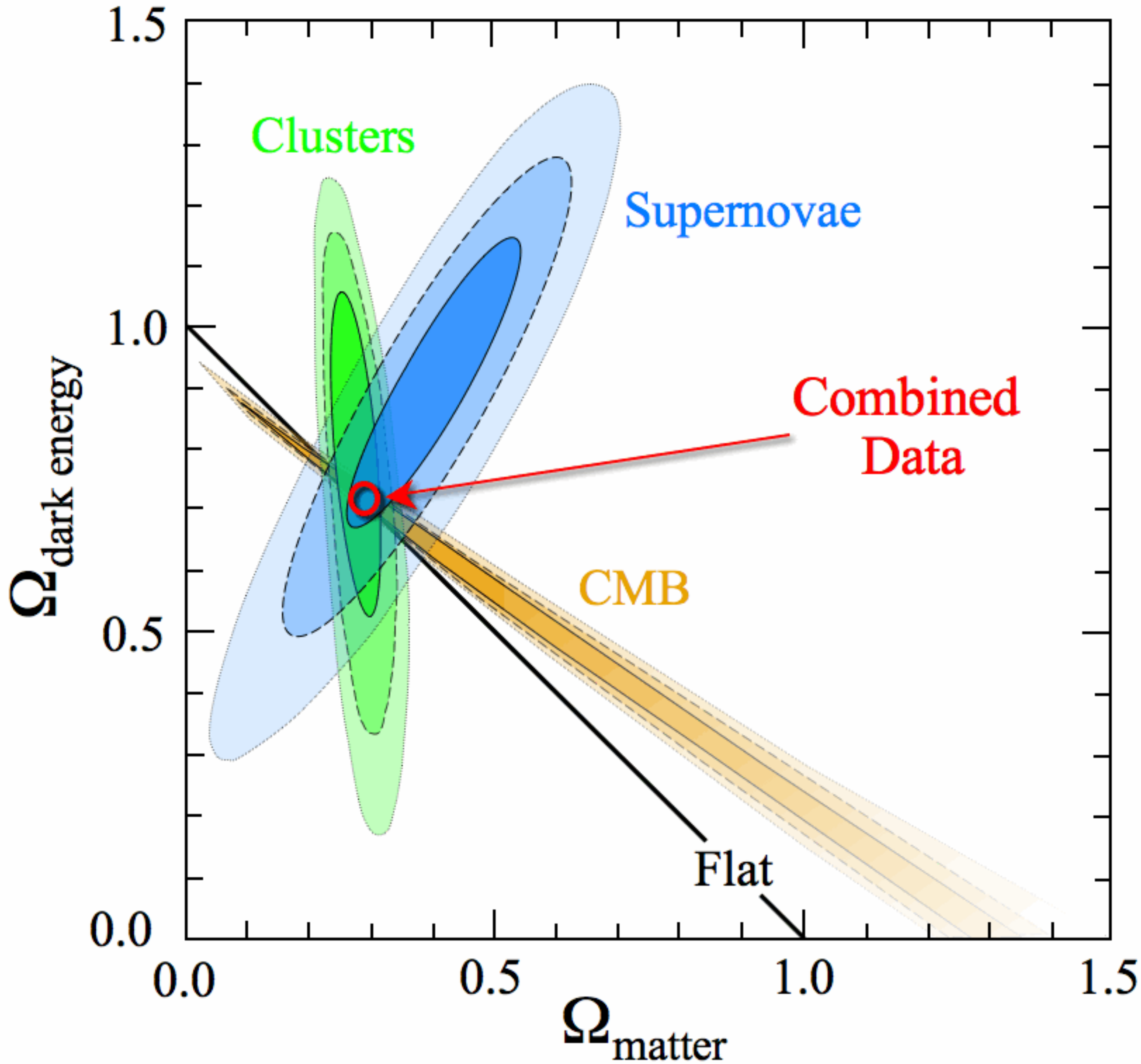
Ay 127, Winter 2017: Cosmological Tests



Tests for the Expansion of the Universe

- Tolman surface brightness (SB) test
  - In a stationary, Euclidean universe $SB = const.$
  - In an expanding universe, $SB \sim (1+z)^{-4}$
  - In a "tired light" model, $SB \sim (1+z)^{-1}$
- Time dilation of Supernova light curves
  - Time stretches by a factor of $(1+v/c) = (1+z)$
- The match between the energy density and T^4 for the blackbody and the CMBR
  - For a blackbody, energy density $u \sim T^4$
  - In an expanding universe, for photons, energy density is $u \sim (1+z)^4$, and since $T \sim 1/\lambda \sim (1+z)$, $u \sim T^4$

The Tolman Test

Surface brightness is flux per unit solid angle: $B = \dfrac{f}{d\omega}$

This is the same as the luminosity per unit area, at some distance D. In cosmology,

$$B = \frac{L}{D_L^2}\frac{D_A^2}{dl^2}$$

In a stationary, Euclidean case, $D = D_L = D_A$, so the distances cancel, and $SB = const$. But in an expanding universe, $D_L = D\,(1+z)$, and $D_A = D\,/\,(1+z)$, so:

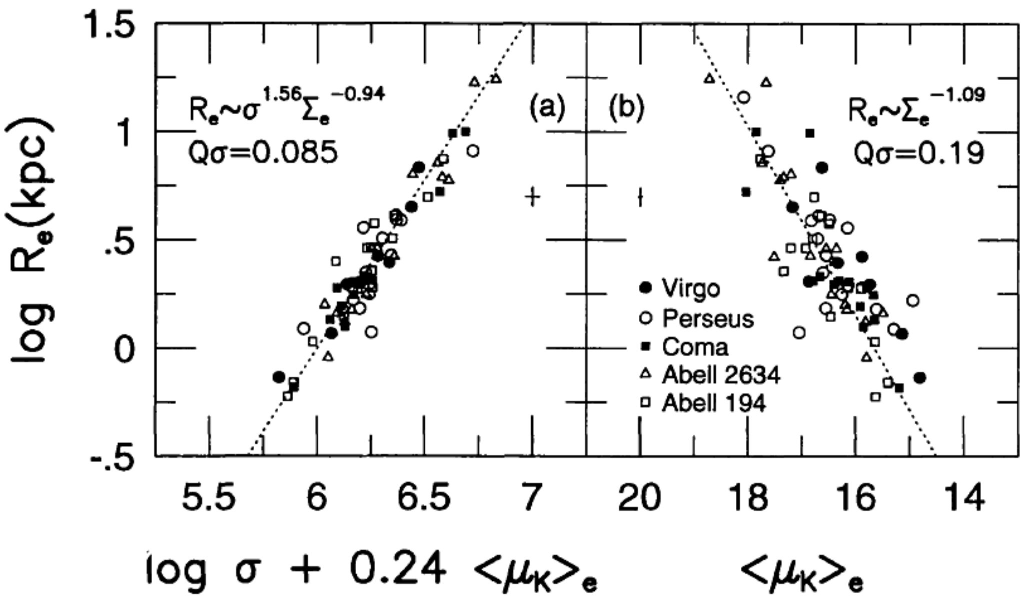
$$B = \frac{L}{dl^2}\frac{D_A^2}{D_L^2} = \frac{L}{dl^2}(1+z)^{-4}$$

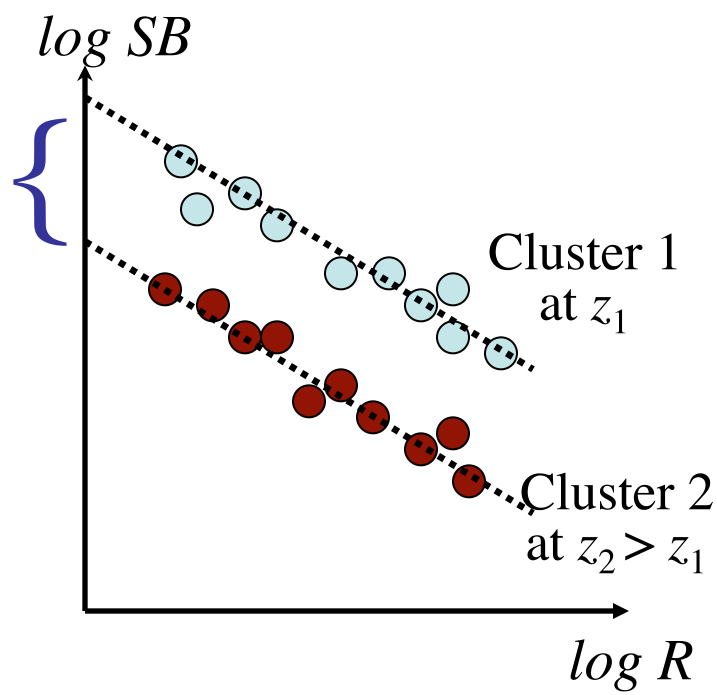
Note that this is independent of cosmology!

Performing the The Tolman Test

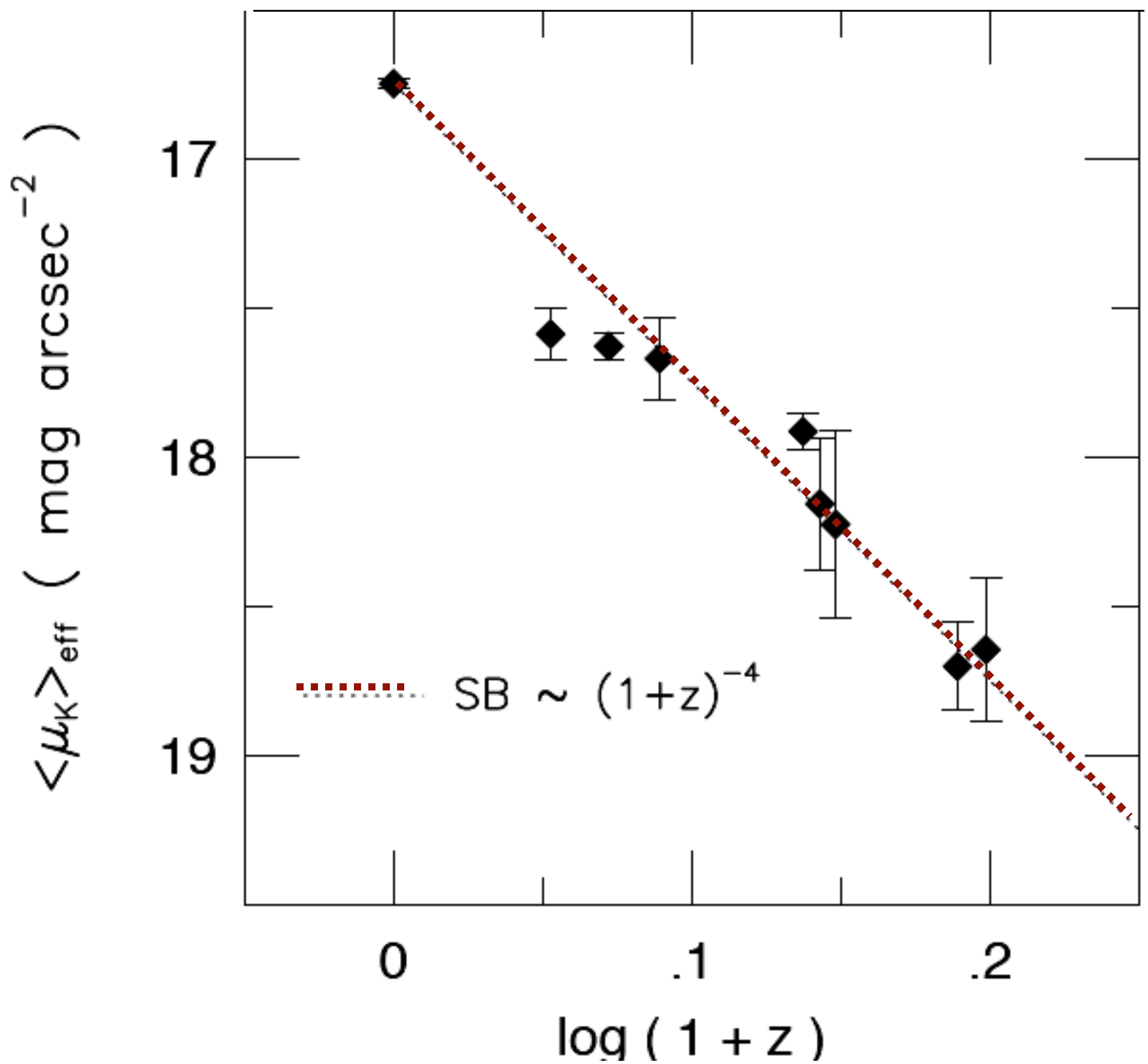
We need a standard (constant) unit of surface brightness = luminosity/area, to observe at a range of redshifts (a "standard fuzz"?)

A good choice is the intercept of surface brightness scaling relations for elliptical galaxies in clusters





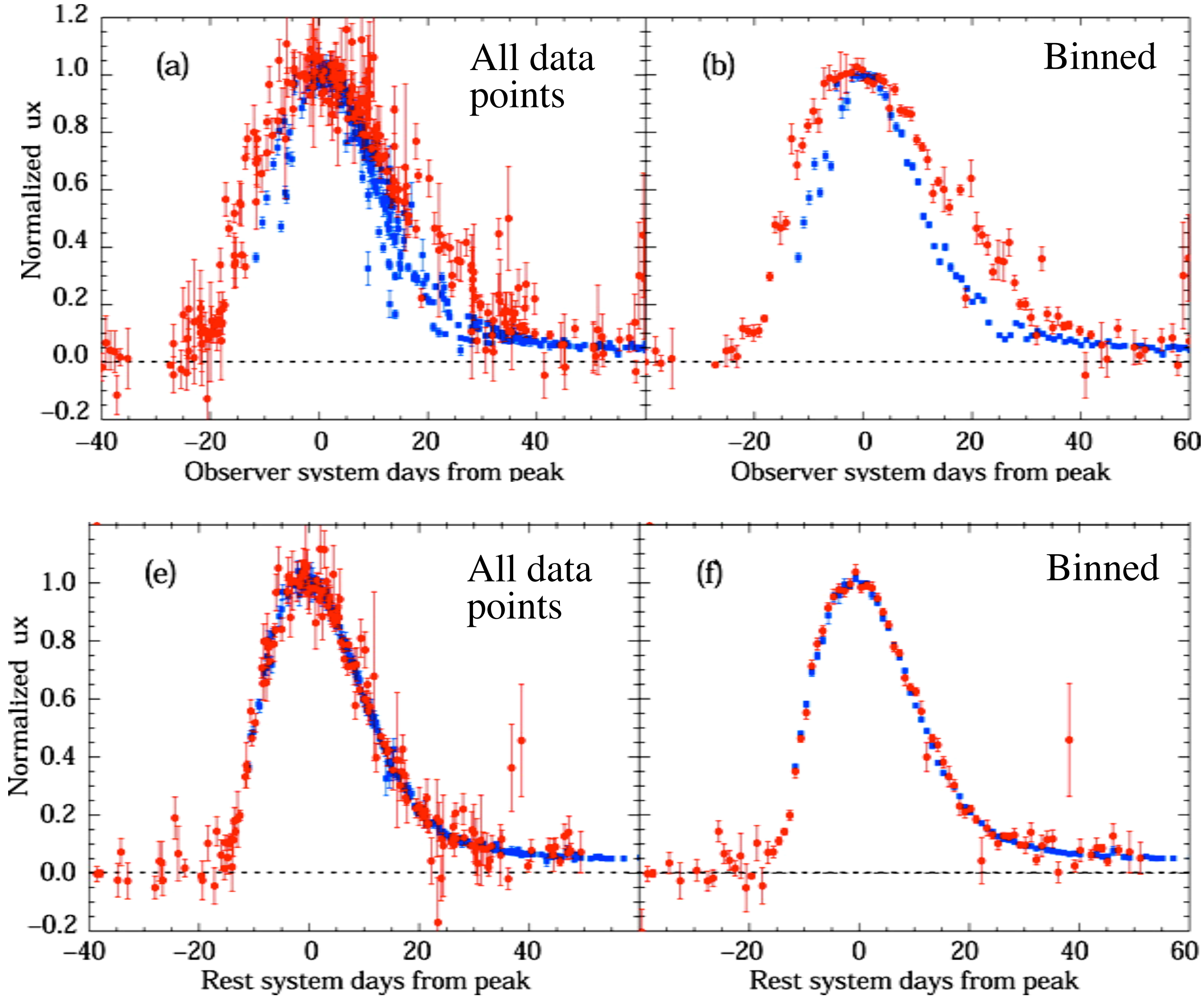
The Tolman Test Results



Surface brightness intercept of the Fundamental Plane correlation, for elliptical galaxies in clusters out to z ~ 0.6. It assumes a reasonable galaxy evolution model correction.

(from Pahre et al.)

Time Dilation of Supernova Lightcurves



Blue dots: a low-z dataset

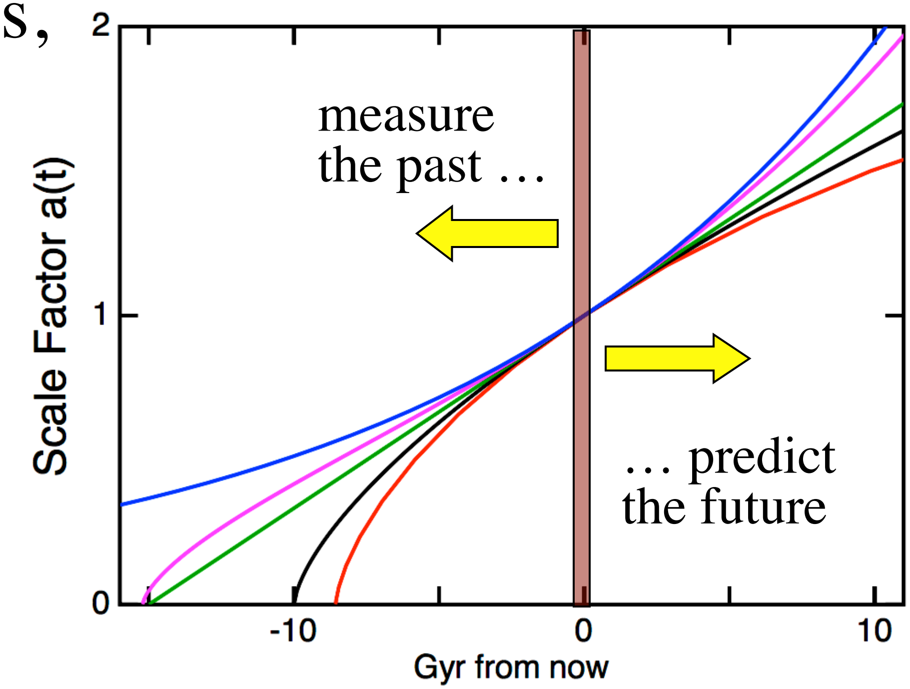
Red dots: a high-z dataset

After applying the proper stretch factor

(Goldhaber et al.)

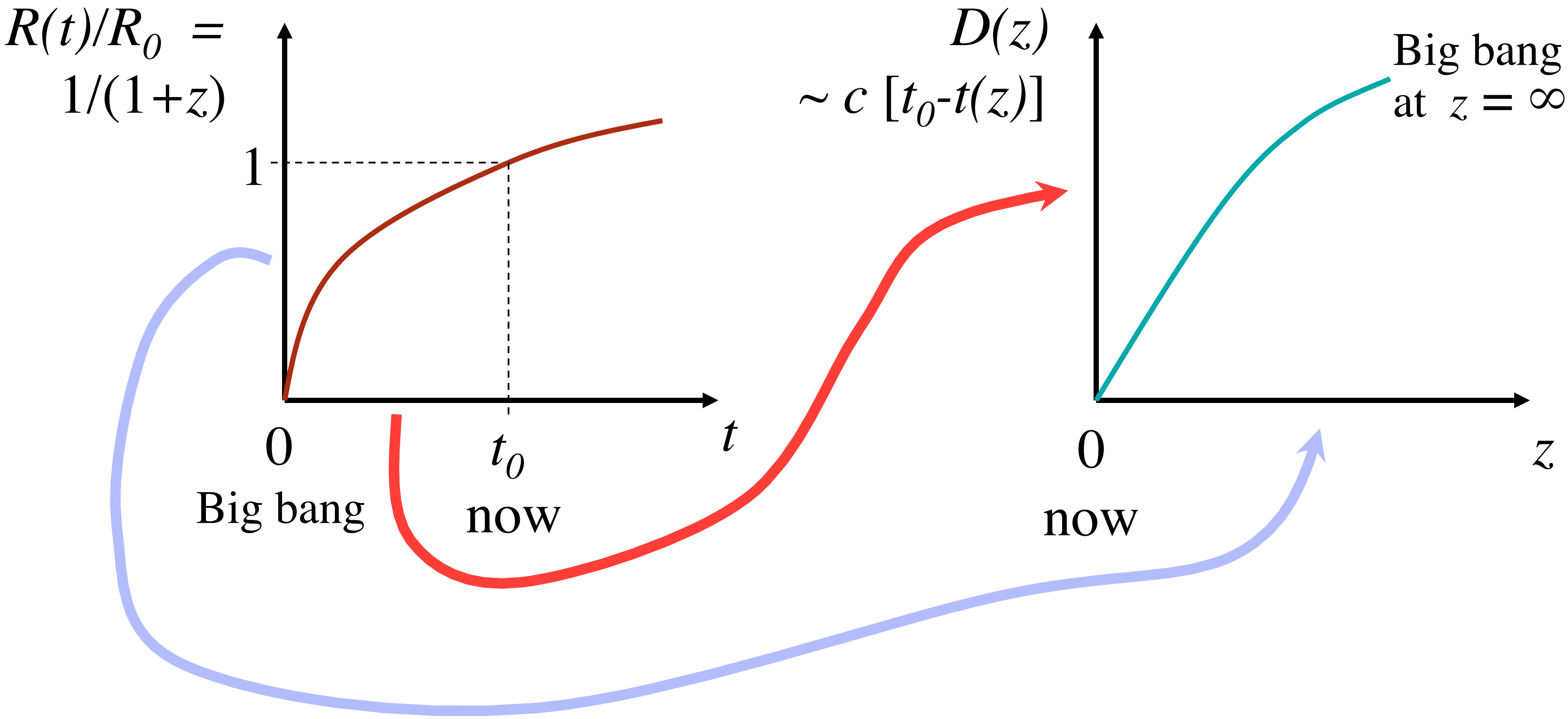
Cosmological Tests: The Why and How

- The goal is to determine the global geometry and the dynamics of the universe, and its ultimate fate
- The basic method is to somehow map the history of the expansion, and compare it with model predictions
- A model (or a family of models) is assumed, e.g., the Friedmann-Lemaitre models, typically defined by a set of parameters, e.g., H_0 , $\Omega_{0,m}$, $\Omega_{0,\Lambda}$, q_0, etc.



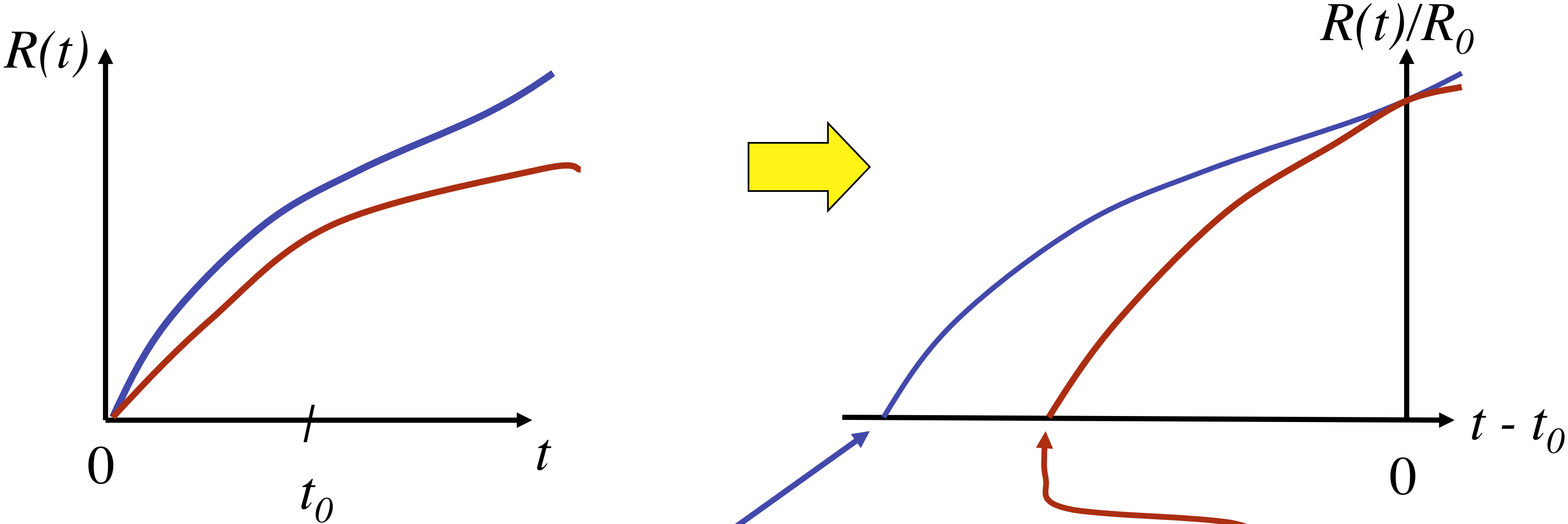
- Model equations are integrated, and compared with the observations

The Basis of Cosmological Tests



All cosmological tests essentially consist of comparing some measure of (relative) distance (or look-back time) to redshift. Absolute distance scaling is given by the H_0.

Cosmological Tests: Expected Generic Behavior of Various Models



Models with a lower density and/or positive Λ expand faster, are thus larger, older today, have more volume and thus higher source counts, at a given *z* sources are further away and thus appear fainter and smaller

Models with a higher density and lower Λ behave exactly the opposite

The Types of Cosmological Tests

- The Hubble diagram: flux (or magnitude) as a proxy for the luminosity distance, vs. redshift - requires "*standard candles*"
- Angular diameter as a proxy for the angular distance, vs. redshift - requires "*standard rulers*"
- Source counts as a function of redshift or flux (or magnitude), probing the evolution of a volume element - requires a population of sources with a constant comoving density - "*standard populations*"
- Indirect tests of age vs. redshift, usually highly model-dependent - "*standard clocks*"
- Local dynamical measurements of the mass density, Ω_{m0}
- If you measure H_0 and t_0 independently, you can constrain a combination of Ω_{m0} and Ω_Λ

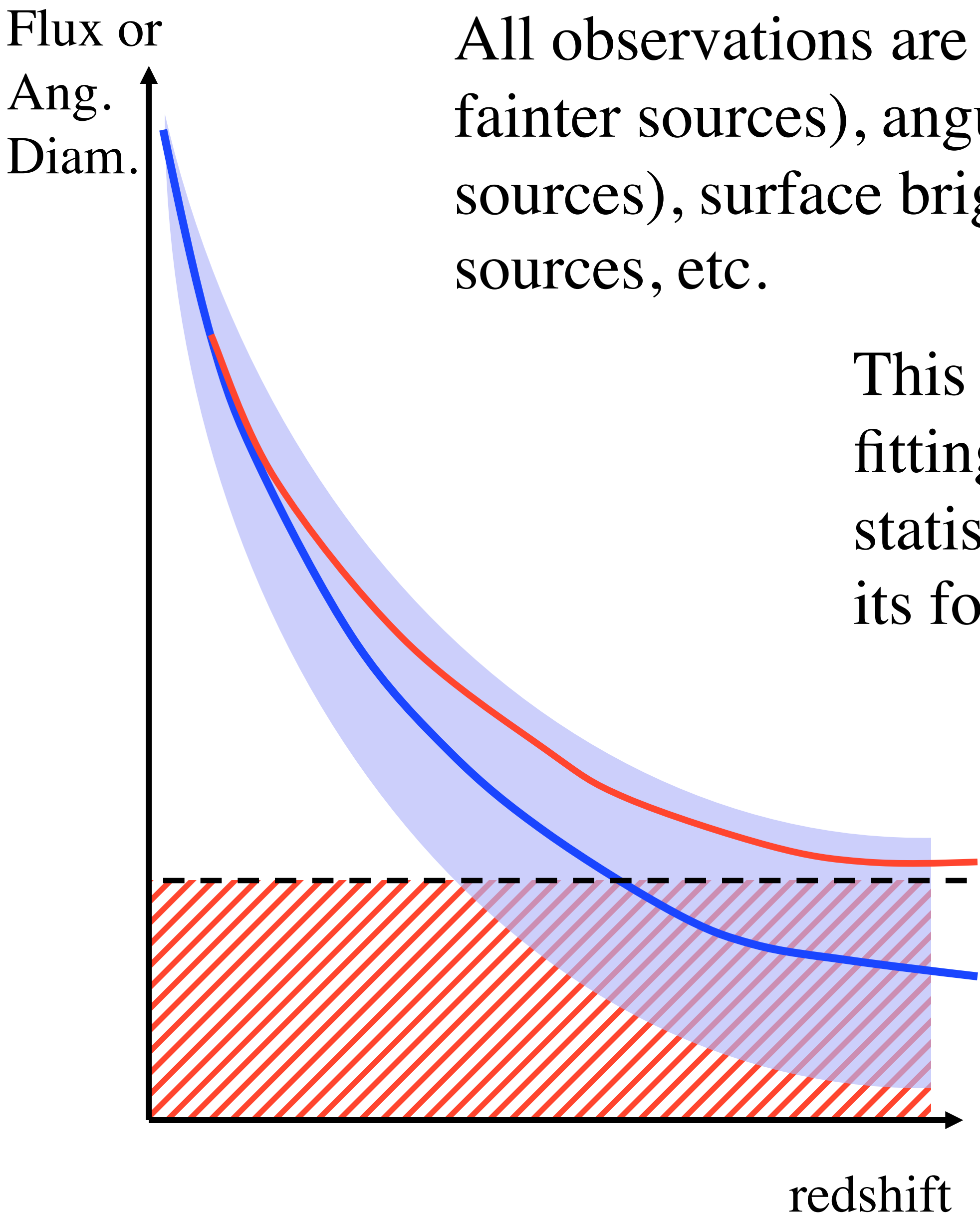
Cosmological Tests: A Brief History

- A program of “classical” cosmological tests (Hubble diagram, angular diameter test, source counts) was initiated by Hubble, and carried out at Palomar and elsewhere by Sandage and others, from 1950s through 1970s
- Galaxies, clusters of galaxies, and radio sources were used as standard candles, rulers, or populations. Unfortunately, all are subject to strong and poorly constrained ***evolutionary effects***, which tend to dominate over the cosmology - this foiled most of the attempted tests, and became obvious by 1980’s
- In the late 1990’s, Supernova Ia Hubble diagram, and especially measurements of CMBR fluctuations power spectra (essentially an angular diameter test) completely redefined the subject
- The cosmological parameters are now known with a remarkable precision - a few percent; this is the era of “***precision cosmology***”

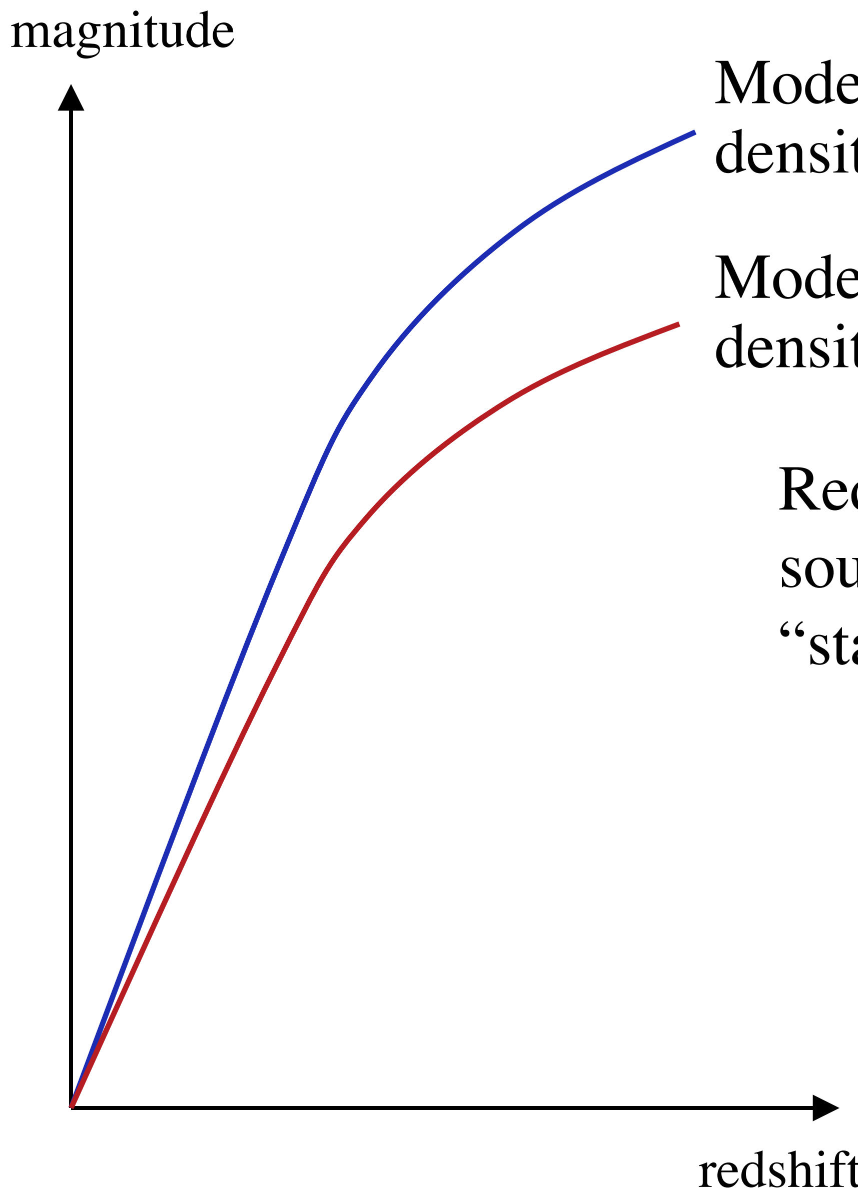
Selection Effects and Biases

All observations are limited in sensitivity (we miss fainter sources), angular resolution (we miss smaller sources), surface brightness (we miss very diffuse sources, etc.

This inevitably introduces a bias in fitting the data, unless a suitable statistical corrrection is made - but its form may not be always known!



The Hubble Diagram



Requires a population on non-evolving sources with a fixed luminosity - “standard candles”. Some candidates:

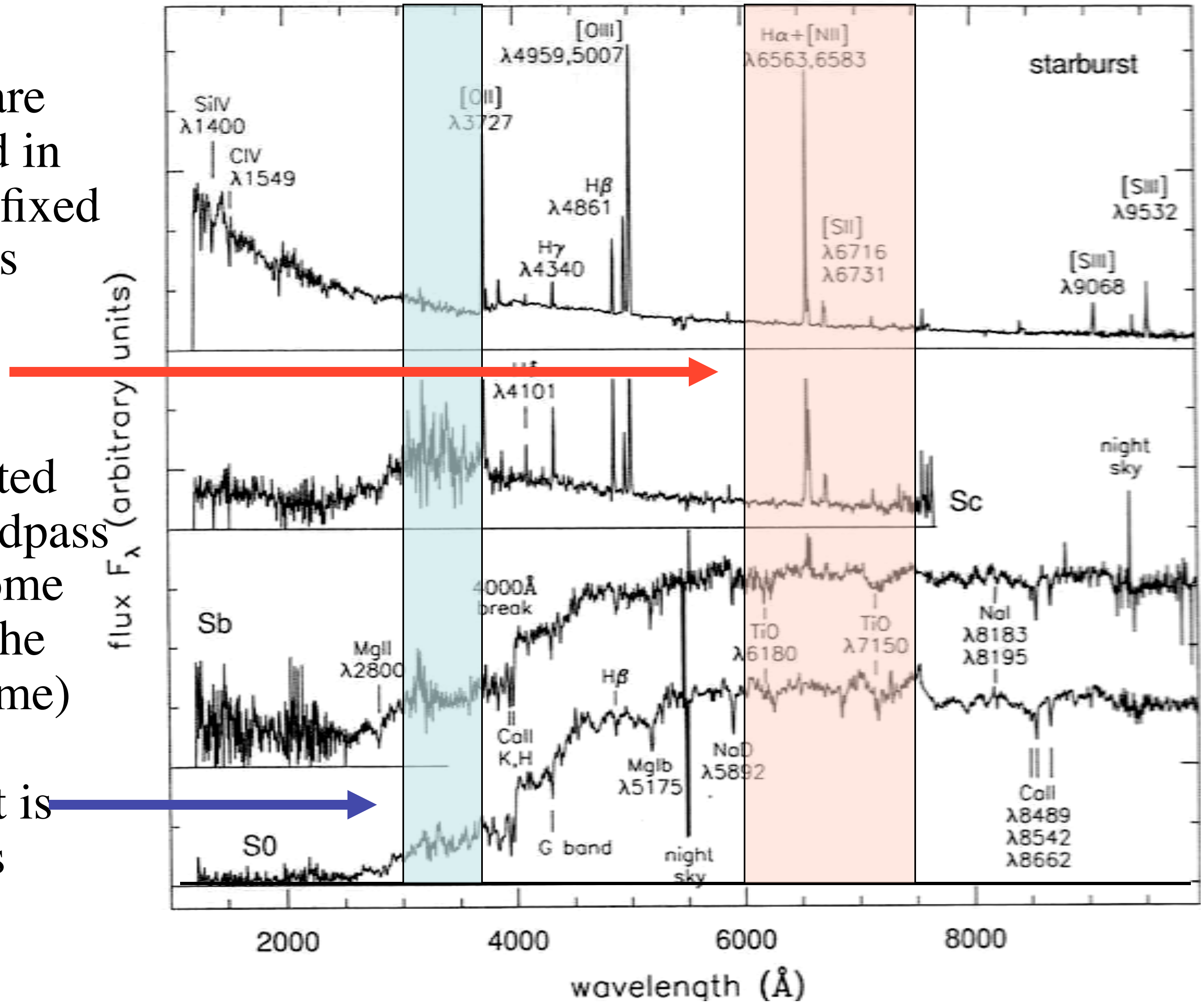
- Brightest cluster ellipticals
- Supernovae of type Ia
- Luminosity functions in clusters
- GRB afterglows ??
- …

The K-Correction

Photometric measurements are always obtained in some bandpass fixed in the observer's frame, e.g., the *U,B,V,R*…

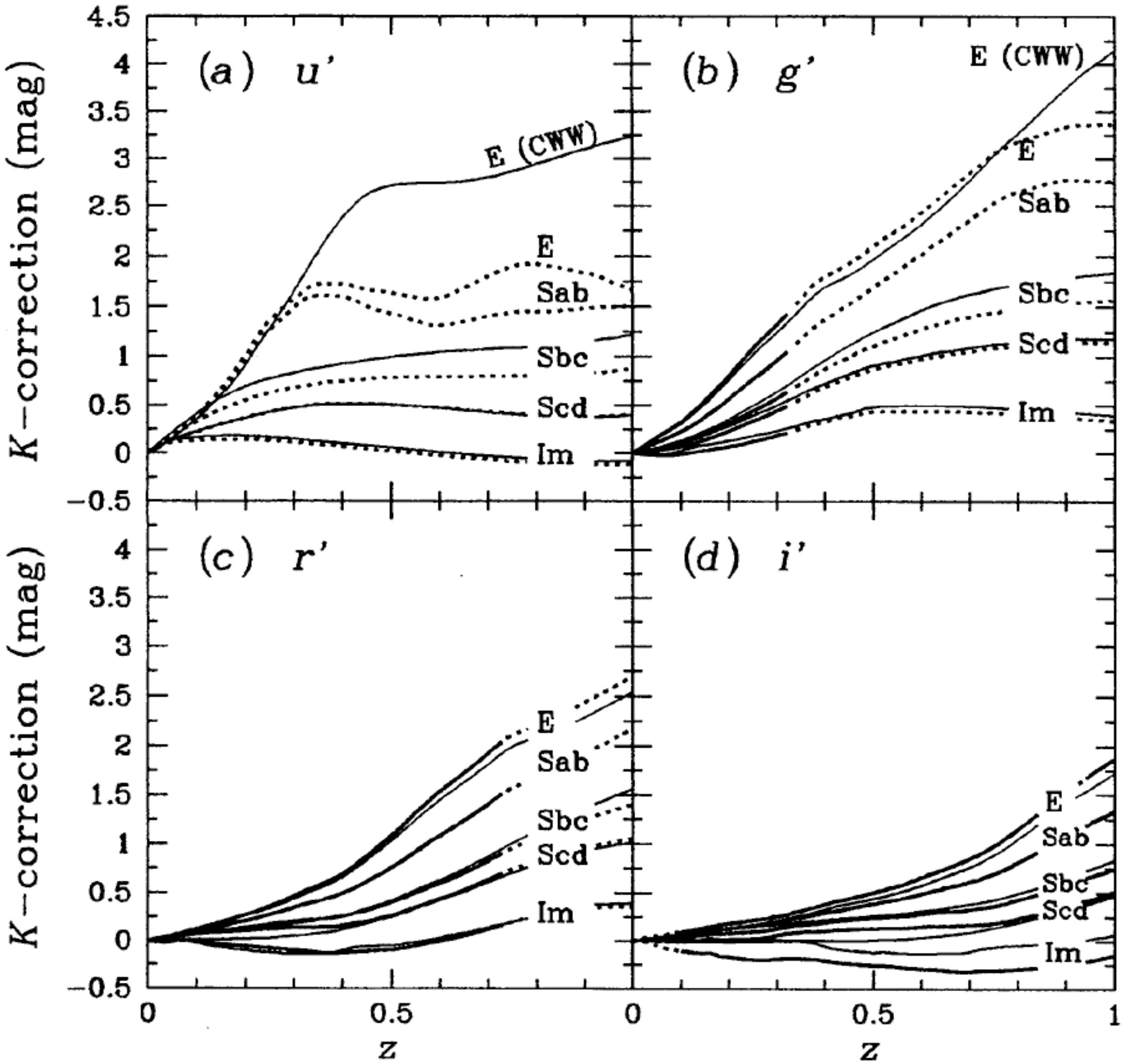
But in a redshifted galaxy, this bandpass now samples some other (bluer in the galaxy's restframe) region of the spectrum, and it is also $(1+z)$ times narrower

Galaxy spectra of different types



The K-Correction

Thus, we integrate the spectrum over the bandpass in the observed frame, and in the galaxy's restframe, take a ratio, express it in magnitudes, and that is the **K-correction**

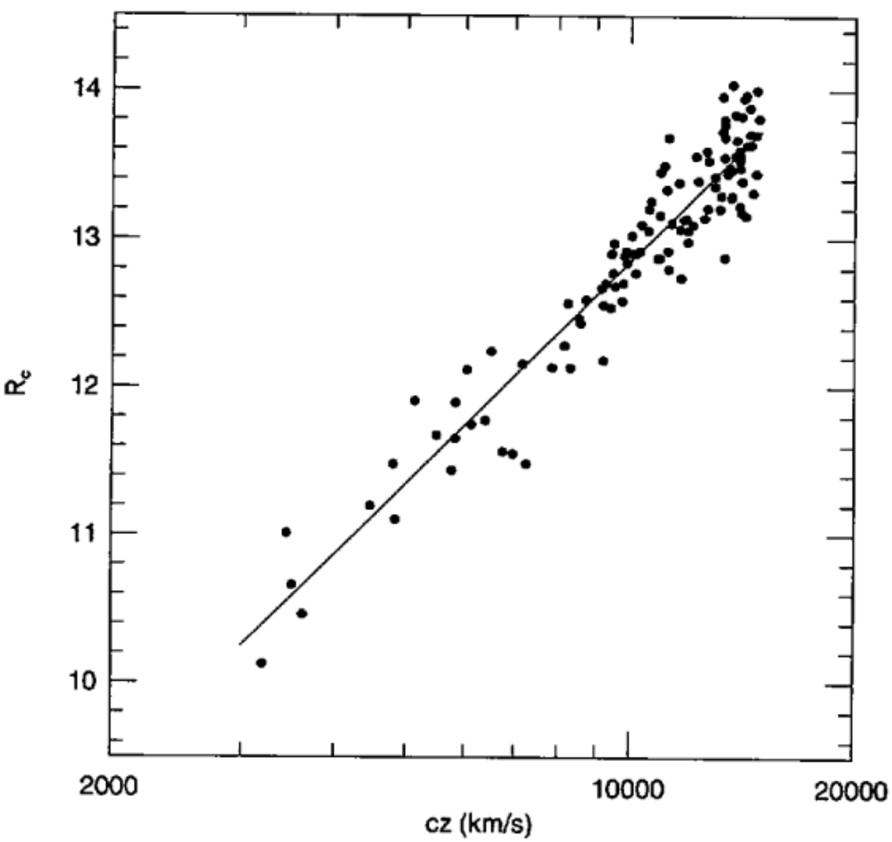


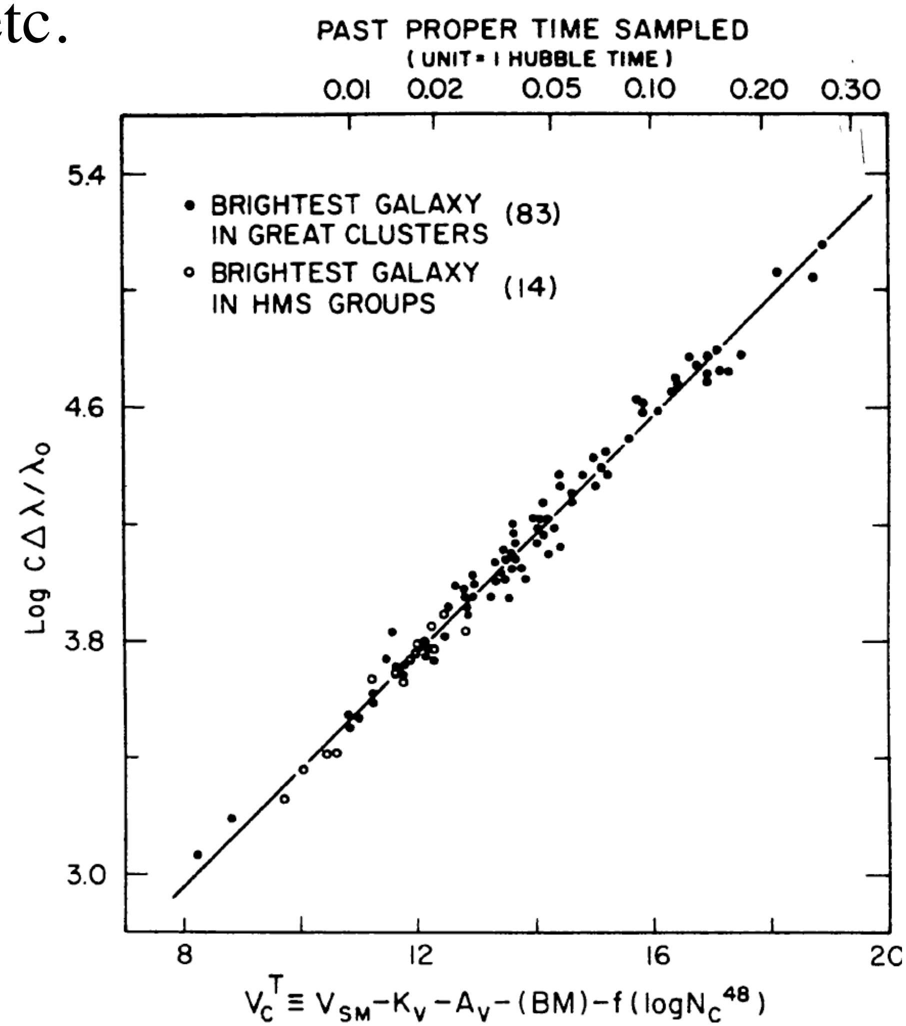
It has to be done for all different types of galaxy spectra, as it depends on the star formation rates, and it varies with bandpass

(Fukugita *et al.*)

The Hubble Diagram: Early Work

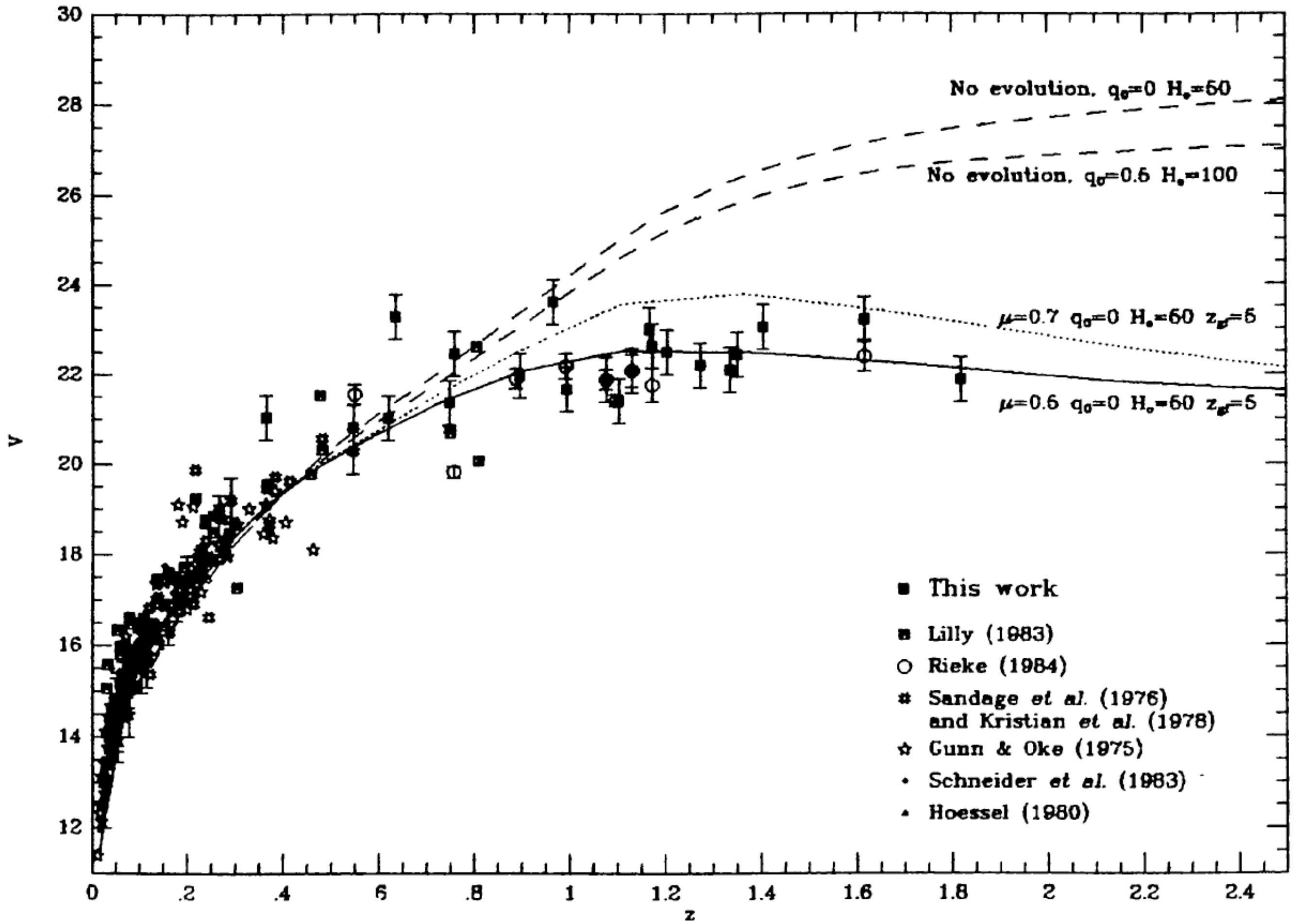
- Mostly done at Palomar by Sandage and collaborators, and by Gunn and collaborators, using brightest cluster ellipticals, with corrections for cluster richness etc.
- Foiled by galaxy evolution!





Effects of Galaxy Evolution

- Alas, galaxies were generally brighter in the past, since there was more star formation, and young, luminous, massive stars have short lifetimes
- This tends to overwhelm the cosmological effects, especially in the bluer bands

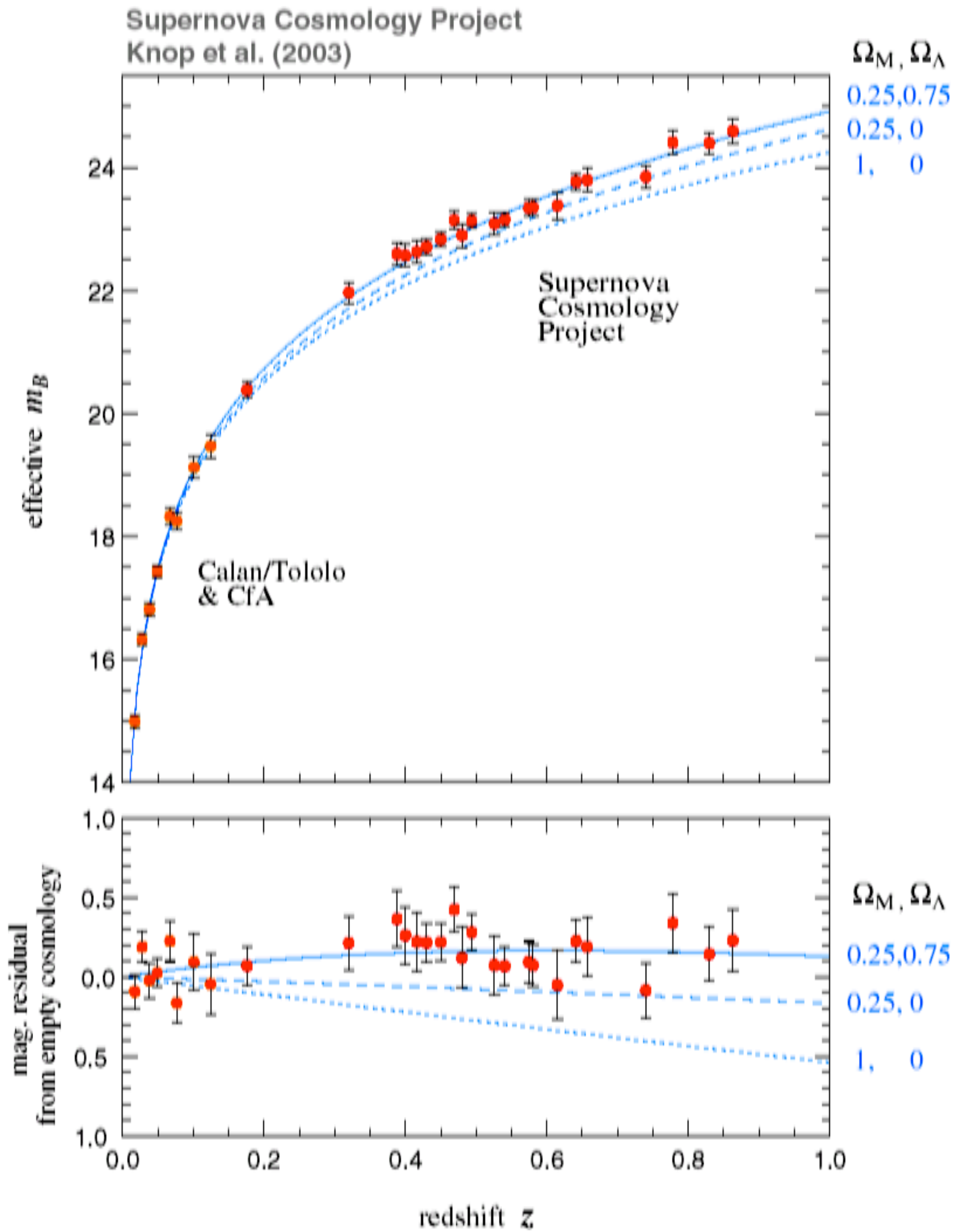


The Hubble diagram ➞ for powerful radio galaxies (Djorgovski et al.)

The Supernova Ia Hubble Diagram

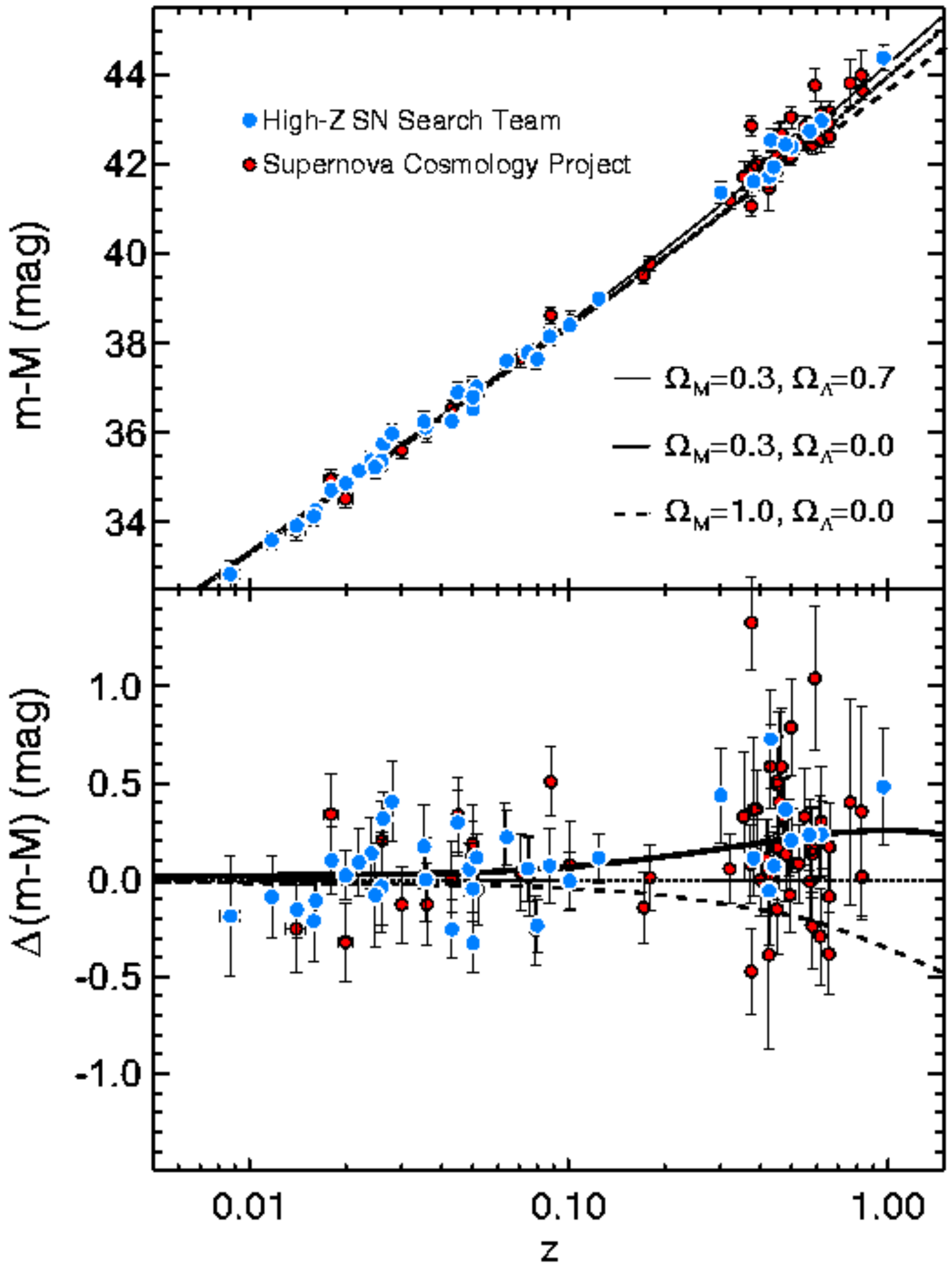
- The field was reborn with the advent of the SN Ia Hubble diagram, following the standardization of their peak brightness using light curve shapes
- There are still some unknowns:
  - Explosions not fully understood; many possible models: Chandrasekhar-mass models, deflagrations vs. detonations
  - Progenitor systems not known: white dwarfs yes, but double degenerate vs. single degenerate binaries …
- SN Ia are not really standard candles …
  - There are large variations in light curve shapes, colors, spectral evolution, and some clear outliers; possible differences in physical parameters, e.g, Ni mass
- But they ***are*** good distance indicators, after the empirical correction for light curve shapes
- Do they evolve (e.g., due to metallicity)? Maybe...

This yielded the **evidence for an accelerating universe and the positive cosmological constant,** independently and simultaneously by two groups: The Supernova Cosmology Project at LBL (Perlmutter et al.), and …



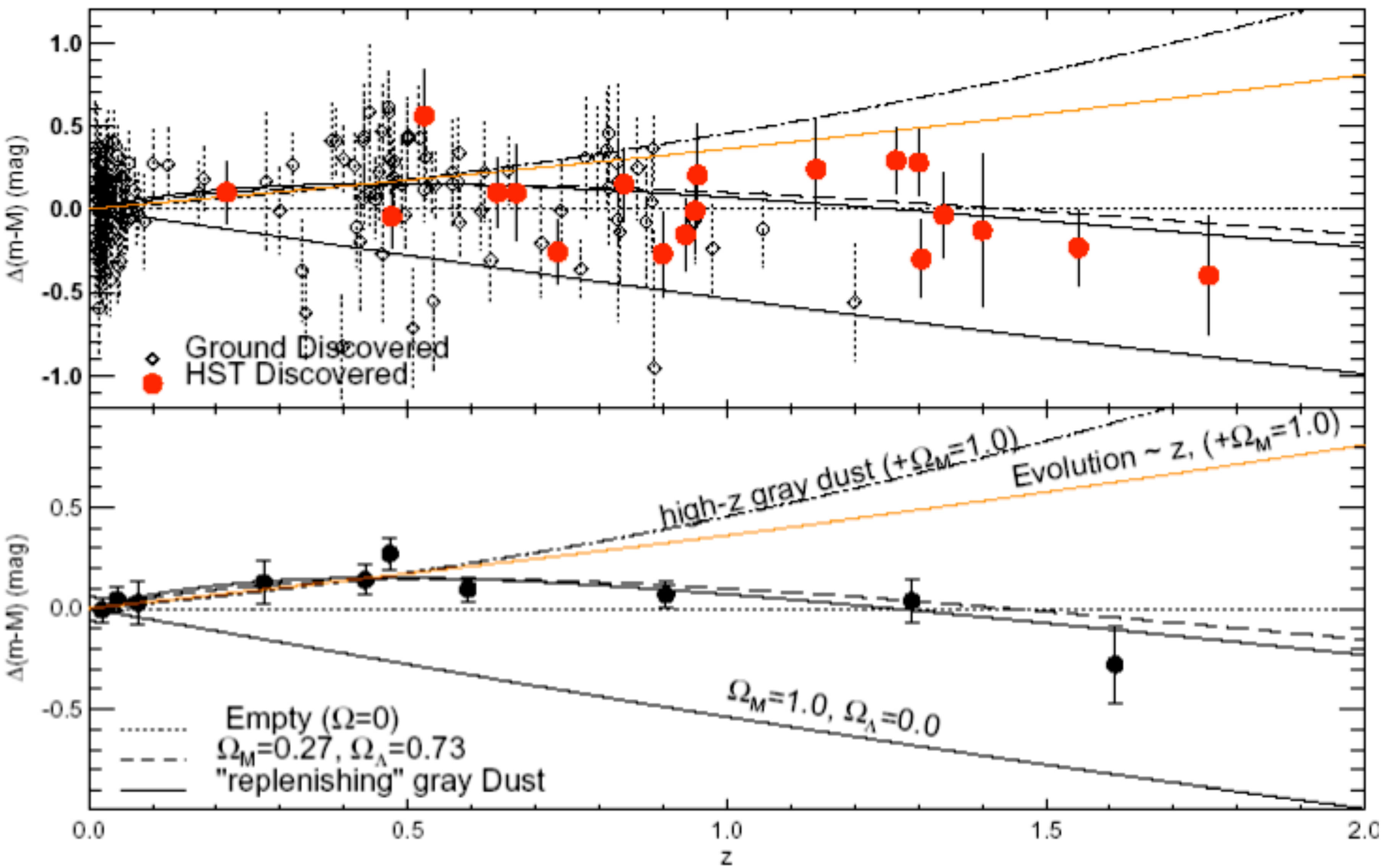
… and by the High-Z Supernova Team

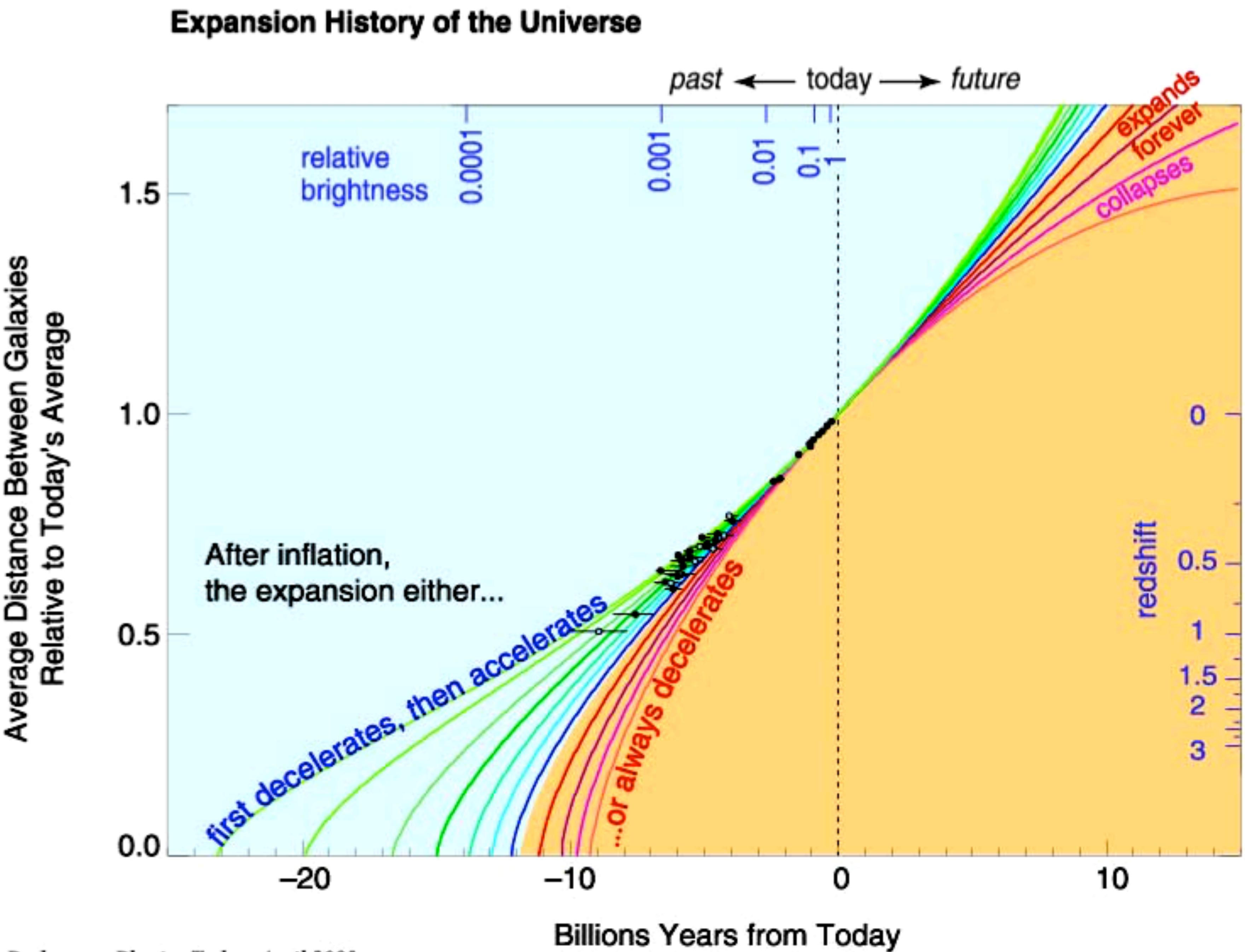
(B. Schmidt, A. Riess, et al.)



Both teams found very similar results …

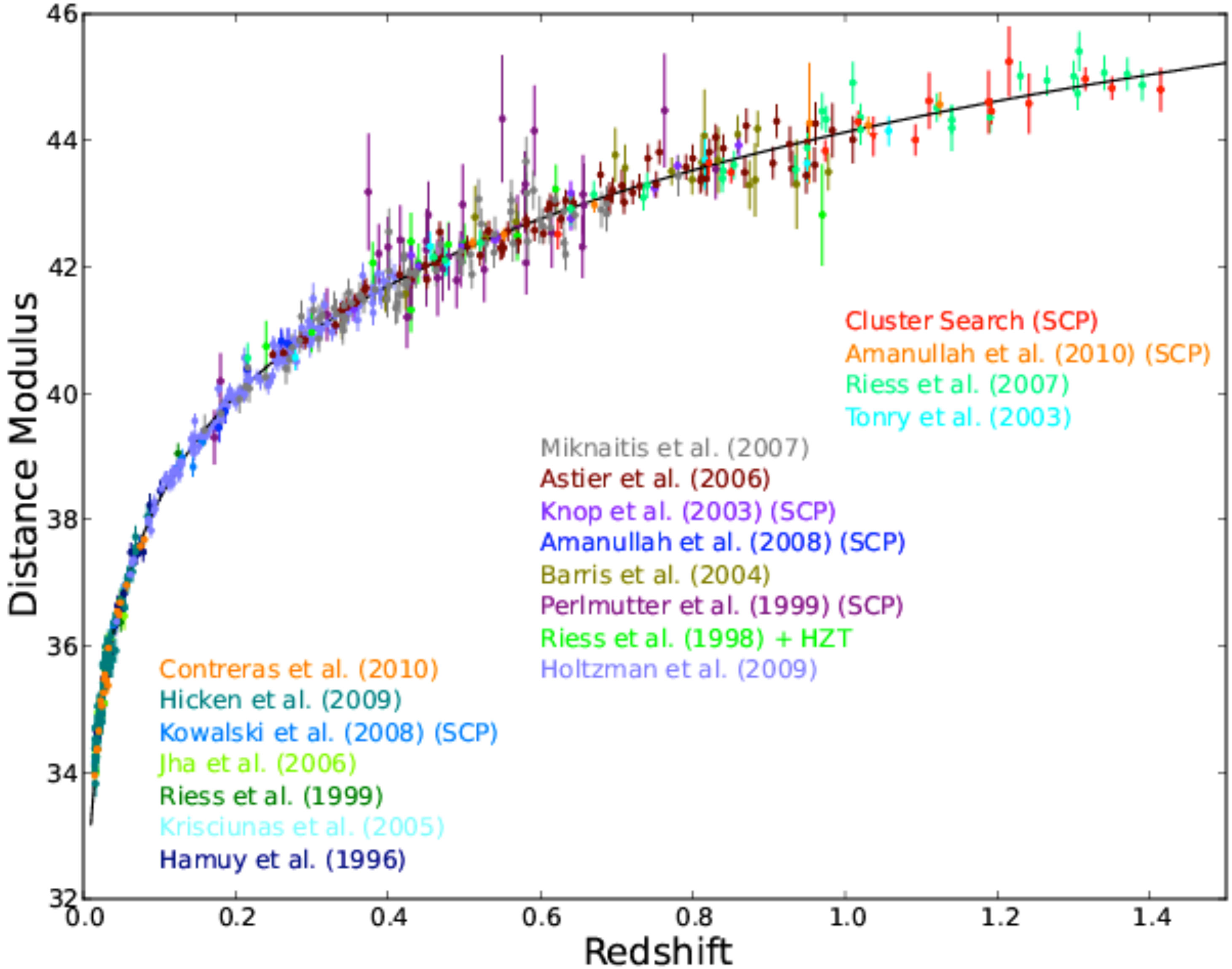
Current evidence points to $\Omega_\Lambda \sim 0.7$

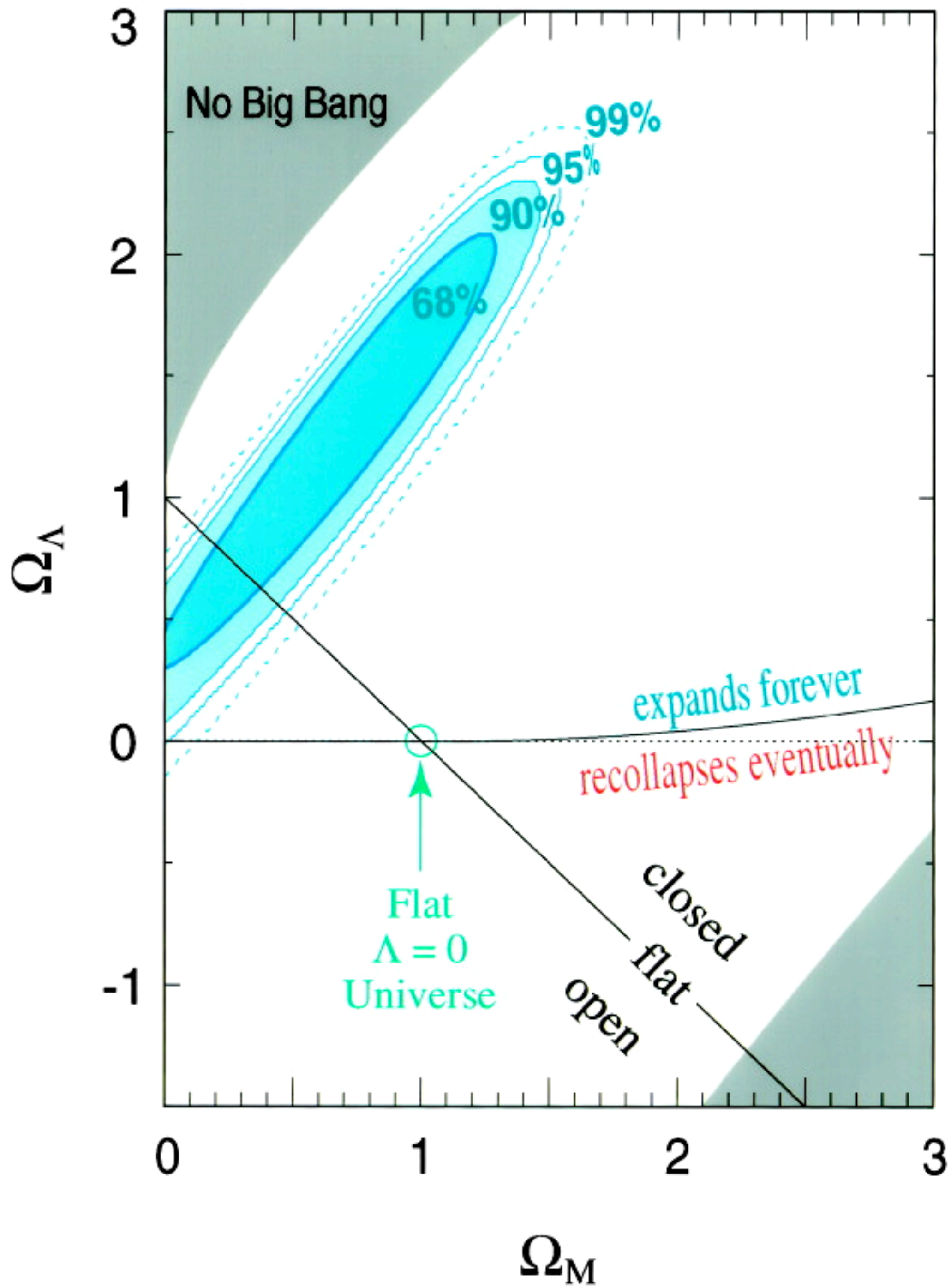




Perlmutter, Physics Today, April 2003

A Modern Version of the SN Hubble Diagram



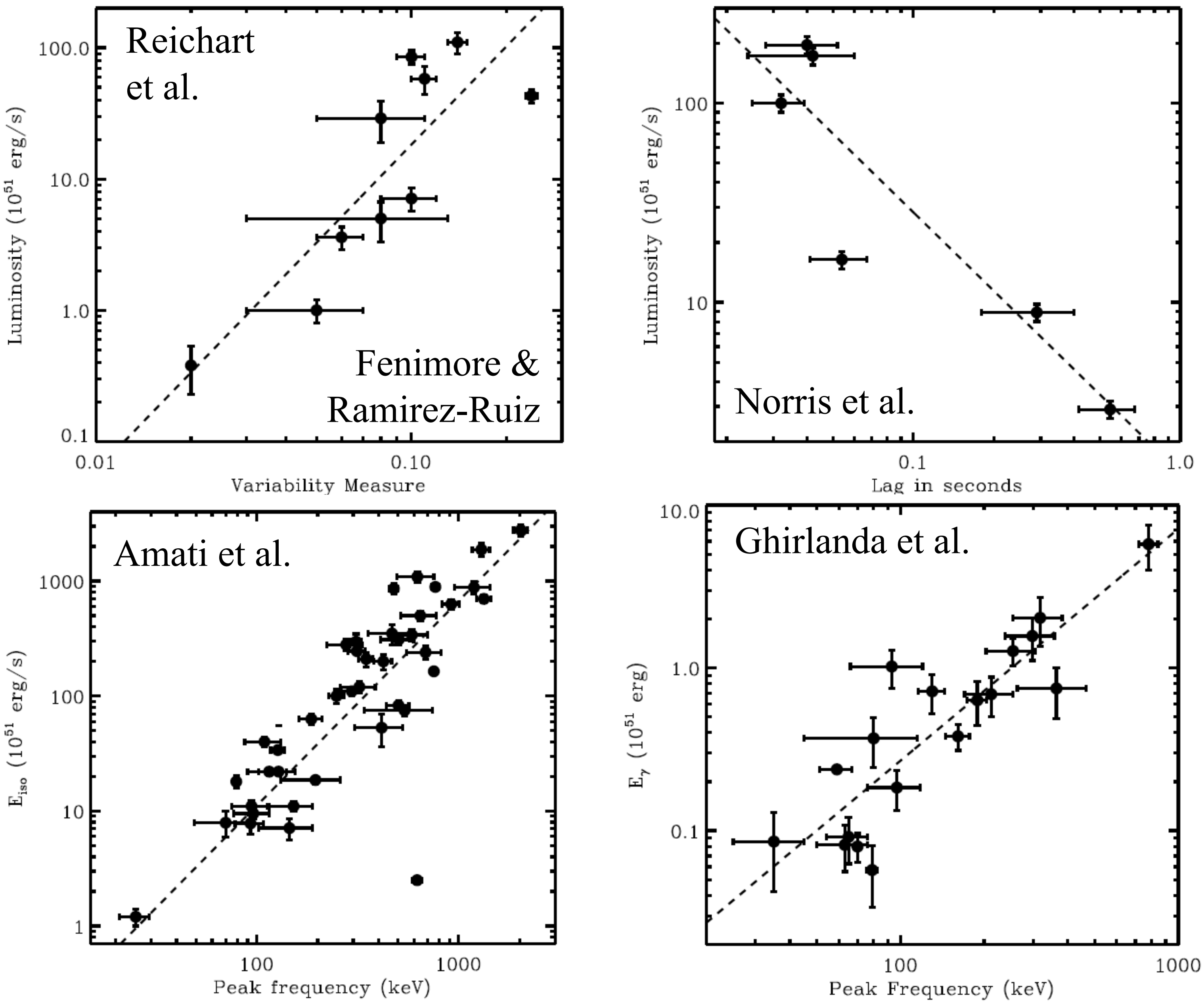


SN measurements on their own actually define an allowed region in the plane of [Ω_m , Ω_Λ]

Example of **degeneracy**: distinct universes produce identical results for this cosmological test

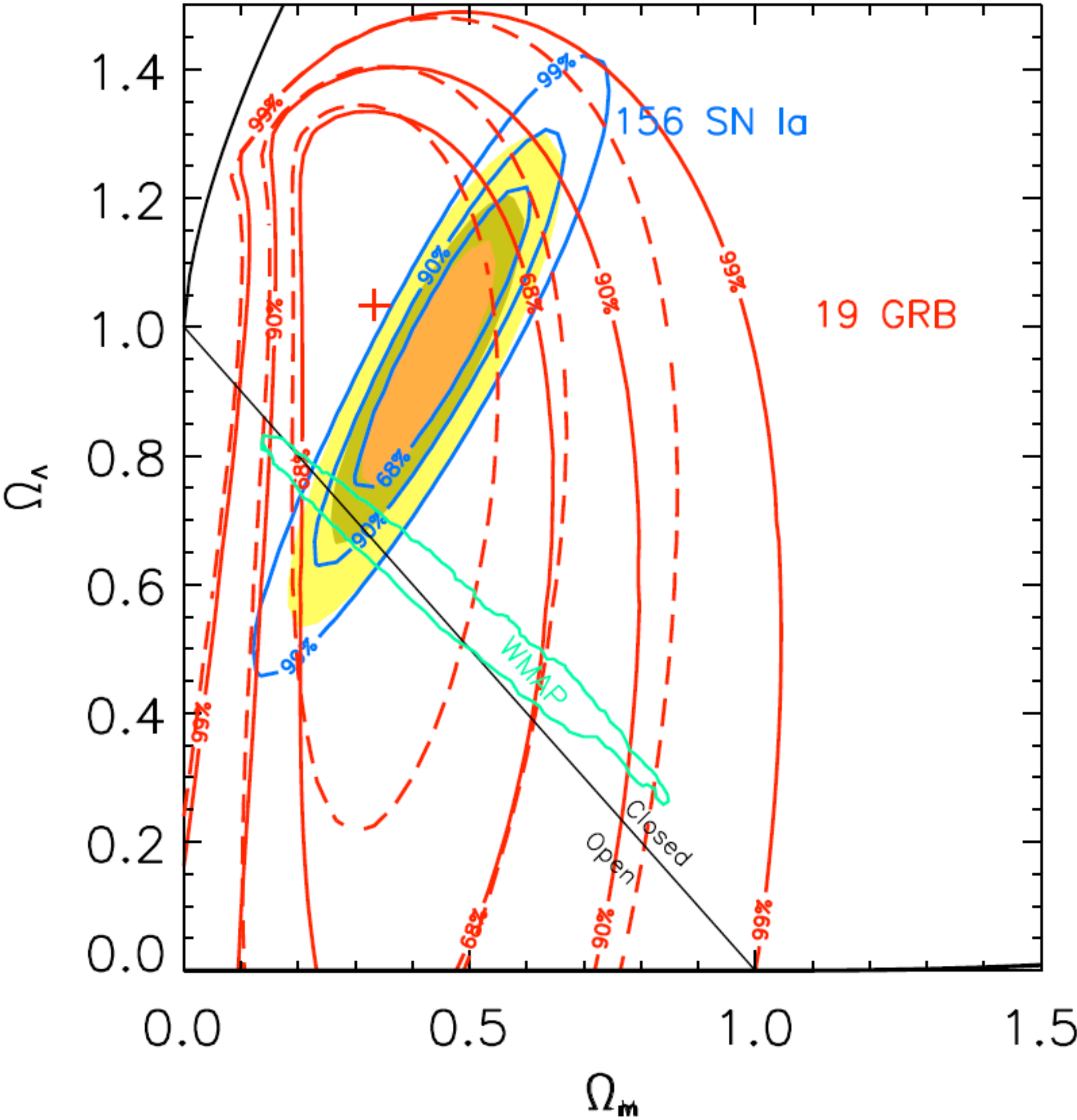
We need some additional, constraints (e.g., flatness) to pin down the actual value of Ω_Λ

GRBs as Standard Candles?



Various distance-independent burst parameters correlate with the total apparent isotropic energy or luminosity

GRBs as Standard Candles?



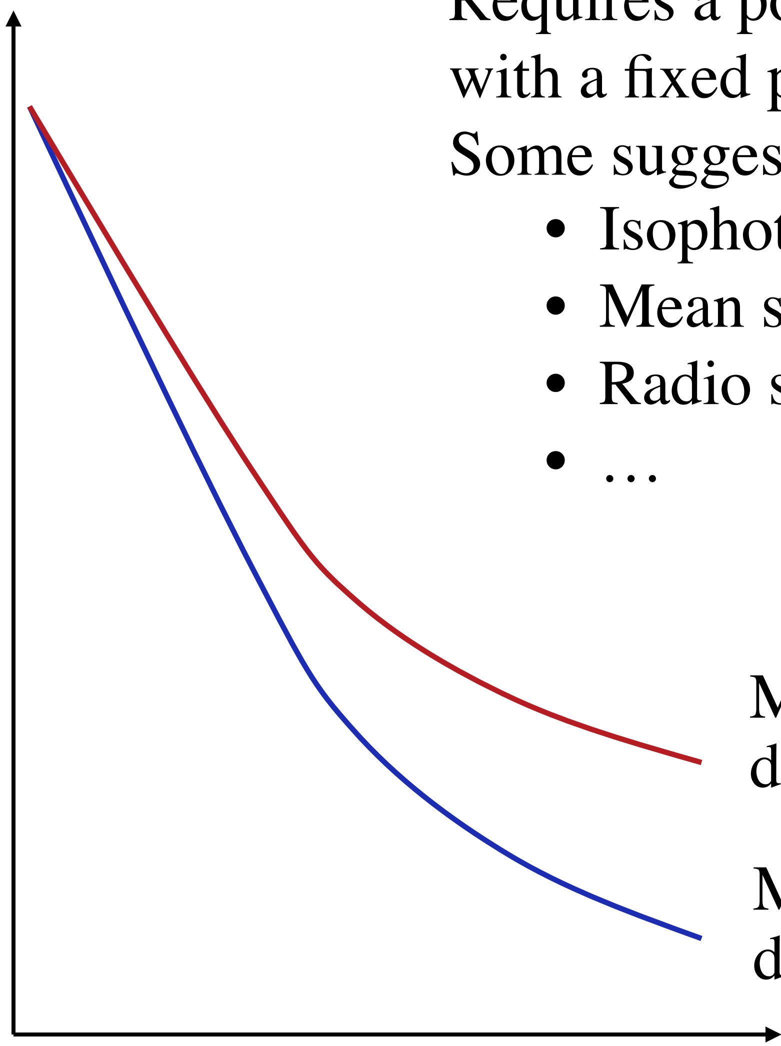
Not quite competitive with SNe yet, but there is a promise…

(Figure from Lazzati et al.)

The Angular Diameter Test

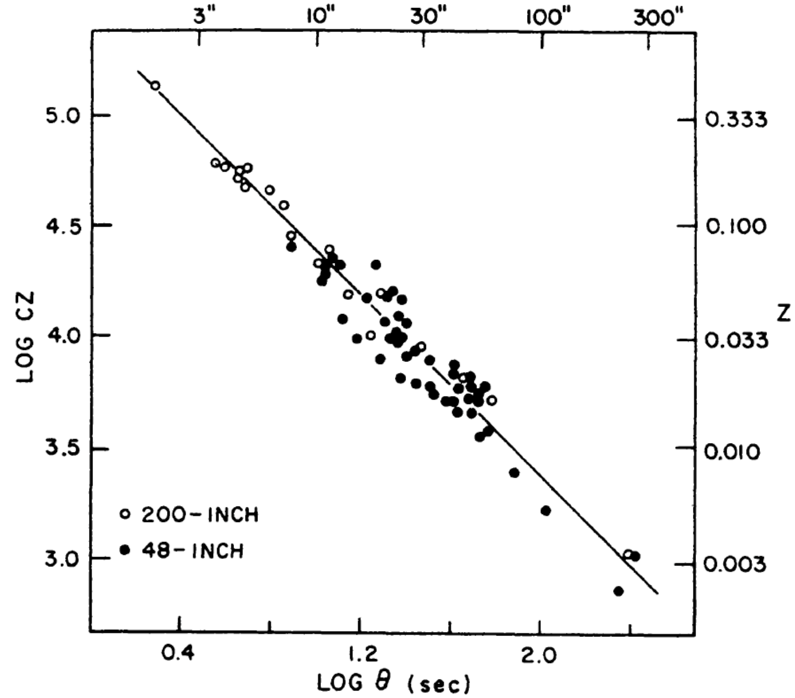
Requires a population on non-evolving sources with a fixed proper size - “standard rulers”.
Some suggested candidates:

- Isophotal diameters of brightest cluster gal.
- Mean separation of galaxies in clusters
- Radio source lobe separations
- …



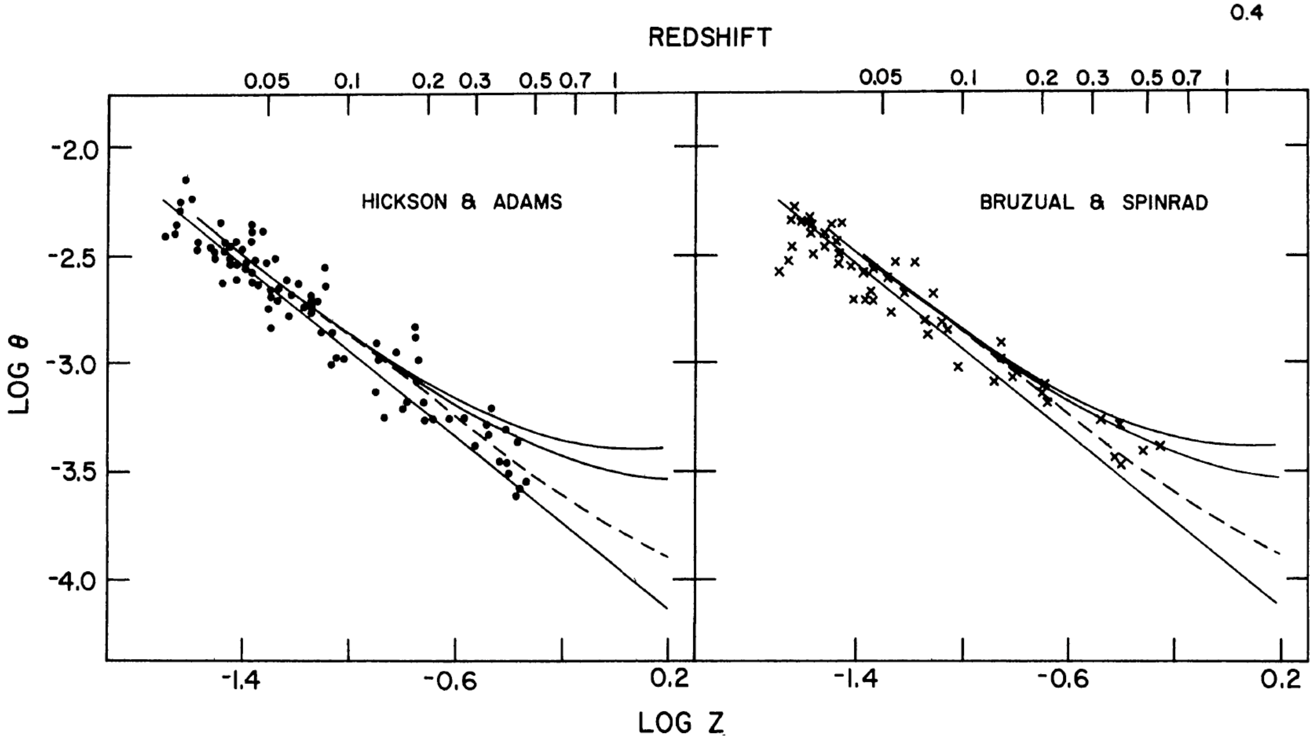
The Angular Diameter Test: Some Early Examples

Brightest cluster ellipticals →



Clusters of galaxies

↓

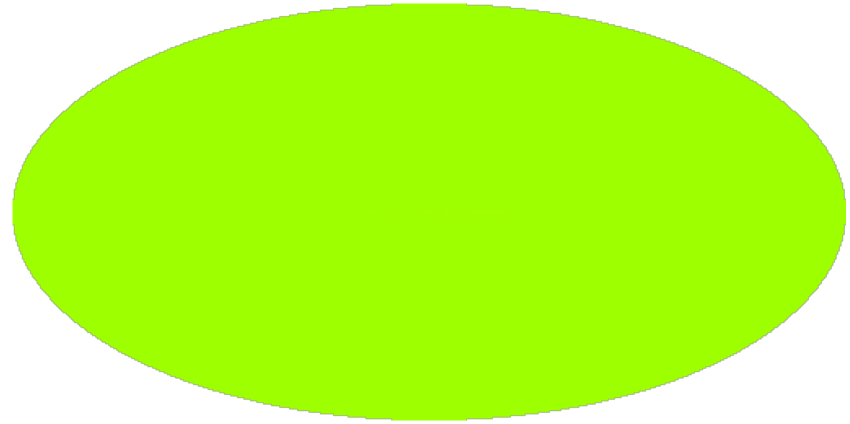


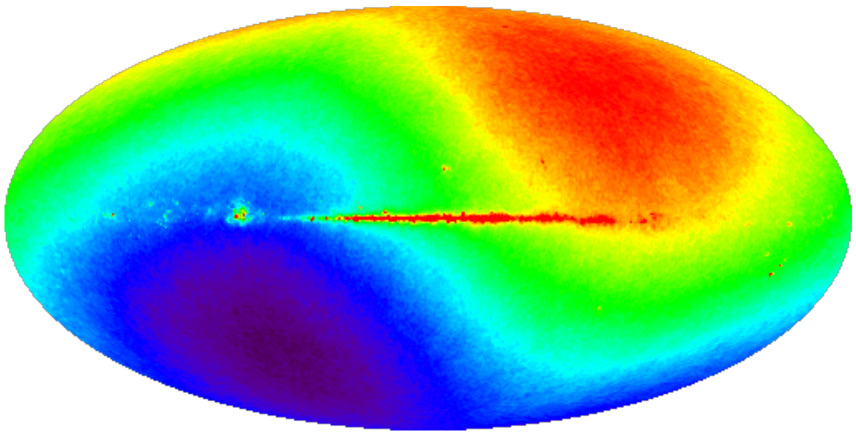
Again, **evolution** overwhelms the cosmological effects …

The Modern Angular Diameter Test: CMBR Fluctuations

- Uses the size of the particle horizon at the time of the recombination (the release of the CMBR) as a standard ruler
- This governs the largest wavelength of the sound waves produced in the universe then, due to the infall of baryons into the large-scale density fluctuations
- These sound waves cause small fluctuations in the temperature of the CMB ($\Delta T/T \sim 10^{-5} - 10^{-6}$) at the appropriate angular scales (~ a degree and less)
- They are measured as the angular power spectra of temperature fluctuations of the CMBR

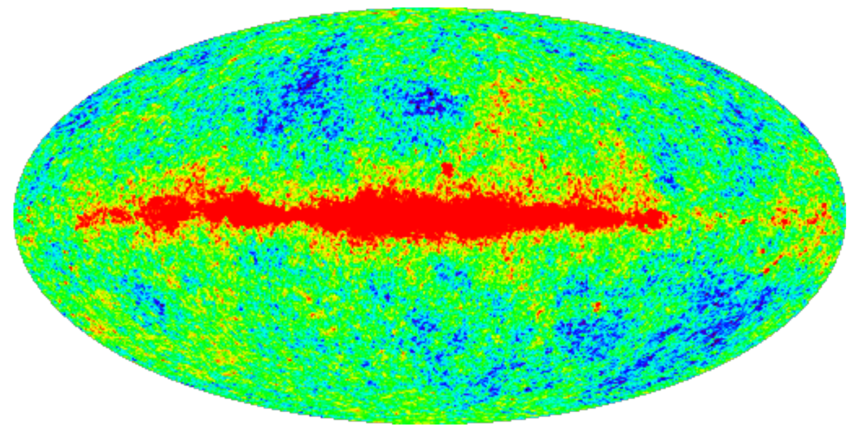
The CMBR sky from WMAP ➡

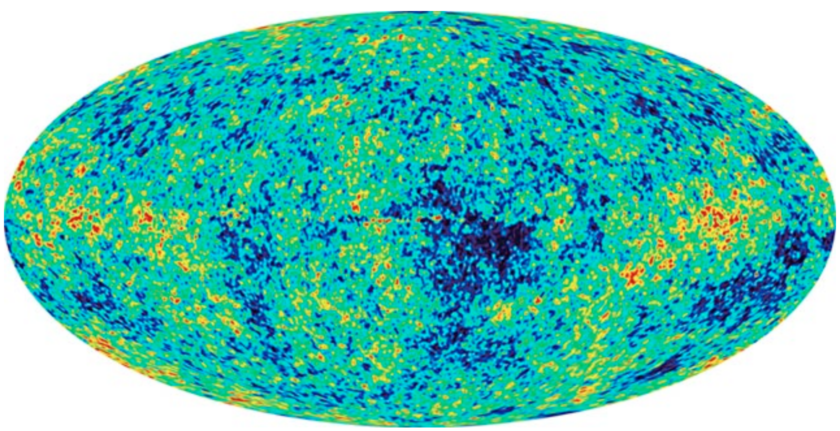




⬅ Enhance the conttrast by 10^3

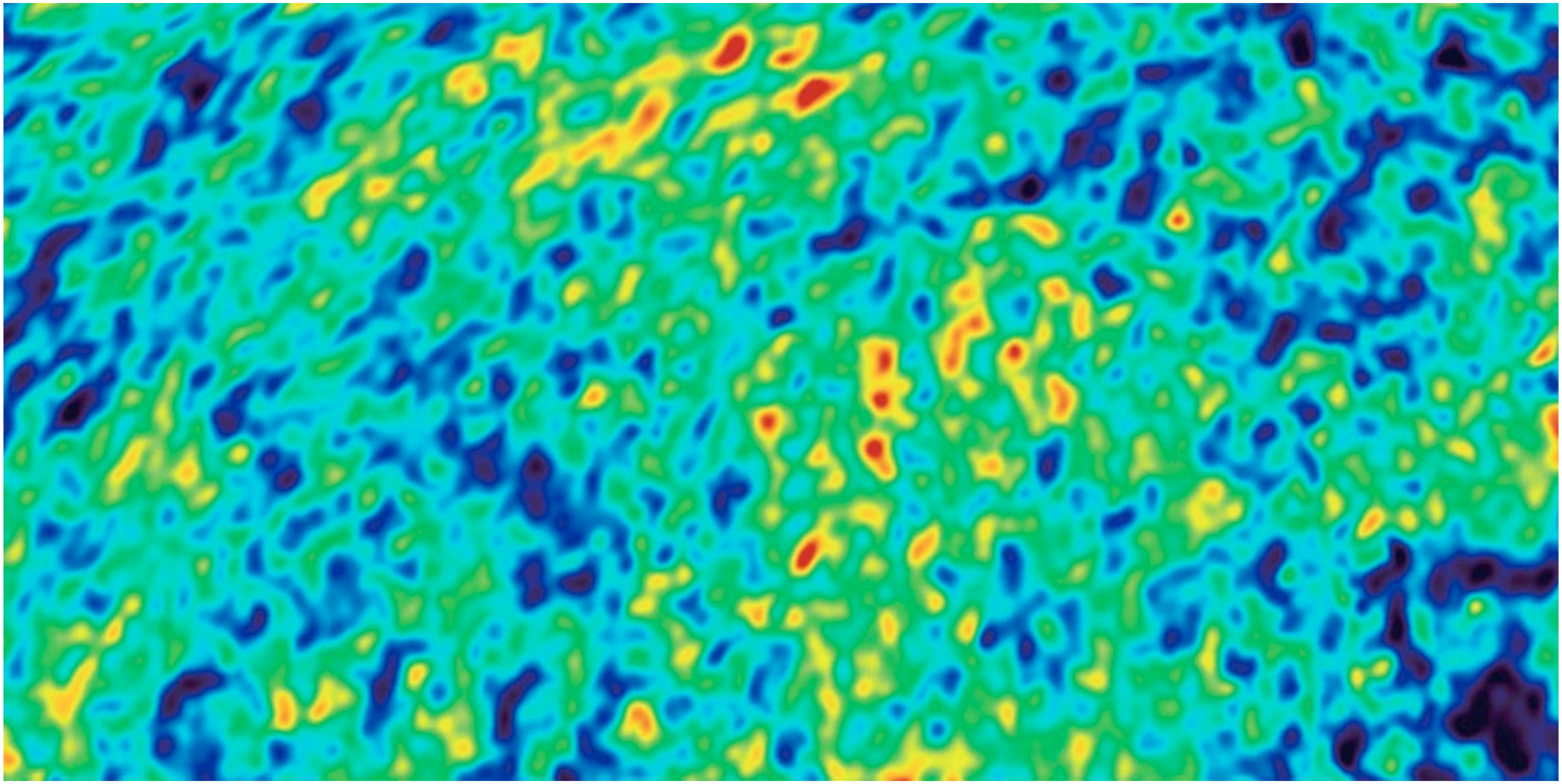
Remove the dipole and enhance the contrast to 10^5 ➡





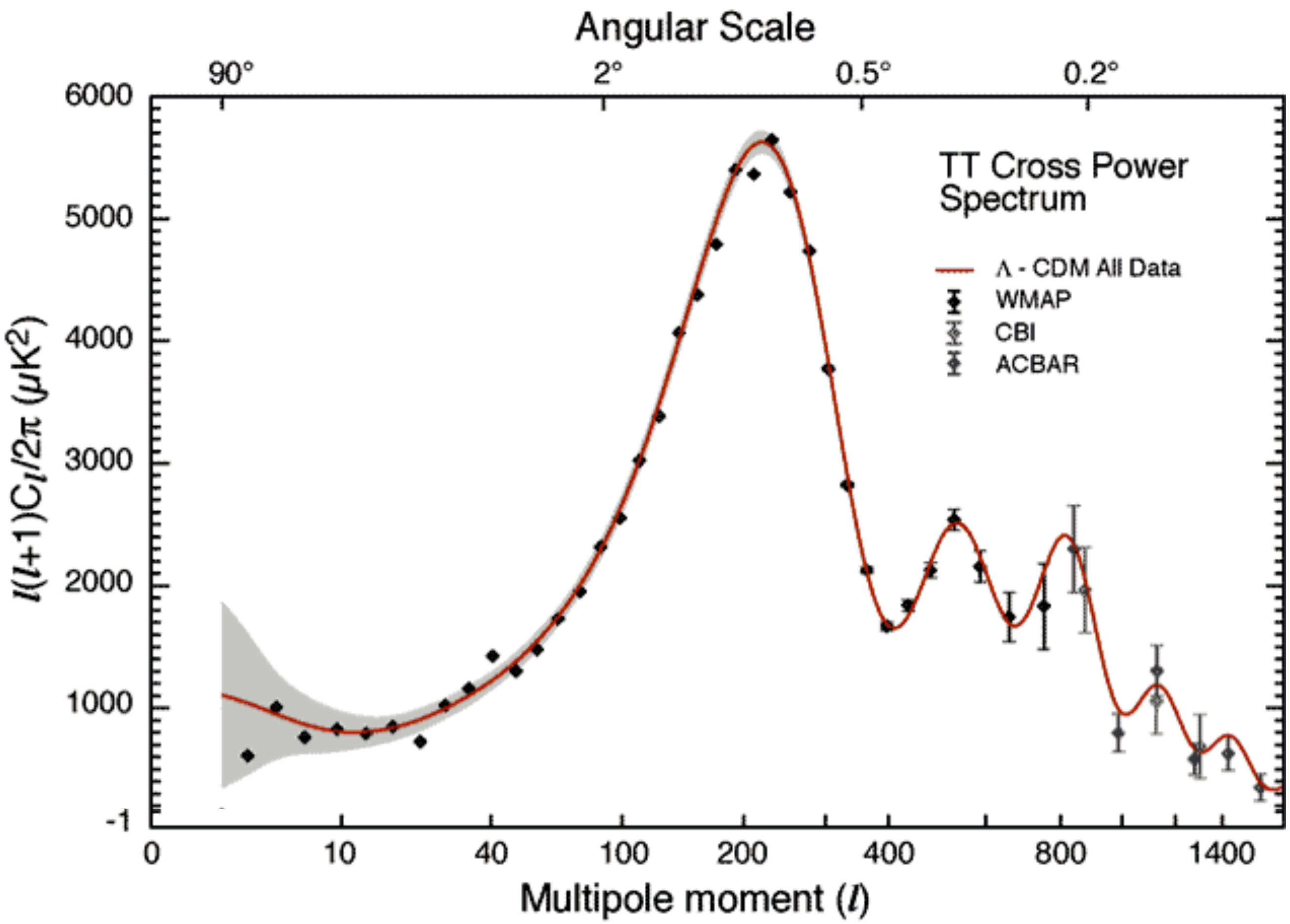
⬅ Remove the Galaxy, the contrast is 10^5 and see the primordial density fluctuations

A characteristic Fluctuation Scale Exists of ~ 1 degree



This corresponds to the size of the particle horizon at the decoupling, and thus to the longest sound wavelength which can be present

The results look like this:



WMAP, angular power spectrum, Bennett et al. 2003

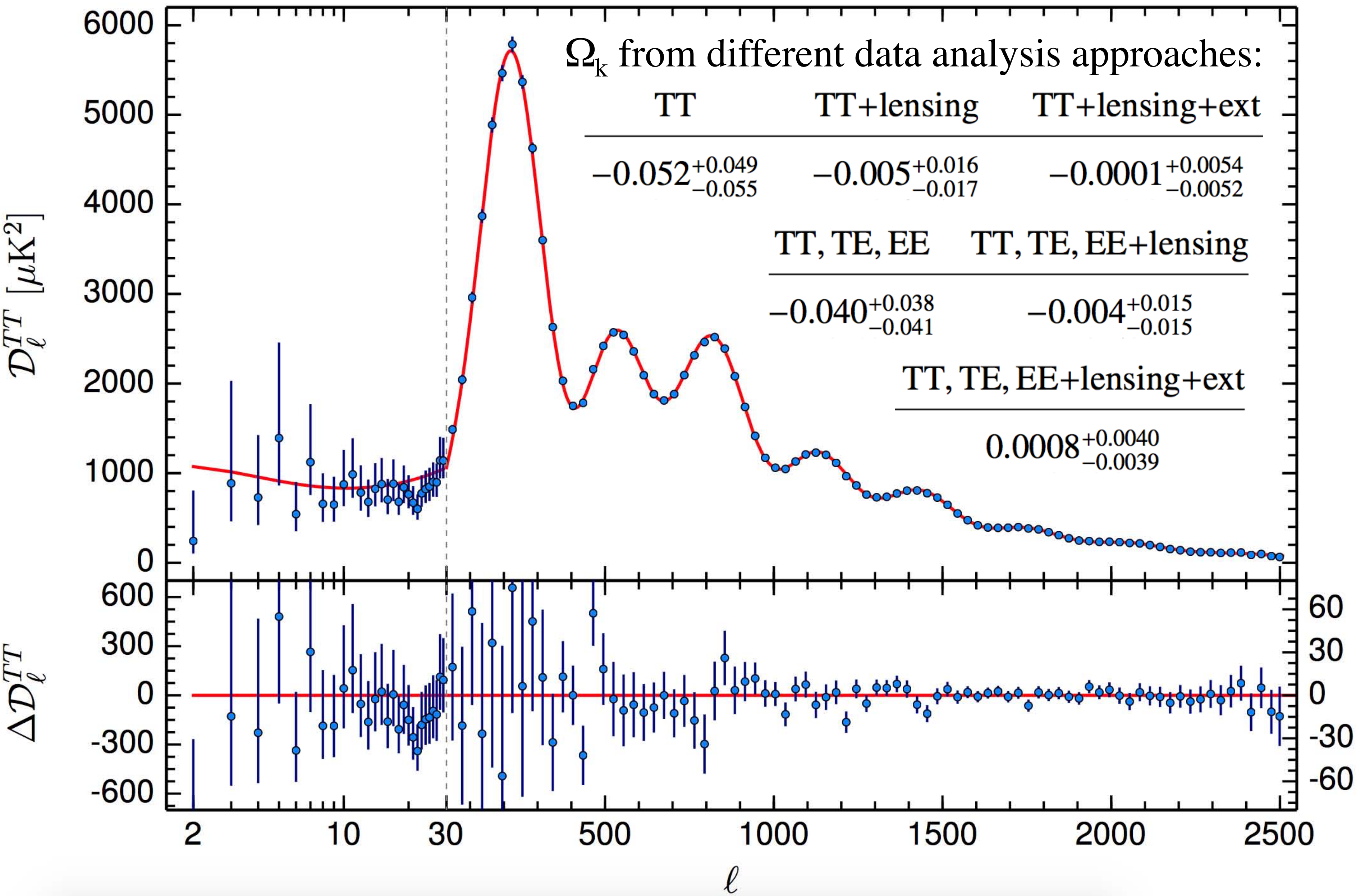
Observed position of the first peak is at:

$$l = 220 \Rightarrow \Omega_{total} = 1.00 \pm 0.01$$

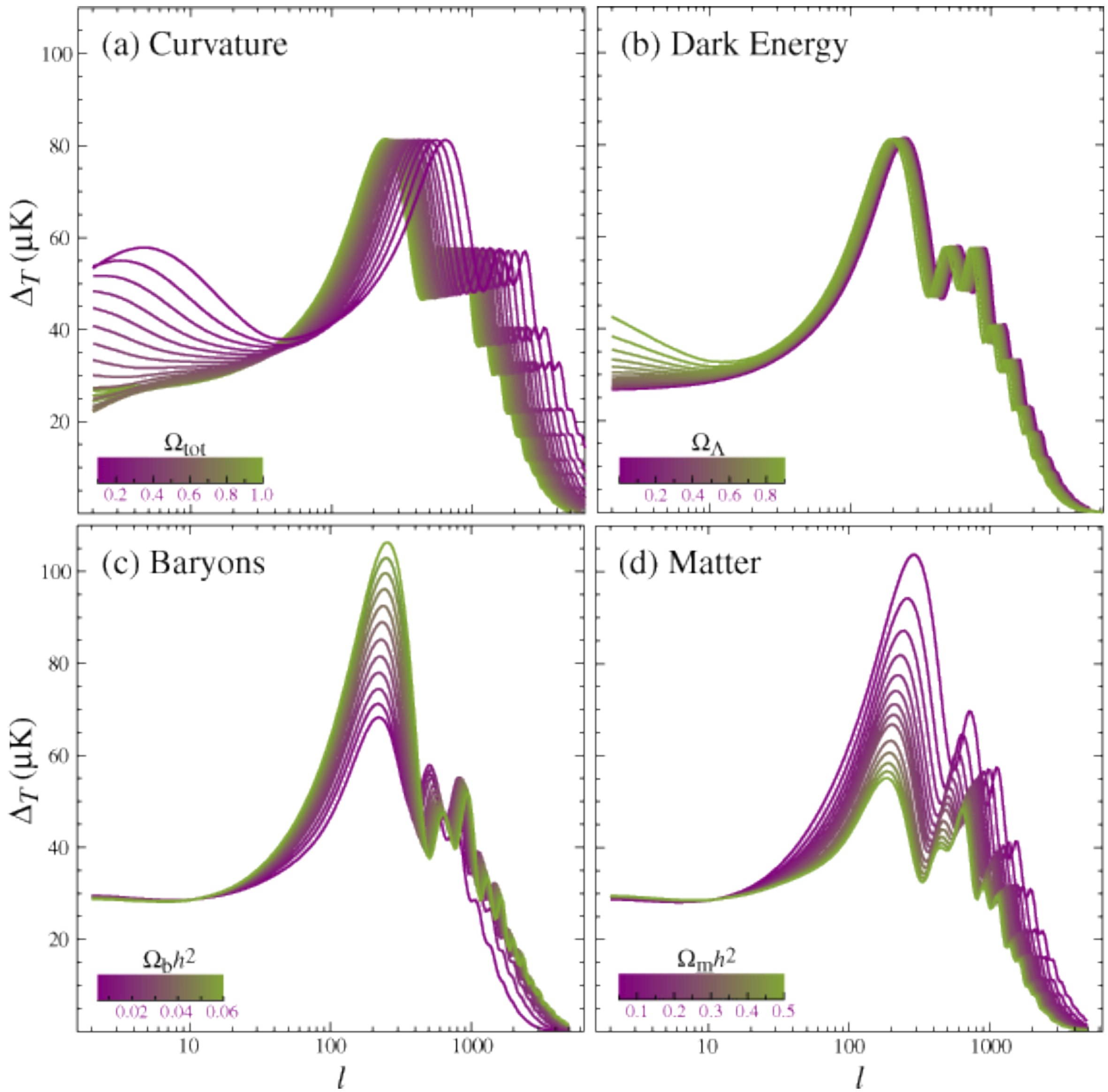
i.e., **the Universe is flat (or very close to being flat)**

Planck Results (2015)

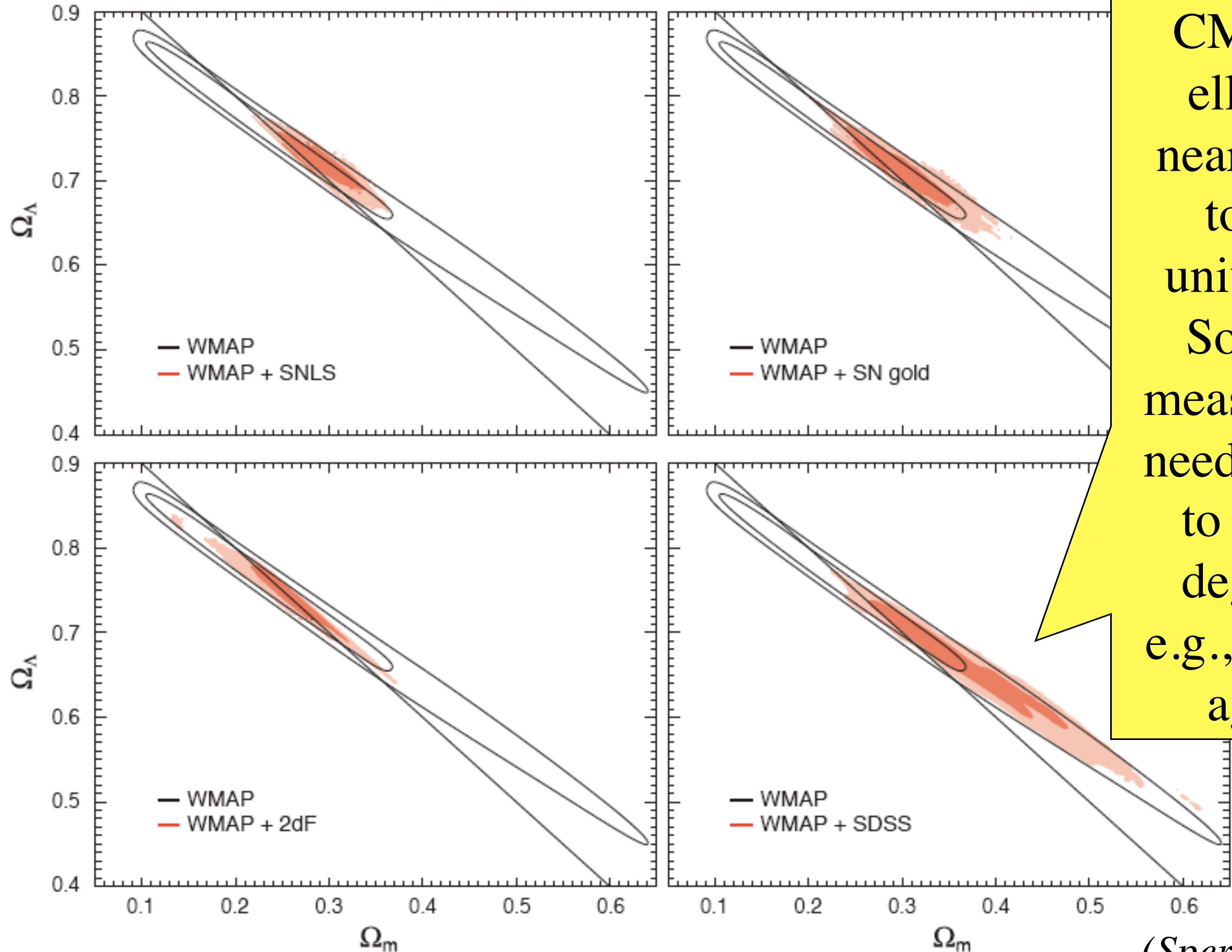
A&A 594, A13 (2016)



Positions and amplitudes of peaks depend on a variety of cosmological parameters in a complex fashion



CMBR Parameter Degeneracy

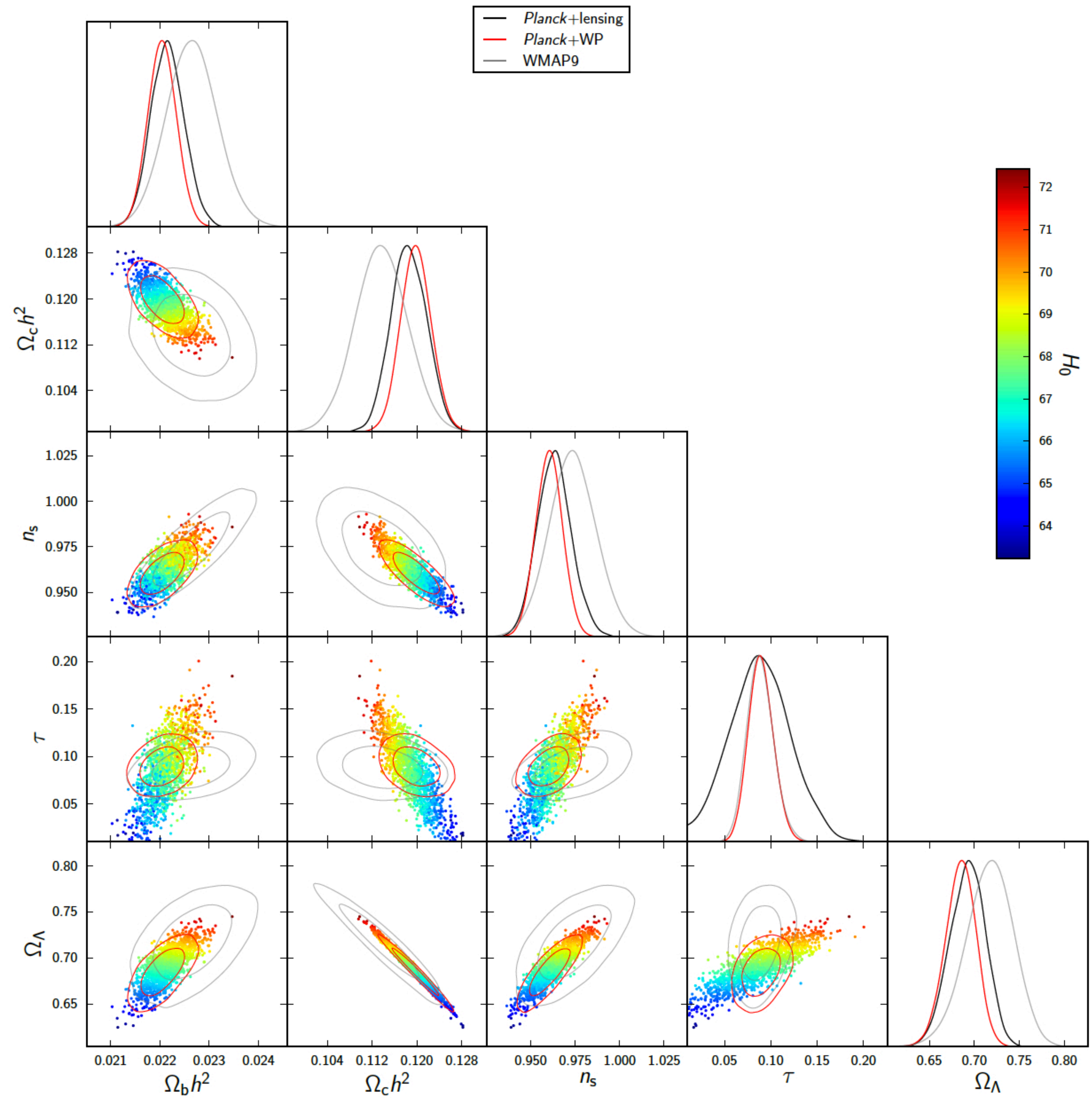


CMBR error ellipses are nearly parallel to the flat universe line. Some other measurement is needed in order to break the degeneracy, e.g., SNe, LSS, ages, etc.

(Spergel et al. 2006)

Estimating Cosmological Parameters

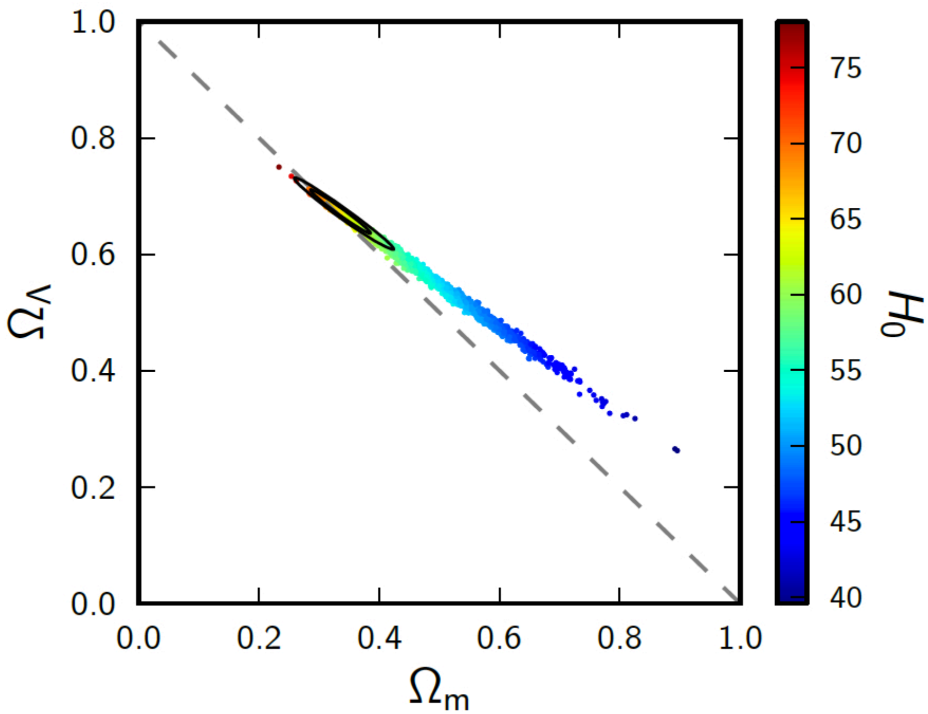
- Many observables depend on complicated combinations of individual cosmological parameters; this is especially true for the analysis of CMB experiments
- Thus, one really gets probability contours or distributions in a multi-dimensional parameter space, which can then be projected on any given parameter axis
- Generally this entails a very laborious and computationally intensive parameter estimation
- It helps if one can declare some of the parameters to be fixed *a priori*, on the basis of our knowledge or prejudices, e.g., "We'll assume that the univese is flat", or "we'll assume the value of H_0 from the HST Key Project", etc.

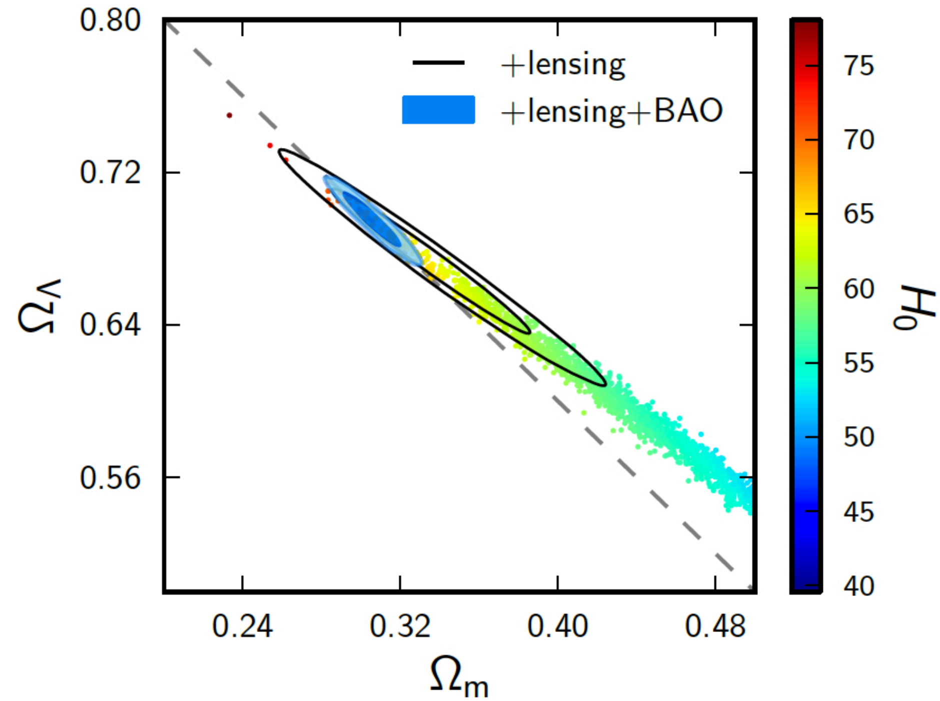


Examples of probability distributions of the various cosmological parameters, from a joint analysis of Planck and other data

Some *Planck* Results (2013)

Matter density and vacuum energy (cosmological constant), for different values of the H_0





Best fit:

$$\Omega_m = 0.317 \pm 0.020$$

$$\Omega_\Lambda = 0.683 \pm 0.020$$

Some *Planck* Results (2013)

Curvature Ω_k and the EOS parameter w

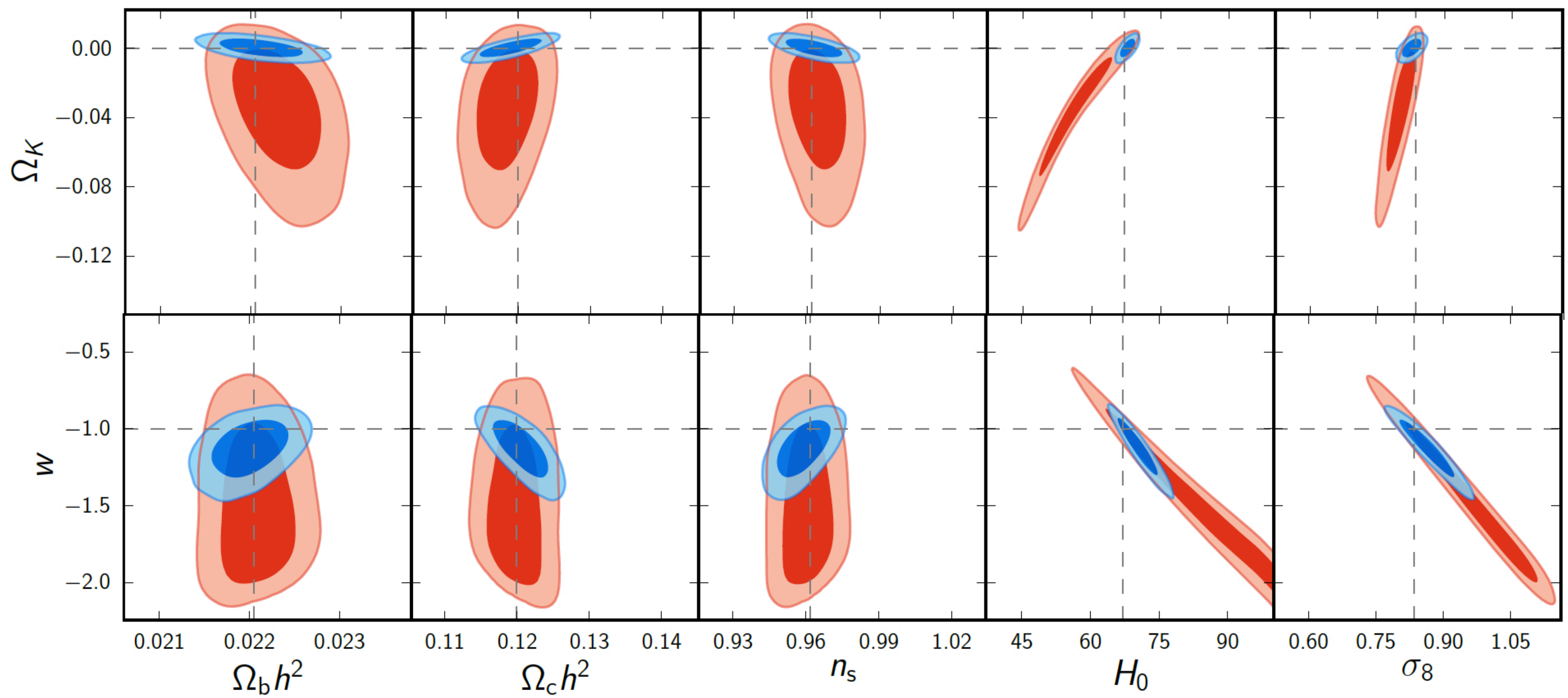
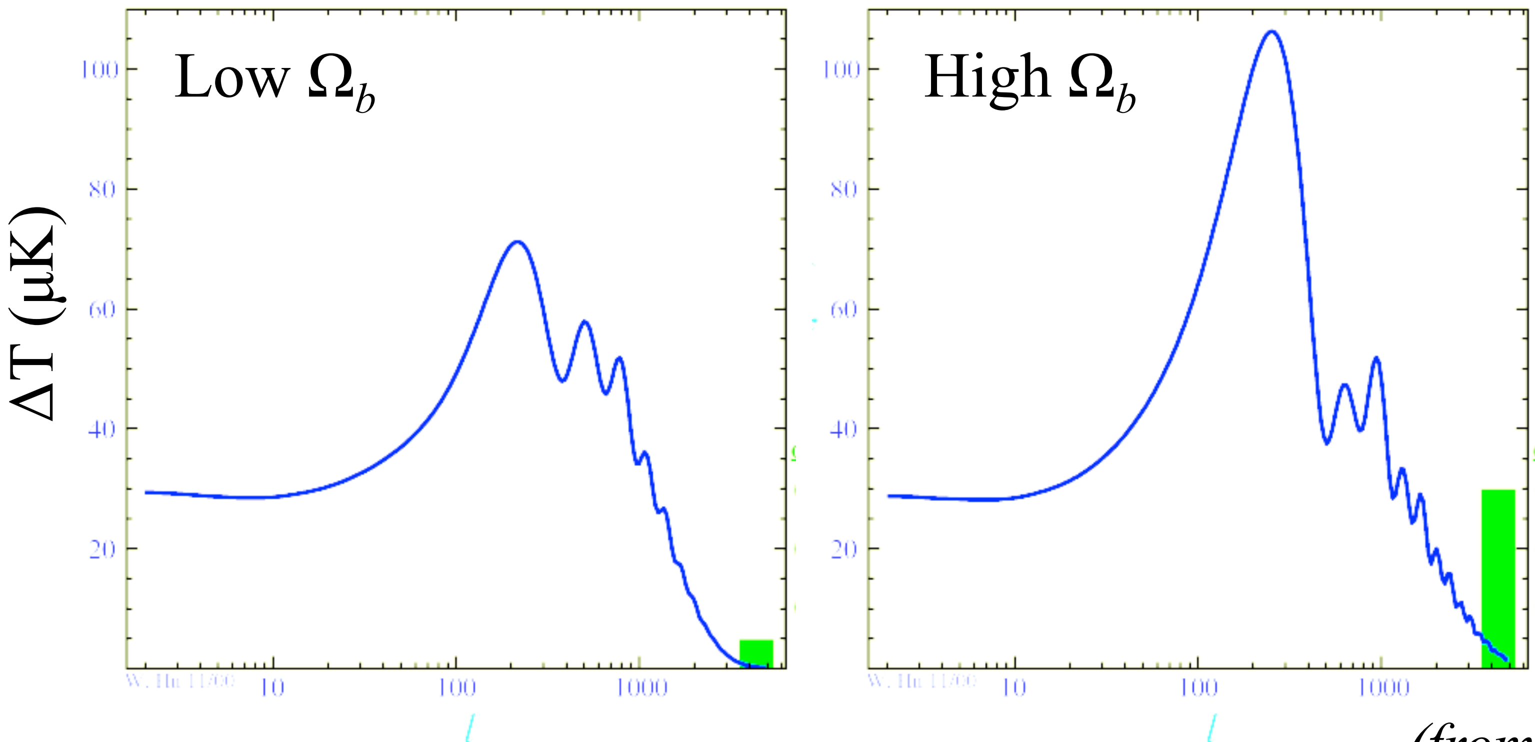


Fig. 21. 68% and 95% confidence regions on one-parameter extensions of the base ΛCDM model for *Planck*+WP (red) and *Planck*+WP+BAO (blue). Horizontal dashed lines correspond to the fixed base model parameter value, and vertical dashed lines show the mean posterior value in the base model for *Planck*+WP.

Baryon Content of the Universe



(from W. Hu)

Increasing the fraction of baryons:

- Increases the amplitude of the Doppler peaks
- Changes the *relative* strength of the peaks - odd peaks become stronger relative to the even peaks (compressions/rarefactions)

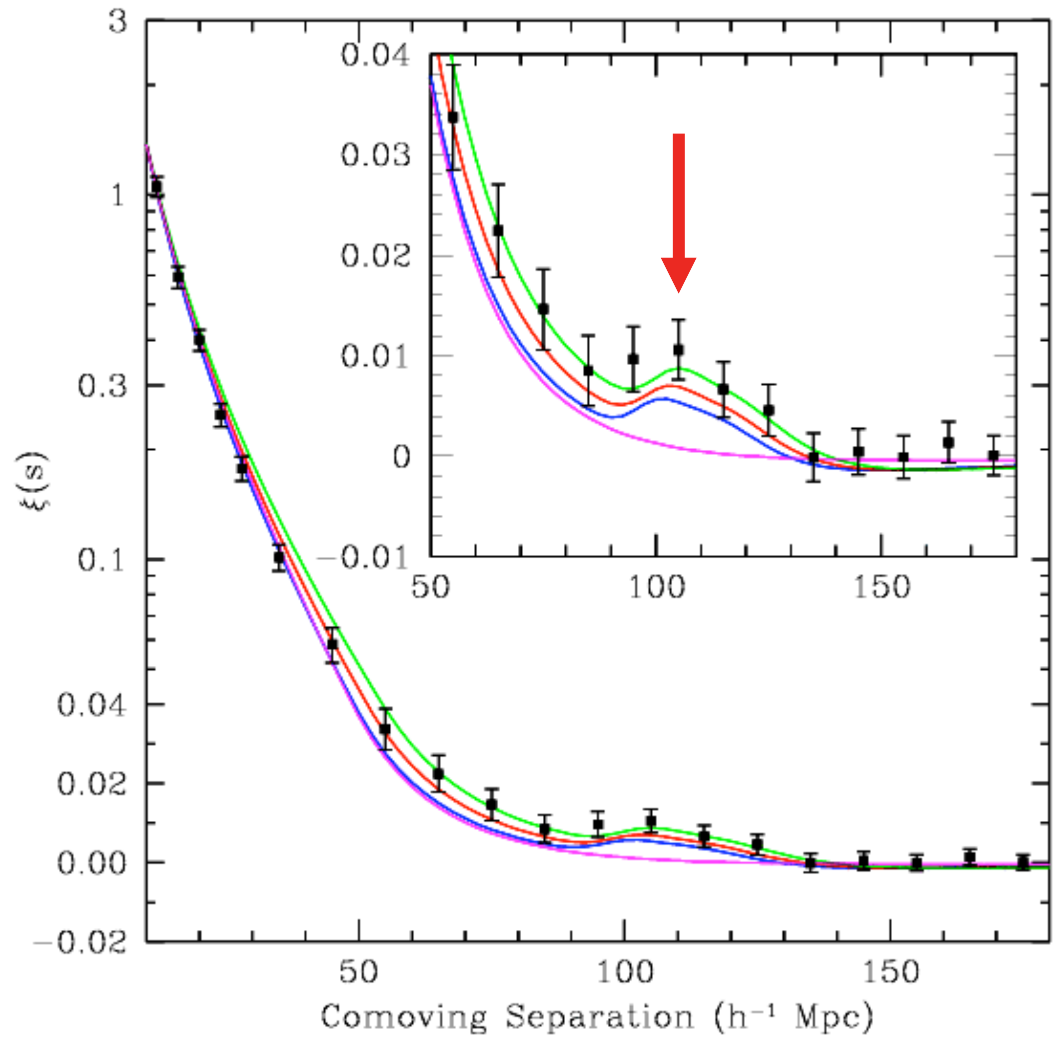
Planck results: $\Omega_b h^2 = 0.022068 \pm 0.00033$

Planck results, 2015

Parameter	TT+lowP 68% limits	TT+lowP+lensing 68% limits	TT+lowP+lensing+ext 68% limits	TT,TE,EE+lowP 68% limits	TT,TE,EE+lowP+lensing 68% limits	TT,TE,EE+lowP+lensing+ext 68% limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	0.1413 ± 0.0011	0.1427 ± 0.0014	0.1422 ± 0.0013	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023

Baryon Acoustic Oscillations (BAO)

Eisenstein et al. 2005 (using SDSS red galaxies); also seen by the 2dF redshift survey



The 1st Doppler peak seen in the CMBR imprints a preferred scale for clustering of galaxies.

Detection of this feature in galaxy clustering at $z \sim 0.3$ gives us another instance of a "standard ruler" for an angular diameter test, at redshifts $z < 1100$

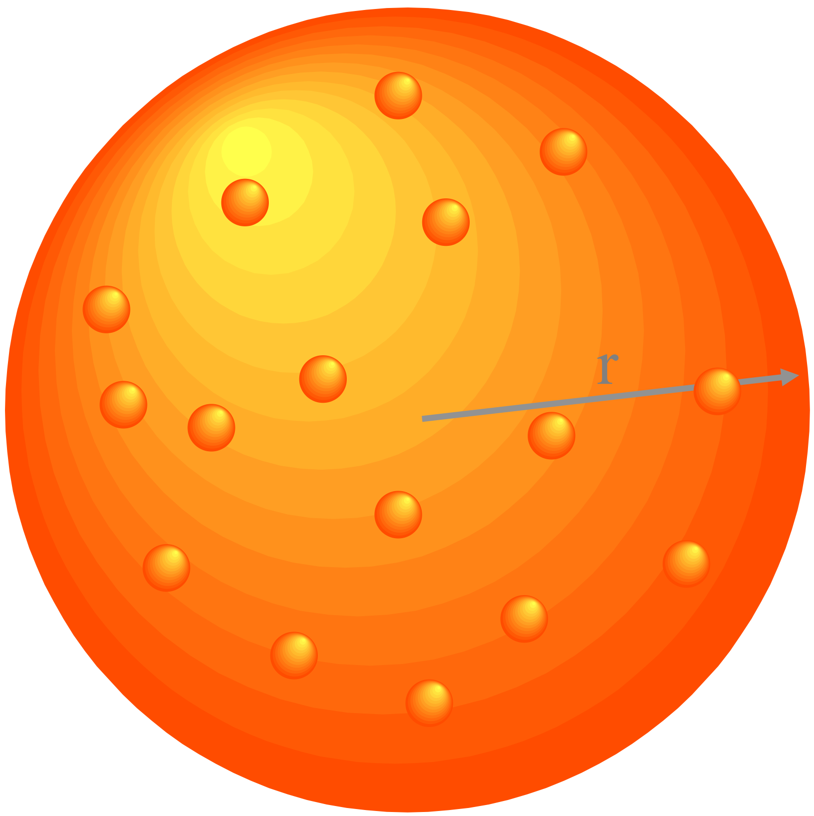
Future redshift surveys can do much better yet

The Number Counts

- Essentially a volume vs. redshift test in disguise; use luminosity distance as a proxy for redshifts
- If one can measure lots of reshifts (expensive!), one could also do a more direct test of source counts per unit comoving volume, as a $f(z)$
- Usually assume that the comoving number density of sources being counted is non-evolving (aha!)
- In radio astronomy, done as a source counts as a function of limiting flux; in optical-IR astronomy, as galaxy counts as a *f*(magnitude)
- Nowadays, the evolution effect, flux limits, etc., are included in modeling predicted counts, which are then compared with the observations

Euclidean Number Counts

Assume a class of objects with luminosities L, which down to some limiting flux f are visible out to a distance r. Then, the observed number N is:



$$N \propto V \qquad V \propto r^3 \qquad \Rightarrow N \propto r^3$$

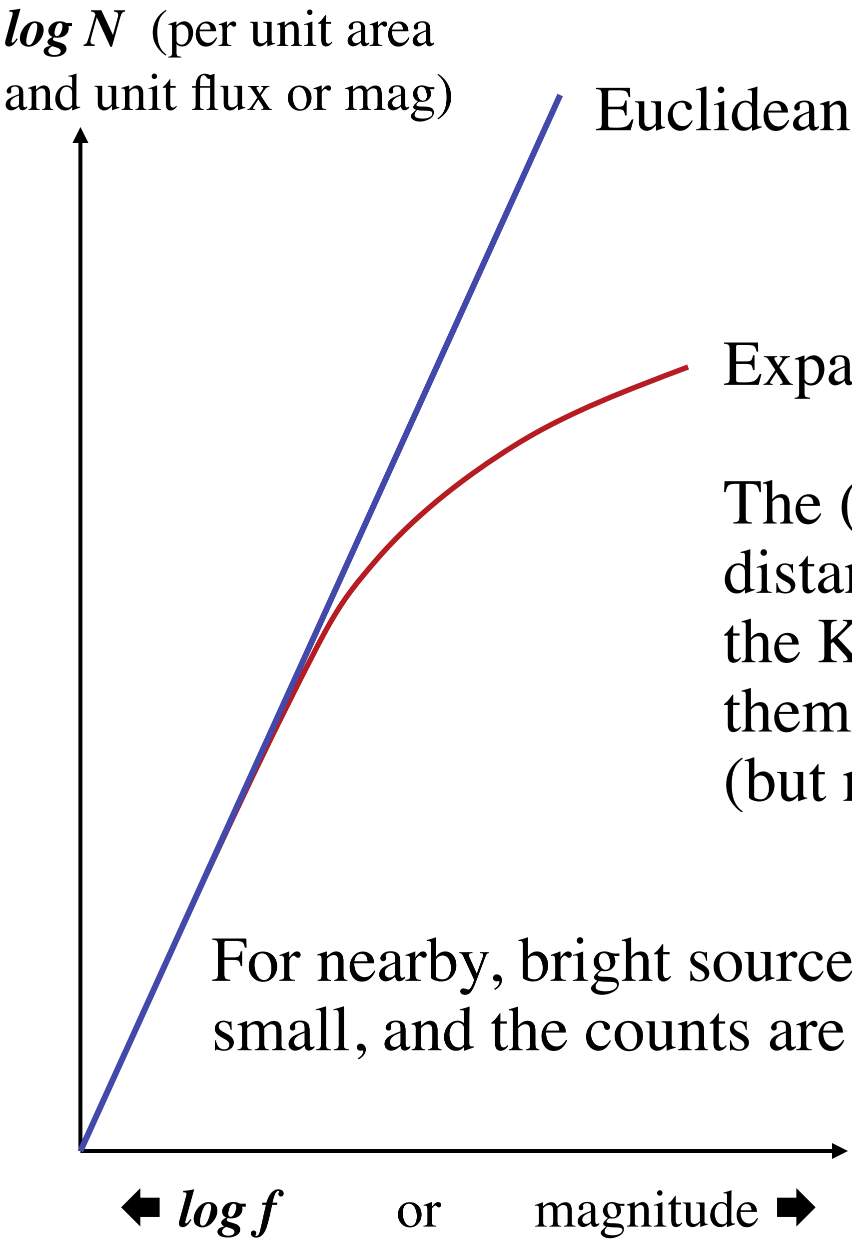
Since the flux f follows the inverse square law,

$$f \propto \frac{1}{r^2}$$

$$r^3 \propto f^{-3/2}$$

Thus we have: $\boxed{N \propto f^{-3/2}}$

Source Counts: The Effect of Expansion



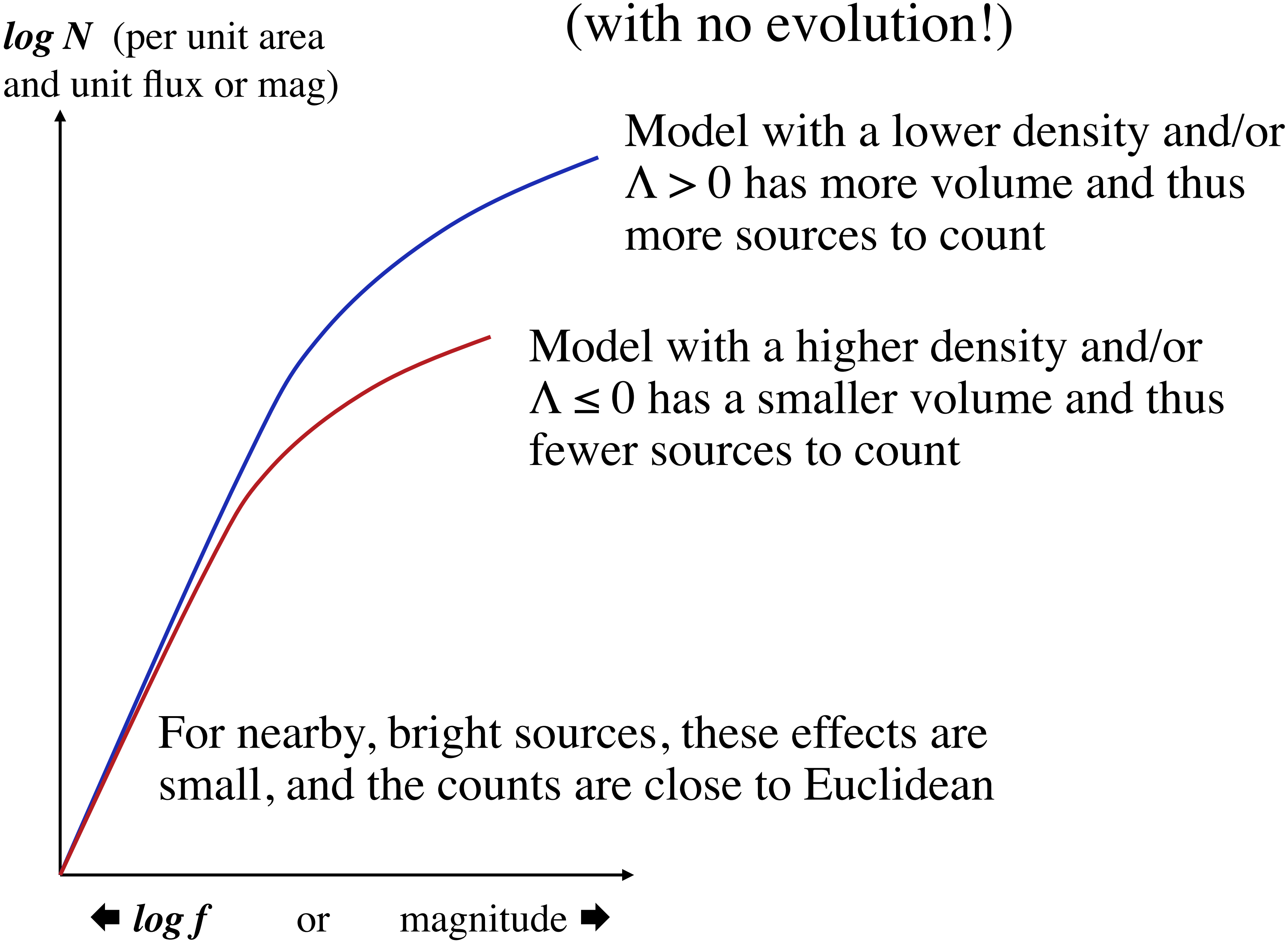
Expanding universe:

The $(1+z)^2$ factor in D_L makes more distant sources fainter, and the K-correction also tends to make them dimmer (but not always - e.g., in sub-mm)

For nearby, bright sources, these effects are small, and the counts are close to Euclidean

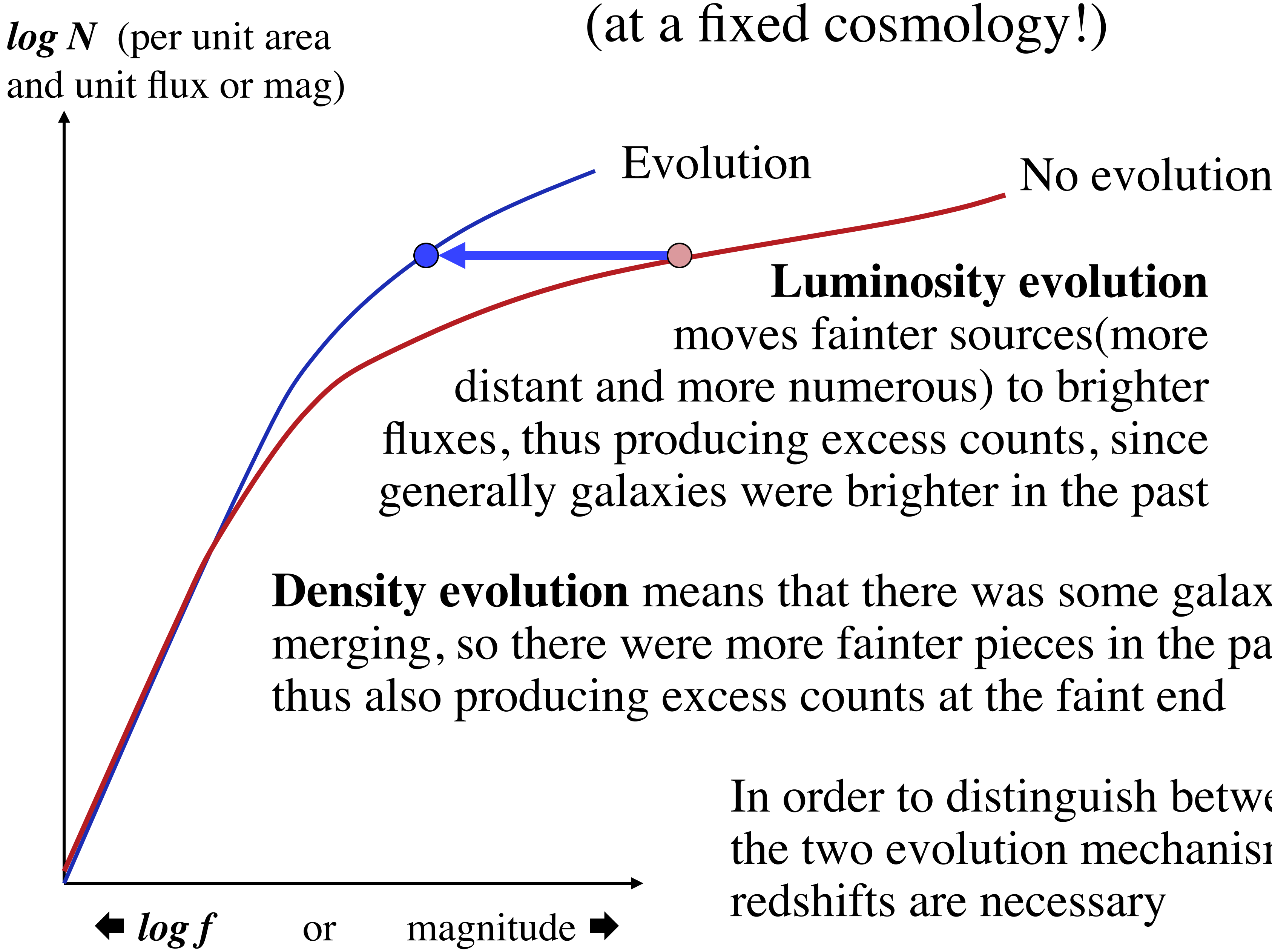
Source Counts: The Effect of Cosmology

(with no evolution!)



Source Counts: The Effect of Evolution

(at a fixed cosmology!)



Luminosity evolution moves fainter sources(more distant and more numerous) to brighter fluxes, thus producing excess counts, since generally galaxies were brighter in the past

Density evolution means that there was some galaxy merging, so there were more fainter pieces in the past, thus also producing excess counts at the faint end

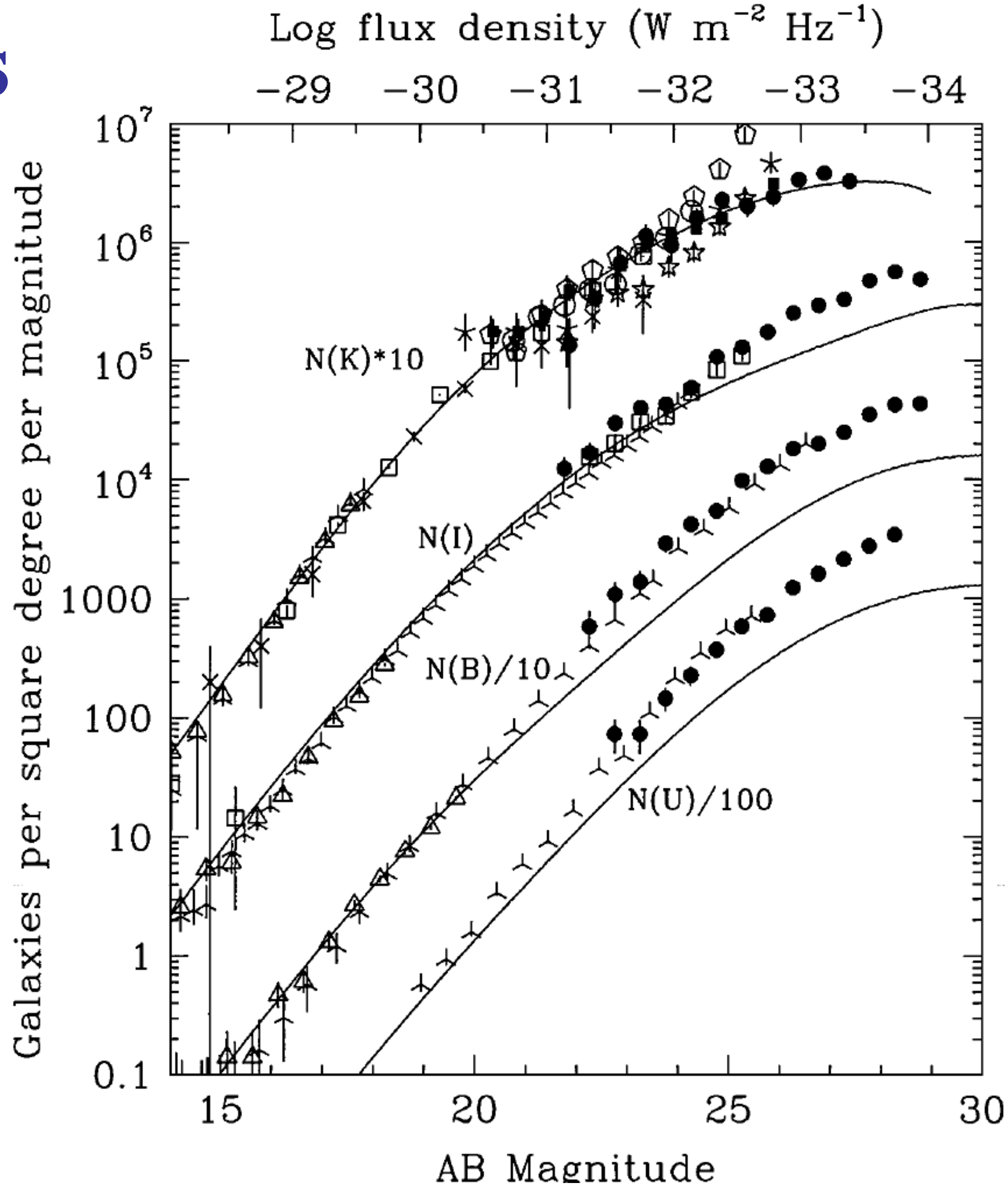
In order to distinguish between the two evolution mechanisms, redshifts are necessary

Galaxy Counts in Practice

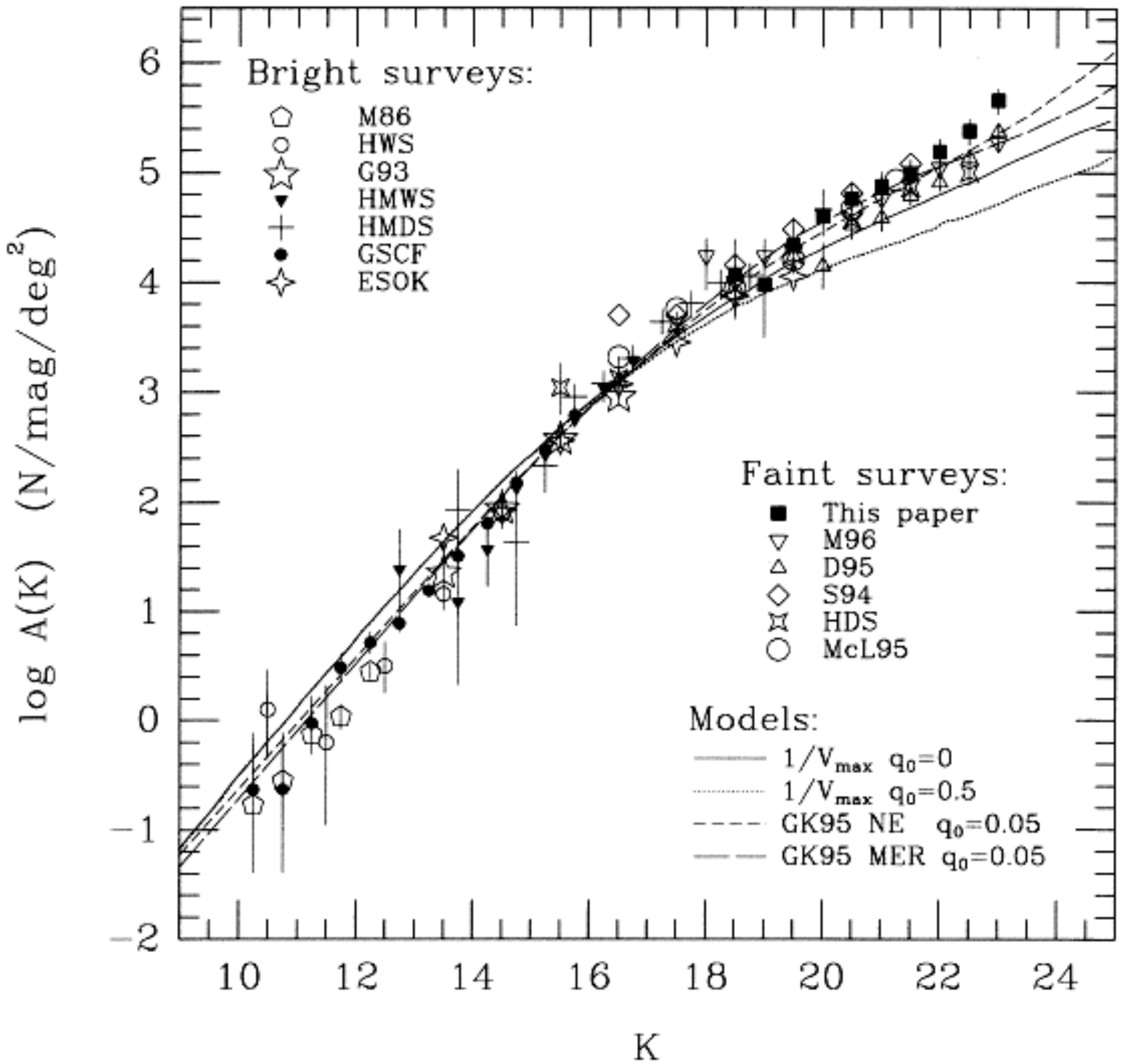
The deepest galaxy counts to date come from HST deep and ultra-deep observations, reaching down to ~ 29th mag

All show excess over the no-evolution models, and more in the bluer bands

The extrapolated total count is ~ 10^{11} galaxies over the entire sky



Galaxy Counts in Practice

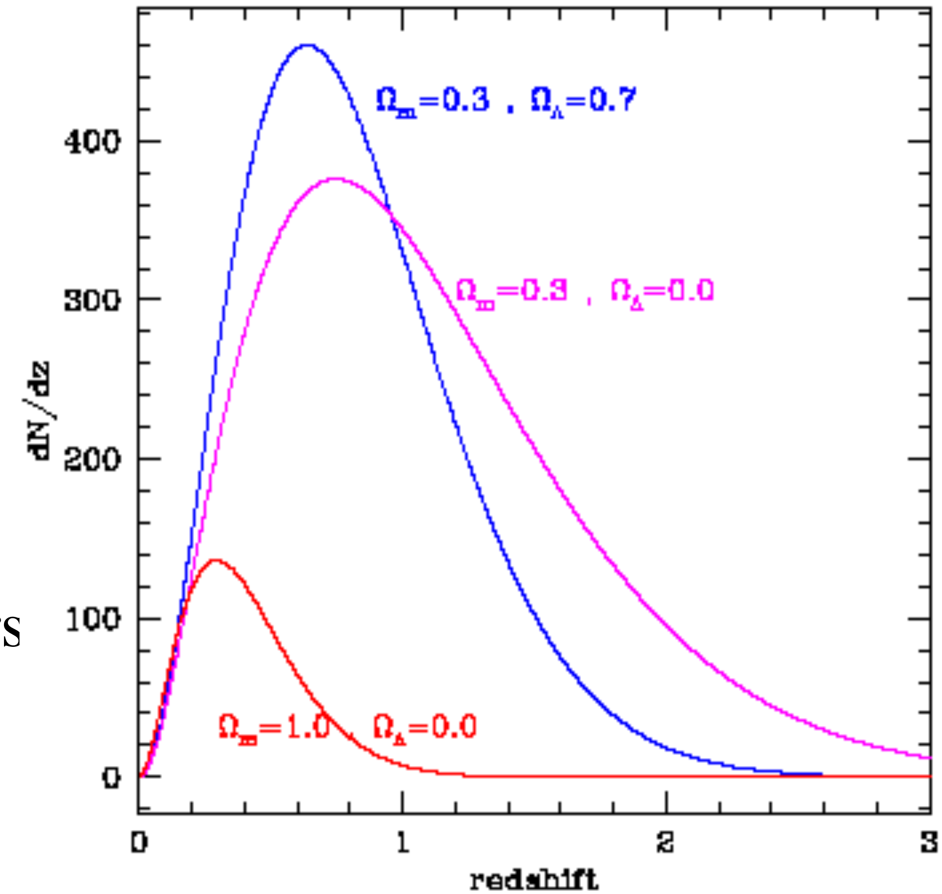


These effects are less prominent, but still present in the near-IR bands, where the effects of unobscured star formation should be less strong, as the light is dominated by the older, slowly evolving red giants

Abundance of Rich Galaxy Clusters

- Given the number density of nearby clusters, we can calculate how many distant clusters we expect to see
- In a high density universe, clusters are just forming now, and we don't expect to find any distant ones
- In a low density universe, clusters began forming long ago, and we expect to find many distant ones

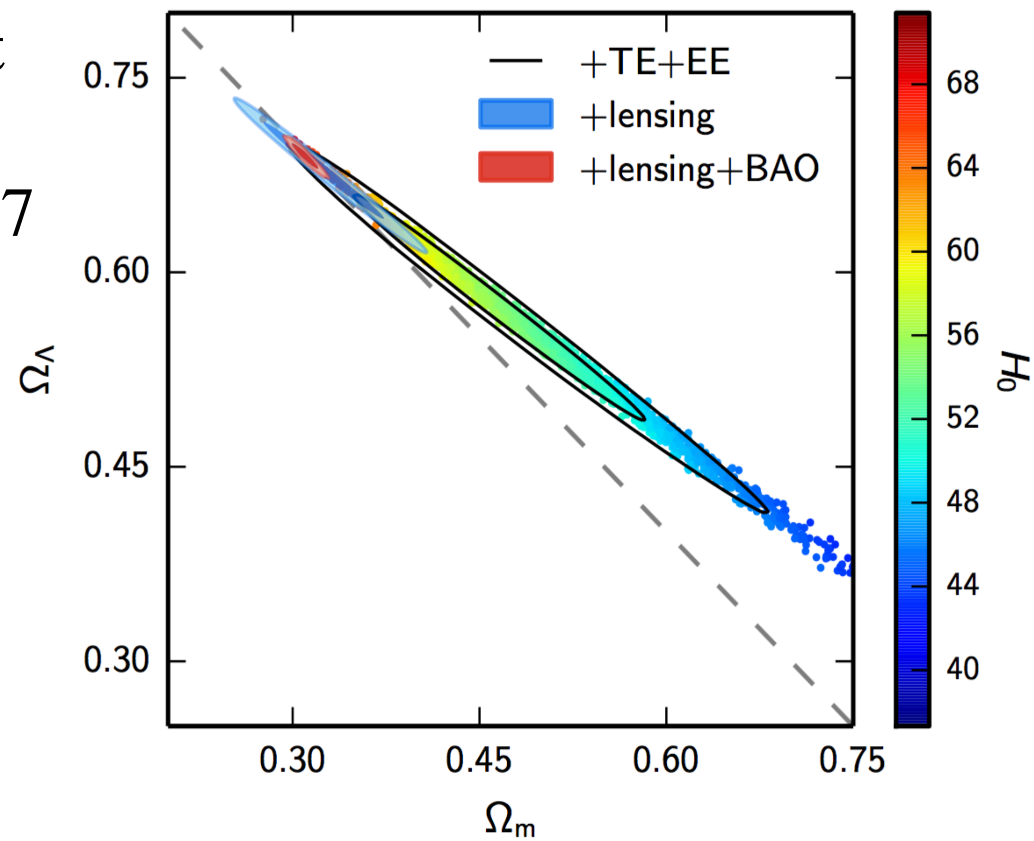
Evolution of Cluster Abundances



- Evolution of cluster abundances:
  - Structures grow more slowly in a low density universe, so we expect to see less evolution when we probe to large distances
  - Expected number in survey grows because volume probed within a particular spot on the sky increases rapidly with distance

The Dark Energy

- The **dominant component** of the observed matter/ energy density: $\Omega_{0,DE} \approx 0.7$
- Causes the accelerated expansion of the universe
- May affect the growth of density perturbations
- Effective only at cosmological distances



- Its physical nature is as yet ***unknown***; this may be the biggest outstanding problem in physics today
- ***Cosmological constant*** is just one special case; a more general possibility is called ***quintessence***

Cosmological Constant or Quintessence?

- **Cosmological constant:** energy density constant in time and spatially uniform
  - Corresponds to the energy density of the physical vacuum
  - A coincidence problem: why is $\Omega_\Lambda \sim \Omega_m$ just now?
- **Quintessence:** time dependent and possibly spatially inhomogeneous; e.g. scalar field rolling down a potential
- Both can be described in the equation of state formalism:

$$P = w\,\rho$$

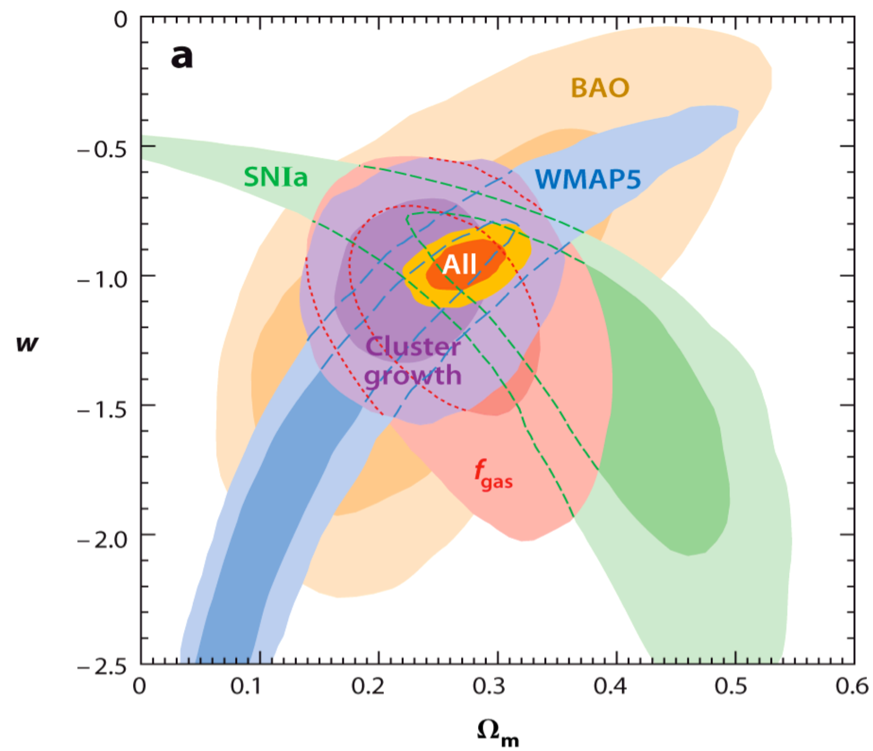
$$\rho \sim R^{-3(w+1)}$$

Cosmological constant: $w = const. = -1,\ \rho = const.$

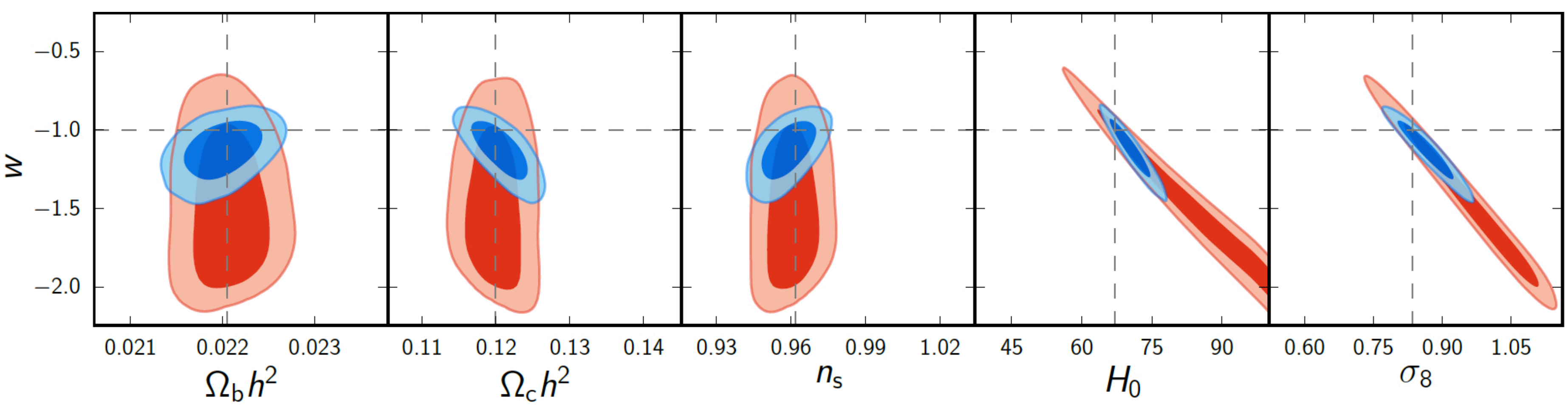
Quintessence: w can have other values and change in time

Observational Constraints on w

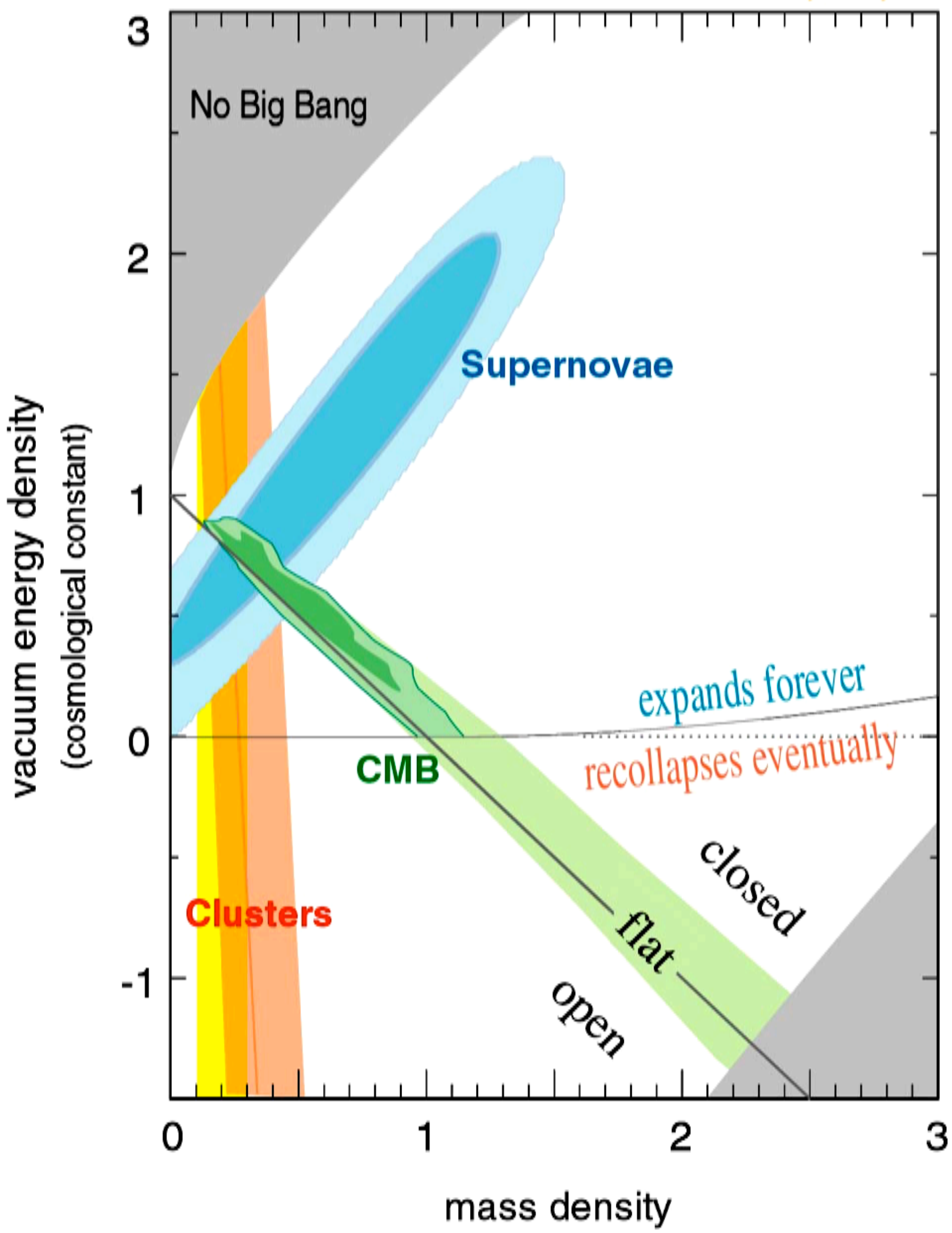
Strongly favor values of $w \sim -1$, i.e., cosmological constant. Some models can be excluded, but there is still room for $\rho_{vac} \neq const.$ models



Planck + WMAP (red) + BAO (blue)



The Cosmic Concordance



Supernovae alone

⇒ **Accelerating expansion**

⇒ $\Lambda > 0$

CMB alone

⇒ **Flat universe**

⇒ $\Lambda > 0$

Any two of SN, CMB, LSS

⇒ **Dark energy ~70%**

Also in agreement with the age estimates (globular clusters, nucleocosmochronology, white dwarfs)

Today's Best Guess Universe

Age:

$t_0 = 13.82 \pm 0.05$ Gyr

Best fit CMB model - consistent with ages of oldest stars

Hubble constant:

$H_0 = 69$ km s^{-1} Mpc^{-1}

CMB + HST Key Project to measure Cepheid distances

Density of ordinary matter:

$\Omega_{baryon} = 0.045$

CMB + comparison of nucleosynthesis with Lyman-a forest deuterium measurement

Density of all forms of matter:

$\Omega_{matter} = 0.31$

Cluster dark matter estimate
CMB power spectrum

Cosmological constant:

$\Omega_{\Lambda} = 0.69$

Supernova data, CMB evidence for a flat universe plus a low matter density

The Component Densities

at z ~ 0, in critical density units, assuming $h \approx 0.7$

Total matter/energy density: $\Omega_{0,tot} \approx 1.00$ From CMB, and consistent with SNe, LSS

Matter density: $\Omega_{0,m} \approx 0.31$ From local dynamics and LSS, and consistent with SNe, CMB

Baryon density: $\Omega_{0,b} \approx 0.045$ From cosmic nucleosynthesis, and independently from CMB

Luminous baryon density: $\Omega_{0,lum} \approx 0.005$ From the census of luminous matter (stars, gas)

Since: $\Omega_{0,tot} > \Omega_{0,m} > \Omega_{0,b} > \Omega_{0,lum}$

There is baryonic dark matter

There is non-baryonic dark matter

There is dark energy

Cosmological Tests Summary

- Tests of the global geometry and dynamics: correlate redshifts (~ scale factors) with some relative measure of distance (~ look back time); could use:
  - "standard candles" (for luminosity distances; e.g., SNe)
  - "standard rulers" (for angular diameter dist's; e.g., CMBR fluc's)
  - "standard abundances" (for volume-redshift test; e.g., rich clusters)
- Get matter density from local dynamics or LSS
- Combine with constraints from the H_0, ages
- There are often parameter couplings and degeneracies, especially with the CMB alone
- Multiple approaches provide cross-checks, break degeneracies
- Concordance cosmology is now fairly well established

Appendix:
Supplementary Slides

This is Not Exactly New …

Nature Vol. 257 October 9 1975

An accelerating Universe

James E. Gunn*
Hale Observatories, California Institute of Technology, Carnegie Institution of Washington, Pasadena, California 91125

Beatrice M. Tinsley*†

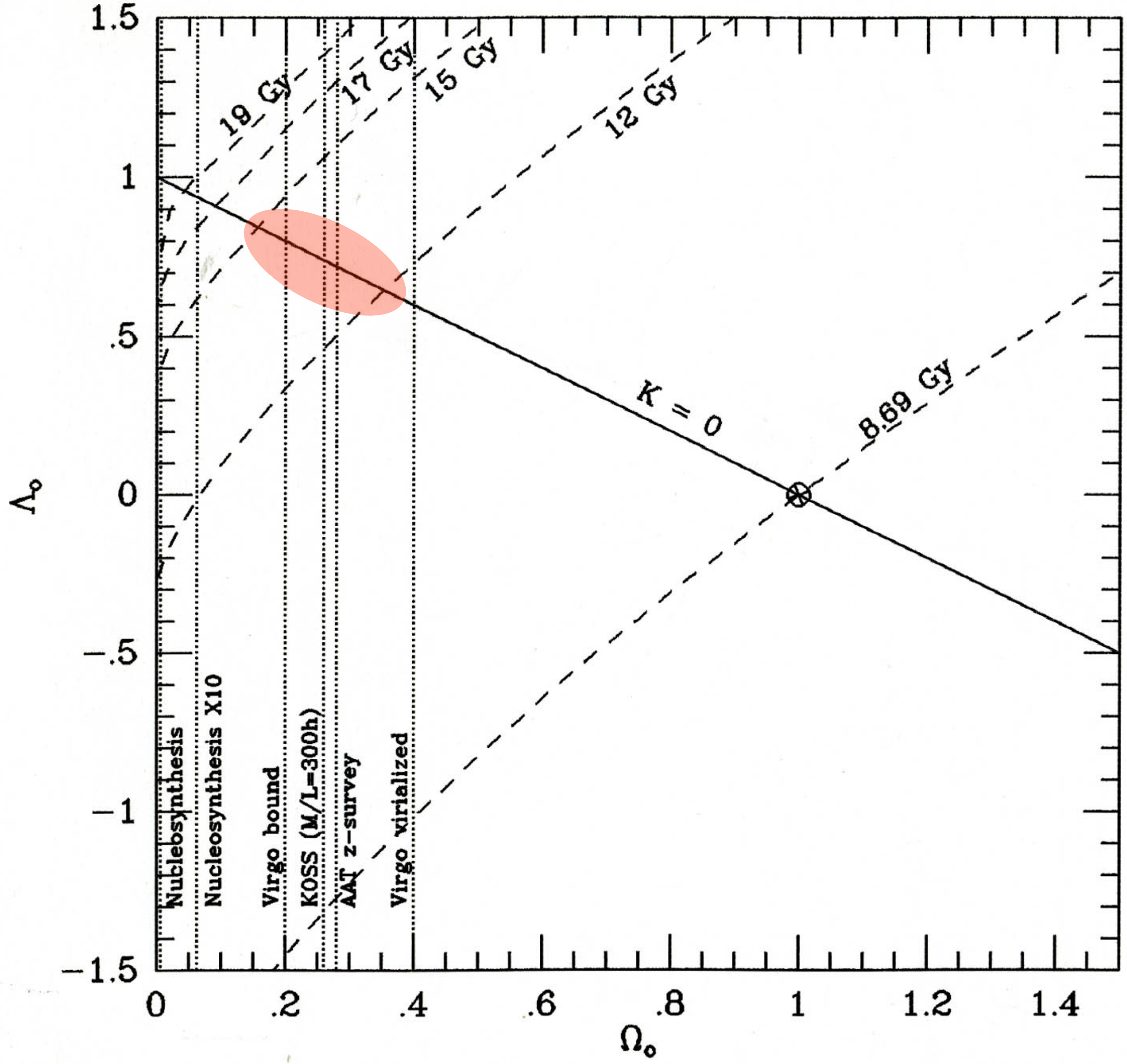
B. Tinsley, *Nature* Vol. 273 18 May 1978

Accelerating Universe revisited

They were driven to this conclusion by the combination of data on the Hubble constant, ages of globular clusters, Hubble diagram, and density measurements … just like today

For the next 20 years, cosmological constant was invoked mainly as a means to solve the apparent conflict between the ages of globular clusters and chemical elements, and the age of the universe derived from the H_0 and density parameter

Concordance Cosmology, Circa 1985



Globular cluster ages, dynamical measurements of matter density, and H_0, all consistent with the newly fashionable, flat (k=0) inflationary universe

(Djorgovski 1985, unpublished)