Ay 127: Problem Set 2 Solution

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1. (a) The peculiar velocity is $v_{pec} = v_{obs} - H_0 d = cz - H_0 d$. Assuming that our redshift measurement is very good (not a bad assumption), we have $\Delta v_{pec} = H_0 \Delta d = 0.2 H_0 d$. Also assume that v_{pec} is small, we get
$$\Delta v_{pec} = 0.2cz = 1000\,\mathrm{km\,s^{-1}}$$
   (b) We want to measure v_{pec} to 10%, which is down at $\Delta v_{pec} = 50\,\mathrm{km\,s^{-1}}$ level. We need $\Delta d = \Delta v_{pec}/H_0 = 0.7\,\mathrm{Mpc}$, which is a $\Delta d/d = H_0 \Delta d/cz = 50/3000 = 0.02$, a 2% measurement.

2. (a) Mass can be estimated via the virial theorem $M\sigma^2 = GM^2/R$:
$$M = \frac{\sigma^2 R}{G} = 7.8 \times 10^{14}\,\mathrm{M_\odot}$$
   for the dispersion velocity $\sigma = 1500\,\mathrm{km\,s^{-1}}$ and radius 1.5 Mpc.

   (b) The total luminosity of the cluster is $500 \times 10^{10}\,\mathrm{L_\odot}$ so the mass-to-light ratio is $M/L = 160$.

   (c) Gas temperature assuming all hydrogen having the same velocity dispersion, $m_p\sigma^2/2 = \frac{3}{2}kT$, is $T = \frac{1}{3}m_p\sigma^2/k = 9 \times 10^7\,\mathrm{K}$.

   (d) With $E_{photon} = kT = hc/\lambda$, this is 7.8 keV (1.6 Å), which is high energy X-ray.

3. (a) Recall that the correlation relation tells you the excess probability of finding a source at radius r in comparison with what you would expect from a uniform distribution. Formally, it is
$$\frac{N_{obs}}{N_{exp}} - 1 = \xi(r)$$
   For the uniform distribution, $\xi(r) = 0$ by definition.

   (b) If all galaxies are in a sheet, $N_{obs} \propto r^2$ while $N_{exp} \propto r^3$. Hence, $\xi(r) \propto r^{-1}$.

   (c) If all galaxies are in a filament, $N_{obs} \propto r$ and $\xi(r) \propto r^{-2}$.

4. (a) The free fall timescale is the time taken for a uniform sphere of mass M and radius $r = R$ to collapse to $r = 0$. This can be computed by integrating
$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2}$$
   Multiply both sides by $\dot{r}$ and notice that $\dot{r}\ddot{r} = \frac{1}{2}\frac{d\dot{r}^2}{dt}$ and $-\frac{\dot{r}}{r^2} = \frac{d}{dt}\frac{1}{r}$. So we get
$$\frac{1}{2}\dot{r}^2 = GM\left(\frac{1}{r} - \frac{1}{r_0}\right)$$
$$r\dot{r}^2 = 2GM\left(1 - \frac{r}{r_0}\right)$$
$$x\dot{x}^2 r_0^3 = 2GM(1 - x)$$

where $r = xr_0$.

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\begin{aligned}
\dot{x} &= -\sqrt{\frac{2GM}{r_0^3}\left(\frac{1}{x}-1\right)} \\
-\int_1^0 \sqrt{\frac{x}{1-x}}dx &= \sqrt{\frac{2GM}{r_0^3}}t \\
t &= \frac{\pi}{2\sqrt{2GM}}r_0^{3/2}
\end{aligned}
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This is the free-fall time.

(b) Balance gravity and centrifugal acceleration, we have

$$\frac{GM}{R^2} = \frac{v_{\text{circ}}^2}{R}$$

so $M = Rv_{\text{circ}}^2/G = 5.6 \times 10^{11} M_\odot$.

(c) We know the final radius of the halo, which is 50 kpc, what was the original size? If the collapse was dissipationless, the energy is conserved. The total energy from virial theorem is $KE + PE = -PE/2 = -GM^2/2R_{\text{max}}$. Initially the halo was not spinning, so the energy was $-GM^2/R_0$ and we get the initial radius $R_0 = 2R_{\text{max}} = 100$ kpc. The collapse time calculated from (a) is $t = 700$ Myr.

(d) If the collapse began immediately after big bang then the formation was complete when the Universe was 700 Myr old.