Ay 127 – Winter 2017 – Final Exam

Distributed on Sun. March 12, due by 5 pm on Friday, March 17

(after which you can go to your St. Pat's party)

The Rules: Closed book, closed notes, no web access, *closed everything* (except your minds) … but you can use tables of physical and astronomical constants or units (attached). You can use a pocket calculator, but not if it has display of formulas and such. You *cannot* discuss the problems with anyone until after everyone turns in their exams.

You have a maximum of 5 hours (it should take less) from the moment you start until the moment you finish. Please mark your exam with the start and stop times. You have to turn it in *in person*, either to the Prof or the TA; do not just leave it in a mailbox.

Please write legibly – it is in your own best interest. Be careful about the units, and double-check your numbers. It is always a good idea to write down the formulas first, *then* plug in the numbers. Good luck!

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Note: Always state your assumptions, e.g., when making any approximations or estimates. If you don't know some number that you need and cannot compute it easily, make the best estimate you can, state why, and move on. We'll give you a credit for the physical reasoning, not just the numerical results.

Problem 1 [25 points]

The observed local energy density of the cosmic far-infrared background (CIRB) is $\boldsymbol{u}_{\text{CIRB}} \approx 7\times10^{-15}$ erg cm^{-3}. Assume that the background is generated by a population of obscured starbursts with a mean luminosity $\langle L\rangle \approx 10^{12}\, L_\odot$, with all energy emerging in the mid/far IR, with a mean redshift $\langle z\rangle \approx 3$, each lasting on average $\Delta t \approx 3\times10^7$ yr.

a. How many such starbursts should there be in the entire observable universe, in order to account for the CIRB? [10 points]

b. Estimate the comoving number density of their progeny at $z \sim 0$, and compare it with the estimated comoving number density of normal galaxies today. [5 points]

c. Assuming the yield of nuclear reactions of 7 MeV/nucleon, how much helium was generated by these starbursts? How much metals, assuming the ratio of helium to metals production by mass $\Delta Y/\Delta Z = 5$? If each of these starbursts was in a galaxy with a baryonic mass $M \approx 10^{11}\, M_\odot$, what is the mean metallicity of the resulting stars? [10 points]

Problem 2 [30 points]

Suppose that all DM is in the form of black holes with a mass $M_\bullet \approx 10^6\, M_\odot$, distributed uniformly in the comoving volume.

a. What is their local number density per Mpc^3? What is the probability of one of them passing through the Milky Way halo now? [5 points]

b. Roughly how many of them are there on the sky? State your assumptions. [10 points]

c. Estimate how many quasars are on the sky. Make reasonable assumptions about the redshifts. Estimate how often with there be gravitational lensing events, with the black holes lensing the background quasars? Again, state your assumptions. [15 points]

Problem 3 [20 points]

The observed brightness of the cosmic X-ray background (CXRB) is $\nu I_\nu \approx 3\times10^{-10}$ W m^{-2} sr^{-1}.

a. Compute the corresponding volume energy density. Compare with the energy density of the CMB, assuming $T_{\text{CMB}} = 2.735$°K. [4 points]

b. It is estimated that on average there is a SMBH with $M_{\text{bh}} \sim 10^7\, M_\odot$ per average ($L \sim L_*$) galaxy. Assuming that the efficiency of accretion in converting the rest mass into energy was ~ 10% (i.e., 90% ends in the SMBH, 10% is radiated away), and that the mean redshift of emission was $\langle z\rangle \sim 3$, compute the energy density today, generated by the making of these SMBHs. Compare it with the numbers for the CXRB and CMBR computed in (a). (Note: you will need to estimate the comoving density of L_* galaxies today.) [16 points]

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Problem 4 [25 points]

a. Derive the formula for a growth of a BH powered by an Eddington-limited accretion, with a radiative efficiency of 10%. [10 points]

b. What would be the value of the e-folding time (aka the Salpeter time)? [10 points]

c. If a SMBH starts growing by accretion when the universe was ~100 Myr old ($z \sim 30$), and reaches the mass of $10^9\ M_\odot$ when the universe was ~ 1 Gyr old ($z \sim 6$), what would the mass of the seed BH have to be? [5 points]

Problem 5 [15 points]

Consider a quasar at $z \approx 6$, powered by a SMBH with a mass $M_\bullet \approx 10^9\ M_\odot$ generated through an accretion with a radiative efficiency of 10%, and assume that ~20% of that energy was emitted in the form of Hydrogen ionizing photons. Assuming an IGM with a uniform density, what would be the radius of the resulting Strömgren sphere?

Problem 6 [10 points]

Assuming the values of $\Omega_m \approx 0.3$ and $\Omega_{CMB} \approx 9 \times 10^{-5}$ today, and that the age at the recombination ($z \approx 1100$) was ~ 380,000 yr,

a. Estimate the redshift z_{eq} and the age at the epoch of matter-radiation equality. [5 points]

b. Estimate the redshift and the age at the onset ($T \sim 10^{10}$ K) and the end ($T \sim 10^9$ K) of the cosmic nucleosynthesis. [5 points]

(Note: you can assume a sharp transition of the expansion law at z_{eq}).

Appendix C
Physical and Astronomical Constants and Unit Conversions

Table C.1 Physical constants.

Quantity	Symbol	Value	Units
Speed of light	c	$2.997\,924\,58 \times 10^{10}$	cm^{-1}
Gravitational constant	G	$6.672\,59(85) \times 10^{-8}$	$\text{dyn cm}^2\ \text{g}^{-2}$
Planck constant	h	$6.626\,075\,5(40) \times 10^{-27}$	erg s^{-1}
Boltzmann constant	k	$1.380\,658(12) \times 10^{-16}$	erg K^{-1}
Stefan–Boltzmann constant	σ	$5.670\,51(19) \times 10^{-5}$	$\text{erg cm}^{-2}\ \text{K}^{-4}\ \text{s}^{-1}$
Thomson cross-section	σ_T	$0.665\,246\,16 \times 10^{-24}$	cm^2
Electron charge	e	$4.803\,206\,8(15) \times 10^{-10}$	E.S.U.
Electron mass	m_e	$9.109\,389\,7(54) \times 10^{-28}$	g
Proton mass	m_p	$1.672\,623\,1(10) \times 10^{-24}$	g
Neutron mass	m_n	$1.674\,928\,6 \times 10^{-24}$	g
Atomic mass unit	m_u	$1.660\,540\,2 \times 10^{-24}$	g
Electron volt	eV	$1.602\,173\,3 \times 10^{-12}$	erg

A Panchromatic View of Galaxies, First Edition. Alessandro Boselli.

Table C.2 Astronomical constants.

Quantity	Symbol	Value	Units
Astronomical unit	AU	1.496×10^{13}	cm
Parsec	pc	3.086×10^{18}	cm
Solar mass	$M_{\odot}$	1.989×10^{33}	g
Solar radius	$R_{\odot}$	6.955×10^{10}	cm
Solar luminosity	$L_{\odot}$	3.845×10^{33}	$\mathrm{erg\,s^{-1}}$
Solar absolute bolometric magnitude	$M_{\mathrm{bol},\odot}$	4.72	mag
Solar absolute B magnitude	$M_{B,\odot}$	5.48	mag
Solar absolute V magnitude	$M_{V,\odot}$	4.83	mag
Solar absolute J magnitude	$M_{J,\odot}$	3.71	mag
Solar absolute H magnitude	$M_{H,\odot}$	3.37	mag
Solar absolute K magnitude	$M_{K,\odot}$	3.35	mag

Table C.3 Unit conversions.

Quantity	Symbol	Conversion
Angström	Å	$1\,\text{Å} = 10^{-8}$ cm
Micron	μm	$1\,\mu\mathrm{m} = 10^{-4}$ cm
Parsec	pc	$1\,\mathrm{pc} = 3.086 \times 10^{18}$ cm
Light year	ly	$9.460\,530 \times 10^{17}$ cm
Kilo-electron volt	keV	$hc/E = 12.398\,54 \times 10^{-8}$ cm
Jansky	Jy	$10^{-23}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$