The Radiation-Dominated Era

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Outline

1. Radiation-Dominated Plasmas, Expansion of the Universe
2. Neutrino Decoupling
3. Synthesis of the Light Elements
4. Observations of D, He, Li

Radiation-Dominated Plasmas

- Consider blackbody spectrum:

$$I_\nu = \frac{gh\nu^3}{c^2}\frac{1}{e^{h\nu/kT} \pm 1}$$

g = number of polarizations (2 for photons)
− = bosons; + = fermions

- Energy density $u = \frac{4\pi}{c}\int_0^\infty I_\nu d\nu = \frac{\pi^2}{30\hbar^3 c^3} g_*(kT)^4$

g_* = g (bosons) or ⅞ g (fermions)

Thermodynamic properties

- Effective matter density: $\rho = \frac{u}{c^2} = \frac{\pi^2}{30\hbar^3 c^5} g_* (kT)^4$

- Pressure: $p = \frac{1}{3} u = \frac{\pi^2}{90\hbar^3 c^3} g_* (kT)^4$

- Entropy density: $s = \int_0^u \frac{du}{T} = \frac{2\pi^2}{45\hbar^3 c^3} g_* k^4 T^3$

Expansion of Universe

- Friedmann equation relating H to T:

$$H^2 = \frac{8}{3}\pi G\rho = \frac{4\pi^2 G}{45\hbar^3 c^5} g_* (kT)^4$$

- Solution: $H \sim T^2 \sim a^{-2}$ ➔ $a \sim t^{1/2}$ ➔ $H=1/(2t)$.

$$t = \frac{3\sqrt{5}\hbar^{3/2}c^{5/2}}{4\pi G^{1/2} g_*^{1/2} (kT)^2}$$

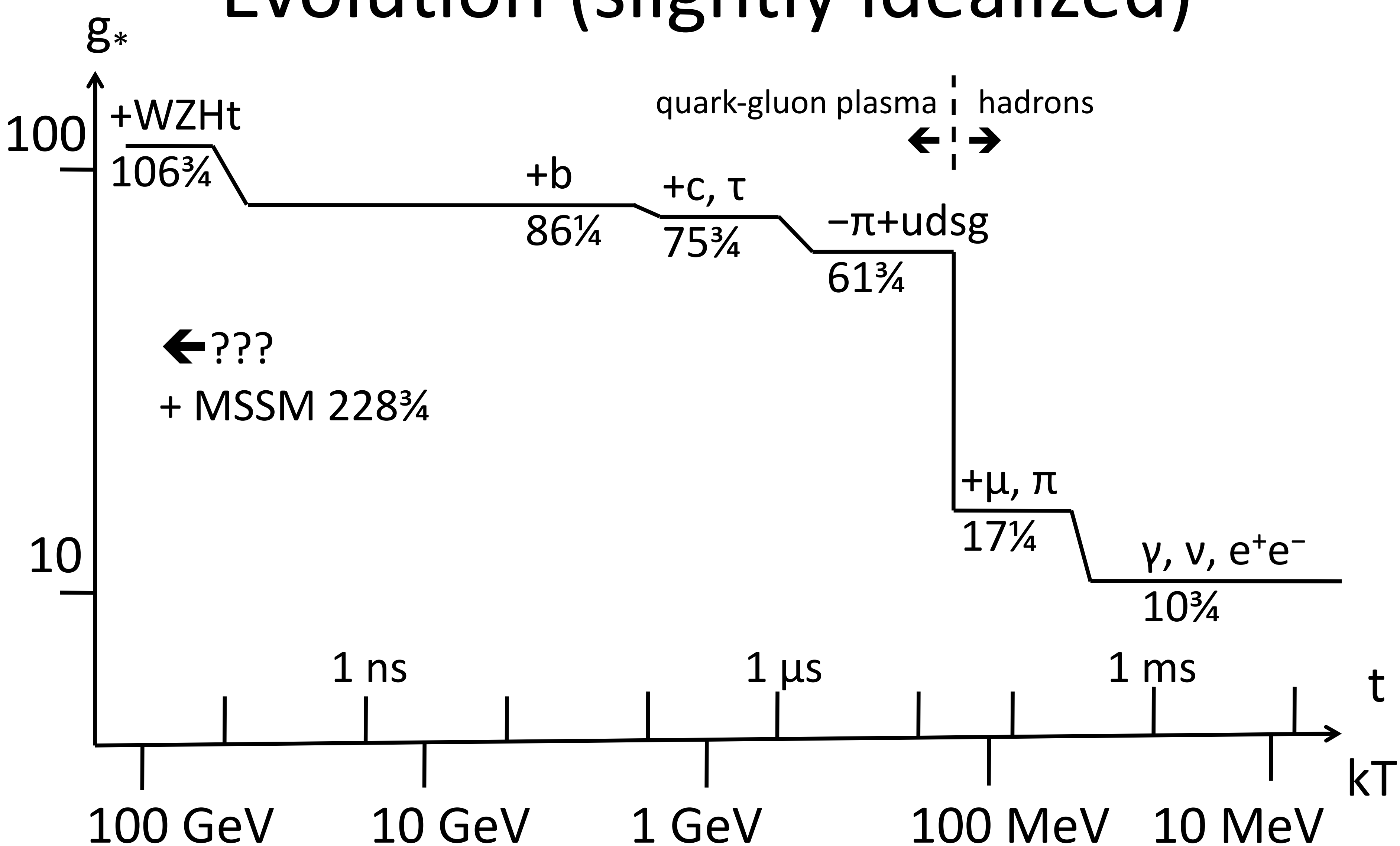
- More convenient form:

$$\boxed{t = \frac{1}{\sqrt{g_*}}\left(\frac{1.56\ \mathrm{MeV}}{kT}\right)^2\ \mathrm{sec}}$$

What is g_*?

- Consider the Universe at t~1 second.
- Photons only: $g_*=2$.
- Also neutrinos: 3 flavors, ×2 for antineutrinos: $\frac{7}{8} \times 3 \times 2 = 21/4$.
- At $3kT \geq m_e c^2$, e^+e^- are "massless." 2 spins each:
  $\frac{7}{8} \times 2 \times 2 = 7/2$.
- At MeV temperatures, $g_* = 43/4$.

Evolution (slightly idealized)



Neutrino Decoupling

- Neutrinos have weak interactions with e^+e^-:

$$n_{\text{e+e-}} \sim 5\times10^{31}(kT)^3_{\text{MeV}}\ \text{cm}^{-3}$$
$$\sigma_{\text{weak}} \sim 10^{-44}\,E^2_{\text{MeV}}\ \text{cm}^2$$

- Typical E ~ 3kT so can find the number of neutrino interactions in the lifetime of the Universe:

$$n\sigma ct \sim 0.1(kT)^5_{\text{MeV}}$$

- At kT~1.6 MeV, universe transitions from being neutrino-opaque to transparent.

Epoch of e^+e^- Annihilation

- At $kT<m_ec^2$ almost all the electrons and positrons annihilate. (~1ppb more e^- than e^+. These few e^- survive.)

$$e^+ + e^- \rightarrow \begin{cases} \geq 2\gamma & \sim 100\% \\ \nu_e + \overline{\nu}_e & \text{(almost) neglgible} \end{cases}$$

- Energy of annihilation heats the photons but not the neutrinos, so $T_\gamma > T_\nu$.
- Can do computation assuming annihilation is adiabatic (conserves entropy of $e^+e^-\gamma$).

Annihilation Part 2

- Entropy before annihilation:

$$s(e^{\pm}\gamma) = \frac{2\pi^2}{45\hbar^3 c^3} g_*(e^{\pm}\gamma) k^4 T_\nu^3 = \frac{11\pi^2}{45\hbar^3 c^3} k^4 T_\nu^3$$

- Entropy after annihilation:

$$s(\gamma) = \frac{2\pi^2}{45\hbar^3 c^3} g_*(\gamma) k^4 T_\gamma^3 = \frac{4\pi^2}{45\hbar^3 c^3} k^4 T_\gamma^3$$

- Equating gives: $\boxed{T_\gamma = \sqrt[3]{\frac{11}{4}} T_\nu}$

(Remains true since both temperatures scale as 1/a.)

Post-annihilation

- Can combine neutrino and photon energy densities to get radiation energy density:

$$u = \frac{\pi^2}{30\hbar^3 c^3}\left[2(kT_\gamma)^4 + \tfrac{21}{4}(kT_\nu)^4\right] = 1.68\frac{\pi^2(kT_\gamma)^4}{15\hbar^3 c^3}$$

- Factor of 1.68 from neutrinos. Affects BBN, CMB, etc.
- Equivalent to "effective" g_*=3.36.
- From temperature ratio, T_ν=1.95(1+z) K.
- They're still here today: ~ 300/cm^3.

Nucleosynthesis

- Universe has net baryon number:

$$\eta = \frac{\text{baryons} - \text{antibaryons}}{\text{photons}} \sim 6 \times 10^{-10}$$

- We don't know why.
- At high temperature the thermodynamically favored state of baryons is p + n.
- At low temperature the favored state is ^{56}Fe.
- Nucleosynthesis is process of forming the heavier nuclei from p+n. Started in Big Bang, continues today in stars.

Primordial nucleosynthesis

- The process:
  1. n/p ratio determined at neutrino decoupling
  2. Universe expands/cools, some neutrons decay
  3. Assembly of D, ^{3}He, ^{4}He, ^{7}Li

- Nuclear processing is not finished after BBN, e.g. heavy elements are produced in stars. This more recent processing must be corrected/avoided to infer primordial abundances.

The initial n/p ratio

- Before neutrino decoupling, neutrons are kept in equilibrium:

$$n + e^+ \leftrightarrow p^+ + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p^+ + e^-$$

- Equilibrium ratio

$$n : p = e^{-Q/kT}$$

where Q = $(m_n - m_p)c^2$ = 1.293 MeV.

- n:p=1:1 at high T, then starts decreasing.

The initial n/p ratio (Part II)

- Neutrino decoupling at kT ~ 1.6 MeV would imply n:p=0.4.
- More detailed calculation gives n:p≈0.15.
- Neutron has a half-life of ~11 minutes.

$$n \rightarrow p^{+} + e^{-} + \bar{\nu}_e$$

- Therefore the neutron abundance declines:

$$n:(n+p) \approx \frac{0.15}{2^{t/11\,\mathrm{min}}}$$

Deuterium (Part I)

- Deuterium is the simplest nucleus. Binding energy is B = 2.22 MeV.

$$n + p^{+} \leftrightarrow \mathrm{D}^{+} + \gamma$$

- Saha-like equation for its abundance:

$$\frac{n(\mathrm{D}^{+})}{n_p n_n} = \frac{3}{4}\left(\frac{2\pi\hbar^2}{m_{\mathrm{red}} kT}\right)^{3/2} e^{B/kT}$$

- Compare with total baryon abundance:

$$n_b = 5.6 \times 10^{20} \Omega_b h^2 T_9^3 \ \mathrm{cm}^{-3}$$

Deuterium (Part II)

- Using notation $X_i = n_i/n_b$, and assuming the universe is mostly p (85%) and n (15%) we get

$$X(\mathrm{D}^+) \approx 5 \times 10^{-14}\,\Omega_b h^2 T_9^{3/2} e^{25.8/T_9}$$

- Deuterium abundance grows exponentially. Would become of order unity at $T_9 \sim 0.7$ (t ~ 3 min) except that deuterium can burn to heavier nuclei.

Helium

- Deuterium consumption:

A=2→3	A=3→4
$D + p \leftrightarrow {}^3He + \gamma$	${}^3He + p \leftrightarrow {}^3H + n$
$D + D \leftrightarrow {}^3H + p$	${}^3H + D \leftrightarrow {}^4He + n$
$D + D \leftrightarrow {}^3He + n$	${}^3He + D \leftrightarrow {}^4He + p$

- When deuterium reaches ~1%, these reactions package most of the available neutrons into the very tightly bound ^{4}He.

What happens next

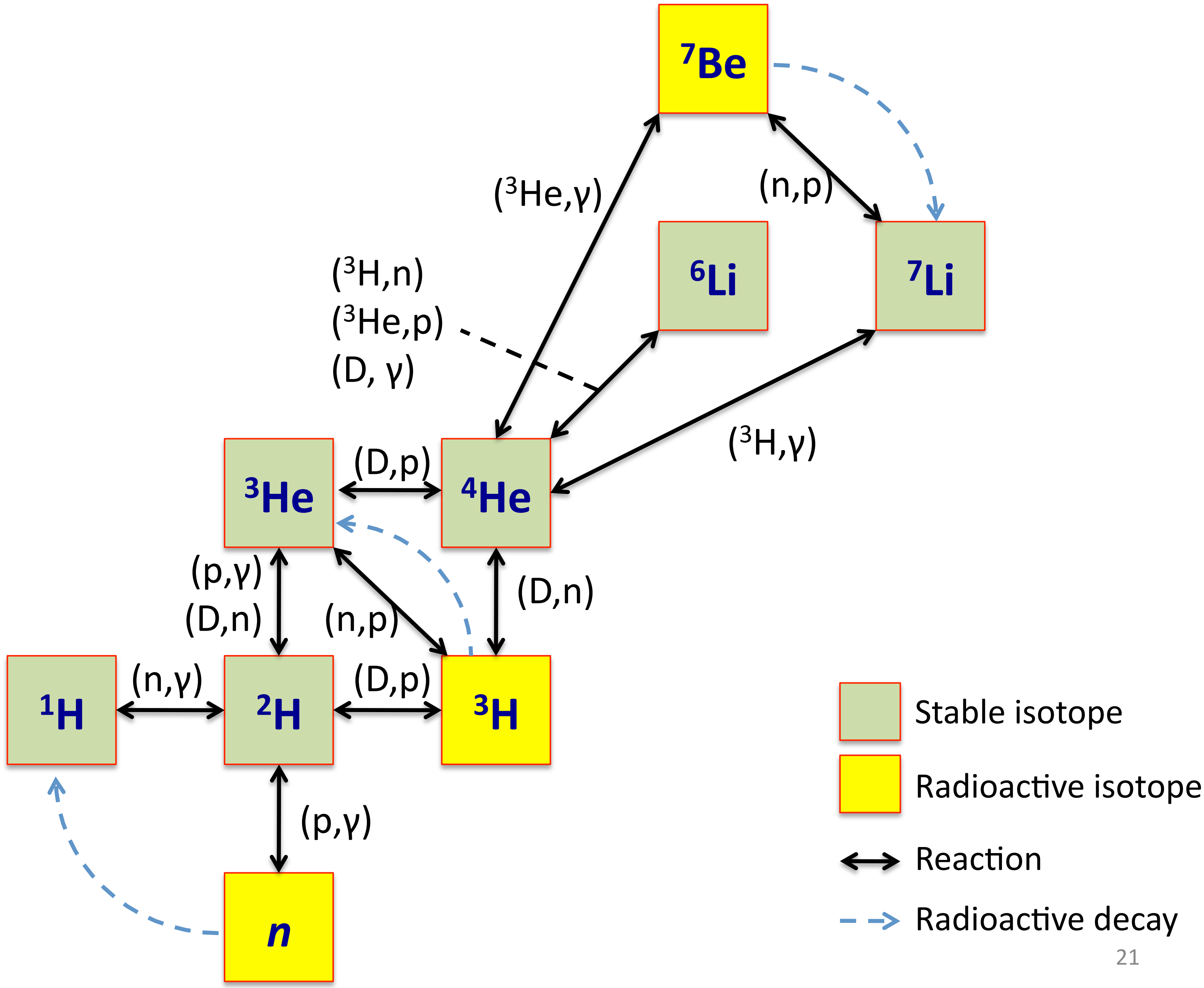
- At t ~ 3 minutes, 13% of baryons are neutrons. Packaged into ^{4}He, we get Y~0.26 (^{4}He fraction by mass). Most of the rest is ^{1}H.

- No new D is produced once free neutrons are gone. Deuterium is burned via D+D and D+p. A small amount, $X_D \sim 2\times10^{-5}$, survives.

- Some ^{3}H and ^{3}He are left over (total $X_3 \sim 10^{-5}$).

End Game

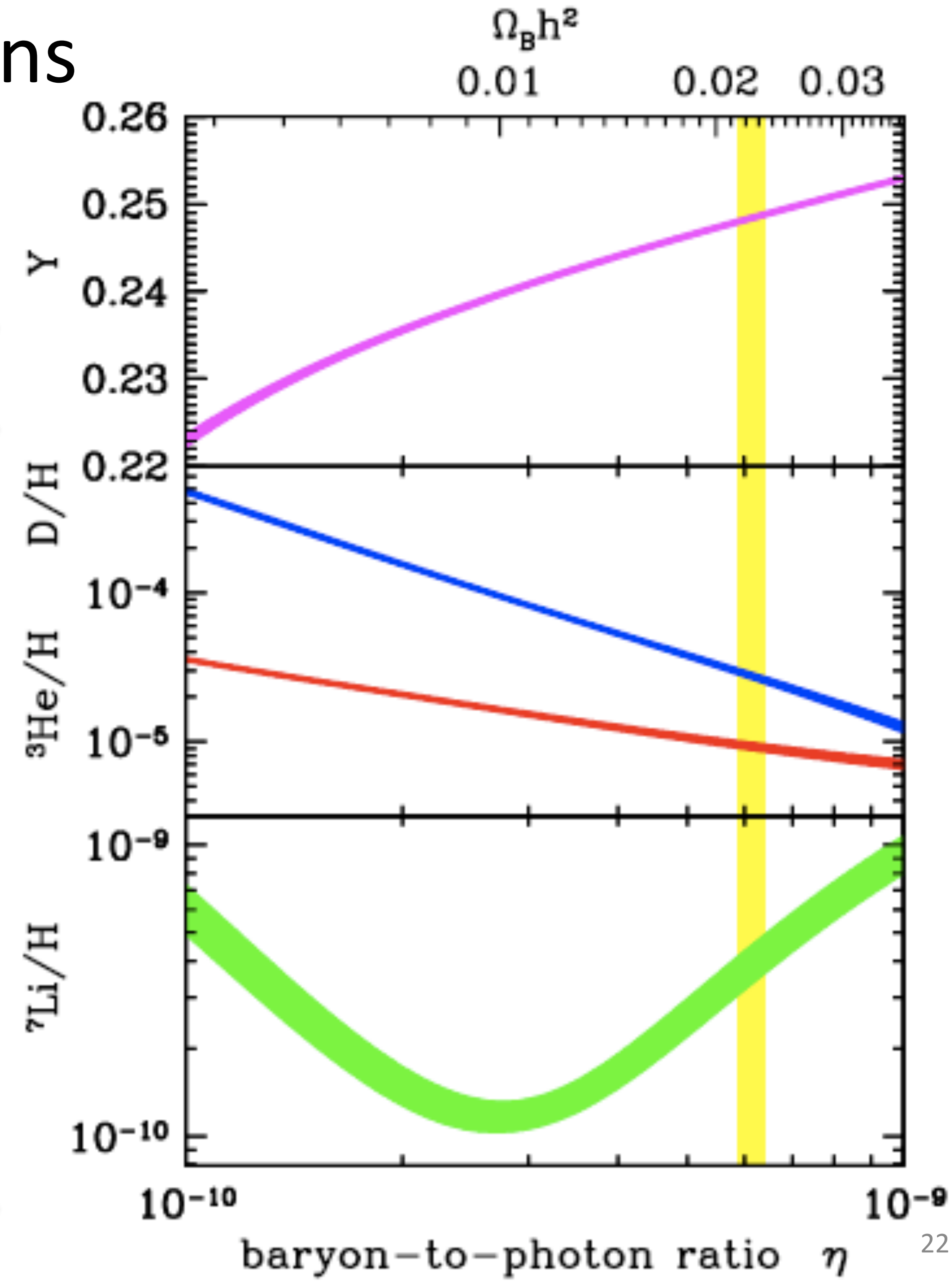
- Some A=7 nuclei ($X_7 \sim 4\times10^{-10}$) are produced by $^3He+^4He$ and $^3H+^4He$.
- ^6Li produced in much smaller amounts since it is fragile ($X_6 \sim 10^{-14}$).
- Heavy element production e.g. $3\alpha \rightarrow {}^{12}C$ negligible at BBN densities (10^{-5} g/cm^3).
- Radioactive nuclei (n, ^3H, ^7Be) decay.



7Be
(3He,γ)
(n,p)
6Li
7Li
(3H,n)
(3He,p)
(D, γ)
(3H,γ)
3He
(D,p)
4He
(p,γ)
(D,n)
(n,p)
(D,n)
1H
(n,γ)
2H
(D,p)
3H
(p,γ)
n
Stable isotope
Radioactive isotope
Reaction
Radioactive decay

BBN Yield Predictions

Cyburt, Fields & Olive 2003
Phys Lett B 567,227

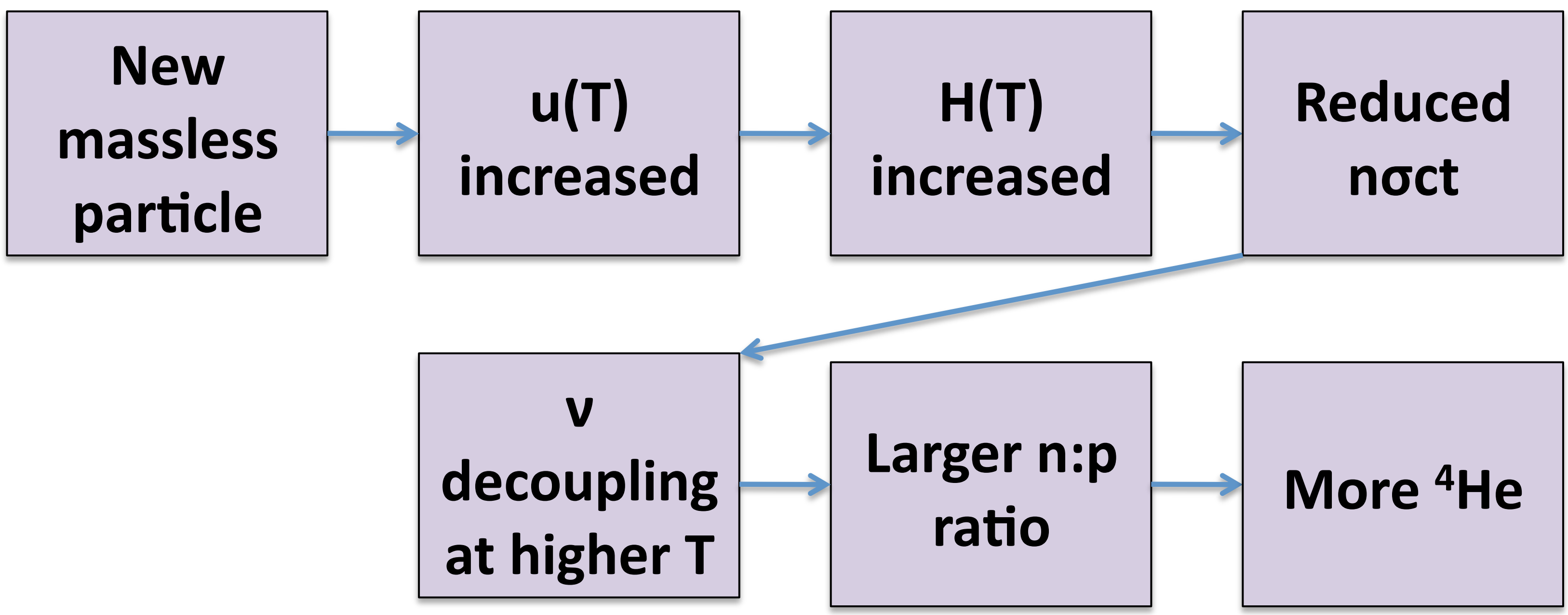


^{4}He fraction by mass

Abundance relative to H by number

Dependence on Cosmology

- ^{4}He: Determined by n:p ratio, so very sensitive to physics at neutrino decoupling, e.g. new massless particles. Slightly sensitive to $\Omega_b h^2$.

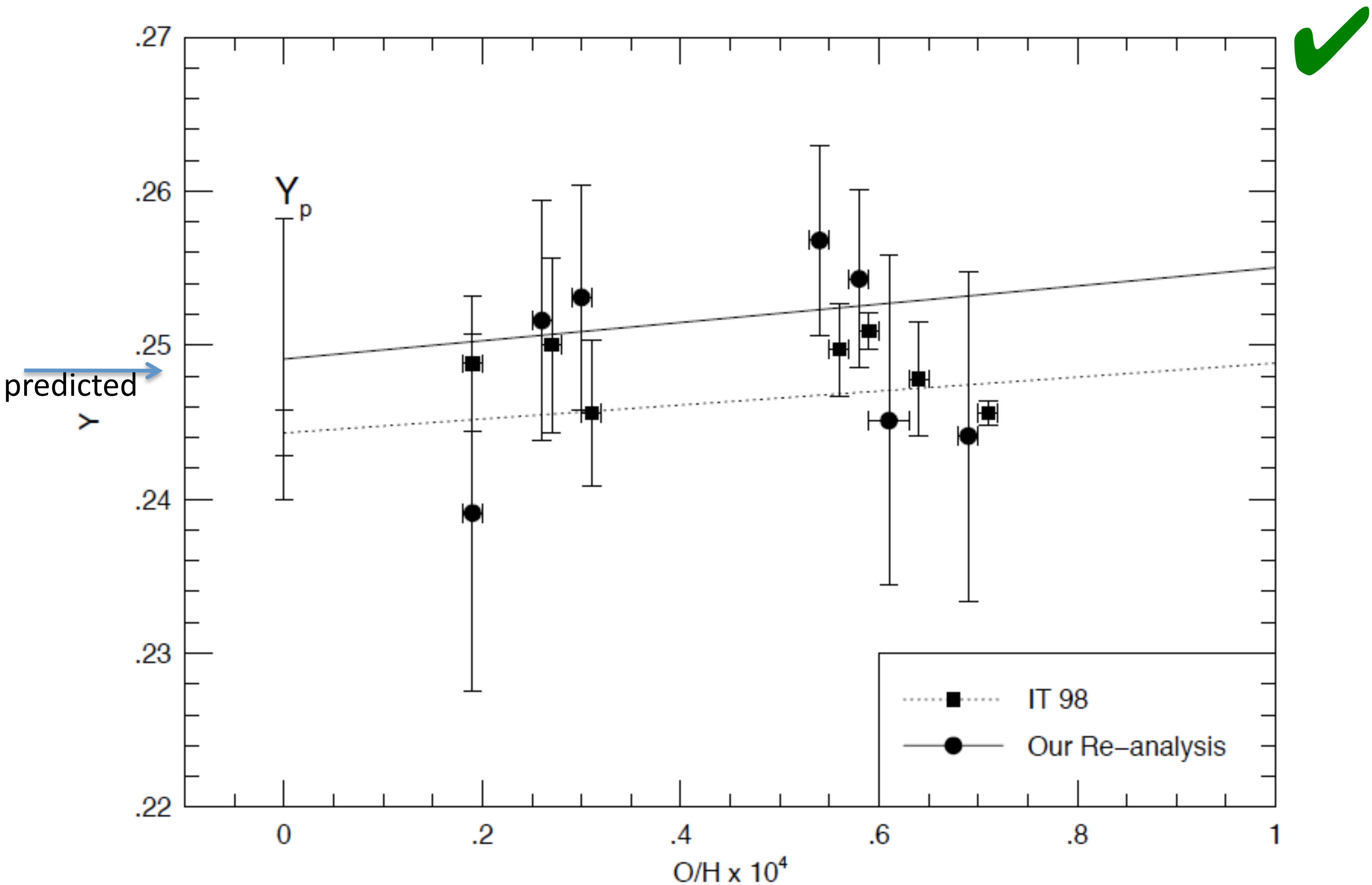


Dependence on Cosmology

- ^{2}H and ^{3}He: Leftovers of incomplete H burning. Decrease for larger baryon density $\Omega_b h^2$.
- ^{7}Li: Non-monotonic behavior in $\Omega_b h^2$:
  - Low $\Omega_b h^2$: direct production of ^{7}Li, destroyed by ^{7}Li+p at high densities.
  - High $\Omega_b h^2$: ^{7}Be can be produced, decays to ^{7}Li.
- ^{6}Li: Primordial contribution should be undetectable.

^{4}He

- Prediction: $Y = 0.2482 \pm 0.0003 \pm 0.0006$
- Usually estimate He abundance from H II region spectra (H I and He I recombination lines).
  - Determine temperature (self-consistent or using [O III]).
  - Model collisional excitation, fluorescence.
  - Optical depth effects. He I 2^3S_1 is metastable.
  - Model reddening.
  - Model stellar He I absorption.
  - Ionization correction factors (H II and He I can coexist).
  - Extrapolate to zero metallicity to get primordial value.



$0.232 < Y_p < 0.258$

Olive & Skillman 2004 ApJ 617,29

Examples of Deuterium Measurements

Location	D/H (ppm)	Method
Earth	150	
Venus	16000±2000	Mass spec. Donahue et al 1982
Mars	900±400	IR lines (HDO/H_2O) Owen et al 1988
Jupiter	22—50 50±20	IR lines (CH_3D/CH_4) Kunde et al 1982 Mass spec. Niemann et al 1996
Saturn	4—29	IR lines (CH_3D/CH_4) Courtin et al 1984
Local ISM	~ 7 – 20	Absorption lines in stellar spectra. Depends on line of sight; see tabulation by Linsky et al 2006
Lyman-α absorbers (IGM)	28±4	H vs D Lyman absorption lines in QSO spectra Kirkman et al 2003

Warnings on D/H

- Not all D/H is the primordial value, or even that at the formation of the solar system.
- Problems:
  - Astration: burning of D to ^{3}He etc. in stars.
  - Chemical fractionation: At low temperatures D binds more tightly to molecules than H due to vibrational zero point energy.

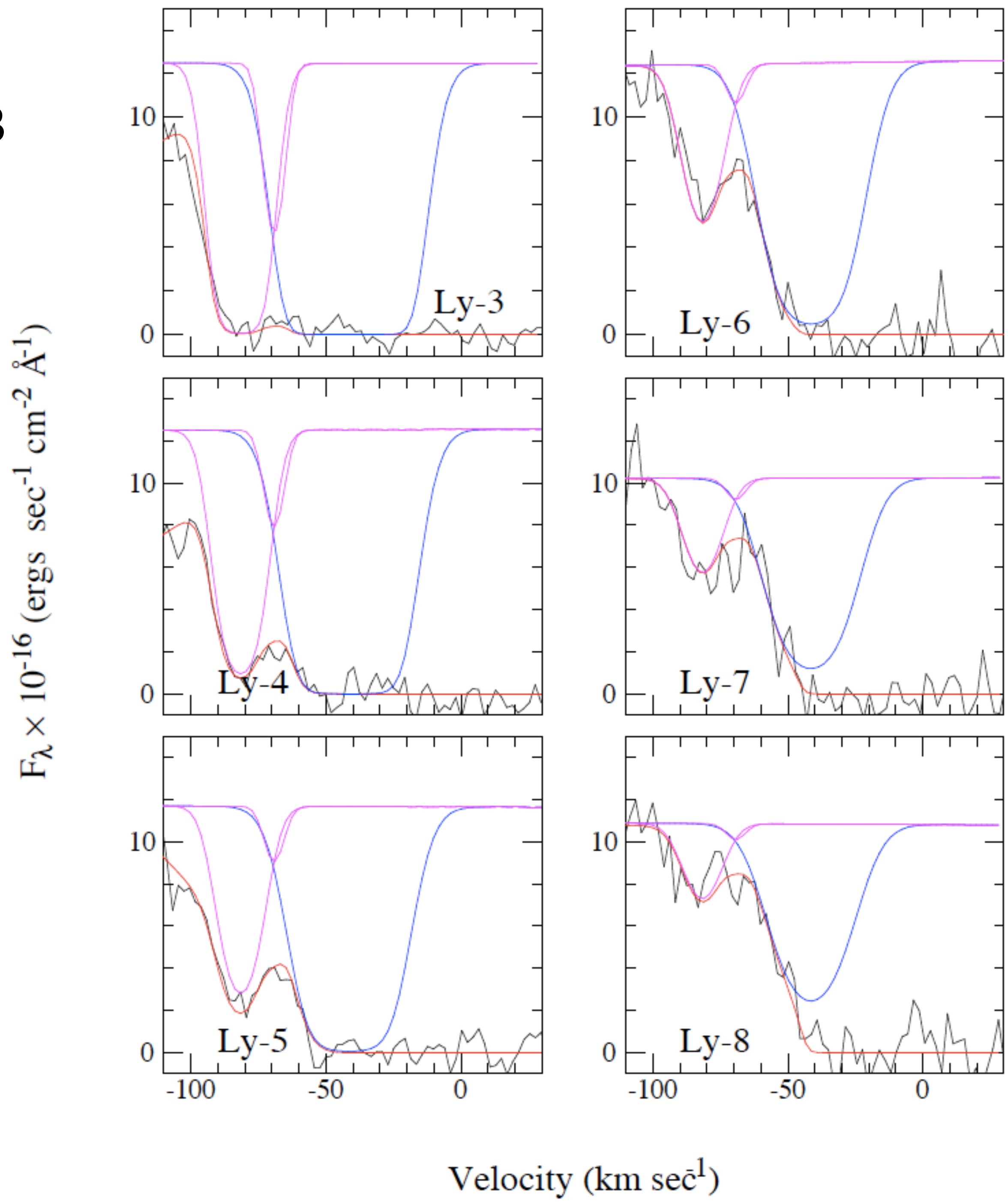
$$XH^+ + D \rightarrow XD^+ + H$$

  - Fractionation due to depth of planetary gravity well. (This is why Jupiter was once used for BBN D/H.)

Intergalactic D/H

- Probably most reliable method is low-metallicity quasar absorption line systems.
  - Little processing by stars
  - Less opportunity to hide deuterium in molecules or dust grains
- Reduced mass of e^-D^+ is greater than e^-p^+ by 1 part in 3600, rescaling all hydrogenic energy levels. Equivalent to velocity shift of $c/3600 = 80$ km/s.
- IGM value 28±4 ppm agrees with predicted 25.7 (+1.7)(−1.3) ppm. ✔

Kirkman et al 2003
ApJS 149, 1
Q1243+3047
z_{abs} = 2.53



^{3}He

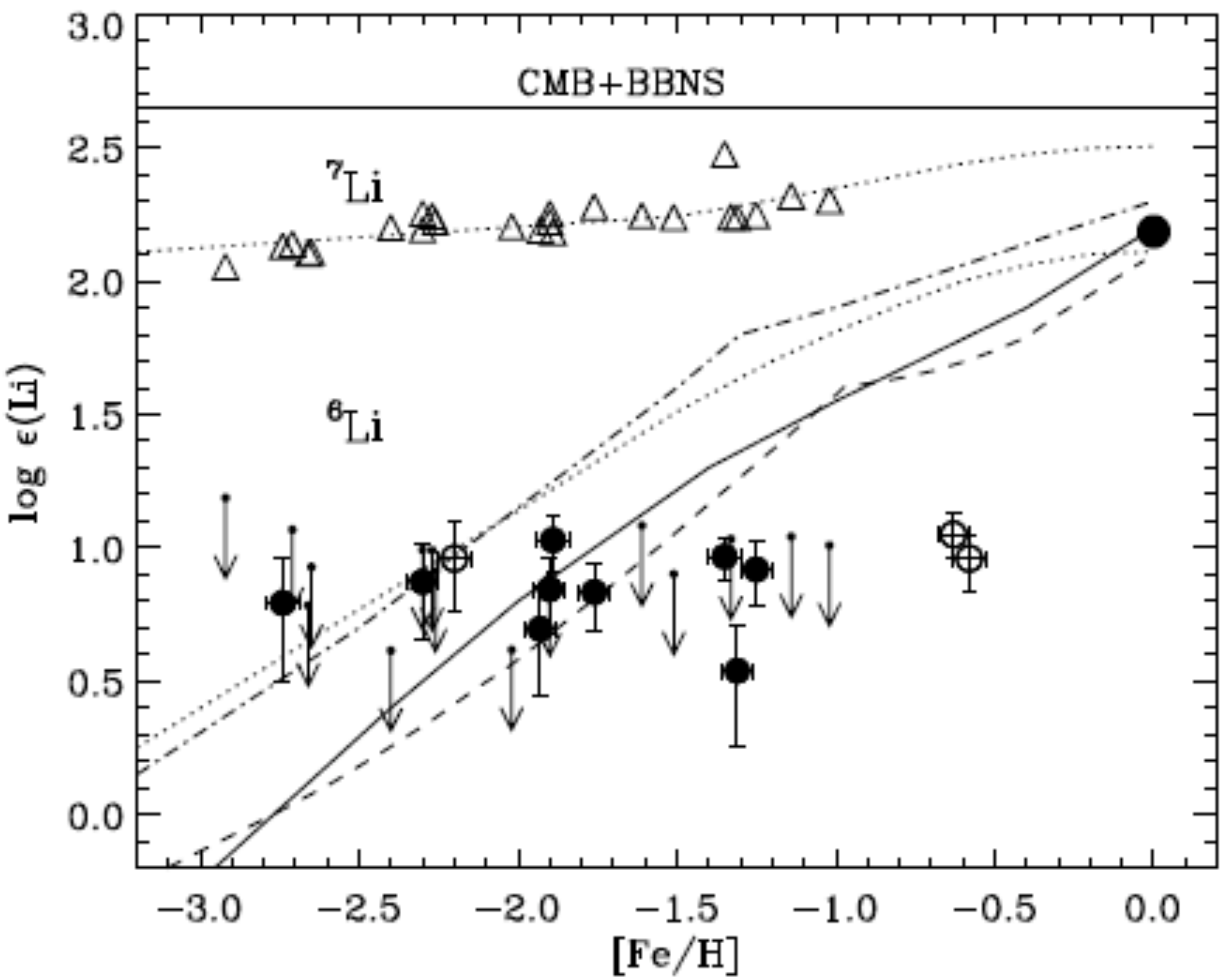
- Difficulties:
  - Can be both produced and destroyed in stars.
  - No IGM measurement.
- Most(?) accepted measurement is $^3He^+$ hyperfine line (λ=3.46cm) low-metallicity H II regions. Bania et al (2002) find $^3He/H < 15$ ppm.
  - Directly observed 11±2 ppm, argue that stars would lead to net increase in ^{3}He.
- Predicted from WMAP baryon abundance: ✓?
  $^3He/H = 10.5 \pm 0.3 \pm 0.3$ ppm.
  - Aside: Jupiter atmosphere ratio $^3He:^4He = (1.1 \pm 0.2) \times 10^{-4}$ [Niemann et al 1996].

Lithium

- Can be measured in low-metallicity stars.
  - Two multiplets available for Li I: 6708Å (2s—2p) and 6104Å (2p—3d).
  - ^{7}Li destroyed at high T, $^7\text{Li} + p \rightarrow 2\,^4\text{He}$. Avoid low-mass stars due to deep convective zone.
  - Slight isotope shift of ^{6}Li vs. ^{7}Li.
- Li can also be produced via cosmic rays:
  - Spallation of CNO elements (also gives Be, B)
  - $^4\text{He} + {}^4\text{He}$ collisions at low energies.

Li abundance evolution

$12 + \log_{10} X_{Li}$



Asplund et al (2006) ApJ 644, 229

The Lithium Problem(s)

- ^{7}Li: Predicted $(5.2\pm0.7)\times10^{-10}$.
  - See update from Cyburt, Fields, Olive 2008.
- Observations give lower values
  - e.g. $(1.1—1.5)\times10^{-10}$ from Asplund et al. 2006.
  - Other determinations are typically $\sim(1—2)\times10^{-10}$. ✗
- ^{6}Li: there have been reports of a plateau at $\sim6\times10^{-12}$, although the accuracy of ^{6}Li/^{7}Li extracted from line profiles has been debated. ✗?

Solutions

- The ^{7}Li problem:
  - Errors in nuclear reaction rates?
  - Depletion in convection zone?
  - Stellar atmosphere modeling?
  - Destroyed in early generation of stars?
  - New physics?