Ay 127 – Homework 1
Due: Friday, April 19, 2013

1. Age of the Universe. [10 points]

(a) Using the Friedmann equation, and assuming that the Universe consists of matter and a cosmological constant and is spatially flat ($\Omega_{\rm tot} = 1$), solve for the time t as a function of scale factor a and the cosmological parameters H_0 and $\Omega_{\rm m}$. You may leave this in the form of an integral since there is not an analytic solution in terms of elementary functions.

(b) Numerically evaluate the integral in (a) and find the present-day age of the Universe for $H_0 = 70$ km/s/Mpc and $\Omega_{\rm m} = 0.3$.[1] What was the age of the Universe at $z = 8$?

2. Intergalactic Thomson scattering. [15 points]

Most of the baryonic matter today is believed to be in the form of diffuse, ionized intergalactic gas. This has been true since the epoch of *reionization*, when ultraviolet radiation (probably) from early galaxies ionized the intergalactic medium. The electrons in this gas should scatter light from distant galaxies. In this problem you will calculate the optical depth due to this effect.

(a) Find the present-day electron density $n_{e,0}$ assuming that the baryonic matter is fully ionized and uniformly distributed, and for the cosmological parameters $\Omega_{\rm b} = 0.05$, $\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70$ km/s/Mpc, and $Y_{\rm He} = 0.25$.

(b) The optical depth between us and any previous epoch can be written as an integral involving the electron density, the Thomson cross section, and the speed of light. Write down this integral, and proceed to numerically evaluate it for [i] a source at $z = 1$, and [ii] a source at $z = 6$. [Be careful of how all the factors in the integral scale with a.]

(c) What does the fact that we can observe the primordial fluctuations in the cosmic microwave background tell us about the redshift at which the Universe was reionized? [In fact, much more can be learned by measuring the polarization caused by scattering of the CMB, but we will cover this later.]

3. Non-production of carbon during Big Bang Nucleosynthesis. [12 points]

The isotope ^{12}C is produced in stars via the "triple alpha" process:

$$2\,{}^4\text{He} \to {}^8\text{Be} \qquad {}^8\text{Be} + {}^4\text{He} \to {}^{12}\text{C}. \tag{1}$$

The rate of carbon production by this mechanism, neglecting electron screening, is

$$\dot{X}_{\rm C} = 2 \times 10^{-57} \frac{e^{-4.294/T_9}}{T_9^3} \left(\frac{n_b}{\text{cm}^3}\right)^2 X_{\rm He}^3\,\text{s}^{-1}, \tag{2}$$

where n_b is the physical baryon density.

Assuming the helium abundance is negligible until $T_9 = 0.8$, and reaches its final value thereafter, estimate the primordial carbon abundance produced by this process. (An order of magnitude estimate should suffice. You will need to work out the relation between temperature, time, and baryon density.)

[1] You may use any method you prefer to evaluate the integral – a C/Fortran program, Mathematica, your graphing calculator, ...