the subject. One of the earliest identified and most easily accessible S0s is NGC 3115. Oort (1940) already as early as during the thirties(!) considered its dynamics; his motivation was to study issues of stability as he was interested in the origin and maintenance of spiral structure (the first part of the paper concerns the origin of the deviation of the vertex as caused by spiral arms). Although his photometry shows evidence for a disk component (‘concentration of light near the major axis’) he does not treat it as a separate component. The velocity data (only a few points of the rotation curve by Humason as reported in the annual report of the Mount Wilson Observatory) was insufficient for a significant treatment. Oort concludes that, if these data are correct, the distribution of mass does not correspond to that of the light and quotes mass-to-light ratios of order 250. It took two decades before some advancement took place. Minkowski (1960) reported in a review at meeting on “*Les Recherches Galactiques et Extragalactiques et la Photographie Électronique*” in Paris in 1959 a new rotation curve, which showed an initial rise, then a secondary minimum followed by another strong rise. In the discussion after Minkowski’s paper, Oort reported that he was able to reproduce this behavior on the assumption of a proportionality of mass to light and a large velocity dispersion, which was reported also by Minkowski.<sup>25</sup>

Oort urged Maarten Schmidt to remeasure the rotation of NGC 3115 and together they took spectra in 1968 on the 200-inch Hale Telescope. It took until 1974 before Williams (1975) reduced the data. Oort never used this for a detailed dynamical study, his interests having turned to problems of galactic nuclei and cosmology (Oort, 1977, 1981, 1983). In the mean time, measurements of the light distribution improved (Miller & Prendergast, 1968; Strom et al., 1977, 1978), but although color information seemed to support the existence of color variations and the rotation curve allowed an estimate of the mass of the disk as a fraction of the total ( $\lesssim 0.4$ ), no comprehensive dynamical model was possible without better spectroscopy. The detailed surface photometry study of Tsikoudi (1979) constituted the first attempt to separate photometric components. The most accurate measurements of the stellar kinematics, confirming the flat shape of the stellar rotation and providing also evidence for a supermassive black hole in the center, has been presented by Kormendy & Richstone (1992).

The kinematic data necessary for a detailed dynamical modelling started to become available only in the eighties, first in the central regions (Rubin, Peterson & Ford, 1980) and then over a more extended region (Illingworth & Schechter, 1982). In the latter study, the kinematics of the disk were estimated from a decomposition of the contributions to the observed velocities and velocity dispersions. The main result of the study was the important deduction, that later was proved to be more general, that in bulges of disk galaxies rotation plays a bigger role in supporting its shape and density distribution than in ellipticals.

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<sup>25</sup>Oort was, however, not satisfied with his solution and never published it. He did illustrate it during his lectures on ‘Stellar Dynamics’ in Leiden, as the notes of one of us –PCK is a student of Oort– show, and mentioned there that he felt that the rotation curve might very well be wrong and needed confirmation.

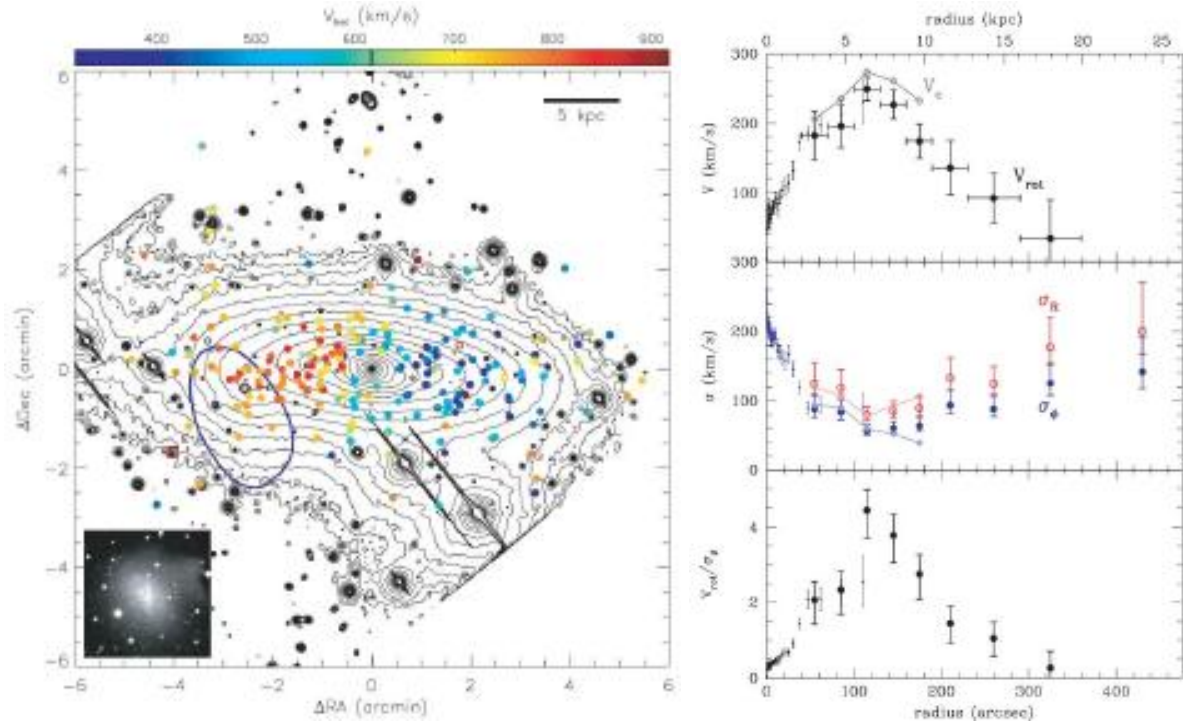


Figure 24: Kinematics of the S0 galaxy NGC 1023 from velocity measurements of planetary nebulae. On the left the distribution over the face of the system and in color the pattern of rotation. On the right the results of an analysis of the measurement, with from top to bottom the rotation velocities, the dispersions (red radial and blue tangential), and the ratio between rotation and velocity dispersion. the rotation velocities. (From Noordermeer et al., 2008)

Kormendy (1984a,b) was the first to measure the velocity dispersion in the disk of an S0 galaxy. His main aim was to estimate the Toomre (1964)  $Q$ -parameter for local stability. Both in NGC 1553 and in the barred S0 NGC 936 he was able to estimate  $Q$  and found it to be well above unity (more like 2 or 3). He therefore concluded that S0 galaxies may differ from spirals in that their stellar disks are too hot (in addition to suffering from lack of gas, which would lower the overall  $Q$ ) to form small-scale structure.

For NGC 3115 the observations of photometry and kinematics have become more sophisticated in recent years (*e.g.* Capaccioli, Vietri & Held, 1988; Silva et al., 1989; Capaccioli et al., 1993; Michard, 2007), but the issue has become much more complicated. A simple answer to the question on whether an S0 originated as a spiral that was swept of its gas or a system that formed its disk with little gas that was not replenished, has not emerged. GMOS multi-object long-slit spectroscopy by Norris, Sharples & Kuntscher (2006) shows that there is a difference between the  $[\alpha/\text{Fe}]$  in the bulge ( $\sim 0.3$ ) and the disk (close to solar), while the average age of stars in the disk (5–8 Gyr) is significantly less than in the bulge (10–12 Gyr). The fact that the disk is bluer than the bulge would then primarily be an age difference. Star formation in the disk of this archetypal S0 has proceeded at least for some time after the formation of the bulge.

We note that the advent of new techniques has greatly improved the observational possibilities. SAURON can measure kinematic data using integral field spectroscopy, such as in the study of the S0 galaxy NGC 7332 by Falc3n-Barroso et al. (2004). In this system, the stellar populations in the disk (but also in the bulge) are again relatively young, so that star formation must have proceeded in the disk until fairly recently. In addition there is evidence for a strong influence by a bar.

Another recent development is the use of planetary nebulae (PNe) to measure the kinematics. This has the advantage that velocities and dispersions can be measured in faint outer parts of galaxies (*e.g.* Coccato et al., 2009). Comparing photometry, absorption-line and PNe kinematics shows that there is good agreement between the PNe number density distribution and the stellar surface brightness and also a good agreement between PNe and absorption line kinematics. An application of this technique to an S0 galaxy is the study of NGC 1023 by Noordermeer et al. (2008); fig. 24 shows that PNe can be measured with a very good distribution over the face of the system. They show a clear pattern of rotation. The left-hand panel shows the accuracy with which velocities and velocity dispersions can be measured. With these data, it is possible to test whether an S0 could result from a minor merger such that its kinematics become dominated by random motions. The inner parts are fitted quite well with a disk that is rotationally supported, but the outer parts would suggest a minor merger event. Information like this on more systems is required to answer the basic questions.

Finally, an interesting approach is that of Aragón-Salamanca, Bedregal & Merrifield (2006) and Barr et al. (2007), who use globular clusters to measuring the fading of S0 galaxies. They do this by comparing the specific frequency of globular clusters (their number per unit luminosity,  $S_N$ ) between S0s and normal spirals. This innovative and powerful method has led to the picture in which the disk has faded by a factor of three or so. S0s with younger ages have values for  $S_N$  that are more like those of spirals. This is consistent with the view that S0s are formed as a result of removal of gas from normal spirals.

Although the kinematics of S0s would suggest that their disks have been present on a long timescale, the evidence for relatively recent star formation indicates that their history is more complicated than previously thought.

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