

The Hubble Constant

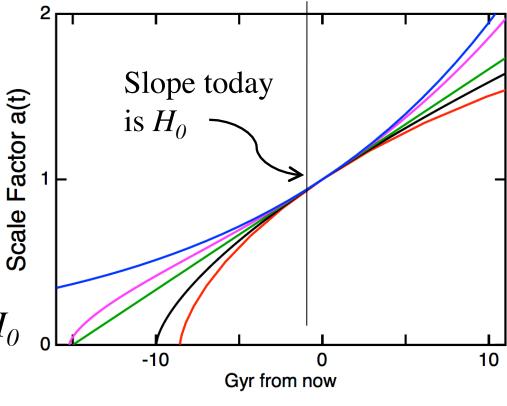
It is the derivative of the expansion law, R(t):

$$H \equiv \frac{\dot{R}}{R}$$

Hubble time: $t_H = 1 / H_0$

Hubble radius: $D_H = c / H_0$

Often written as:



$$h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1}), \text{ or } h_{70} = H_0 / (70 \text{ km s}^{-1} \text{ Mpc}^{-1})$$

 $D_H = c / H_0 = 4.283 h_{70}^{-1} \text{ Gpc} = 1.322 \times 10^{28} h_{70}^{-1} \text{ cm}$
 $t_H = 1 / H_0 = 13.98 h_{70}^{-1} \text{ Gyr} = 4.409 \times 10^{17} h_{70}^{-1} \text{ s}$
At low z's, distance $D \approx z D_H$

The Scale of the Universe

- The **Hubble length**, $D_H = c/H_0$, and the **Hubble time**, $t_H = 1/H_0$ give the approximate spatial and temporal scales of the universe
- H_0 is independent of the "shape parameters" (expressed as density parameters) Ω_m , Ω_Λ , Ω_k , w, etc., which govern the global geometry and dynamics of the universe
- Distances to galaxies, quasars, etc., scale linearly with H_0 , $D \approx cz/H_0$. They are necessary in order to convert observable quantities (e.g., fluxes, angular sizes) into physical ones (luminosities, linear sizes, energies, masses, etc.)

Measuring the Scale of the Universe

• The only clean-cut distance measurements in astronomy are from trigonometric parallaxes. Everything else requires physical modeling and/or a set of calibration steps (the "distance ladder"), and always some statistics:

Use parallaxes to calibrate some set of distance indicators

- → Use them to calibrate another distance indicator further away
 - → And then another, reaching even further
 - → etc., etc.
 - → Until you reach a "pure Hubble flow"
- The age of the universe can be constrained independently from the H_0 , by estimating ages of the oldest things one can find around (e.g., globular clusters, heavy elements, white dwarfs)

The Hubble's Constant Has a Long and Disreputable History ...

THE VELOCITY-DISTANCE RELATION AMONG EXTRA-GALACTIC NEBULAE¹

By EDWIN HUBBLE AND MILTON L. HUMASON (1931, ApJ 74, 43)

The new data extend out to about eighteen times the distance available in the first formulation of the velocity-distance relation, but the form of the relation remains unchanged except for the revision of the unit of distance. The relation is

Vel.=
$$\frac{\text{Dist. (parsecs)}}{1790}$$
, $H_0 = 560 \text{ km/s/Mpc}$

and the uncertainty is estimated to be of the order of 10 per cent.

Since then, the value of the H_0 has shrunk by an order of magnitude, but the errors were always quoted to be about 10% ...

Generally, Hubble was estimating $H_0 \sim 600$ km/s/Mpc. This implies for the age of the universe $\sim 1/H_0 < 2$ Gyr - which was a problem!

Distance Ladder

Methods yielding absolute distances:

Parallax (trigonometric, secular, and statistical)
The moving cluster method - has some assumptions
Baade-Wesselink method for pulsating stars
Expanding photosphere method for Type II SNe

Model

Sunyaev-Zeldovich effect Gravitational lens time delays

Secondary distance indicators: "standard candles", requiring a calibration from an absolute method applied to local objects - the distance ladder:

Pulsating variables: Cepheids, RR Lyrae, Miras

Main sequence fitting to star clusters

Brightest red giants

Planetary nebula luminosity function

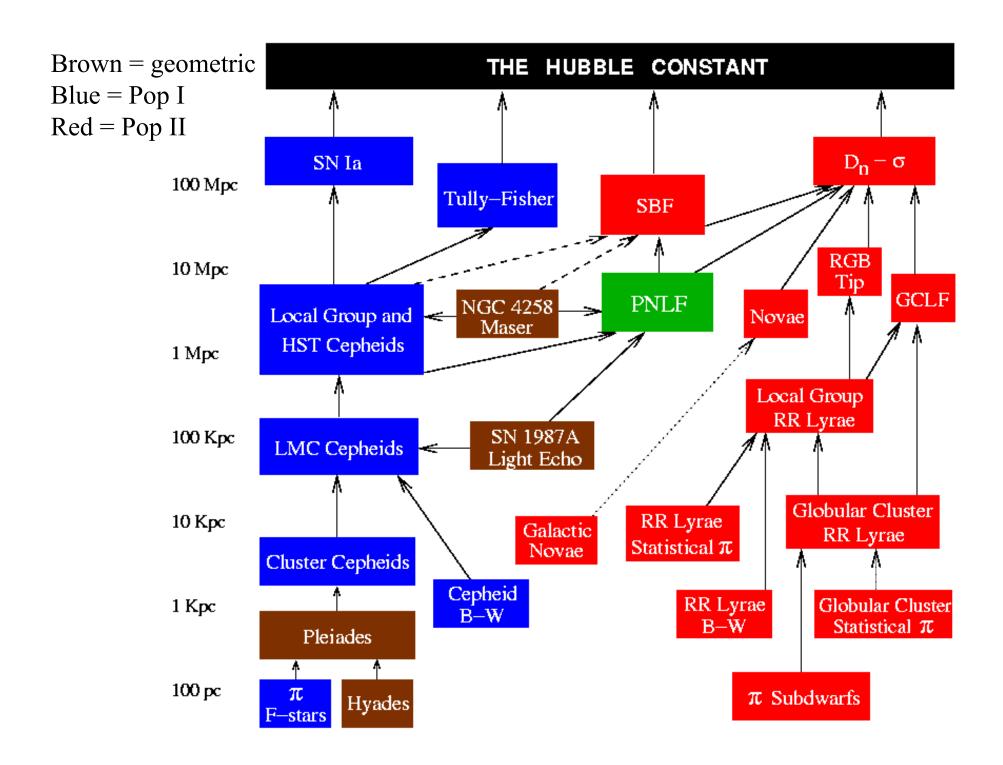
Globular cluster luminosity function

Surface brightness fluctuations

Tully-Fisher, D_n-σ, FP scaling relations for galaxies

Type Ia Supernovae

... etc.



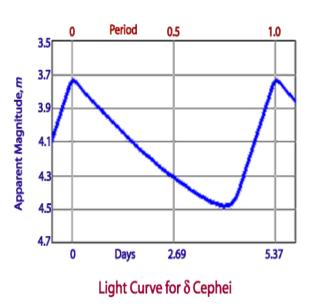
Galactic Distance Scale is the Basis for the Cosmological Distance Scale

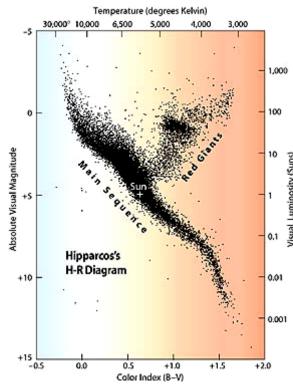
• Trig. Parallaxes are *the only* model-independent, non-statistical) method - the foundation of the distance scale: $\mathbf{D} [pc] = 1 / \pi [arcsec]$

 The moving cluster method is geometric, but statistical

Parallaxes calibrate the rest of the Galactic

distance scale, which is then transferred to the nearby galaxies, sometimes via star clusters: CMDs, pulsating variables





Cepheids

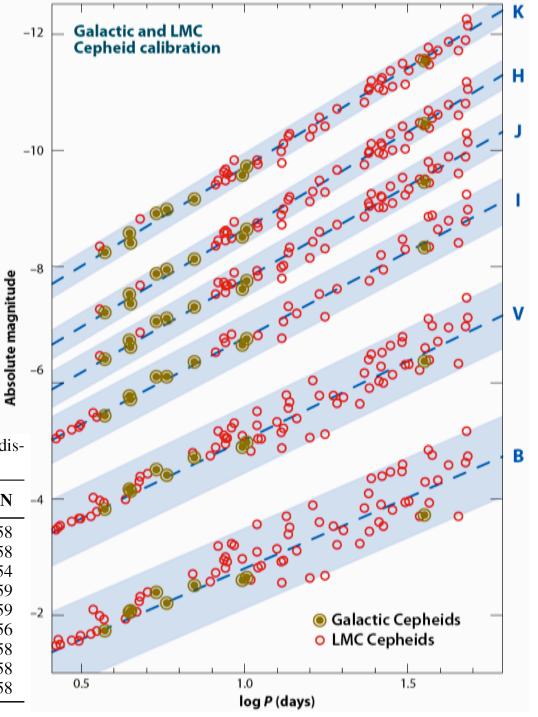
- Luminous ($M \sim -4$ to -7 mag), pulsating variables, evolved high-mass stars on the instability strip in the H-R diagram
- Shown by Henrietta Leavitt in 1912 to obey a period-luminosity relation (P-L) from her sample of Cepheids in the SMC: brighter Cepheids have longer periods than fainter ones
- Advantages: Cepheids are bright, so are easily seen in other galaxies, the physics of stellar pulsation is well understood
- **Disadvantages:** They are relatively rare, their period depends (how much is still controversial) on their metallicity or color (P-L-Z or P-L-C) relation; multiple epoch observations are required; found in spirals (Pop I), so extinction corrections are necessary
- P-L relation usually calibrated using the distance to the LMC and now using Hipparcos parallaxes. This is the biggest uncertainty now remaining in deriving the H_0 !
- With HST we can observe to distances out to ~25 Mpc

Cepheid P-L Rel'n in different photometric bandpasses

Amplitudes are larger in bluer bands, but extinction and metallicity corrections are also larger; redder bands may be better overall

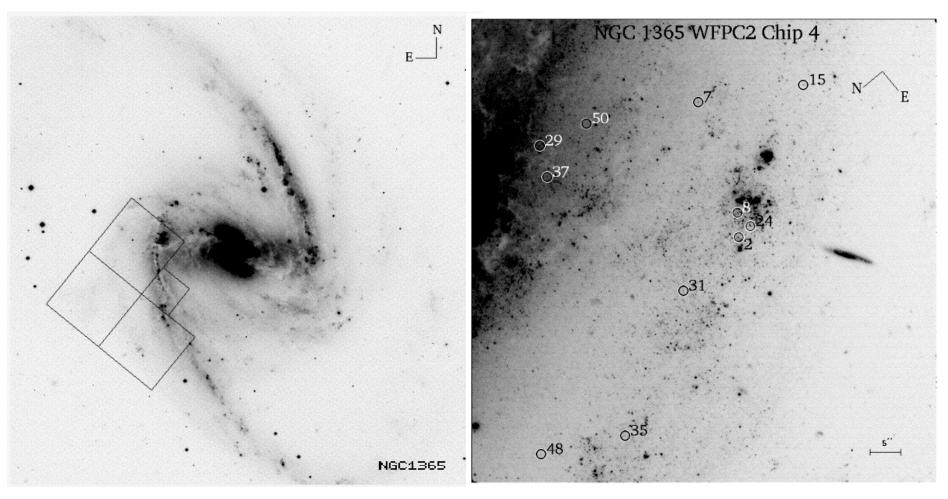
TABLE 3. Galactic Leavitt Laws from fundamental distances. Table adapted from Fouqué *et al.* 2007.

Band	Slope	Intercept	σ	N
\overline{B}	-2.289 ± 0.091	-0.936 ± 0.027	0.207	58
V	-2.678 ± 0.076	-1.275 ± 0.023	0.173	58
R_{c}	-2.874 ± 0.084	-1.531 ± 0.025	0.180	54
I_c	-2.980 ± 0.074	-1.726 ± 0.022	0.168	59
J	-3.194 ± 0.068	-2.064 ± 0.020	0.155	59
H	-3.328 ± 0.064	-2.215 ± 0.019	0.146	56
K_{S}	-3.365 ± 0.063	-2.282 ± 0.019	0.144	58
W_{vi}	-3.477 ± 0.074	-2.414 ± 0.022	0.168	58
W_{bi}	-3.600 ± 0.079	-2.401 ± 0.023	0.178	58



The HST H_0 Key Project

Sample images for discovery of Cepheids



The HST H_0 Key Project

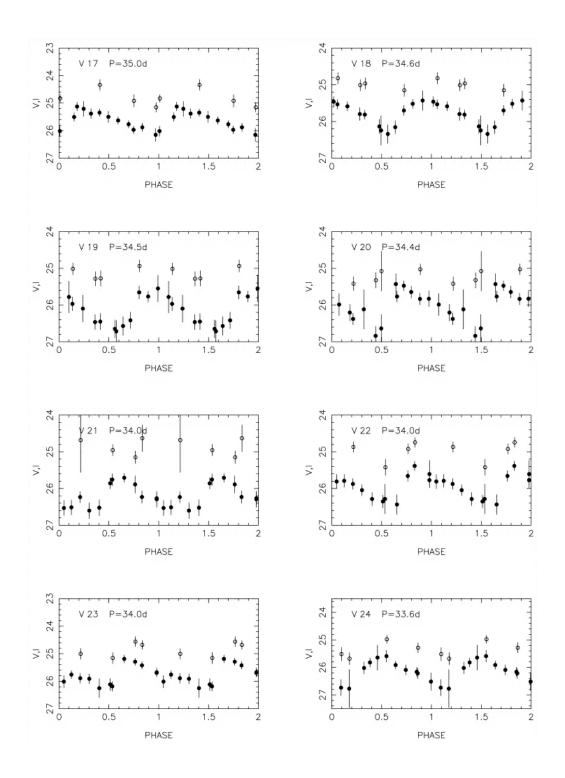
- Started in 1990, final results in 2001! Leaders include W. Freedman, R. Kennicutt, J. Mould, J. Huchra, and many others (reference: Freedman *et al.* 2001, ApJ, 553, 47)
- Observe Cepheids in ~18 spirals to test the universality of the Cepheid P-L relation and greatly improve calibration of other distance indicators
- Their Cepheid P-L relation zero point is tied directly to the distance to the LMC (largest source of error for the H_0 !)
- Combining different estimators, they find:

$H_0 = 72 \pm 3$ (random) ± 7 (systematic) km/s/Mpc

• Since then, the Cepheid calibration has improved, and other methods yield results in an excellent agreement

The HST H_0 Key Project

Sample Cepheid light curves for NGC 1365

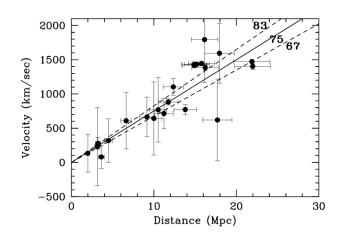


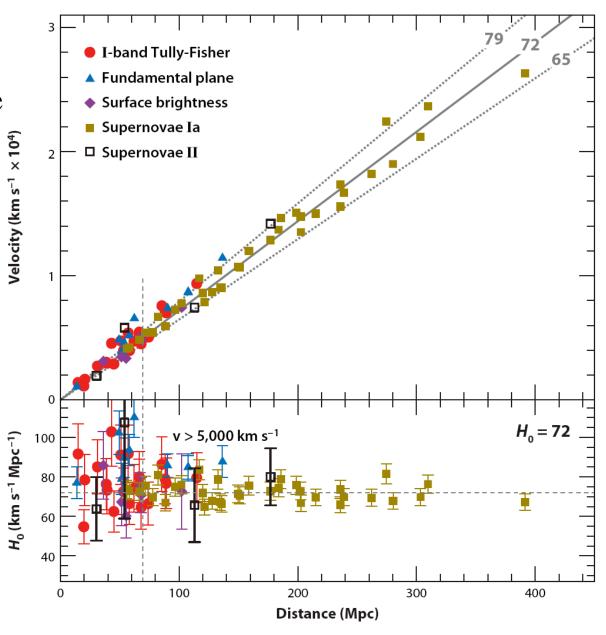
The HST H_0 Key Project Results

Overall Hubble diagram, from all types of distance indicators →

$$H_0 = 72 \pm (3)_r \pm [7]_s$$

From Cepheid distances alone 1

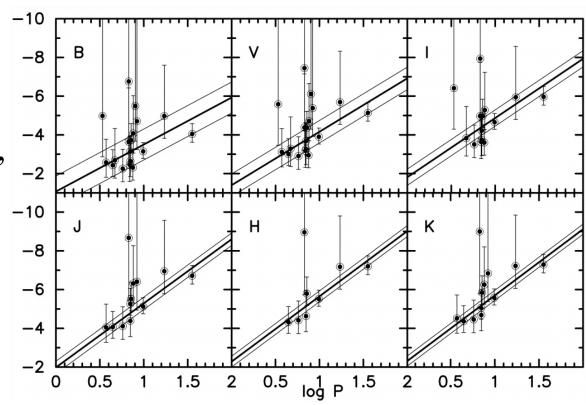




Hipparcos Calibration of the Cepheid Period-Luminosity Relation

P-L relations for Cepheids with measured parallaxes, in different photometric bands

(from Freedman & Madore)



Typical

fits give: $\langle M_V \rangle = -2.76 \log P - 1.45$

$$\langle M_I \rangle = -2.96 \log P - 1.88$$

... with the estimated errors in the range of ~5% - 20%

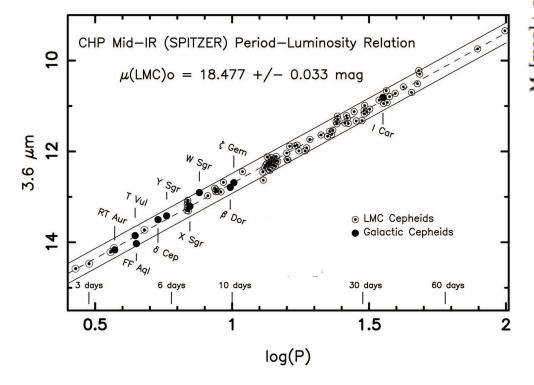
Recent Progress on H₀

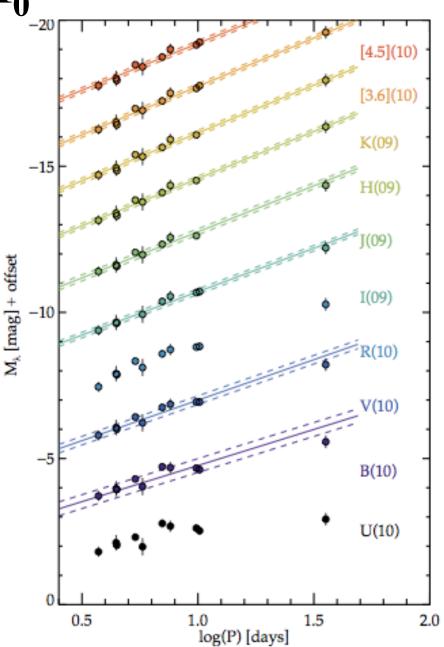
(from W. Freedman)

Zero point: HST parallaxes for Milky Way Cepheids

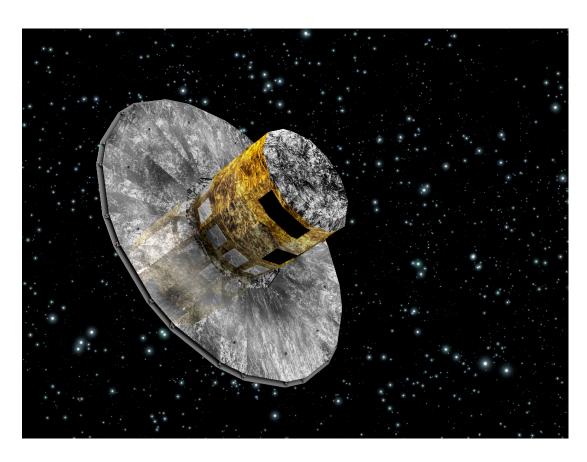
Published UV/optical/near-IR + Spitzer mid-IR data

$$\sigma = 0.10 \text{ mag}$$





Gaia – Data Release 1



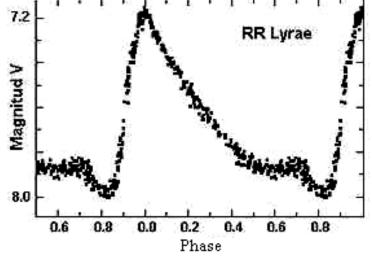
- September 14, 2016
- 1 year of data
- 1 billion stars
- 300 µas typical accuracy

When complete (2022):

- $\sigma_{\pi} / \pi < 1\%$ out to several kiloparsecs
- Cepheids, RR Lyrae, red giants in Milky Way to <<1%
- Distance to LMC to 1-2%

RR Lyrae Stars

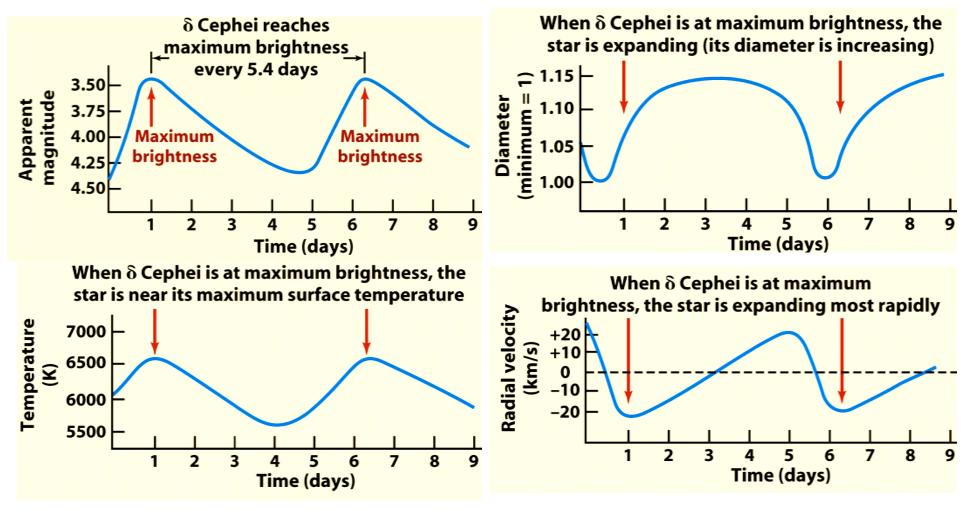
- Pulsating variables, evolved old, low mass, low metallicity stars
 - Pop II indicator, found in globular clusters, galactic halos
- Lower luminosity than Cepheids, $M_V \sim 0.75 + /-0.1$
 - There may be a metallicity dependence
- Have periods of 0.4 0.6 days, so don't require as much observing to find or monitor
- Advantages: less dust, easy to find
- **Disadvantages:** fainter (2 mag fainter than Cepheids). Used for Local Group galaxies only. The calibration is still uncertain (uses globular cluster distances from



their main sequence fitting; or from Magellanic Clouds clusters, assuming that we know their distances)

Physical Parameters of Pulsating Variables

Star's diameter, temperature (and thus luminosity) pulsate, and obviously the velocity of the photosphere must also change



Baade-Wesselink Method

Consider a pulsating star at minimum, with a measured temperature T_1 and observed flux f_1 with radius R_1 , then:

 $f_1 = \frac{4\pi R_1^2 \sigma T_1^4}{4\pi D^2}$

Similarly at maximum, with a measured temperature T_2 and observed flux f_2 with radius R_2 :

$$f_2 = \frac{4\pi R_2^2 \sigma T_2^4}{4\pi D^2}$$

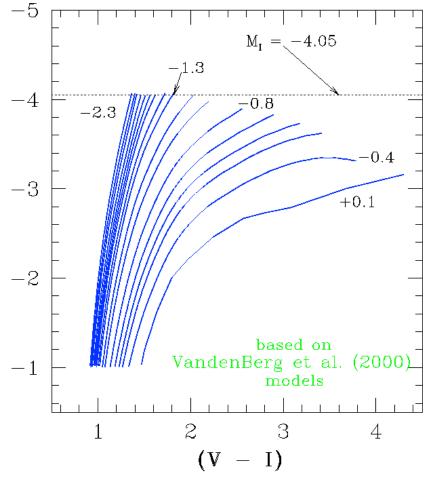
Note: T_1 , T_2 , f_1 , f_2 are directly observable! Just need the radius... So, from spectroscopic observations we can get the photospheric velocity v(t), from this we can determine the $R_2 = R_1 + \Delta R = R_1 + \int_{t_1}^{t_2} v(t) \, dt$ change in radius, ΔR :

 \rightarrow 3 equations, 3 unknowns, solve for R_1, R_2 , and D!

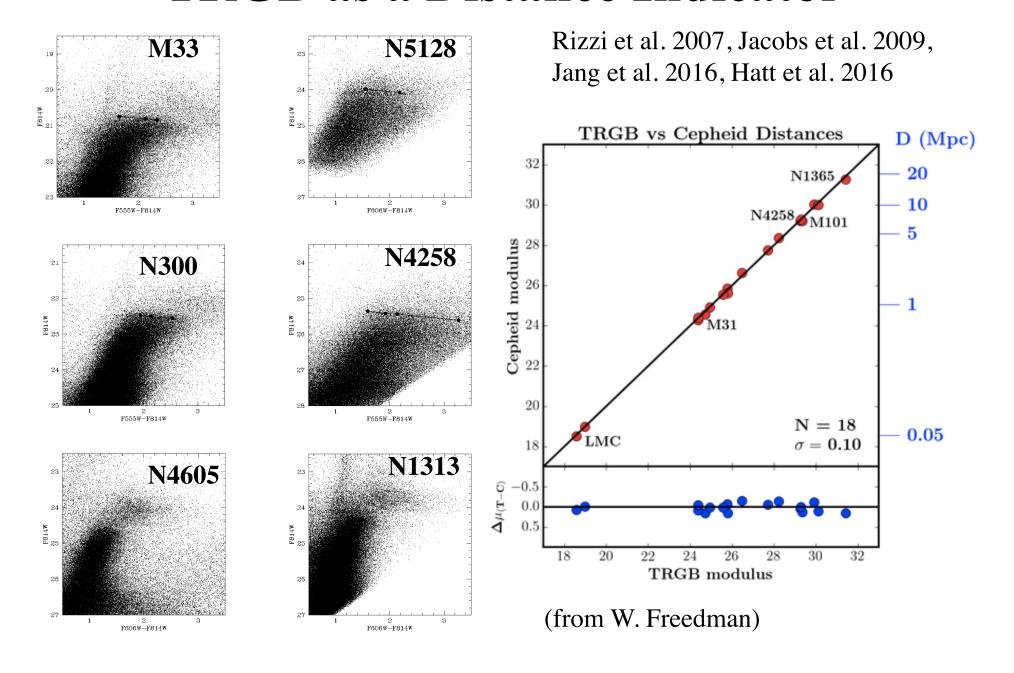
Difficulties lie in modeling the effects of the stellar atmosphere, and deriving the true radial velocity from what we observe.

Tip of the Red Giant Branch

- Brightest stars in old stellar populations are red giants
- In I-band, $M_I = -4.1 \pm 0.1 \approx$ constant for the tip of the red giant branch (TRGB) if stars are old and metal-poor ([Fe/H] < -0.7)
- These conditions are met for dwarf ⁻⁵ galaxies and galactic halos
- Advantages: Relatively bright, reasonably precise, RGB stars are plentiful. Extinction problems are reduced
- **Disadvantages:** Only good out to ~20 Mpc (Virgo), only works for old, metal poor populations
- Calibration from subdwarf parallaxes from Hipparcos and distances to galactic GCs

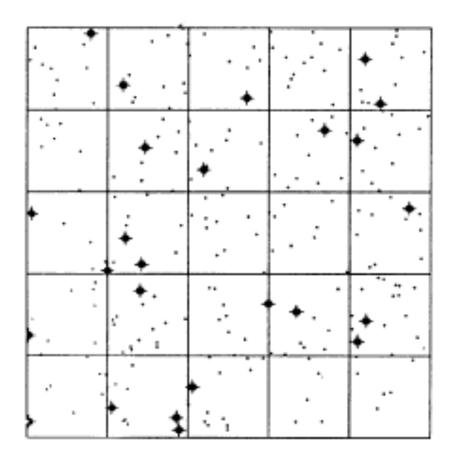


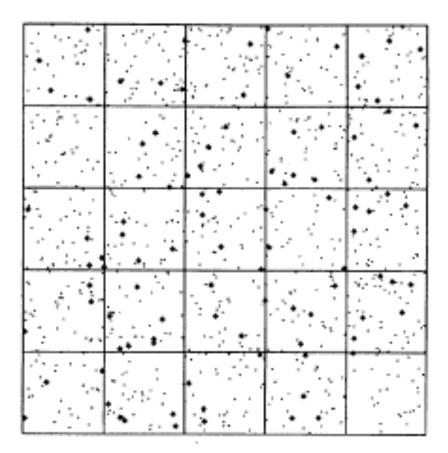
TRGB as a Distance Indicator



Surface Brightness Fluctuations

Consider stars projected onto a pixel grid of your detector:





Nearby Galaxy

A galaxy twice farther away is "smoother"

Surface Brightness Fluctuations

- Surface brightness fluctuations for old stellar populations (E's, SO's and bulges) are based primarily on their giant stars
- Assume typical average flux per star < f >, the average flux per pixel is then N < f >, and the variance per pixel is N < f >. But the number of stars per pixel N scales as D and the flux per star decreases as D . Thus the variance scales as D and the RMS scales as D . Thus a galaxy twice as far away appears twice as smooth. The average flux < f > can be measured as the ratio of the variance and the mean flux per pixel. If we know the average L (or M) we can measure D
- <*M*> is roughly the absolute magnitude of a giant star and can be calibrated empirically using the bulge of M31, although there is a color-luminosity relation, so <*M*_I $> = -1.74 + 4.5 [(V-I)_0 -1.15]$
- Have to model and remove contamination from foreground stars, background galaxies, and globular clusters
- Can be used out to ~ 100 Mpc in the IR, using the HST

Pushing Into the Hubble Flow

- Hubble's law: $D = H_0 v$
- But the total observed velocity v is a combination of the cosmological expansion, and the *peculiar velocity* of any given galaxy, $v = v_{cosmo} + v_{pec}$
- Typically $v_{pec} \sim$ a few hundred km/s, and it is produced by gravitational infall into the local large scale structures (e.g., the local supercluster), with characteristic scales of tens of Mpc
- Thus, one wants to measure H_0 on scales greater than tens of Mpc, and where $v_{cosmo} >> v_{pec}$. This is the Hubble flow regime
- This requires *luminous standard candles* galaxies or Supernovae

Galaxy Scaling Relations

- Once a set of distances to galaxies of some type is obtained, one finds correlations between distance-dependent quantities (e.g., luminosity, radius) and distance-independent ones (e.g., rotational speeds for disks, or velocity dispersions for ellipticals and bulges, surface brightness, etc.). These are called *distance indicator relations*
- Examples:
 - Tully-Fisher relation for spirals (luminosity vs. rotation speed)
 - Fundamental Plane relations for ellipticals
- These relations must be calibrated locally using other distance indicators, e.g. Cepheids or surface brightness fluctuations; then they can be extended into the general Hubble flow regime
- Their origins and thus their universality are not yet well understood. Caveat emptor!

The Tully-Fisher Relation

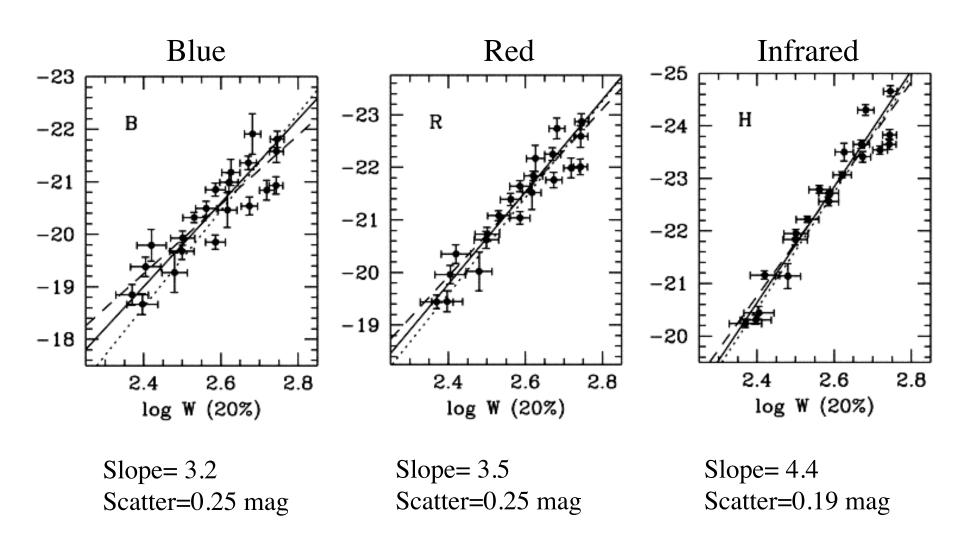
• A well-defined luminosity vs. rotational speed (often measured as a H I 21 cm line width) relation for spirals:

$$L \sim v_{\rm rot}^{\gamma}$$
, $\gamma \approx 4$, varies with wavelength

Or: $M = b \log(W) + c$, where:

- *M* is the absolute magnitude
- W is the Doppler broadened line width, typically measured using the HI 21cm line, corrected for inclination $W_{\text{true}} = W_{\text{obs}} / \sin(i)$
- Both the slope b and the zero-point c can be measured from a set of nearby spiral galaxies with well-known distances
- The slope b can be also measured from any set of galaxies with roughly the same distance - e.g., galaxies in a cluster - even if that distance is not known
- Scatter is $\sim 10-20\%$ at best, which limits the accuracy
- Problems include dust extinction, so working in the redded bands is better

Tully-Fisher Relation at Various Wavelengths



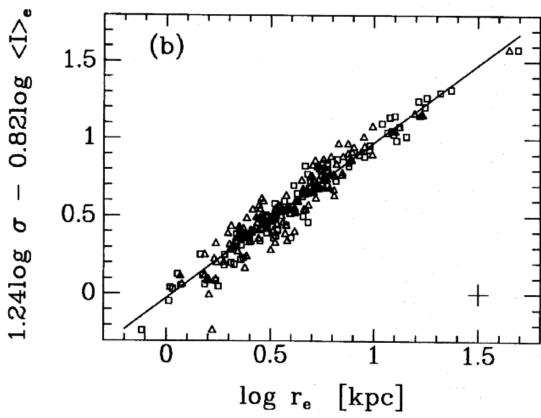
Fundamental Plane Relations

• A set of bivariate scaling relations for elliptical galaxies, including relations between distance dependent quantities such as radius or luminosity, and a combination of two distance-independent ones, such as velocity dispersion or surface

brightness

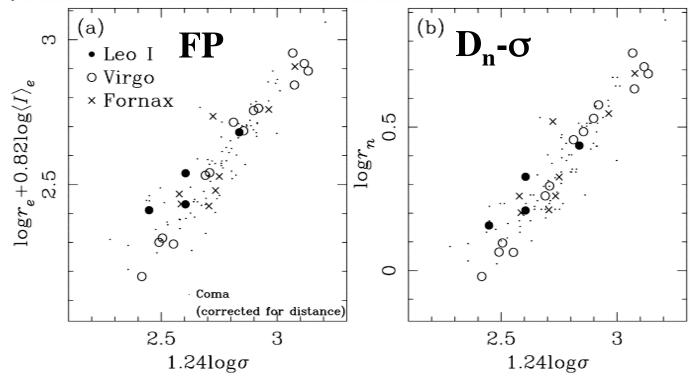
• Scatter ~ 10%, but it could be lower?

 Usually calibrated using surface brightness fluctuations distances



The D_n-\sigma Relation

- A projection of the Fundamental Plane, where a combination of radius and surface brightness is combined into a *modified isophotal* diameter called D_n which is the angular diameter that encloses a mean surface brightness in the B band of $\langle \mu_B \rangle = 20.75$ mag/arcsec²
- D_n is a *standard yardstick*, and can be used to measure relative distances to ellipticals
- Also calibrated using SBF

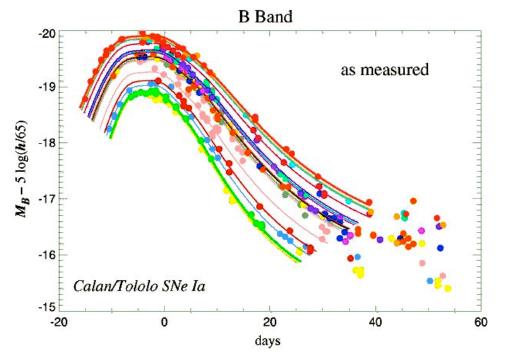


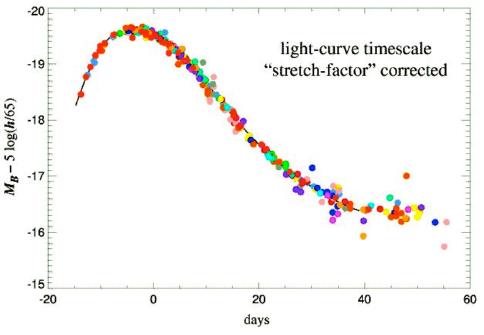
Supernovae as Standard Candles

- Bright and thus visible far away
- Two different types of supernovae are used as standard candles:
 - Type Ia from a binary WD accreting material and detonating
 - * These are pretty good standard candles, peak $M_{\rm V} \sim$ -19.3
 - * There is a diversity of light curves, but they can be standardized to a peak magnitude scatter of $\sim 10\%$
 - Type II = hydrogen in spectrum, from collapse of massive stars (also Type Ib)
 - * These aren't good standard candles, but we can measure their distances using the "Expanding Photosphere Method" (EPM), essentially the Baade-Wesselink method of measuring the expansion of the outer envelope
 - * Not as bright as Type Ia's

SNe Ia as Standard Candles

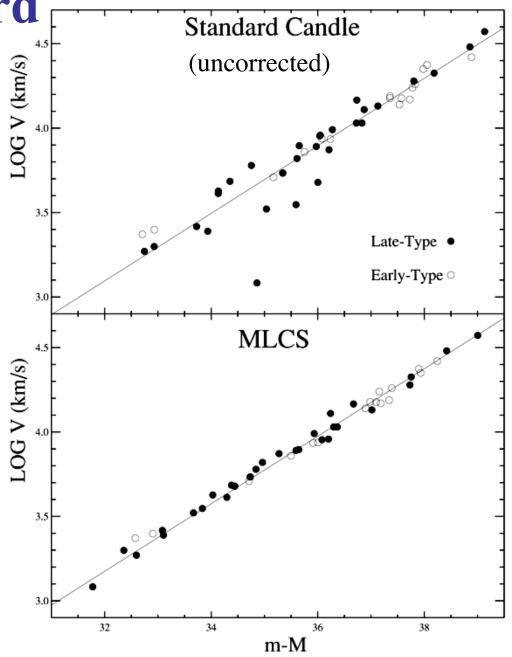
- The peak brightness of a SN Ia correlates with the shape of its light curve (steeper → fainter)
- Correcting for this effect standardizes the peak luminosity to ~10% or better
- However, the absolute zeropoint of the SN Ia distance scale has to be calibrated externally, e.g., with Cepheids





SNe Ia as Standard Candles

- A comparable or better correction also uses the color information (the Multicolor Light Curve method)
- This makes SNe Ia a superb cosmological tool (note: you only need relative distances to test cosmological models; absolute distances are only needed for the H_0)



The Expanding Photosphere Method (EPM)

- One of few methods for a direct determination of distances; unfortunately, it is somewhat model-dependent
- Uses Type II SNe could cross-check with Cepheids
- Based on the Stefan-Boltzmann law, $L \sim 4\pi R^2 T^4$

If you can measure T (distance-independent), understand the deviations from the perfect blackbody, and could determine R, then from the observed flux F and the inferred luminosity L you can get the distance D

EPM assumes that SNII radiate as dilute blackbodies

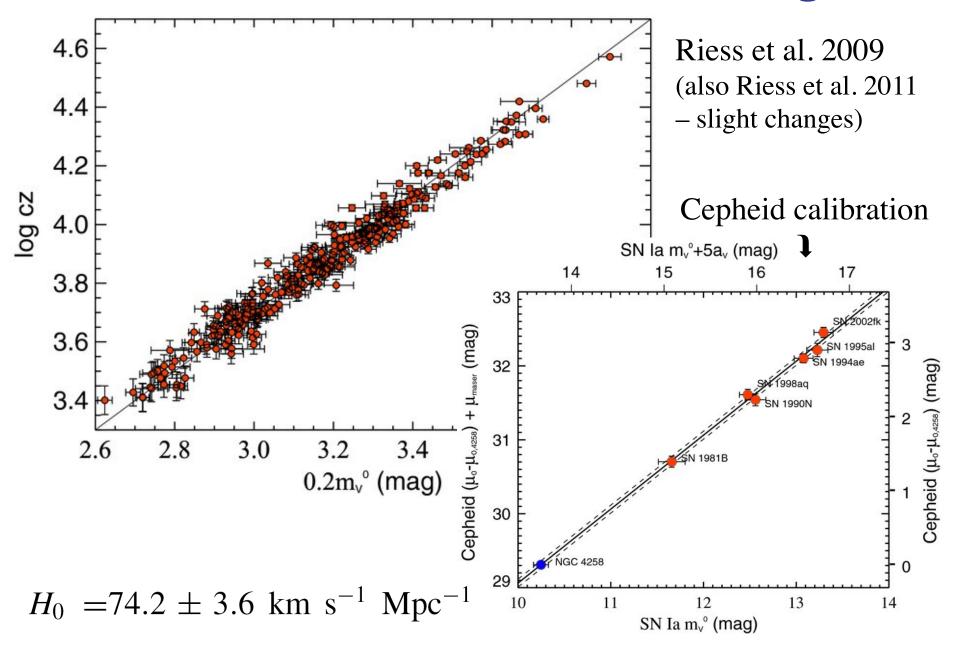
Apparent
$$\theta_{ph} = \frac{R_{ph}}{D} = \sqrt{\frac{F_{\lambda}}{\zeta^2 \pi B_{\lambda}(T)}},$$

Fudge factor to account for the deviations from blackbody, from spectra models

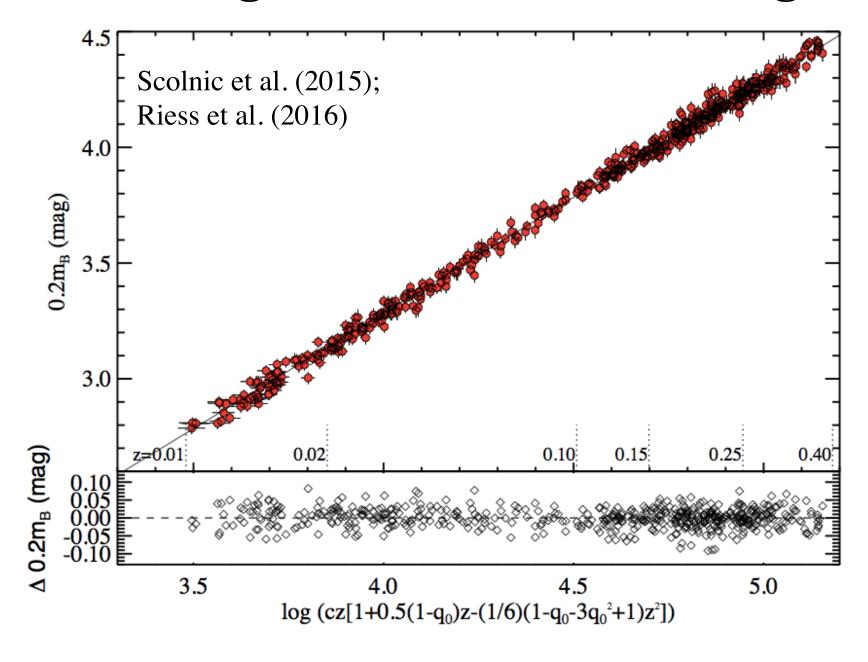
Determine the radius by monitoring the expansion velocity

$$R_{ph}=v_{ph}(t-t_0)+R_0,$$
 And solve for the distance! $t=D\left(rac{ heta_{ph}}{v_{ph}}
ight)+t_0$

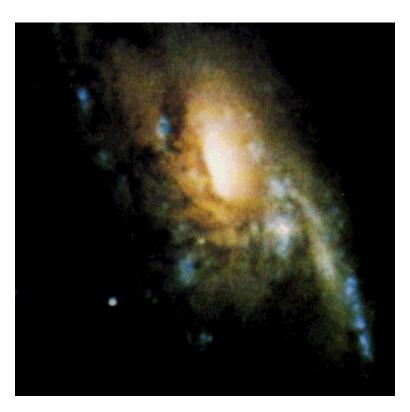
The Low-Redshift SN Ia Hubble Diagram

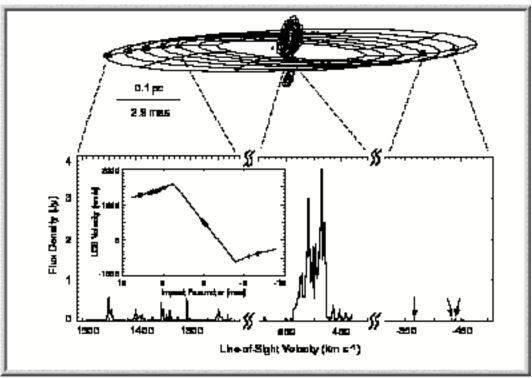


Recent Progress on SN Hubble Diagram



Another Test: Nuclear Masers in NGC 4258

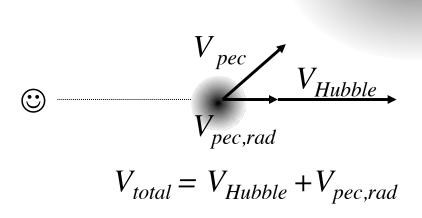


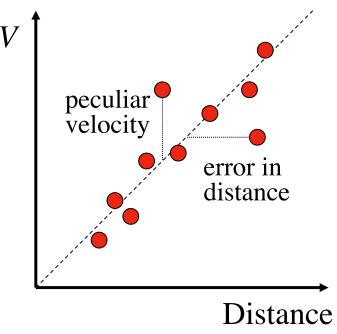


Herrnstein *et al.* (1999) have analyzed the proper motions and radial velocities of NGC 4258's nuclear masers. The orbits are Keplerian and yield a distance of 7.2 ± 0.3 Mpc, or $(m-M)_0 = 29.29 \pm 0.09$. This is inconsistent with the Cepheid distance modulus of 29.44 ± 0.12 at the $\sim 1.2\sigma$ level.

Another Problem: Peculiar Velocities

- Large-scale density field inevitably generates a peculiar velocity field, due to the acceleration over the Hubble time
- Note that we can in practice only observe the radial component
- Peculiar velocities act as a noise (on the V=cz axis, orthogonal to errors in distances) in the Hubble diagram and could thus bias the measurements of the H_0 (which is why we want "far field" measurements)





Bypassing the Distance Ladder

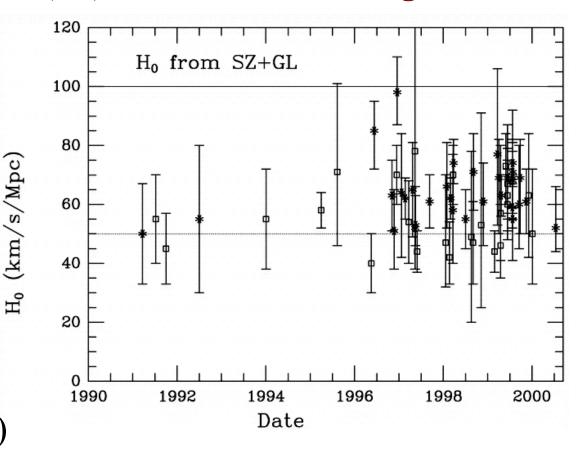
There are a two methods which can be used to large distances, which don't depend on local calibrations:

- 1. Gravitational lens time delays
- 2. Synyaev-Zeldovich (SZ) effect for clusters of galaxies

Both are very *model-dependent!*

Both tend to produce values of H_0 somewhat lower than the HST Key Project

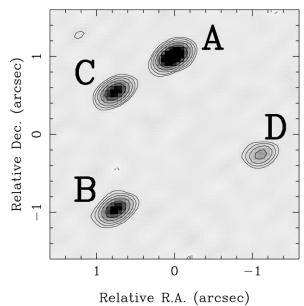
... And finally, the CMB fluctuations (more about that later)

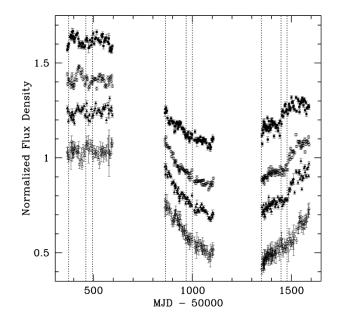


Gravitational Lens Time Delays

- Assuming the mass model for the lensing galaxy of a gravitationally lensed quasar is well-known (!?!), the different light paths taken by various images of the quasar will lead to time delays in the arrival time of the light to us. This be can be traced by the quasar variability
- If the lensing galaxy is in a cluster, we also need to know the mass distribution of the cluster and any other mass distribution along the line of sight. The modeling is complex!

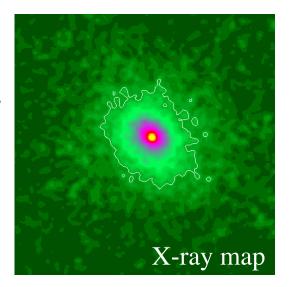
Images and lightcurves for the lens B1608+656 (from Fassnacht *et al*. 2000)





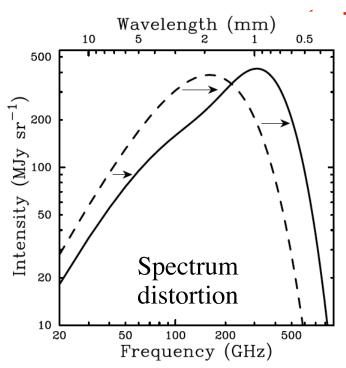
Synyaev-Zeldovich Effect

- Clusters of galaxies are filled with hot X-ray gas
- The electrons in the intracluster gas will scatter the background photons from the CMBR to higher energies (frequencies) and distort the blackbody spectrum



Outgoing CMB photon

Incoming CMB photon



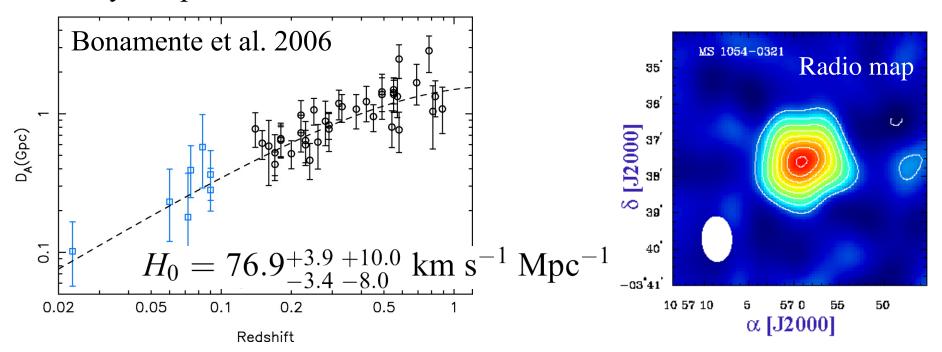
Galaxy Cluster with hot gas

For every photon scattered away from observer, there is another scattered towards.

This is detectable as a slight temperature dip or bump (depending on the frequency) in the radio map of the cluster, against the uniform CMBR background

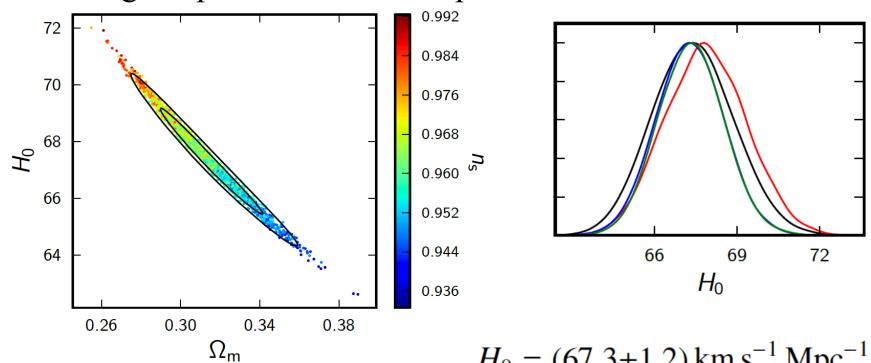
H_0 From the Synyaev-Zeldovich Effect

- From the amplitude of the CMB dip and X-ray data estimate the electron density and temperature of the X-ray gas along the line of sight and thus estimate the path length along the line of sight
- Assume on average depth ~ width, from the projected angular diameter we can determine the distance to the cluster
- Potential uncertainties include cluster substructure or shape. Also,
 X-ray temperature measurements are difficult



H_0 From the CMB

• Bayesian solutions from model fits to CMB fluctuations – cosmological parameters are coupled

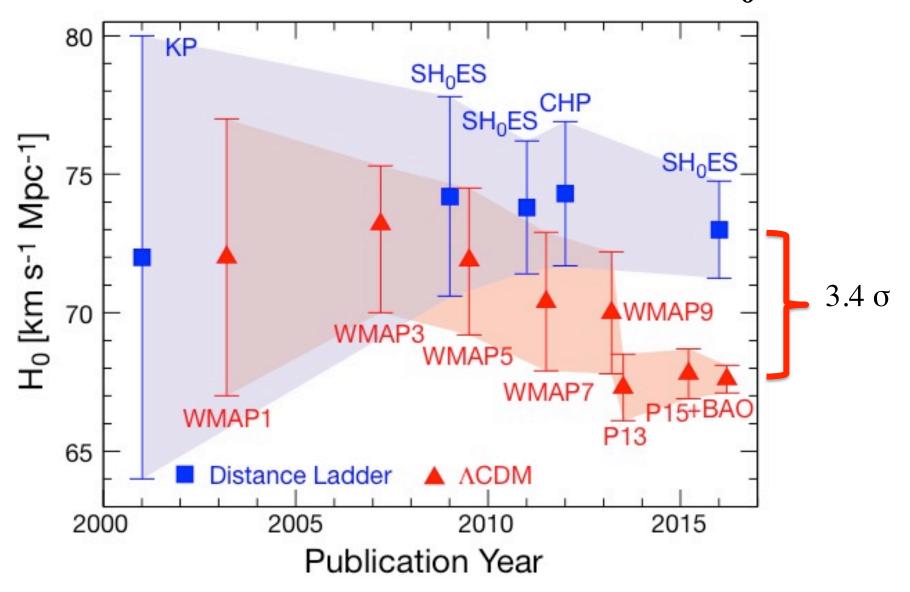


Planck (2013) results:

 $H_0 = (67.3 \pm 1.2) \,\mathrm{km \, s^{-1} \, Mpc^{-1}}$

	Planck+WP		Planck+WP+highL		Planck+lensing+WP+highL		Planck+WP+highL+BAO	
Parameter	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
H_0	67.04	67.3 ± 1.2	67.15	67.3 ± 1.2	67.94	67.9 ± 1.0	67.77	67.80 ± 0.77
Age/Gyr	13.8242	13.817 ± 0.048	13.8170	13.813 ± 0.047	13.7914	13.794 ± 0.044	13.7965	13.798 ± 0.037

The Current Tension in H₀



(Slide from W. Freedman)

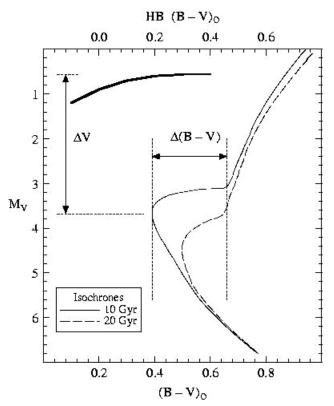
Measuring the Age of the Universe

- Related to the Hubble time $t_H = 1/H_0$, but the exact value depends on the cosmological parameters
- Could place a *lower limit* from the ages of astrophysical objects (pref. the oldest you can find), e.g.,
 - Globular clusters in our Galaxy; known to be very old. Need stellar evolution isochrones to fit to color-magnitude diagrams
 - White dwarfs, from their observed luminosity function,
 cooling theory, and assumed star formation rate
 - Heavy elements, produced in the first Supernovae; somewhat model-dependent
 - Age-dating stellar populations in distant galaxies; this is very tricky and requires modeling of stellar population evolution, with many uncertain parameters

Ages of Globular Clusters

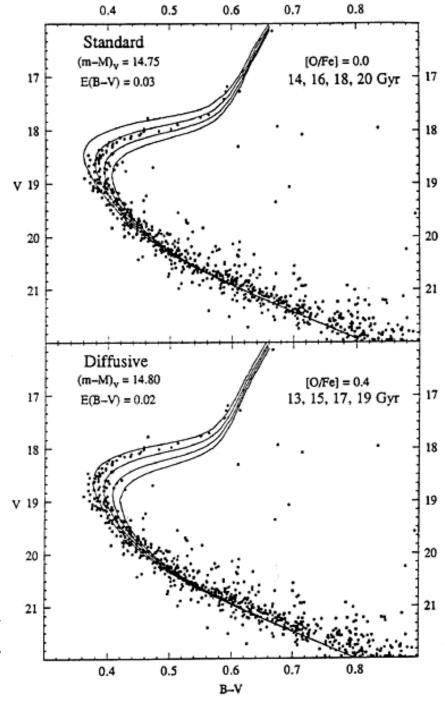
- We measure the age of a globular cluster by measuring the magnitude of the main sequence turnoff or the difference between that magnitude and the level of the horizontal branch, and comparing this to stellar evolutionary models of which estimate the surface temperature and luminosity of a stars as a function of time
- There are a fair number of uncertainties in these estimates, including errors in measuring the distances to the GCs and uncertainties in the isochrones used to derive ages (i.e., stellar evolution models)
- Inputs to stellar evolution models include: oxygen abundance [O/Fe], treatment of convection, He abundance, reaction rates of $^{14}N + p \rightarrow ^{15}O + \gamma$, He diffusion, conversions from theoretical temperatures and luminosities to observed colors and magnitudes, and opacities; and especially *distances*

Globular Cluster Ages

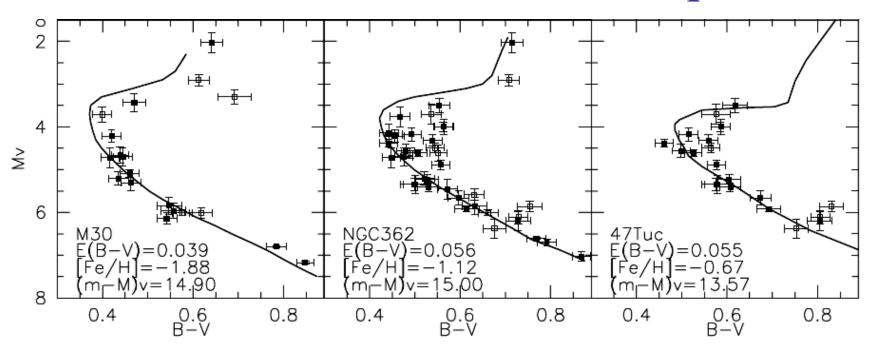


Schematic CMD and isochrones

Examples of actual model isochrones fits



Globular Cluster Ages From Hipparcos Calibrations of Their Main Sequences



Examples of g.c. main sequence isochrone fits, for clusters of a different metallicity (Graton et al.)

The same group has published two slightly different estimates of the mean age of the oldest clusters:

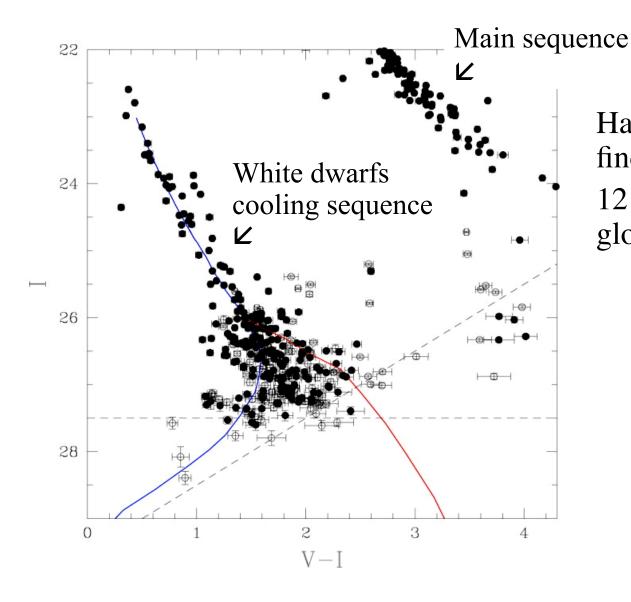
Age =
$$11.8^{+2.1}_{-2.5}$$
Gyr

$$Age = 12.3^{+2.1}_{-2.5} Gyr$$

White Dwarf Cooling Curves

- White dwarfs are the end stage of stellar evolution for stars with initial masses $< 8 M_{\odot}$
- They are supported by electron degeneracy pressure (not fusion) and are slowly cooling and fading as they radiate
- We can use the luminosity of the faintest WDs in a cluster to estimate the cluster age by comparing the observed luminosities to theoretical cooling curves
- Theoretical curves are subject to uncertainties related to the core composition of white dwarfs, detailed radiative transfer calculations which are difficult at cool temperatures
- White dwarfs are faint so this is hard to do. Need deep HST observations
- Only been done for one globular cluster, consistent with the ages of GCs found from the main sequence turnoff luminosities, would be nice if there were more

An Example: White Dwarf Sequence of M4



Hansen *et al.* (2002) find an age of 12.7 ± 0.7 Gyr for the globular cluster M4

> Blue = hydrogen atmosphere models Red = helium atmosphere models for a $0.6 M_{\odot} WD$

Nucleocosmochronology

- Can use the radioactive decay of elements to age date the oldest stars in the galaxy
- Has been done with 232 Th (half-life = 14 Gyr) and 238 U (half-life = 4.5 Gyr) and other elements
- Measuring the ratio of various elements provides an estimate of the age of the universe given theoretical predictions of the initial abundance ratio
- This is difficult because Th and U have weak spectral lines so this can only be done with stars with enhanced Th and U (requires large surveys for metalpoor stars) and unknown theoretical predictions for the production of r-process (rapid neutron capture) elements

Nucleocosmochronology:

An Example Isotope Ratios and Ages for a Single Star

CHRONOMETRIC AGE ESTIMATES FOR BD +17°3248

Chronometer Pair	Predicted	Observed	Age (Gyr)	Solara	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	≥13.4	2.32	11.0
U/Ir	0.05036	0.0045	≥15.5	0.0369	13.5
U/Pt	0.013	0.0019	≥12.4	0.01846	14.6

a From Burris et al. 2001.

(from Cowan et al. 2002)

Mean = 13.8 + /-4, but note the spread!

Summary: The Key Points

- Measurements of the H_0 are now good to ~5%, and may be improved in the future; various methods are in a good agreement, but some tension remains
- The concept of the distance ladder; many uncertainties and calibration problems, model-dependence, etc.
- Cepheids as the key local distance indicator
- SNe as a bridge to the far-field measurements
- Far-field measurements (SZ effect, lensing, CMB)
- Ages of the oldest stars (globular clusters), white dwarfs, and heavy elements are consistent with the age inferred from the H_0 and other cosmological param's