Ay 127 – Winter 2017 – Midterm Exam

Distributed on Fri. Feb. 3, due by 5 pm on Thursday, Feb. 9

The Rules: Closed book, closed notes, no web access, *closed everything* (except your minds) … but you can use tables of physical and astronomical constants or units (attached). You can use a pocket calculator, but not if it has display of formulas and such. You *cannot* discuss the problems with anyone until after everyone turns in their exams.

You have a maximum of 4 hours (it should take less) from the moment you start until the moment you finish. Please mark your exam with the start and stop times. You have to turn it in *in person*, either to the Prof or the TA; do not just leave it in a mailbox.

Please write legibly – it is in your own best interest. Be careful about the units, and double-check your numbers. It is always a good idea to write down the formulas first, *then* plug in the numbers. Good luck!

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1. Derive (in a Newtonian approximation) the Friedmann equation for:
   a. A universe with a non-relativistic matter only (no radiation or cosmological constant). [3/30 points]
   b. Now do the same for a fully radiation dominated universe. [2/30 points]
2. Now assume that the matter density is exactly equal to the critical value $\Omega_0 = \Omega_m = 1$ (aka the Einstein – de Sitter model).
   a. Derive the age (*not* the look-back time!) vs. redshift relation in this universe? [2/30 points]
   b. What is its present age in the units of $t_H = 1 / H_0$? [1/30 points]
   c. Derive the equation for the distance as a function of redshift for this universe. [3/30 points]
   d. What is the distance to $z = 1$ in units of $D_H = c / H_0$? [1/30 points]
   e. Now do the same as in (a) – (d) for a fully radiation dominated universe. [7/30 points]
3. Assume that we live in a universe with $h = 0.7$, $\Omega_m \approx 0.3$, $\Omega_{vac} \approx 0.7$, and $T_{CMB} = 2.7°$ K.
   a. Estimate the value of the critical density ρ_0. [1/30 points]
   b. Estimate the Ω_r for the CMB today, using the blackbody energy density formula, $u = 4\sigma T^4/c$, where σ is the Stefan-Boltzmann constant. [1/30 points]
   c. Estimate the redshift of the transition from the radiation-dominated to the matter-dominated universe (i.e., when $\Omega_m \approx \Omega_r$). What was T_{CMB} then? [1/30 points]
   d. Estimate the redshift of the transition from the matter-dominated to the dark energy dominated universe (i.e., when $\Omega_m \approx \Omega_{vac}$). [1/30 points]
   e. If the age of the universe was 380,000 yrs at the recombination ($z \approx 1100$), estimate its age and the temperature at the redshift computed in (c). Hint: would Einstein – de Sitter model be a good approximation then? [1/30 points]
   f. If the cosmic nucleosynthesis happens when $T \approx 10^{10}$ ° K, estimate the redshift and the age when this happens. [2/30 points]
4. Assume for simplicity that we live in an Einstein – de Sitter universe that is 10 Gyr old, and that remarkably, the first acoustic peak is at the angular scale of 1°.
   a. Derive the formula for the angular diameter distance in this universe (you can refer to what you did in 2c). [2/30 points]
   b. What is the physical size in Mpc (specify: commoving or proper) that corresponds to the first acoustic peak? [1/30 points]
   c. What would be the angular scale for observing the BAO at $z = 0.5$ in this universe? [1/30 points]

Appendix C
Physical and Astronomical Constants and Unit Conversions

Table C.1 Physical constants.

Quantity	Symbol	Value	Units
Speed of light	c	$2.997\,924\,58 \times 10^{10}$	cm^{-1}
Gravitational constant	G	$6.672\,59(85) \times 10^{-8}$	$dyn\,cm^2\,g^{-2}$
Planck constant	h	$6.626\,075\,5(40) \times 10^{-27}$	$erg\,s^{-1}$
Boltzmann constant	k	$1.380\,658(12) \times 10^{-16}$	$erg\,K^{-1}$
Stefan–Boltzmann constant	σ	$5.670\,51(19) \times 10^{-5}$	$erg\,cm^{-2}\,K^{-4}\,s^{-1}$
Thomson cross-section	σ_T	$0.665\,246\,16 \times 10^{-24}$	cm^2
Electron charge	e	$4.803\,206\,8(15) \times 10^{-10}$	E.S.U.
Electron mass	m_e	$9.109\,389\,7(54) \times 10^{-28}$	g
Proton mass	m_p	$1.672\,623\,1(10) \times 10^{-24}$	g
Neutron mass	m_n	$1.674\,928\,6 \times 10^{-24}$	g
Atomic mass unit	m_u	$1.660\,540\,2 \times 10^{-24}$	g
Electron volt	eV	$1.602\,173\,3 \times 10^{-12}$	erg

A Panchromatic View of Galaxies, First Edition. Alessandro Boselli.

Table C.2 Astronomical constants.

Quantity	Symbol	Value	Units
Astronomical unit	AU	1.496×10^{13}	cm
Parsec	pc	3.086×10^{18}	cm
Solar mass	$M_\odot$	1.989×10^{33}	g
Solar radius	$R_\odot$	6.955×10^{10}	cm
Solar luminosity	$L_\odot$	3.845×10^{33}	$\mathrm{erg\,s^{-1}}$
Solar absolute bolometric magnitude	$M_{\mathrm{bol},\odot}$	4.72	mag
Solar absolute *B* magnitude	$M_{B,\odot}$	5.48	mag
Solar absolute *V* magnitude	$M_{V,\odot}$	4.83	mag
Solar absolute *J* magnitude	$M_{J,\odot}$	3.71	mag
Solar absolute *H* magnitude	$M_{H,\odot}$	3.37	mag
Solar absolute *K* magnitude	$M_{K,\odot}$	3.35	mag

Table C.3 Unit conversions.

Quantity	Symbol	Conversion
Angström	Å	$1\,\text{Å} = 10^{-8}$ cm
Micron	μm	$1\,\mu\mathrm{m} = 10^{-4}$ cm
Parsec	pc	$1\,\mathrm{pc} = 3.086 \times 10^{18}$ cm
Light year	ly	$9.460\,530 \times 10^{17}$ cm
Kilo-electron volt	keV	$hc/E = 12.398\,54 \times 10^{-8}$ cm
Jansky	Jy	$10^{-23}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}}$