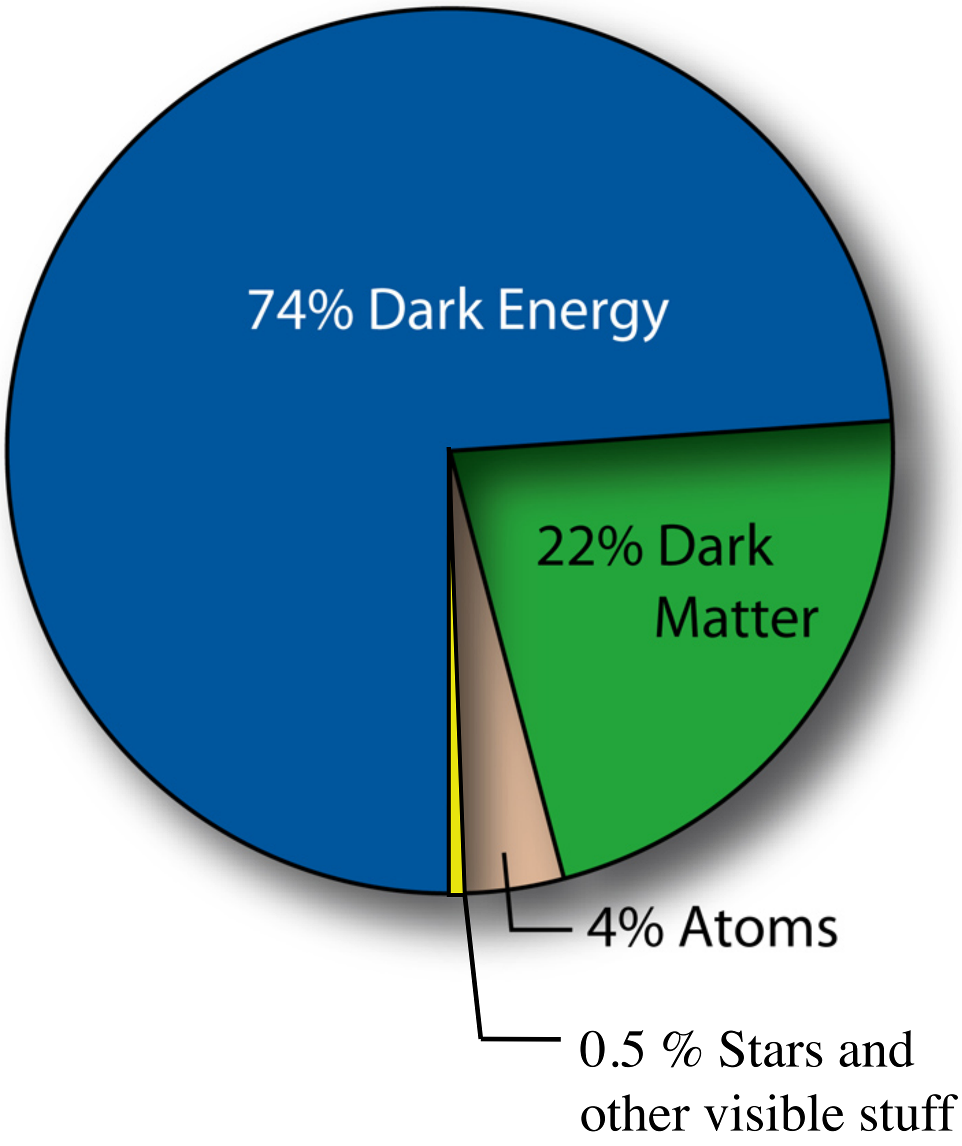

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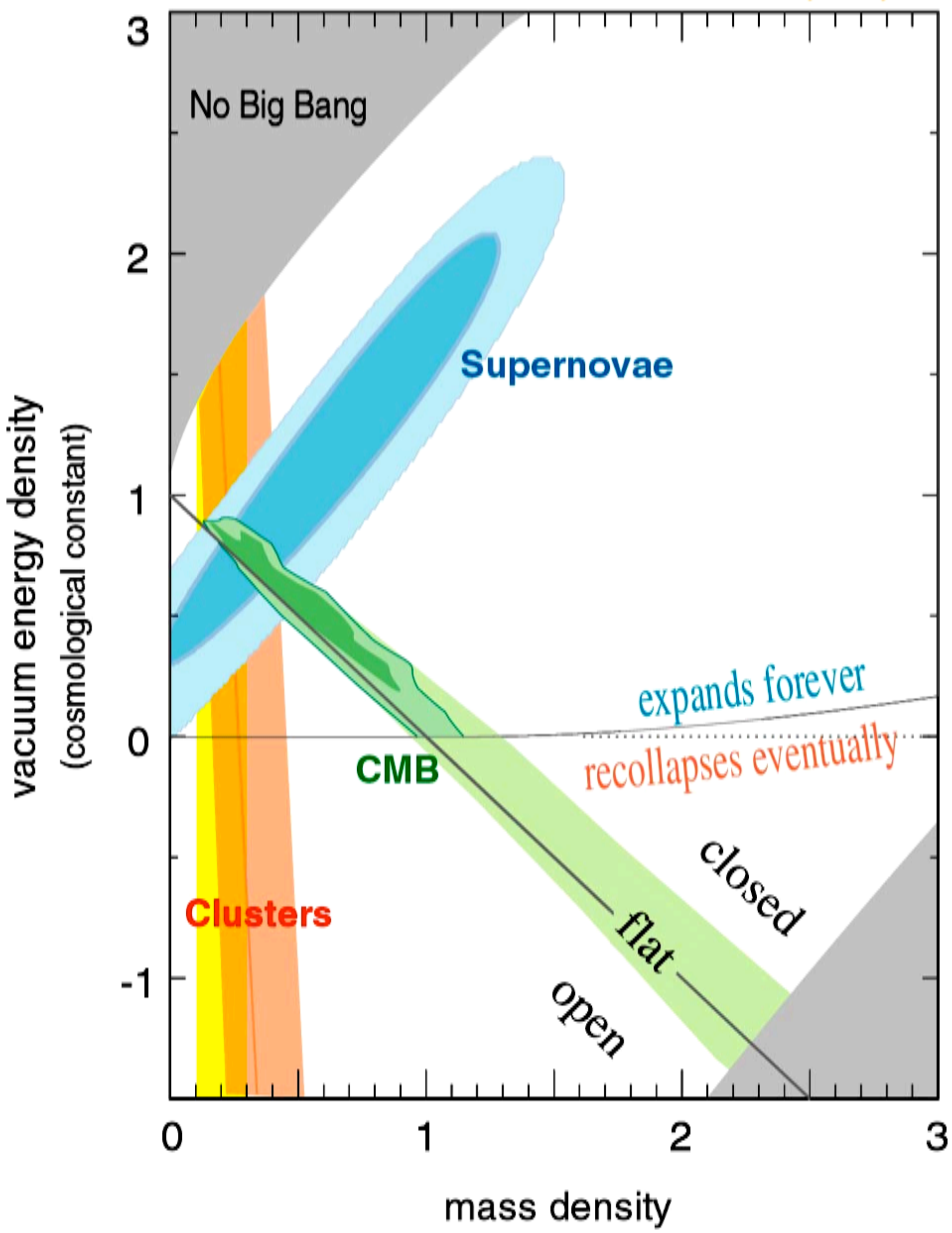
The Contents of the Universe

The Cosmic Concordance



Supernovae alone

⇒ **Accelerating expansion**

⇒ **$\Lambda > 0$**

CMB alone

⇒ **Flat universe**

⇒ **$\Lambda > 0$**

Any two of SN, CMB, LSS

⇒ **Dark energy ~70%**

Also in agreement with the age estimates (globular clusters, nucleocosmochronology, white dwarfs)

Today's Best Guess Universe

Age:

$t_0 = 13.82 \pm 0.05$ Gyr

Best fit CMB model - consistent with ages of oldest stars

Hubble constant:

$H_0 = 69$ km s^{-1} Mpc^{-1}

CMB + HST Key Project to measure Cepheid distances

Density of ordinary matter:

$\Omega_{baryon} = 0.04$

CMB + comparison of nucleosynthesis with Lyman-a forest deuterium measurement

Density of all forms of matter:

$\Omega_{matter} = 0.31$

Cluster dark matter estimate CMB power spectrum

Cosmological constant:

$\Omega_{\Lambda} = 0.69$

Supernova data, CMB evidence for a flat universe plus a low matter density

The Component Densities

at z ~ 0, in critical density units, assuming $h \approx 0.7$

Total matter/energy density: $\Omega_{0,tot} \approx 1.00$ — From CMB, and consistent with SNe, LSS

Matter density: $\Omega_{0,m} \approx 0.31$ — From local dynamics and LSS, and consistent with SNe, CMB

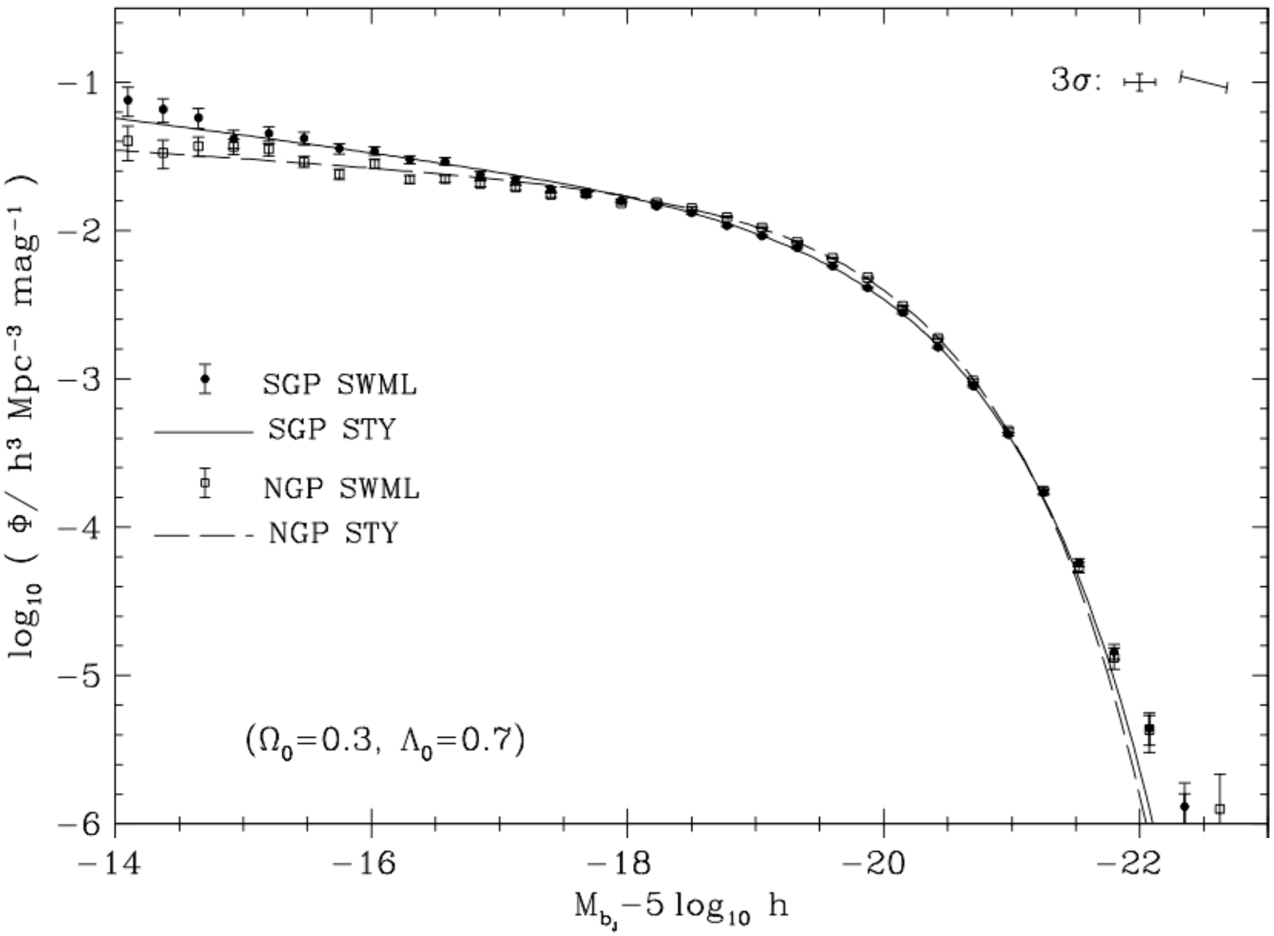
Baryon density: $\Omega_{0,b} \approx 0.045$ — From cosmic nucleosynthesis, and independently from CMB

Luminous baryon density: $\Omega_{0,lum} \approx 0.005$ — From the census of luminous matter (stars, gas)

Since: $\Omega_{0,tot} > \Omega_{0,m} > \Omega_{0,b} > \Omega_{0,lum}$

- $\Omega_{0,b} > \Omega_{0,lum}$ → There is baryonic dark matter
- $\Omega_{0,m} > \Omega_{0,b}$ → There is non-baryonic dark matter
- $\Omega_{0,tot} > \Omega_{0,m}$ → There is dark energy

The Luminosity Density



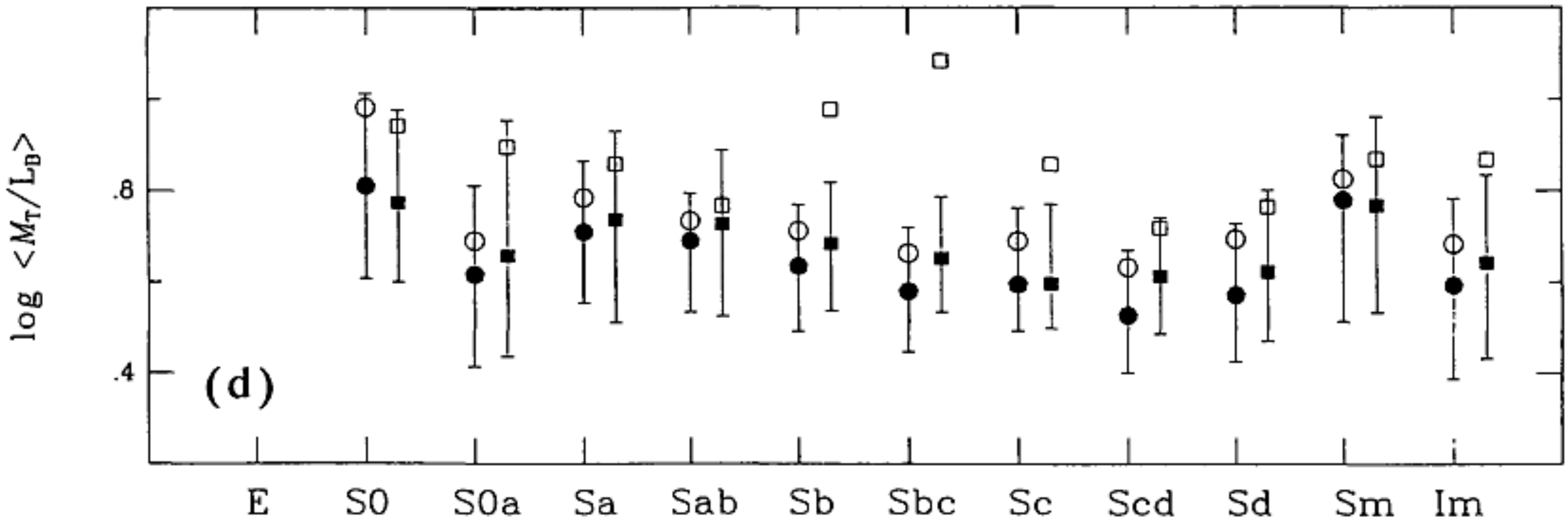
Integrate galaxy luminosity function (obtained from large redshift surveys) to obtain the mean luminosity density at $z \sim 0$

SDSS, *r* band: $\rho_L = (1.8 \pm 0.2) \times 10^8\ h_{70}\ L_\odot/\text{Mpc}^3$

2dFGRS, *b* band: $\rho_L = (1.4 \pm 0.2) \times 10^8\ h_{70}\ L_\odot/\text{Mpc}^3$

Luminosity To Mass

Typical (M/L) ratios in the B band along the Hubble sequence, within the luminous portions of galaxies, are ~ 4 - 5 $M_\odot/L_\odot$



This includes some dark matter - for pure stellar populations, (M/L) ratios should be slightly lower. ISM adds ~ 10%. Note that in the B band, (M/L) ratios are very sensitive to any recent star formation, and to dust extinction.

The Local Mass Density of the Luminous Matter in Galaxies

$$\rho_{\rm lum} = \rho_{\rm L} \times \langle M/L \rangle \times \langle 1 + f_{\rm gas} \rangle \approx (7 \pm 2) \times 10^{8}\, h_{70}\, M_{\odot}/{\rm Mpc}^{3}$$

$$\rho_{\rm lum} \approx (4.7 \pm 1.3) \times 10^{-32}\, h_{70}\ {\rm g\ cm}^{-3}$$

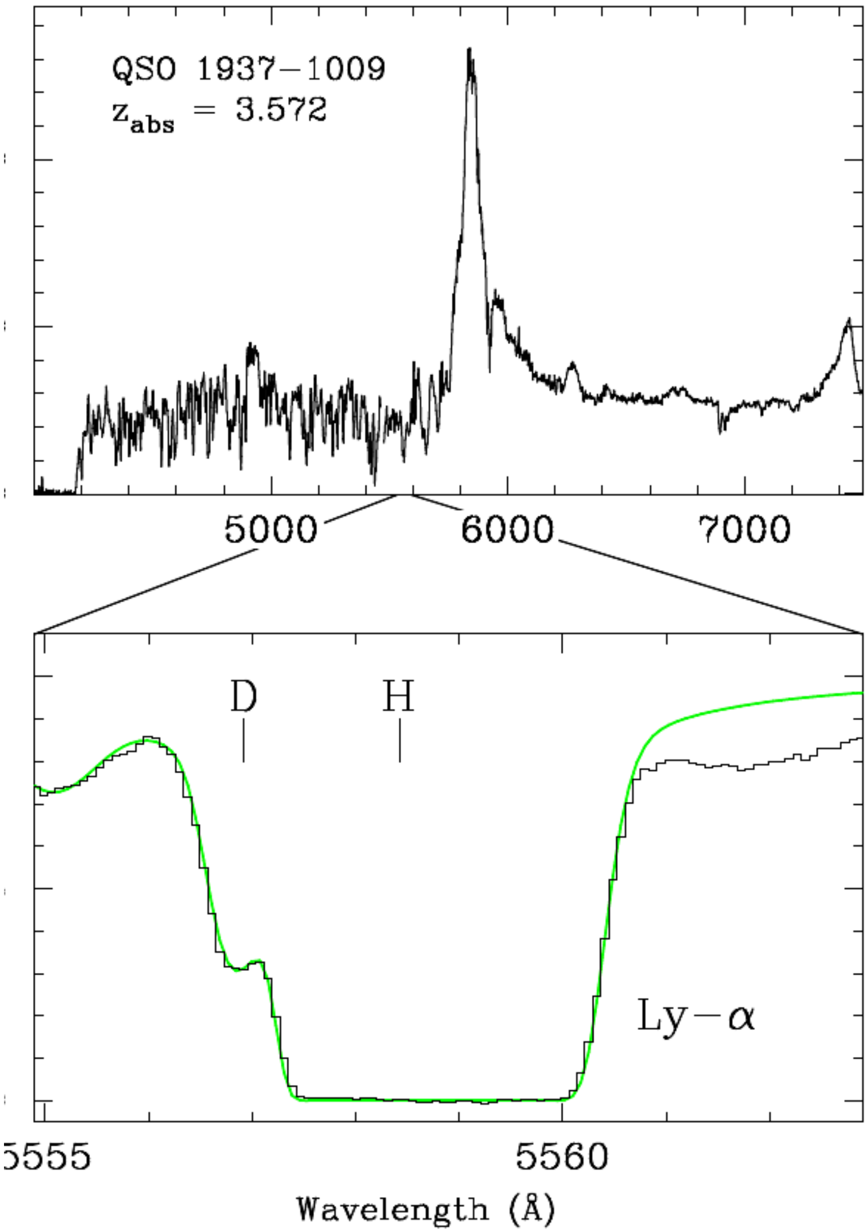
Recall that $\rho_{0,{\rm crit}} = 3H_0^2/(8\pi G) = 0.921 \times 10^{-29}\, h_{70}^{2}\ {\rm g\ cm}^{-3}$

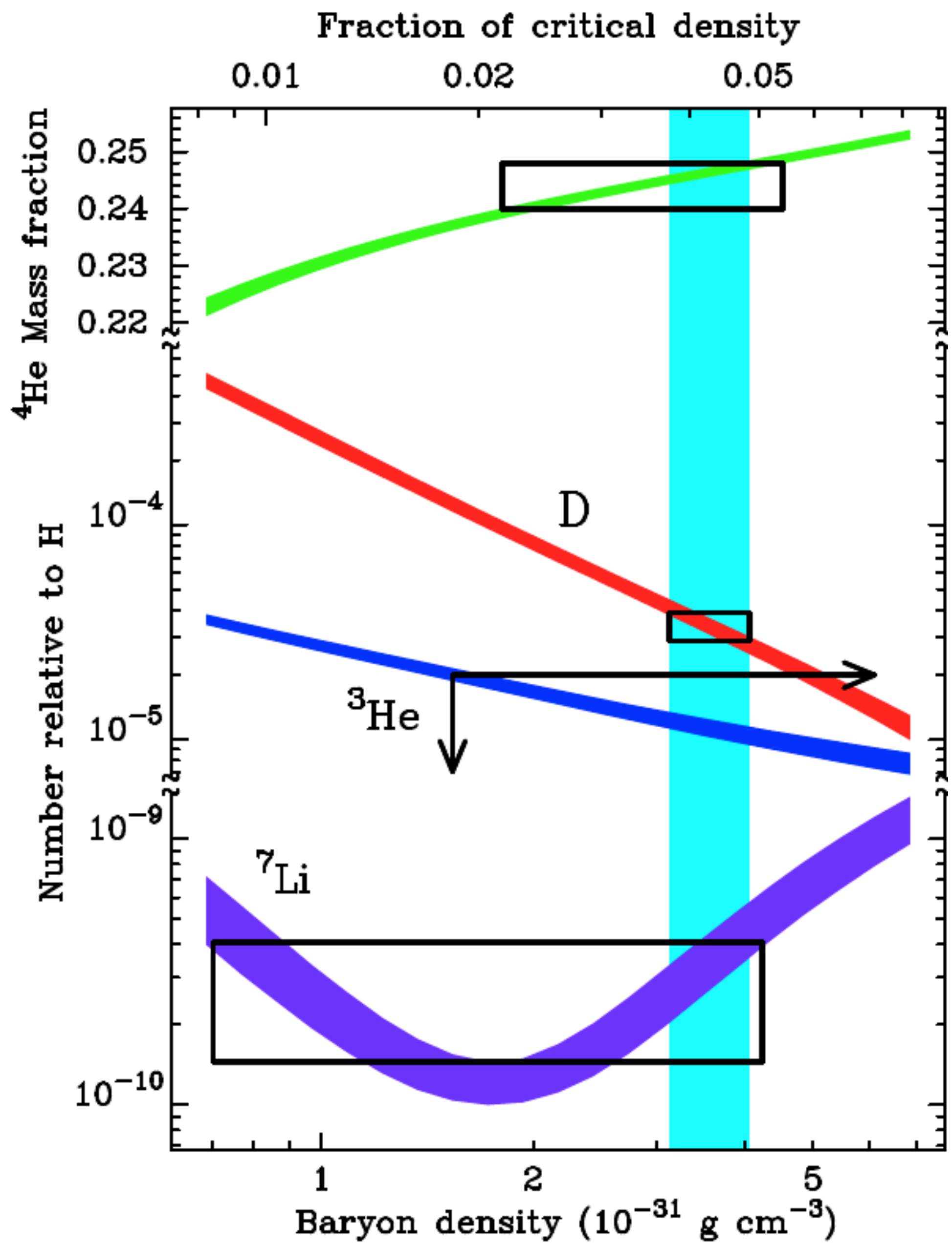
Thus, $\Omega_{0,lum} \approx (0.0051 \pm 0.0015)\, h_{70}^{-1}$

All of the visible matter amounts to only half a percent of the total mass/energy content of the universe!

(Interestingly, this may be comparable to the contribution from the massive cosmological neutrinos…)

Baryon Density From Cosmic Nucleosynthesis





The Total Baryon Density

It is measured in two completely independent ways:

1. The cosmic nucleosynthesis:

- It occurs in the first few minutes after the Big Bang
- Reaction rates are ~ ρ_{baryon}^2, so the residual abundances of D, He, and Li are very sensitive to ρ_{baryon} (especially for D)
- Measured in spectra of distant QSOs (actually Lyα forest clouds), low metallicity starforming dwarfs, halo stars, etc.

Results give: $\Omega_{baryons} h^2 = 0.021 \rightarrow 0.025$

2. Analysis of CMB fluctuations:

Results give: $\Omega_{baryons} h^2 = 0.024 \pm 0.001$

Thus, $\boldsymbol{\Omega_{0,b}} \approx (0.045 \pm 0.002)\ h_{70}^{-2}$

The Baryonic Dark Matter

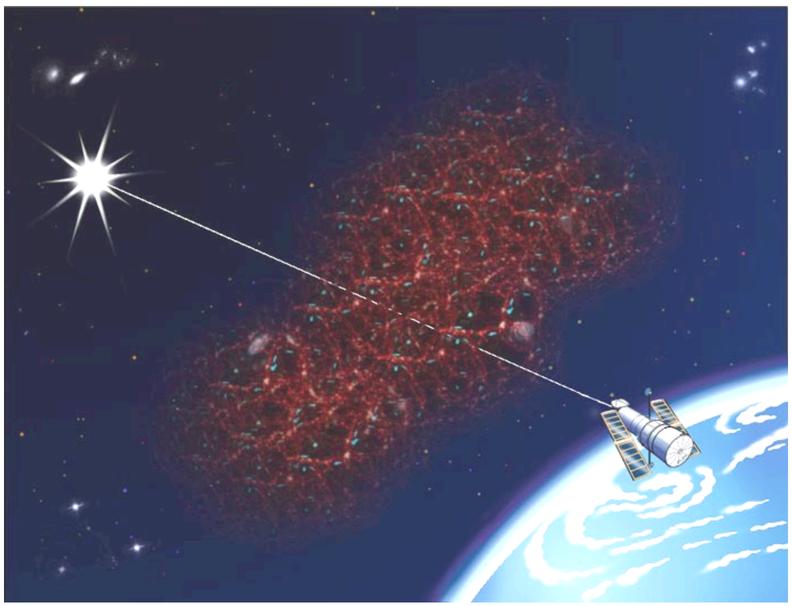
(or just "missing", not necessarily "dark"?)

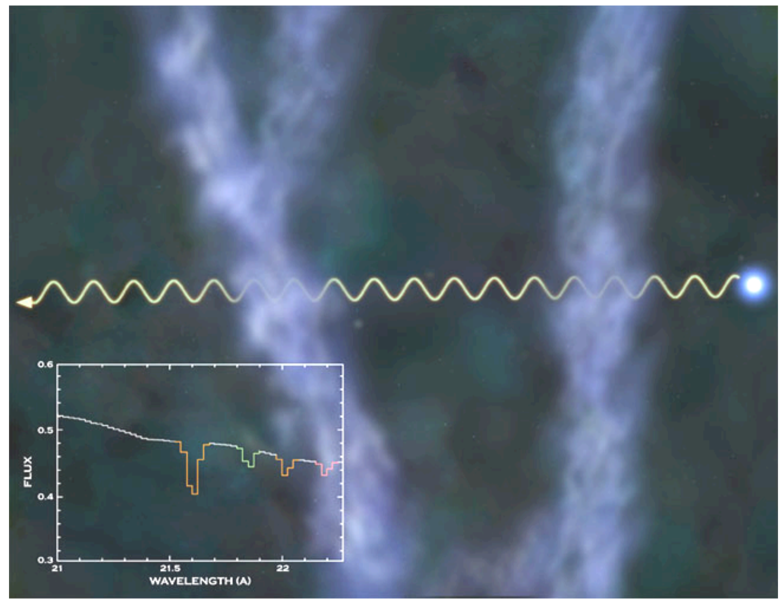
So, where are 90% of baryons hiding? Some possibilities:

- **MAssive Compact Halo Objects (MACHOs)**
  - Very low mass stars, white dwarfs, neutron stars, black holes (produced post-nucleosynthesis, from baryons), brown dwarfs, interstellar comets, slushballs…
- **Cold molecular (H_2) gas clouds**
  - Would have to be compact, dense, low volume fill factor
  - Very hard to detect!
- **Warm/hot gas**, bound to galaxy groups ← The best bet?
  - Leftover gas from IGM, never collapsed to galaxies
  - Virial temperatures ~ 10^5 - 10^6 K, corresponding to the velocity dispersions ~ 300 km/s
  - Very hard to detect! (ISM opaque to FUV/soft-X)

Missing Baryons in Warm/Hot IGM?

This hypothetical Baryon reservoir would have Virial temps. of ~ 10^5 - 10^6 K, where the peak emission is in FUV/soft-X, which is effectively absorbed by the ISM in our Galaxy, and is thus essentially impossible to detect in emission …

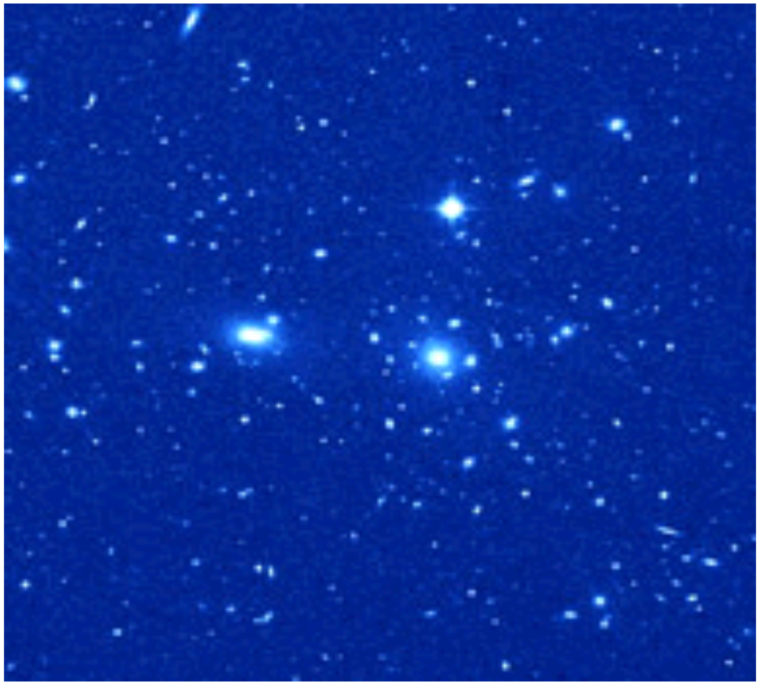




However, it might have been ***detected in absorption*** in the UV (HST and FUSE) and X-Rays (Chandra), using O VI, O VII, and O VIII lines

The Non-Baryonic Dark Matter

Discovered by Zwicky in 1937, by comparing the visible mass in galaxies in the Coma cluster (estimated $M_* \sim 10^{13}$ $M_\odot$), with the virial mass estimates ($M_{vir} \sim 5 \times 10^{14}$ $M_\odot$)



Confirmed by the modern measurements of galaxy dynamics, X-ray gas analysis, and masses derived from gravitational lensing

Virial Masses of Clusters:

Virial Theorem for a test particle (a galaxy, or a proton), moving in a cluster potential well:

$$E_k = E_p / 2 \quad \rightarrow \quad m_g \sigma^2 / 2 = G\, m_g\, M_{cl} / (2\, R_{cl})$$

where σ is the velocity dispersion

Thus the cluster mass is: $M_{cl} = \sigma^2 R_{cl} / G$

Typical values for clusters:

$\sigma \sim 500 - 1500$ km/s

$R_{cl} \sim 3 - 5$ Mpc

Thus, typical cluster masses are $\mathbf{M_{cl} \sim 10^{14} - 10^{15}\, M_{\odot}}$

The typical cluster luminosities (~ 100 - 1000 galaxies) are $L_{cl} \sim 10^{12}\, L_{\odot}$, and thus $(M/L) \sim 200 - 500$ in solar units

→ ***Lots of dark matter!***

Masses of Clusters From X-ray Gas

- Note that for a proton moving in the cluster potential well with a $\sigma \sim 10^3$ km/s, $E_k = m_p\,\sigma^2 / 2 = 5\,k\,T / 2 \sim$ few keV, and $T \sim$ few 10^7 °K → **X-ray gas**
- Hydrostatic equilibrium requires:

$$M(r) = -\,kT/\mu m_H G\,(d\ ln\ \rho\,/d\ ln\ r)\ r$$

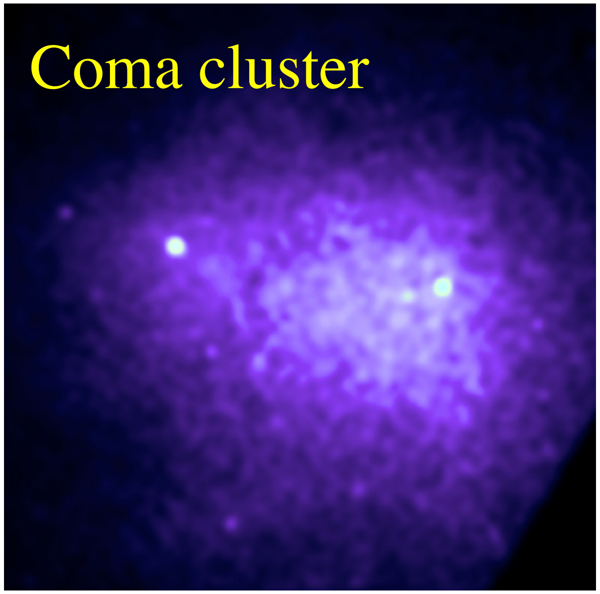
- If the cluster is ~ spherically symmetric this can be derived from X-ray intensity and spectral observations
- Typical cluster mass components from X-rays:

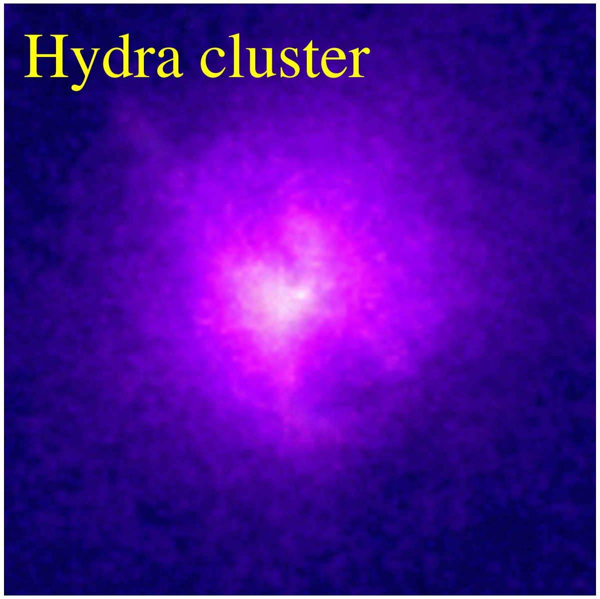
  Total mass: 10^{14} to 10^{15} $M_{\odot}$

  Luminous mass: ~5%

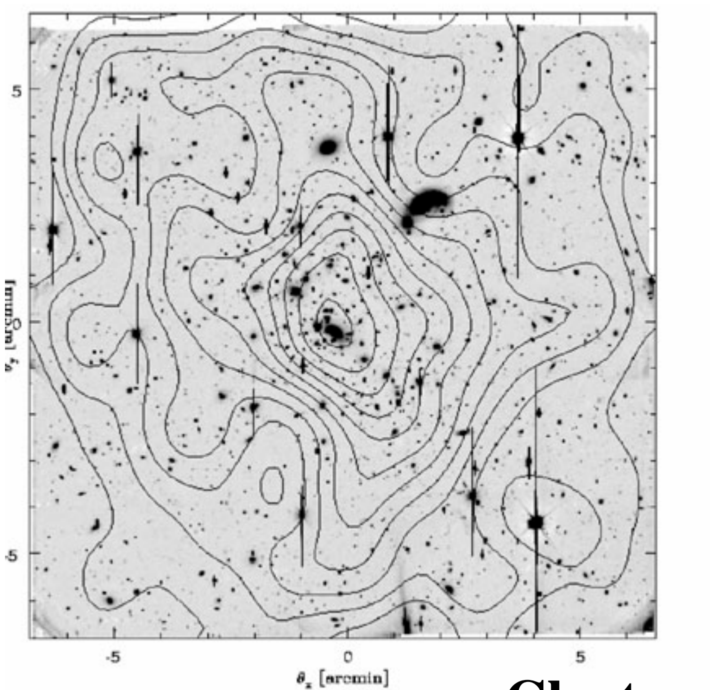
  Gaseous mass: ~ 10%

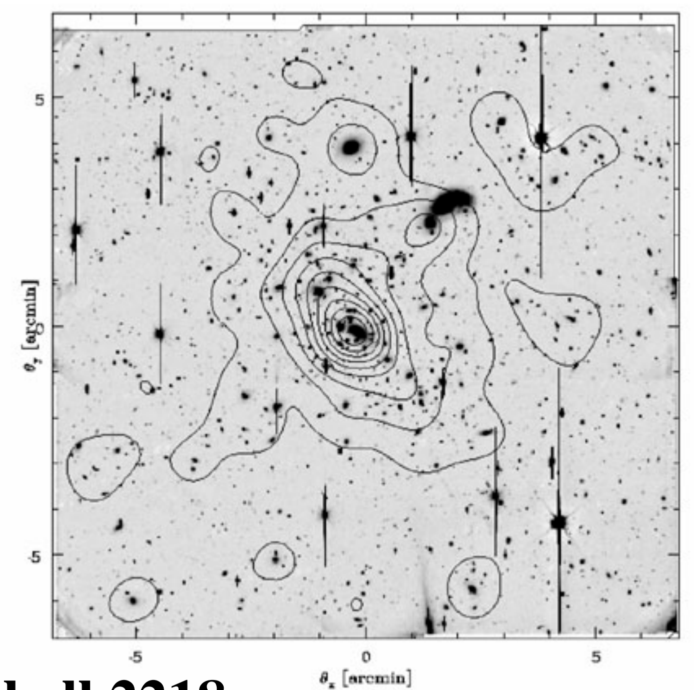
  Dark matter: ~85%





Galaxy number density

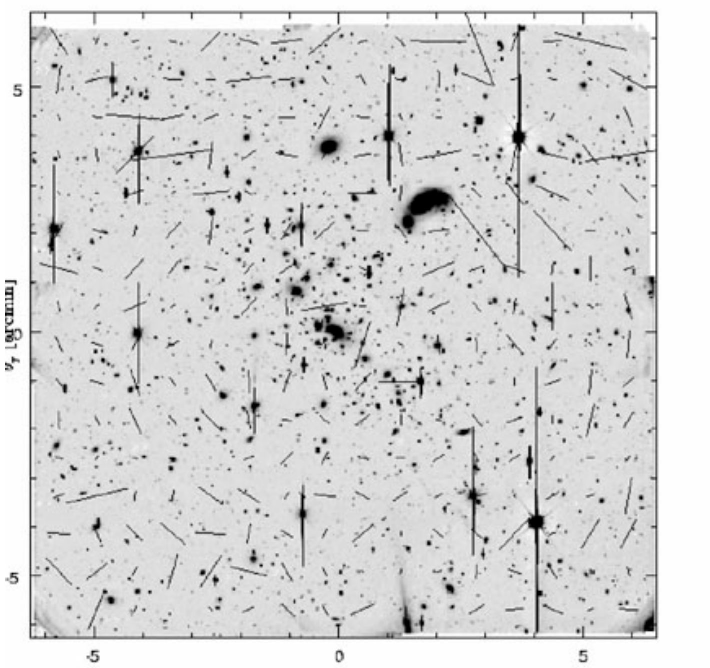


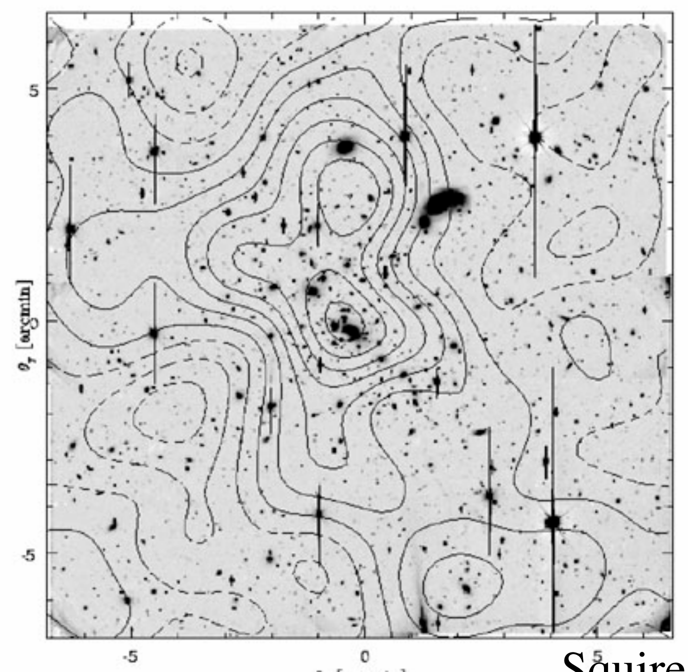


Light

Cluster Abell 2218

Shear map





Mass

Squires et al. 1996

Cluster Masses From Gravitational Lensing

Strong lensing constraints:

A370	$M \sim 5 \times 10^{13} h^{-1} M_\odot$	$M/L \sim 270h$
A2390	$M \sim 8 \times 10^{13} h^{-1} M_\odot$	$M/L \sim 240h$
MS2137	$M \sim 3 \times 10^{13} h^{-1} M_\odot$	$M/L \sim 500h$
A2218	$M \sim 1.4 \times 10^{14} h^{-1} M_\odot$	$M/L \sim 360h$

Weak lensing constraints (a subset):

MS1224	$M/L \sim 800h$
A1689	$M/L \sim 400h$
CL1455	$M/L \sim 520h$
A2218	$M/L \sim 310h$
CL0016	$M/L \sim 180h$
A851	$M/L \sim 200h$
A2163	$M/L \sim 300h$

Lots of dark matter in clusters, in a broad agreement with virial mass estimates

Clusters of galaxies imply $\Omega_{dm} \sim 0.1 - 0.3$


The “Bullet” Clusters

Baryonic Mass Fraction in Clusters

- We can measure the baryonic fraction of galaxy cluster mass

$$f_B = f_{gas} + f_{gal} + f_{db} \qquad\qquad f_B > f_{gas} + f_{gal}$$

- Assume that this is universal, i.e., that clusters provide a fair sample of the Universe. Then taking the value of Ω_B from nucleosynthesis and CMB, we can estimate the total matter density parameter Ω_M :

$$f_U = \frac{\Omega_B}{\Omega_M} \qquad\qquad \text{if } f_U = f_B \text{ then } \Omega_M = \frac{\Omega_B}{f_B}$$

- Gas constitutes ~20% of the total mass in the most massive clusters. This gives a lower limit on f_B, and hard, upper limit on Ω_M:

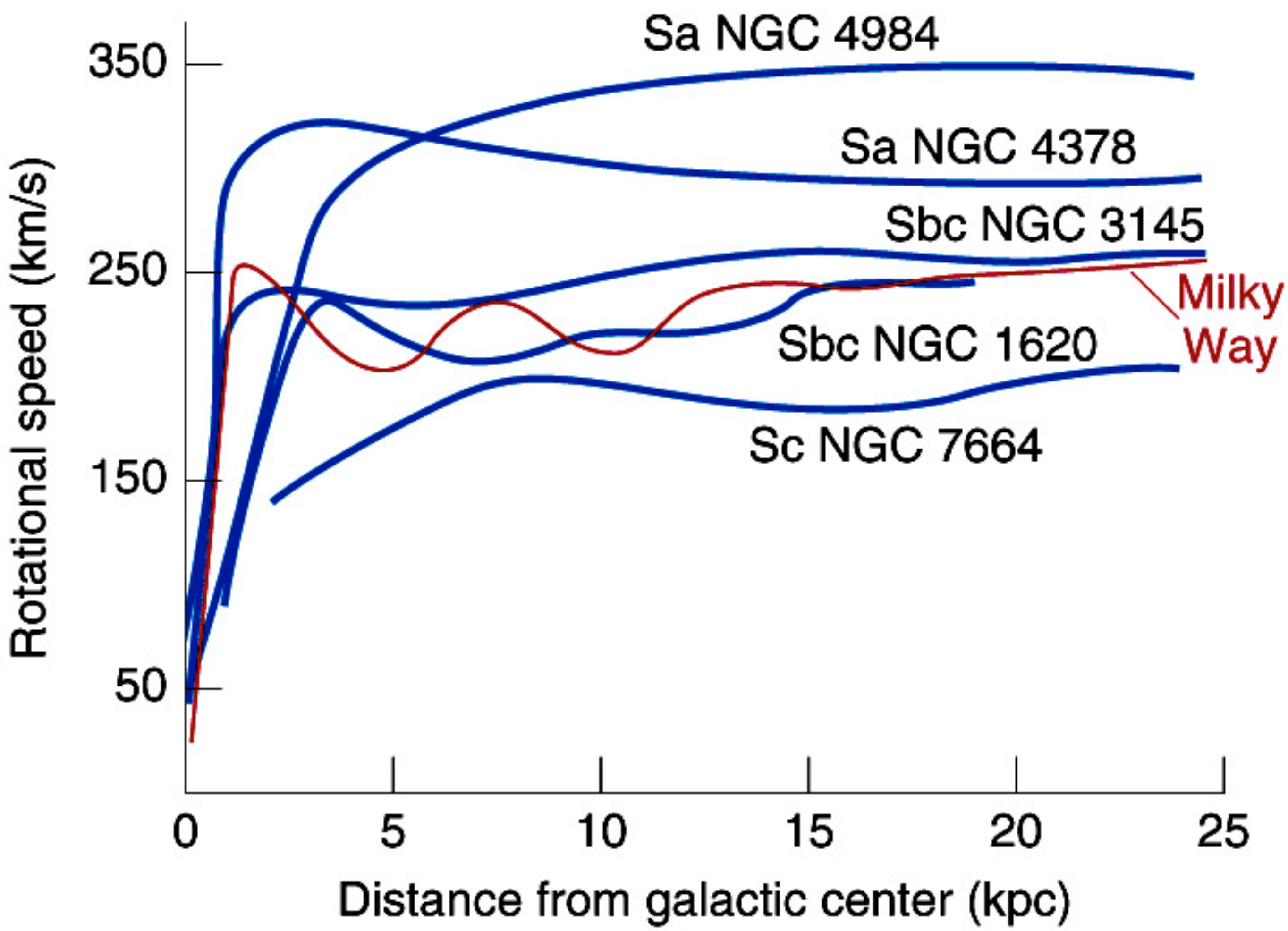
$$\text{because } f_{gas} < f_B \quad \Omega_M < \frac{\Omega_B}{f_{gas}} \qquad\qquad \Omega_M < 0.36 \pm 0.01$$

- Combined with measurements of the galaxy contribution to the cluster mass we get a best estimate of $\Omega_M \approx 0.25$

… in an excellent agreement with other methods!

Flat Rotation Curves of Disk Galaxies:

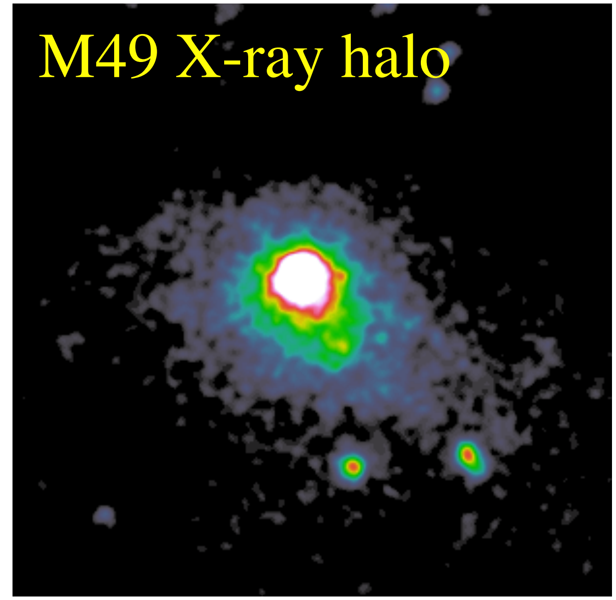
The Other Key Piece of Evidence for the Existence of Dark Matter

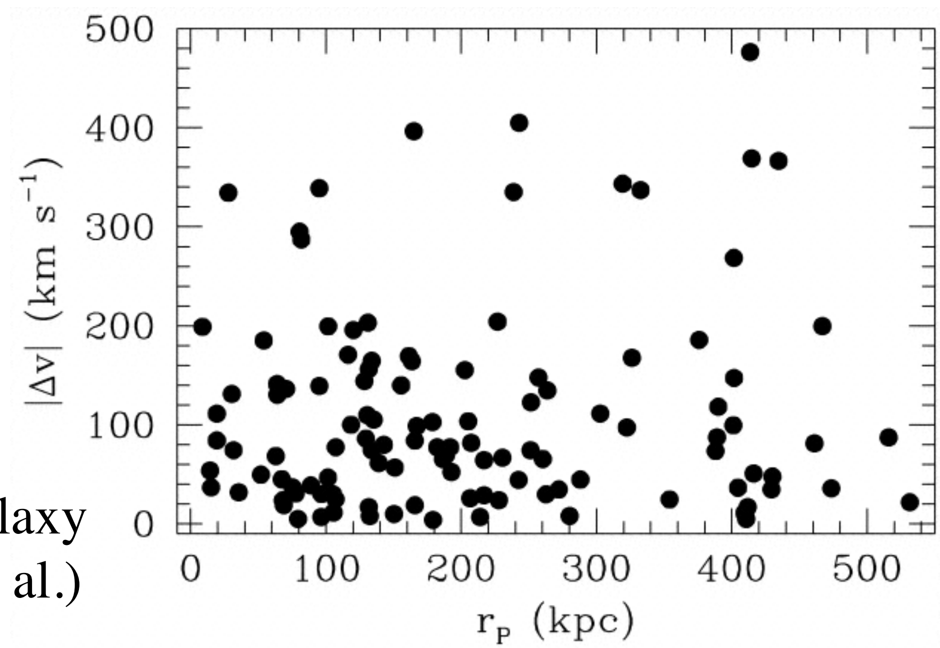


Noted early by Jan Oort and others, but really appreciated since 1970's, due to the work by Rubin, Ford, and others

Dark Matter in Elliptical Galaxies

- Similar to spirals, but using X-ray gas, planetary nebulae, globular clusters, or companion galaxies as test particles to map the velocity field at large radii
- X-ray gas gives the strongest evidence for DM in ellipticals, but mass density in the visible parts is dominated by baryons
- Most of the motions are random, rather than circular, so one can speak of a flat velocity dispersion curve

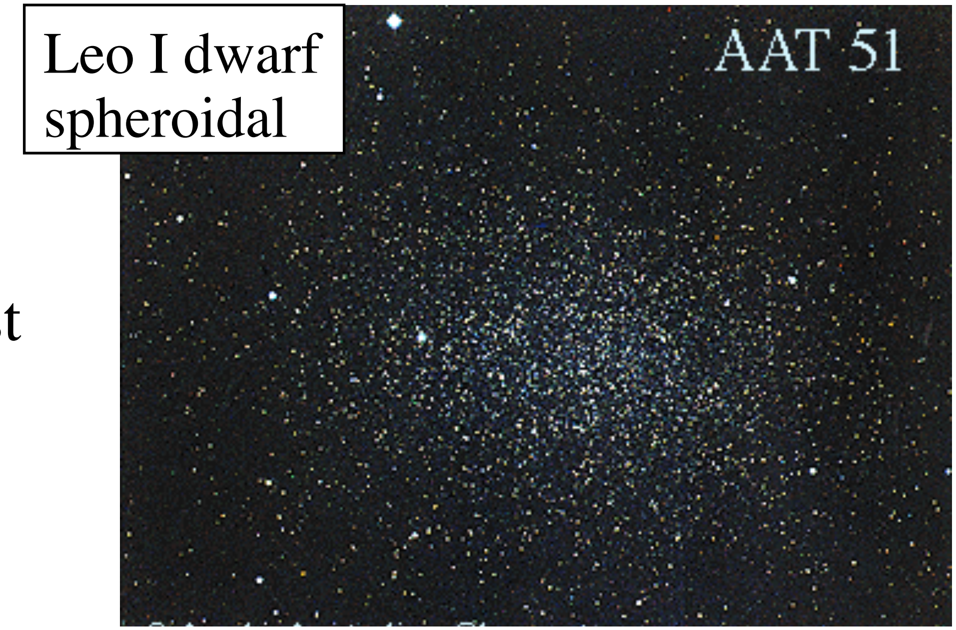


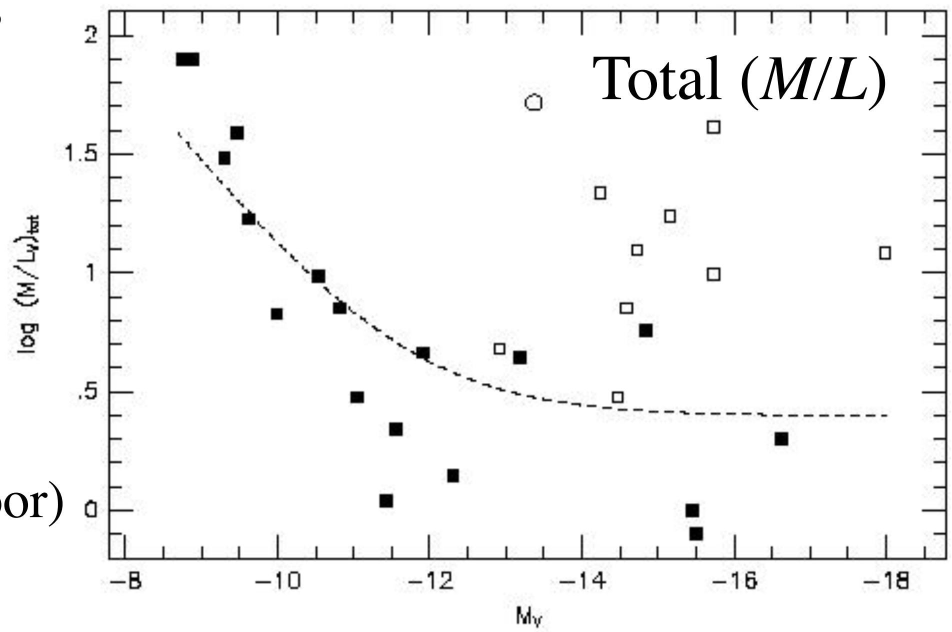


Relative velocities of dwarf galaxy companions of E's (Zaritsky et al.)

Dark Matter in Dwarf Galaxies

- Kinematics of dwarf galaxies suggests copious amounts of DM, especially in the lowest luminosity systems (the smallest systems are the darkest), with (*M*/*L*) ratios reaching ~ 100!
- One theory is that baryons have been expelled by galactic winds in their early star forming stages, while the DM remained

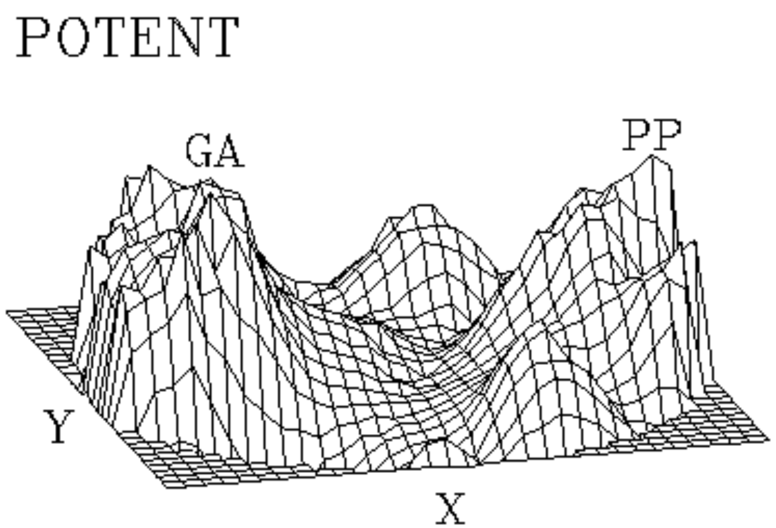




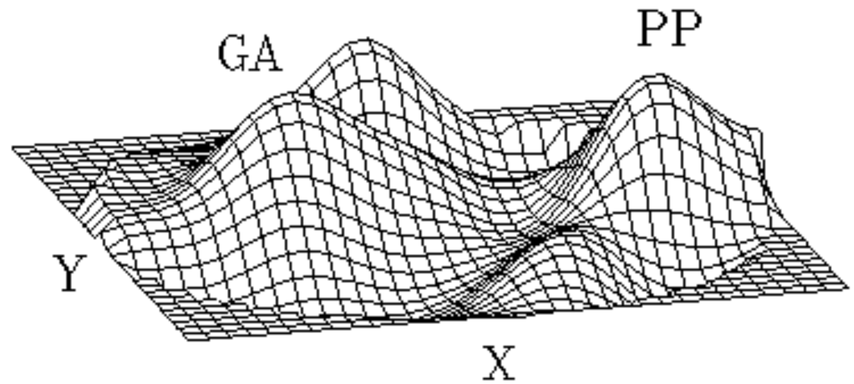
Filled squares = dSph (gas poor)
Open squares= dIrr (gas rich)

Mass Density From Peculiar Velocities

- Assume that the measured galaxy peculiar velocities are generated from nearby large mass concentrations; derive the implied gravitational potential, which implies the mass distribution
- Compare the observed velocity field to a density field (derived from a galaxy redshift survey) and derive the matter density distribution
- Most results favor $\Omega_m < 0.3$



IRAS



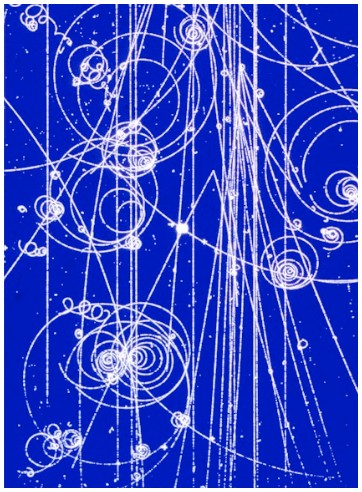
Density contours from POTENT (peculiar velocity analysis) and IRAS redshift survey

Non-Baryonic DM Candidates

- **Massive neutrinos** ← The ***only*** DM constituent actually known to exist!
  - Known to exist and to have mass, but how much?
- **Weakly Interacting Massive Particles (WIMPs)**
  - Not known to exist, but possible
  - A generic category, e.g., the neutralino = the least massive SUSY particle; also inlcude gravitinos, photinos, and higgsino
  - Thermal relics from the Big Bang
  - Possible masses > 10 GeV
  - WIMPzillas: 10^{10} × mass of WIMPS, would have been created just after the Big Bang, and might explain ultra-high-energy cosmic rays
- **Axions**
  - Predicted in some versions of quantum chromodynamics
  - Originate in non-thermal processes
  - Could interact electromagnetically
  - Possible masses 10^{-12} eV to 1 MeV
- **Many (many!) other speculative possibilities …**

The Physical Nature of the DM

We know that ***some of it*** is regular matter, H and He atoms and ions, just hidden; and some is in massive cosmological neutrinos



But we also know that ***most of it*** is composed of some as yet unknown type of particles, or represents some new physics

The proposed possible constituents range from unknown ultra-light particles, to massive black holes and cosmic strings, but the favorite DM particles are WIMPs, or axions

These particles could be detected in laboratory experiments, or with accelerators like the LHC

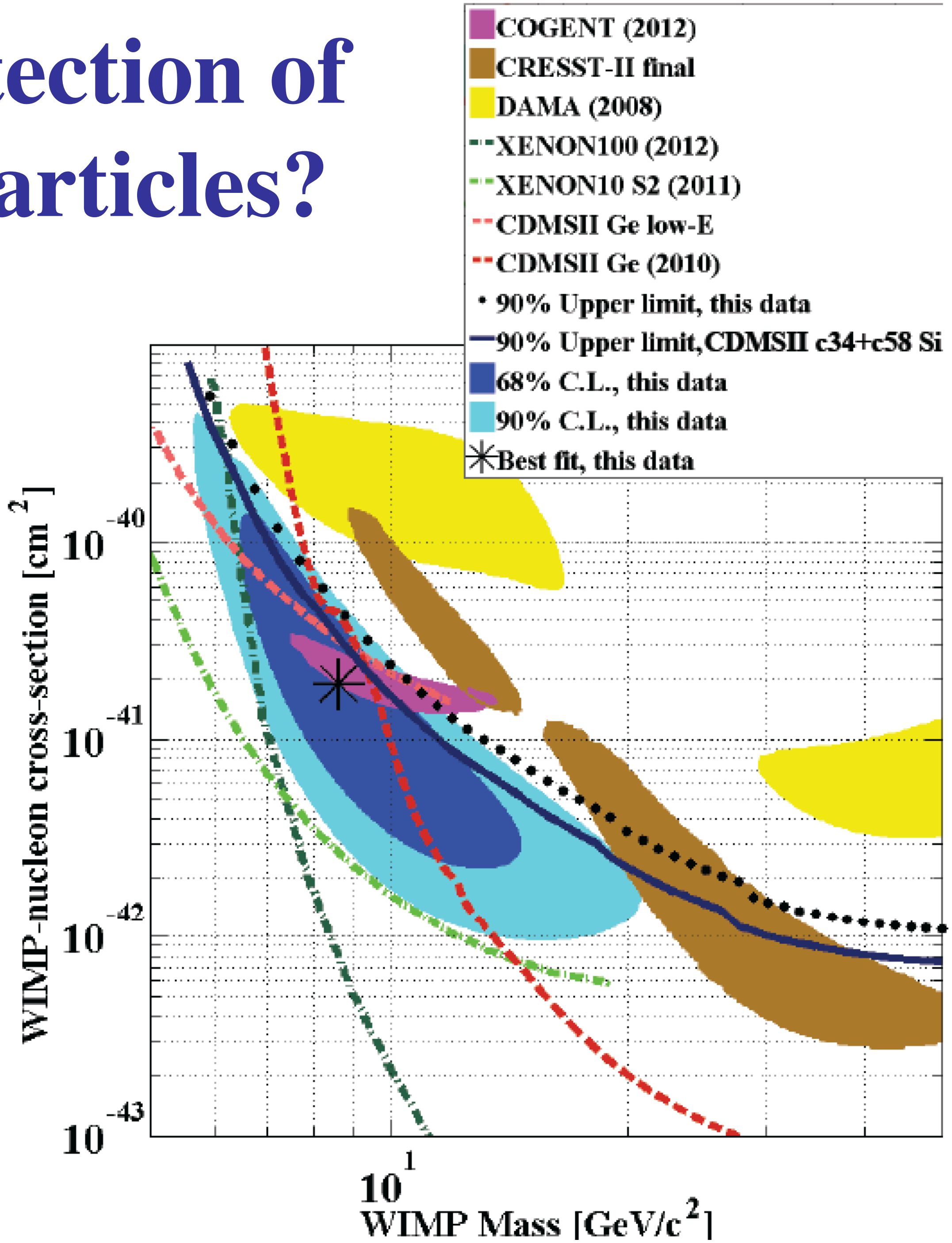
The Types of Non-Baryonic Dark Matter

- DM dominates the density field and thus governs the structure formation in the universe
- **Hot (HDM):** matter is relativistic, so low-mass particles such as neutrinos
  - Their streaming erases the small-scale density fluctuations, so big structures form first, then later fragment. This is "top-down" structure formation
- **Cold (CDM):** matter moves more slowly; includes exotic as yet unknown particles such as axions, WIMPs, etc.
  - Density fluctuations at all scales survive. Small fluctuations collapse first, then larger ones (pulling in the littler ones along the way). This is "bottom-up" structure formation and this is the best match to what we observe
- There is probably a little bit of HDM and a lot of CDM

Laboratory Detection of Dark Matter Particles?

Now pursued by many groups
Usually involves inelastic scattering of a DM particle in an ultracold crystal, and measurement of the deposited kinetic energy.

No convincing results yet.



Is There Really a Dark Matter …
… Or is Newtonian Gravity Wrong?

- Milgrom (1983) proposed a modification to Newtonian gravity, Modified Newtonian Dynamics (MOND), in which

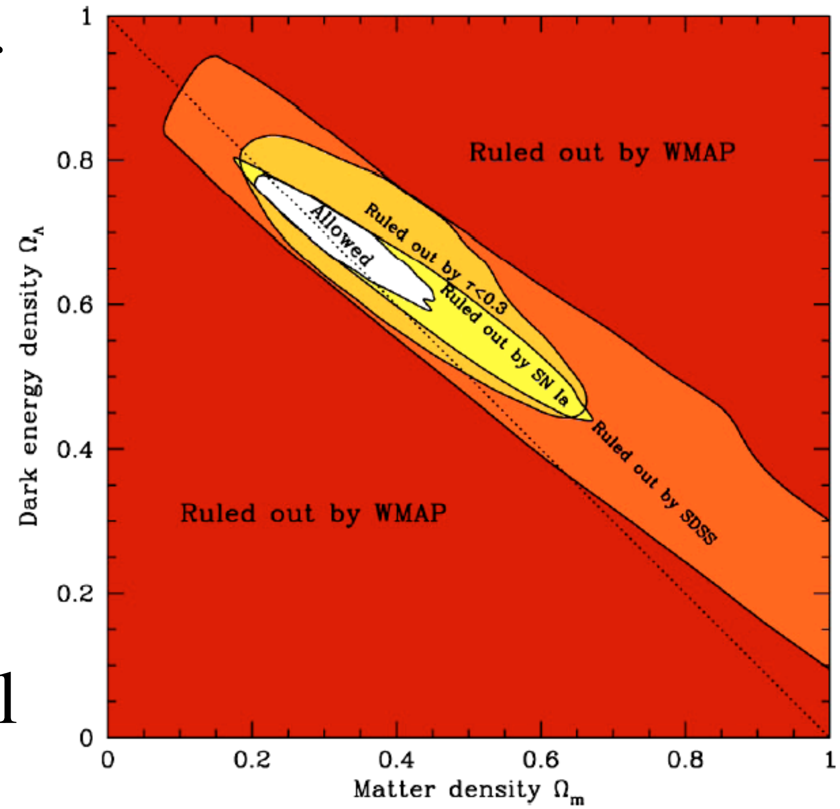
$$F = m\,\mu\,(a/a_0)\,a$$

where $\mu\,(x \gg 1) = 1$ (normal gravity), and $\mu\,(x \ll 1) \sim x$, so MOND would only kick in at low accelerations (what we generally see in galaxy dynamics) $a_0 \sim 10^{-8}$ cm/s^2

- For $a \ll a_0$, $a = (a_0\, g_N)^{1/2}$ there is more acceleration than expected from Newtonian gravity at slow acceleration scales
- MOND *may* explain flat rotation curves and the Tully-Fisher relation, but can't explain extra mass in the cores of big clusters (acceleration scales too big); probably not dwarf galaxies
- It is an *ad hoc* model - no clear physical motivation other than to get rid of the DM - and no other testable predictions
- It could be made consistent with GR, but it is awkward…

The Dark Energy

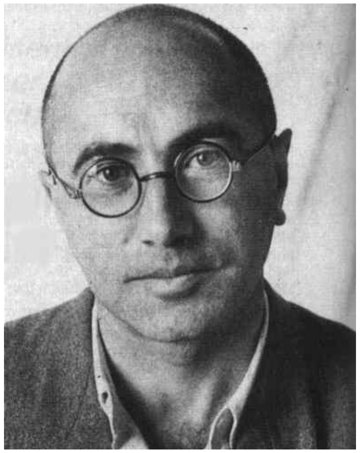
- The **dominant component** of the observed matter/energy density: $\Omega_{0,DE} \approx 0.7$
- Causes the accelerated expansion of the universe
- May affect the growth of density perturbations
- Effective only at cosmological distances

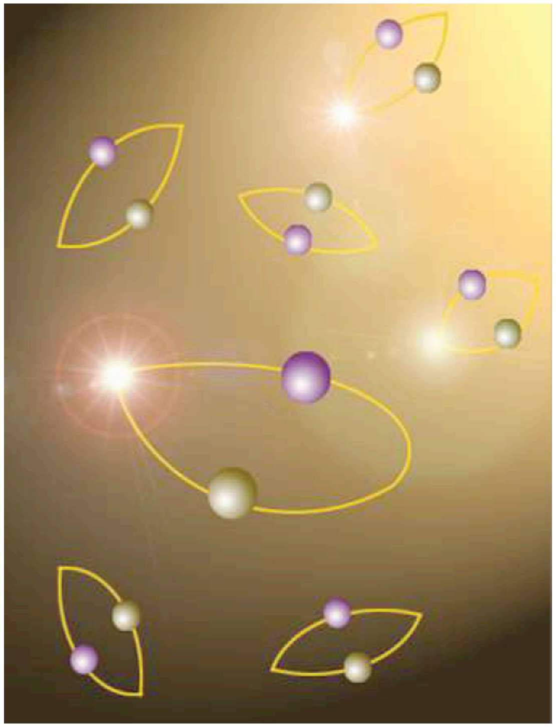


- Its physical nature is as yet ***unknown***; this may be the biggest outstanding problem in physics today
- ***Cosmological constant*** is just one special case; a more general possibility is called ***quintessence***

The Weight of the Vacuum

A key idea was due to Yakov Zel'dovich (1968)





A modern, quantum view of the physical vacuum is that it is not really empty - it is filled with virtual particle-antiparticle pairs. Their fluctuations give rise to a net energy density - a ground(?) state of the physical vacuum.

That would manifest itself as a **cosmological constant**

Unfortunately, we do not yet have a theory which would enable us to calculate this. But eager minds do try…

The Worst Scientific Prediction Ever

- A "natural" Planck system of units expresses everything as combination of fundamental physical constants; the Planck density is:

$$\rho_{Planck} = c^{5} / (\hbar\, G^{2}) = 5.15 \times 10^{93} \text{ g cm}^{-3}$$

- The observed value is:

$$\rho_{vac} = \Omega_{\text{vac}}\, \rho_{crit} \approx 6.5 \times 10^{-30} \text{ g cm}^{-3}$$

Ooops! Off by 123 orders of magnitude …

- This is modestly called *"the fine-tuning problem"*

  (because it requires a cancellation to 1 part in 10^{123})

- The other "natural" value is zero
- So, lacking a proper theory, physicists just declared the cosmological constant to be zero, and went on…

Cosmological Constant or Quintessence?

- **Cosmological constant:** energy density constant in time and spatially uniform
  - Corresponds to the energy density of the physical vacuum
  - A coincidence problem: why is $\Omega_\Lambda \sim \Omega_m$ just now?
- **Quintessence:** time dependent and possibly spatially inhomogeneous; e.g. scalar field rolling down a potential
- Both can be described in the equation of state formalism:

$$P = w\,\rho$$

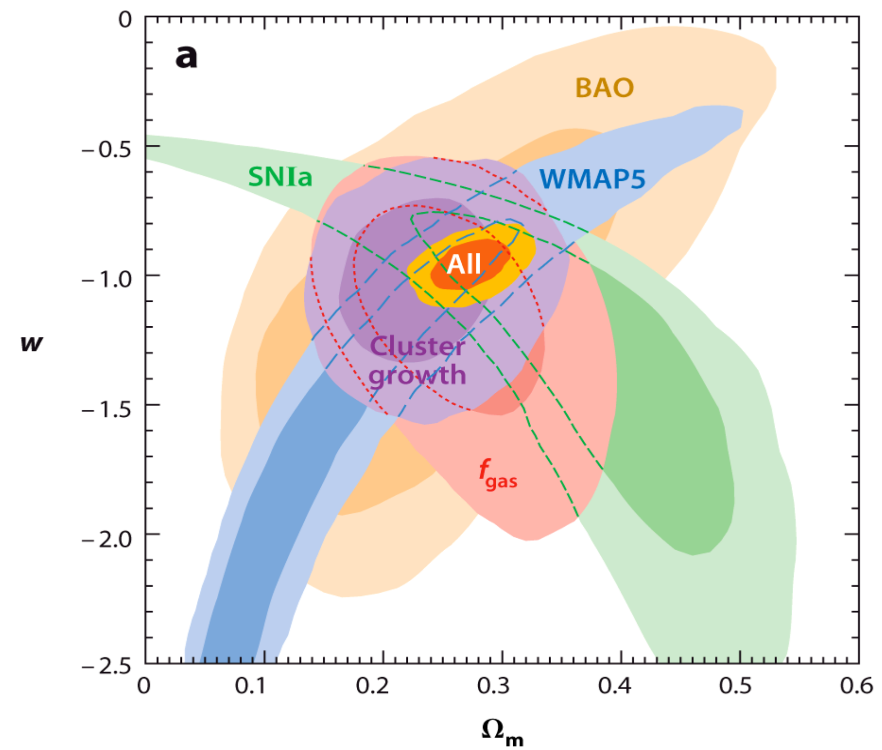
$$\rho \sim R^{-3(w+1)}$$

Cosmological constant: $w = const. = -1,\ \rho = const.$

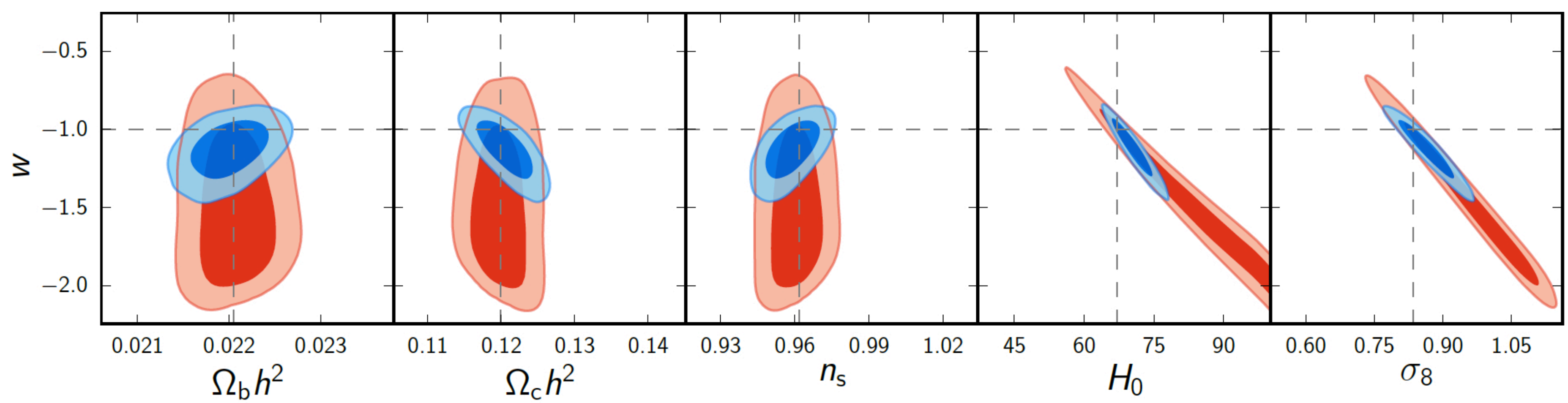
Quintessence: w can have other values and change in time

Observational Constraints on *w*

Strongly favor values of $w \sim -1$, i.e., cosmological constant. Some models can be excluded, but there is still room for $\rho_{vac} \neq const.$ models

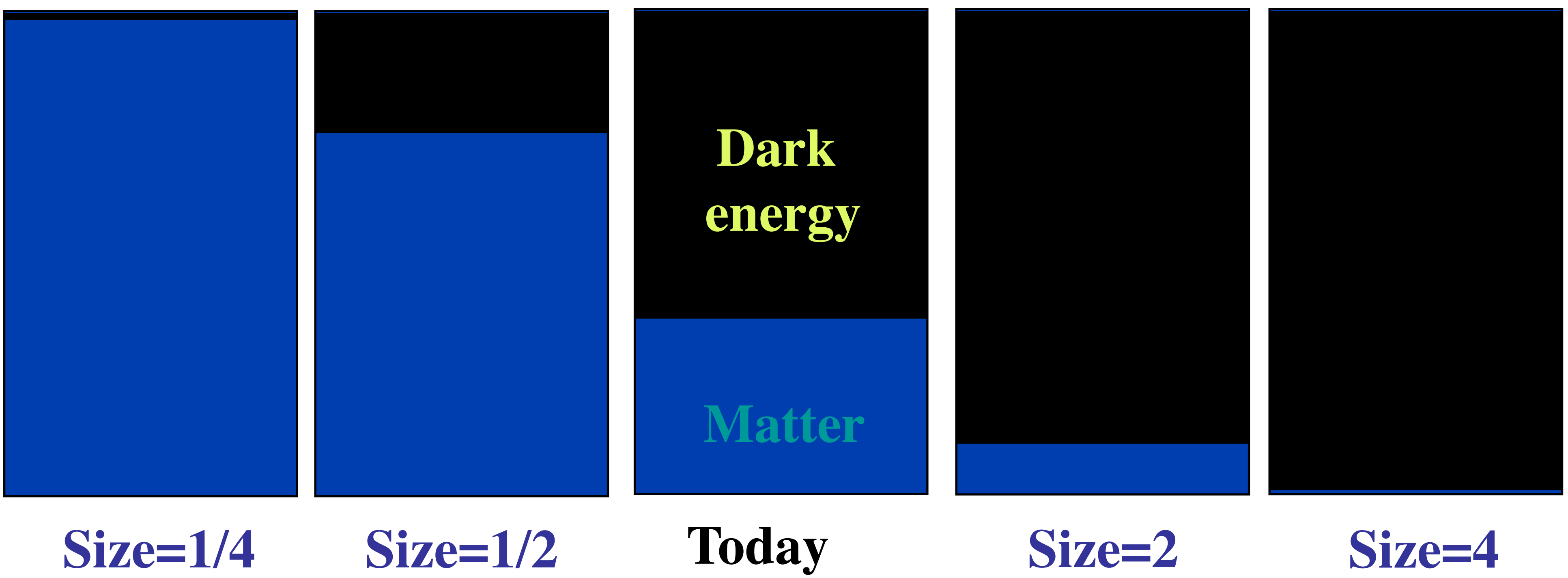


Planck + WMAP (red) + BAO (blue)



The Cosmic Coincidence Problem

If the dark energy is really due to a cosmological constant, its density does not change in time, whereas the matter density does - and they just happen to be comparable today! Seems un-natural …

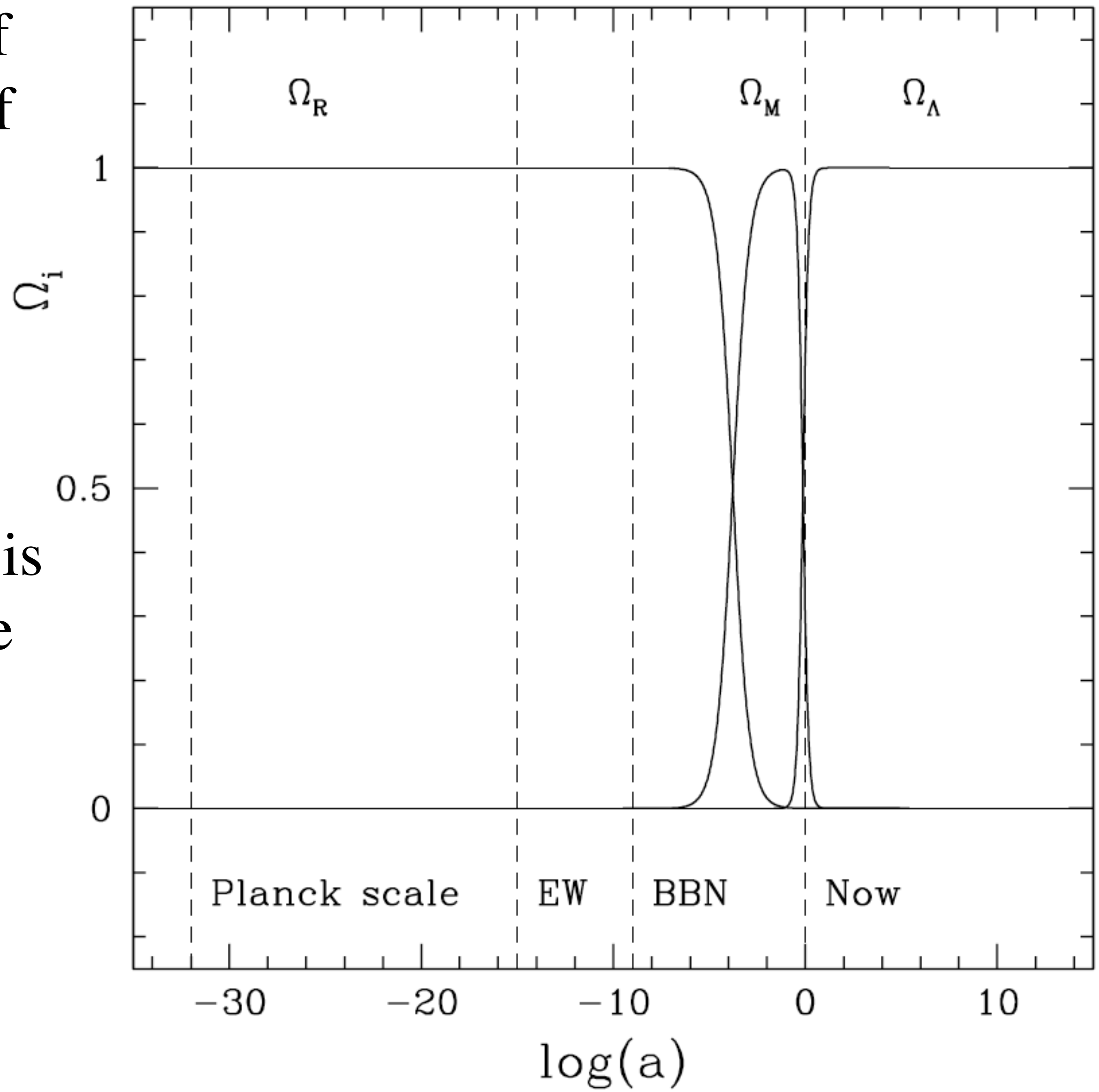


The Cosmological Coincidence Problem

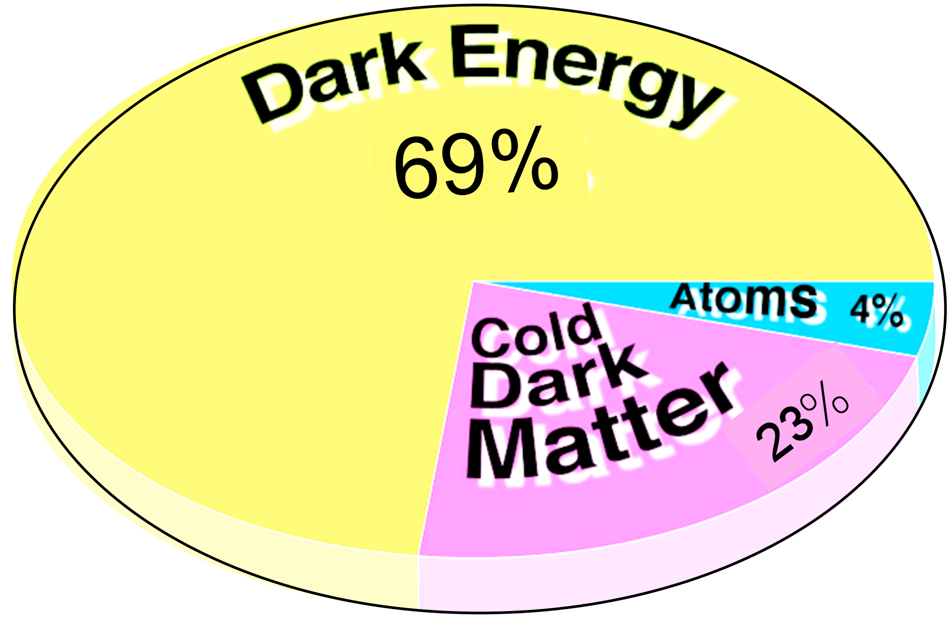
The time dependence of the density parameter of various mass/energy density components:

We seem to live in a special era, when the vacuum energy density is just starting to dominate the dynamics of the universe …

But this may be just an artifact of plotting on a log axis…



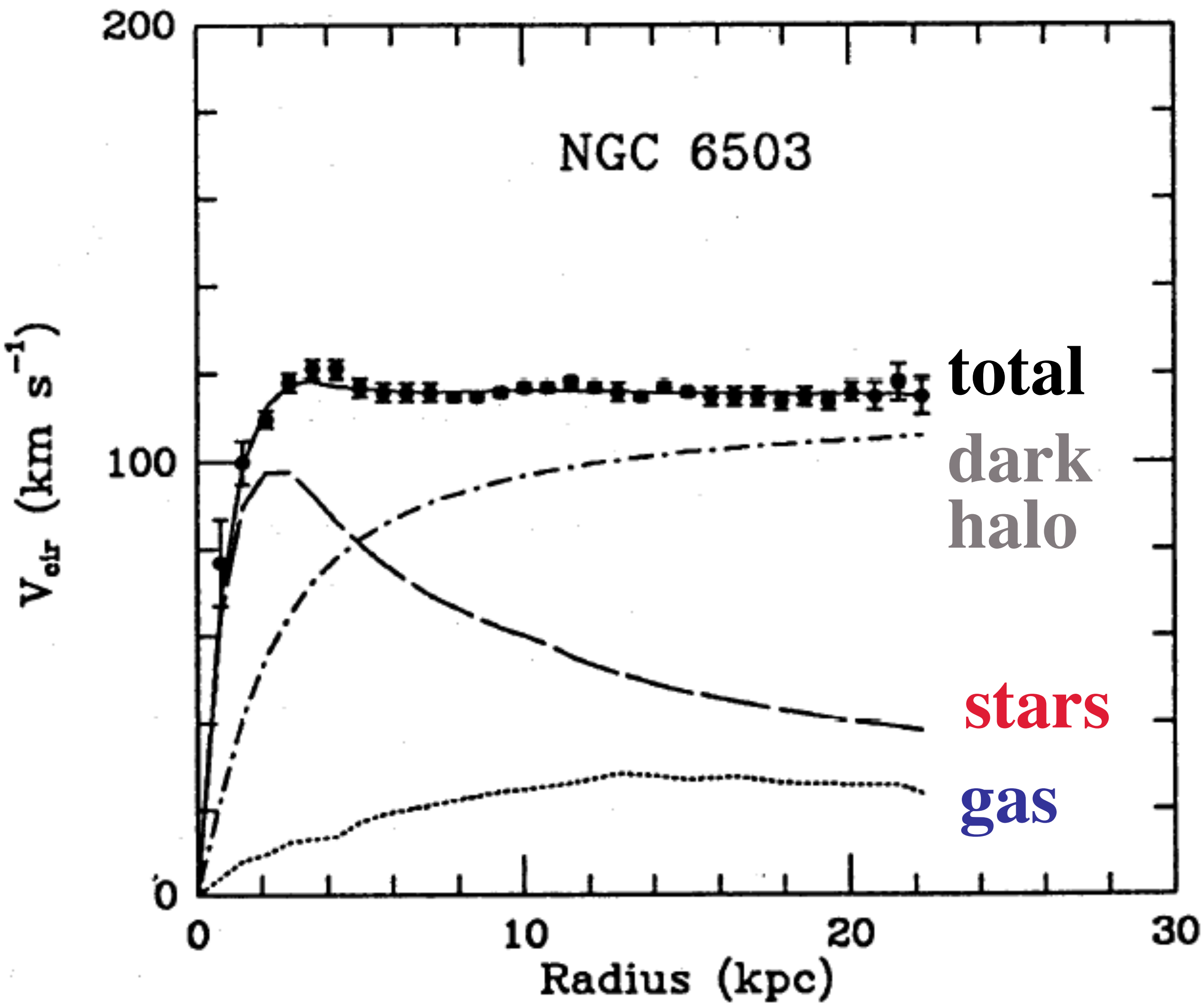
Contents of the Universe: Summary



- $\Omega_0 = 1.00 \pm 0.02$
- $\Omega_m \approx 0.31 \pm 0.02$
  - $\Omega_b \approx 0.045 \pm 10\%$
    - Includes $\Omega_{visible} \approx 0.005$
  - $\Omega_{non\text{-}b} \approx 0.25$
    - Includes $\Omega_\nu < 0.005$
  - $\Omega_{CMBR} \approx 0.0001$
- $\Omega_{de} \approx 0.69 \pm 0.02$
- The baryonic DM is probably (mostly) in the form of a warm gas (~ 10^5 - 10^6 K), associated with galaxies and groups
- The non-baryonic DM may have more than one component, aside from the neutrinos; their nature is as yet unknown, but plausible candidates exist (wimps, axions)
- The physical nature of the DE is currently completely unknown

Supplementary Slides

Disk Galaxy Rotation Curves: Mass Component Contributions



Dark Matter dominates at large radii

It cannot be concentrated in the disk, as it would make the velocity dispersion of stars too high

Interpreting the Rotation Curve

Motions of the stars and gas in the disk of a spiral galaxy are approximately circular (V_R and $V_Z << V_R$).

Define the circular velocity at radius r in the galaxy as *V(r)*. Acceleration of the star moving in a circular orbit must be balanced by gravitational force:

$$\frac{V^2(r)}{r} = -F_r(r)$$

To calculate $F_r(r)$, must in principle sum up gravitational force from bulge, disk and halo. If the mass enclosed within radius *r* is *M(r)*, gravitational force is:

$$F_r = -\frac{GM(r)}{r^2}$$

Thus, from observed *V(r)*, we can infer *M(r)*

Mass Distribution and Rotation Curve

If the density $\rho = const.$, then: $$M(r) = \frac{4}{3}\pi r^3 \rho$$

Implied rotation curve rises linearly with radius; this is about right for central regions of spirals, but fails at the larger radii where $V(r) \sim const.$

$$V(r) = \sqrt{\frac{4\pi G\rho}{3}}\, r$$

Consider instead a power law density profile: $$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}$$

with $\alpha < 3$, the rotation curve is:

$$V(r) = \sqrt{\frac{4\pi G\rho_0 r_0^\alpha}{3-\alpha}}\, r^{1-\alpha/2}$$

$V(r) = const.$ then implies $\rho(r) \sim r^{-2}$. This profile is called a *singular isothermal sphere*. Note that the enclosed mass increases linearly with radius, $M(r) \sim r$! (Where does it stop?)

Some Proposed DM Constituents:

(from Trimble 1987)

Note the range of masses ~ 10^{80} !

Table 3 Summary of nonbaryonic dark matter candidates[a]

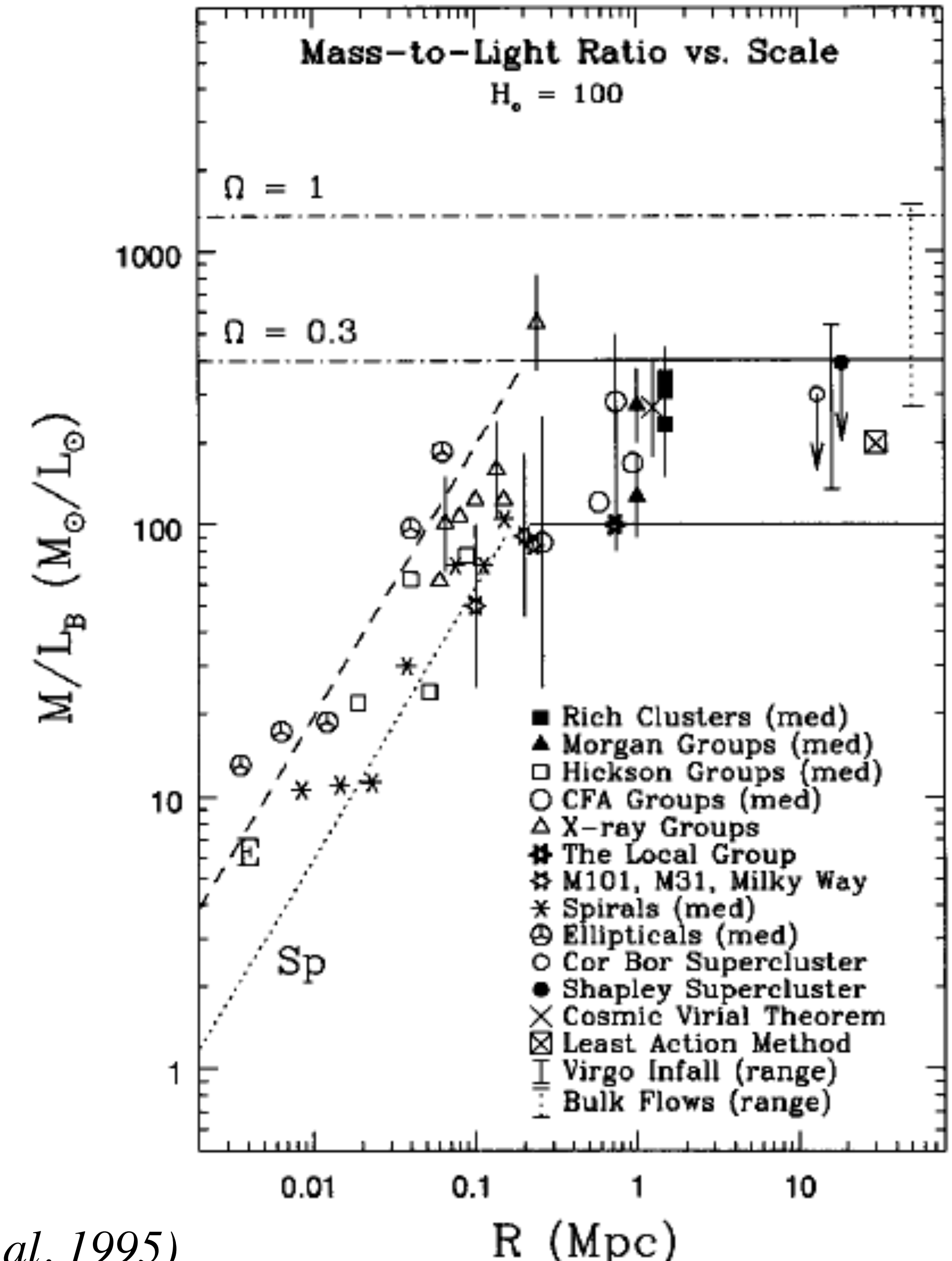
Candidate/particle	Approximate mass	Predicted by	Astrophysical effects
$G(R)$	—	Non-Newtonian gravitation	Mimics DM on large scales
Λ (cosmological constant)	—	General relativity	Provides $\Omega = 1$ without DM
Axion, majoron, goldstone boson	10^{-5} eV	QCD; PQ symmetry breaking	Cold DM
Ordinary neutrino	10–100 eV	GUTs	Hot DM
Light higgsino, photino, gravitino, axino, sneutrino[b]	10–100 eV	SUSY/SUGR	Hot DM
Para-photon	20–400 eV	Modified QED	Hot/warm DM
Right-handed neutrino	500 eV	Superweak interaction	Warm DM
Gravitino, etc.[b]	500 eV	SUSY/SUGR	Warm DM
Photino, gravitino, axino, mirror particle, simpson neutrino[b]	keV	SUSY/SUGR	Warm/cold DM
Photino, sneutrino, higgsino, gluino, heavy neutrino[b]	MeV	SUSY/SUGR	Cold DM
Shadow matter	MeV	SUSY/SUGR	Hot/cold (like baryons)
Preon	20–200 TeV	Composite models	Cold DM
Monopoles	10^{16} GeV	GUTs	Cold DM
Pyrgon, maximon, perry pole, newtorites, Schwarzschild	10^{19} GeV	Higher-dimension theories	Cold DM
Supersymmetric strings	10^{19} GeV	SUSY/SUGR	Cold DM
Quark nuggets, nuclearites	10^{15} g	QCD, GUTs	Cold DM
Primordial black holes	10^{15-30} g	General relativity	Cold DM
Cosmic strings, domain walls	$10^{8-10}\ M_\odot$	GUTs	Promote galaxy formation, but cannot contribute much to Ω

[a] Abbreviations: DM, dark matter; QCD, quantum chromodynamics; PQ, Peccei & Quinn; GUTs, grand unified theories; SUSY, supersymmetric theories; SUGR, supergravity; QED, quantum electrodynamics.

[b] Of these various supersymmetric particles predicted by assorted versions of supersymmetric theories and supergravity, only one, the lightest, can be stable and contribute to Ω, but the theories do not at present tell us which one it will be or the mass to be expected.

Dark Matter Distribution

Dynamical measurements indicate that the (*M*/*L*) ratio increases with the scale, from galaxies to clusters, implying that the DM is distributed more diffusely than light, but then it saturates with a value corresponding to $\Omega_{0,m} \sim 0.25$



(Bahcall et al. 1995)

Planck results, 2013

	Planck		*Planck*+lensing		*Planck*+WP	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8285	0.823 ± 0.018	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14094	0.1414 ± 0.0029	0.14305	0.1426 ± 0.0025
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059	0.09603	0.09593 ± 0.00058	0.09591	0.09589 ± 0.00057
Y_P	0.247710	0.24771 ± 0.00014	0.247785	0.24775 ± 0.00014	0.247695	0.24770 ± 0.00012
Age/Gyr	13.819	13.813 ± 0.058	13.784	13.796 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.01	1090.16 ± 0.65	1090.48	1090.43 ± 0.54