# Transition to the Matter Dominated Era, Recombination, and the Beginning of Chemistry

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# Outline

1. Matter Domination
2. Recombination: Equilibrium Model
3. Recombination: Kinetics
4. The Post-Recombination Universe

# The End of the Radiation Era

- Radiation energy density (photons + neutrinos):

$$\rho_r = 7.8 \times 10^{-34} (1+z)^4 \ \ \mathrm{g/cm^3}$$

- Compare to matter energy density:

$$\rho_m = 1.88 \times 10^{-29} \Omega_m h^2 (1+z)^3 \ \ \mathrm{g/cm^3}$$

- The matter becomes dominant at:

$$1 + z_{\mathrm{eq}} = 24100 \Omega_m h^2 \approx 3100$$

# Recombination

- Recall Saha equation (the original).

$$\frac{n_e n_p}{n(\text{H I})} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT}$$

- I = 13.6 eV, I/k = 158000 K.
- Define $x_e = n_e/n_H$ (all H nuclei). Then:

$$\frac{x_e x_p}{x(\text{H I})} = \frac{1}{n_\text{H}}\left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT}$$

$$n_\text{H} = 1.0\times 10^4 \frac{\Omega_b h^2}{0.023}\frac{1-Y}{0.76} T_4^3 \text{ cm}^{-3}$$

# Recombination, Part 2

- In thermal equilibrium, the balance of ionized (p+e) and neutral (HI) hydrogen is:

$$\frac{x_e x_p}{x(\mathrm{H\,I})} = 2.5\times 10^{17}\,\frac{0.023}{\Omega_b h^2}\,\frac{0.76}{1-Y}\,T_4^{-3/2}e^{-15.8/T_4}$$

- Universe recombines when exponential factor outweighs the $10^{17}$.
- 50% ionization expected @ 3740 K, z=1370.

# Reaction Kinetics

- In the real Universe, recombination is not an equilibrium process. Must identify reactions that lead to recombination.
- First try:

$$\mathrm{H}^{+} + e^{-} \leftrightarrow \mathrm{H}(1s) + \gamma$$

  – Doesn't work. Universe is optically thick to E>13.6eV photons if $x_{HI} > 10^{-8}$.

# Recombination via Excited States

- First recombine to excited states – Universe is optically thick to these photons

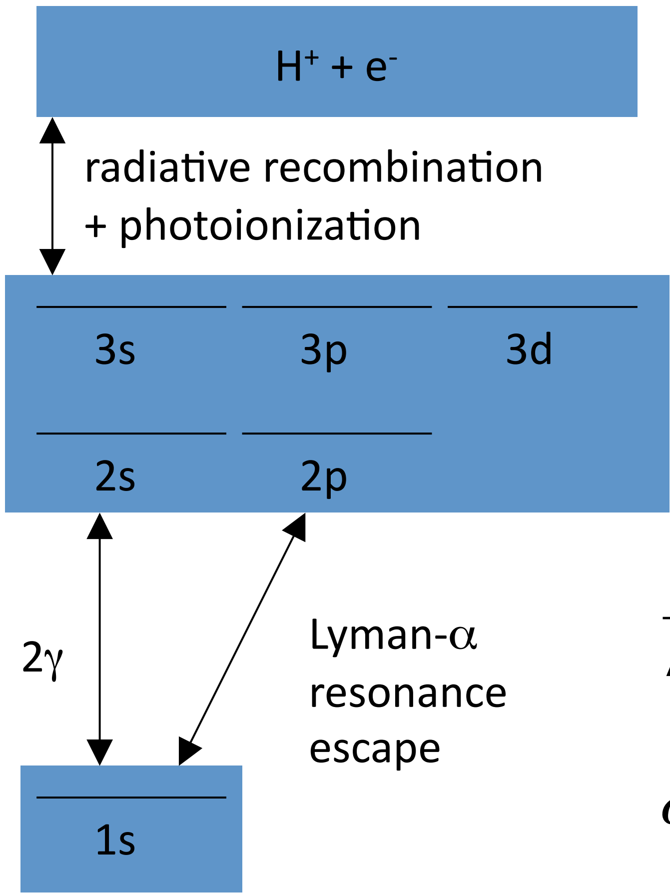
$$H^+ + e^- \leftrightarrow H(nl) + \gamma, \qquad n \geq 2$$

- Decay to ground state:

$$H(2p) \leftrightarrow H(1s) + \gamma_{\mathrm{Ly}\alpha}, \text{ photon redshifts out of line}$$

$$H(2\mathrm{s}) \leftrightarrow H(1s) + \gamma + \gamma$$

# Recombination via Excited States



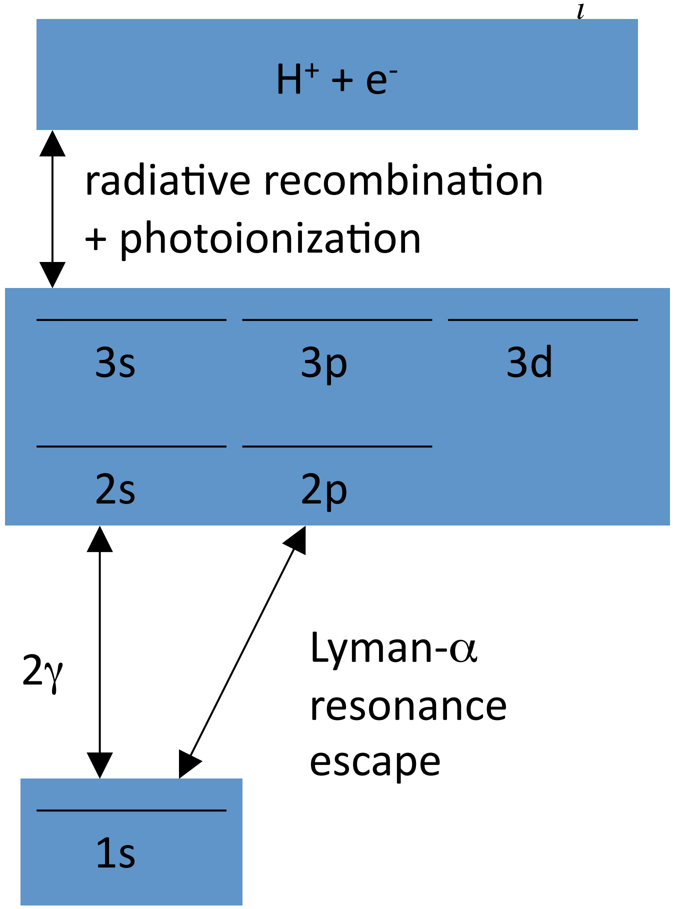

- Effective recombination rate is recombination coefficient to excited states times branching fraction to ground state.
- Must include ionization rate to achieve balance in the case of Saha equilibrium.

$$\frac{\#\,\mathrm{rec}}{\Delta V\ \Delta t} = \frac{2\Lambda + 6A_{Ly\alpha}P_{esc}}{2\Lambda + 6A_{Ly\alpha}P_{esc} + \sum_i g_i e^{-(E_i - E_2)/kT}\beta_i}\alpha_B n_e n_p$$

$$\alpha_B = \sum_{nl,n\geq 2}\alpha_{nl}$$

# Recombination via Excited States

$$\frac{dx_{HI}}{dt} = \frac{2\Lambda + 6A_{Ly\alpha}P_{esc}}{2\Lambda + 6A_{Ly\alpha}P_{esc} + \sum_i g_i e^{-(E_i - E_2)/kT}\beta_i}\,\alpha_B\left[x_e x_p n_H - \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT} x_{HI}\right]$$




$\Lambda$ = 2-photon decay rate from 2s = 8.2 $s^{-1}$

$P_{esc}$ = escape probability from Lyman-$\alpha$ line

$A_{Ly\alpha}$ = Lyman-$\alpha$ decay rate = $6\times10^8$ $s^{-1}$.

$\alpha_e$ = recombination rate to excited states

$g_i$ = degeneracy of level l = $2l_i+1$

$\beta_i$ = photoionization rate from level l

I = ionization energy

# Escape probability

- Total optical depth for a photon redshifting through Lyman-α line:

$$\sigma(\nu) = \frac{3\lambda_{Ly\alpha}^2 A_{Ly\alpha}}{8\pi}\delta(\nu - \nu_{Ly\alpha})$$

$$\tau = \int n_{HI}\sigma c\, dt = n_{HI} c \int \sigma(\nu)\frac{d\nu}{H\nu} = \frac{3\lambda_{Ly\alpha}^3 A_{Ly\alpha} n_H x_{HI}}{8\pi H}$$

- Usual (Sobolev) approximation for $P_{esc}$:
  - Emission and absorption line profiles are the same
  - So to escape a photon has to traverse an optical depth $\tau_{esc}$ uniformly distributed between 0 and τ.

$$P_{esc} = \left\langle e^{-\tau_{esc}} \right\rangle = \frac{1}{\tau}\int_0^\tau e^{-\tau_{esc}} d\tau_{esc} = \frac{1-e^{-\tau}}{\tau}$$

# Simplifications to Recombination Equation

- Escape Probability: practical case: $\tau >> 1$, so

$$P_{esc} \approx \frac{1}{\tau} = \frac{8\pi H}{3\lambda_{Ly\alpha}^3 A_{Ly\alpha} n_H x_{HI}}$$

- Ionization rate: from detailed balance: (recall energy of 2nd level is $-I/4$)

$$\sum_i g_i e^{-(E_i - E_2)/kT} \beta_i = 2\alpha_B \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{I/4kT}$$

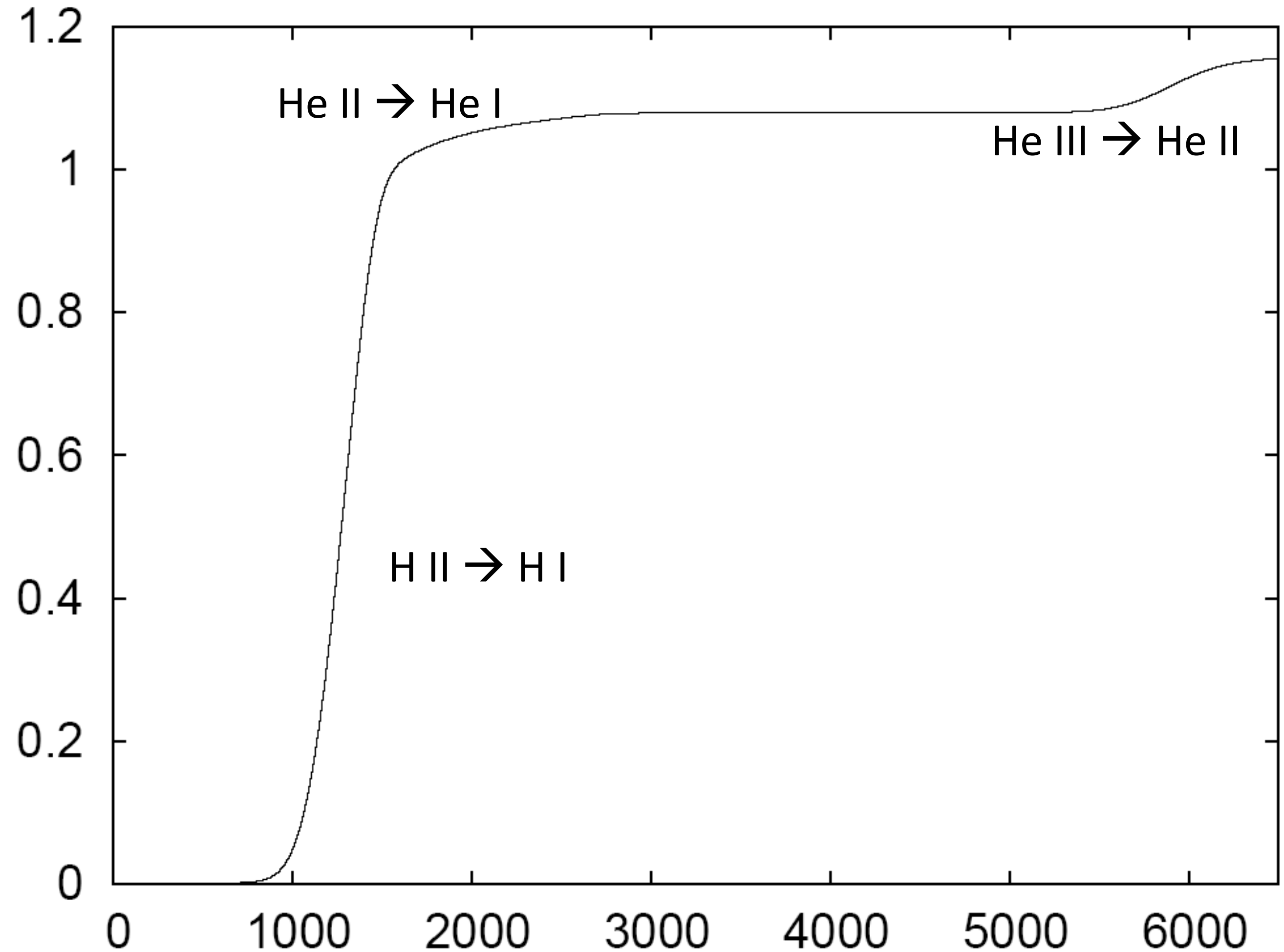
- This leads to the Peebles (1968) or Zel'dovich et al (1968) equation.

# The Peebles/Zel'dovich et al ODE

$$\frac{dx_{HI}}{dt} = \frac{\Lambda + \dfrac{8\pi H}{\lambda_{Ly\alpha}^3 n_H x_{HI}}}{\Lambda + \dfrac{8\pi H}{\lambda_{Ly\alpha}^3 n_H x_{HI}} + \alpha_B \left(\dfrac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{I/4kT}} \alpha_B \left[ x_e x_p n_H - \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT} x_{HI} \right]$$

- Two limiting cases:
  - "Early": Decay from excited states is the bottleneck ($z>850$)
  - "Late": Recombination to excited state is the bottleneck, also ionization negligible ($z<850$).

# Solution




$x_e$

z

**Seager, Sasselov & Scott 1999 – Recfast**

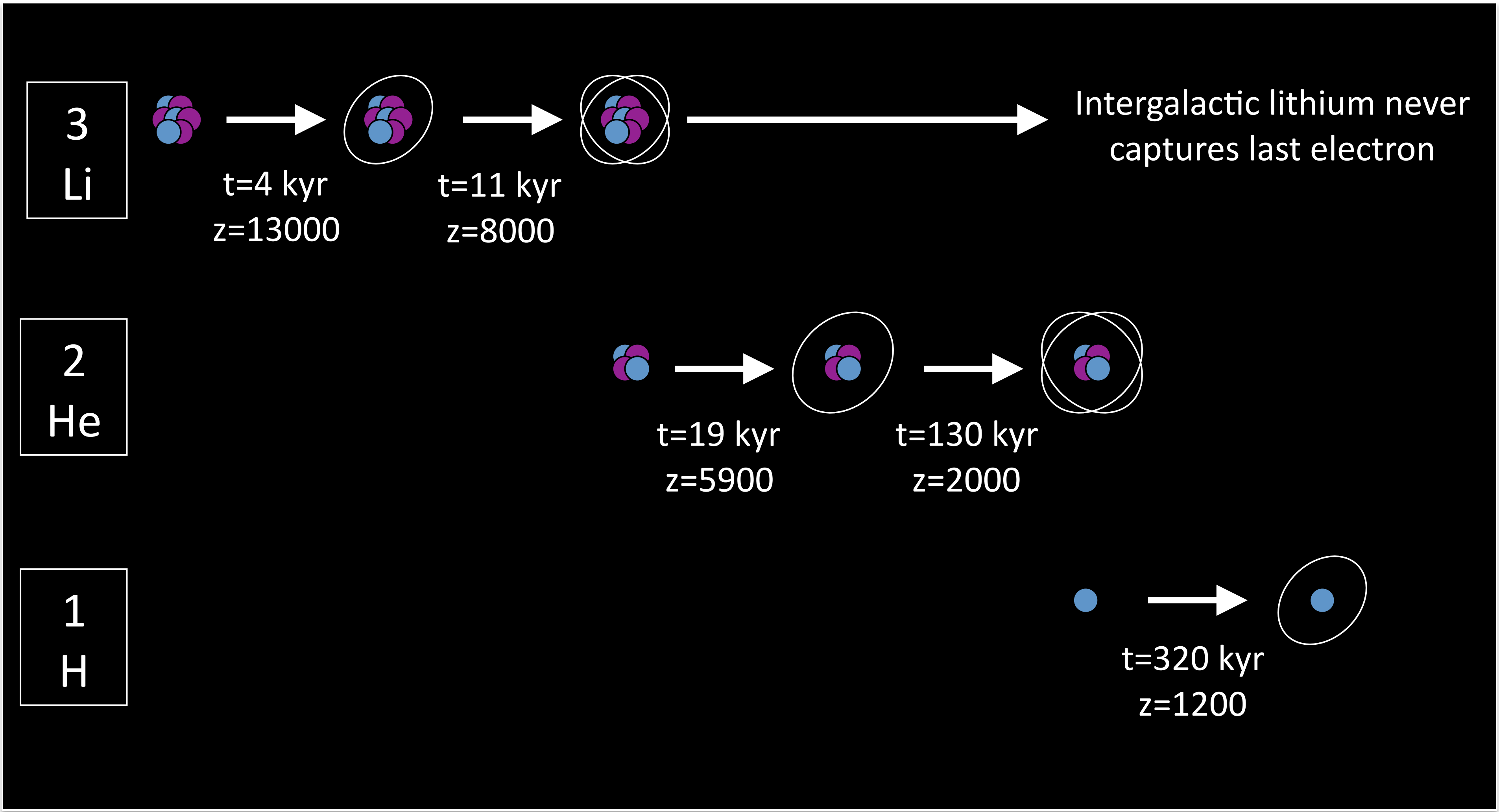
# Helium recombination

- He:H=0.08 by number, so for fully ionized gas $x_e$=1.16.
- Helium recombination similar to hydrogen:
  - He III → He II @ z~5800 (I = 54.4 eV).
  - He II → He I @ z~2000 (I = 24.6 eV) has additional pathways to ground state since at z<2200 there is a small amount of H I present.

$$\mathrm{He}(1s2p, S=0) \leftrightarrow \mathrm{He}(1s^2, S=0) + \gamma(58.4\mathrm{nm})$$

$$\mathrm{H}(1s) + \gamma(58.4\mathrm{nm}) \leftrightarrow \mathrm{H}^+ + e^-$$

# Recombination Timeline



# Implications for CMB

- Optical depth to Thomson scattering per e-fold of expansion is:

$$\frac{d\tau}{d\ln a} = \frac{n_H x_e \sigma_T c}{H} \approx 3.8 \times 10^{-3} (1+z)^{3/2} x_e$$

- At recombination era, $z \sim 10^3$ so this is >>1 when $x_e \sim 1$. Recombination makes the universe transparent.
- Integrated optical depth is unity at z~1100. ("Surface of last scattering.")

# After Recombination

- Tail end of recombination is second-order reaction. Once ionizations become insignificant:

$$\frac{dx_e}{dt} = -\alpha_B n_H x_e^2$$

$$x_e^{-1} = \text{const} + \int \alpha_B n_H dt$$

- Roughly, $\alpha_B$~$10^{-12}$ cm$^3$/s, $n_H$~100 cm$^{-3}$, t~$10^{13}$ s, so expect recombination to end with $x_e$~$10^{-3}$. Detailed calculations give $2\times10^{-4}$ (e.g. Scott & Moss 2009).

# Thermal Decoupling

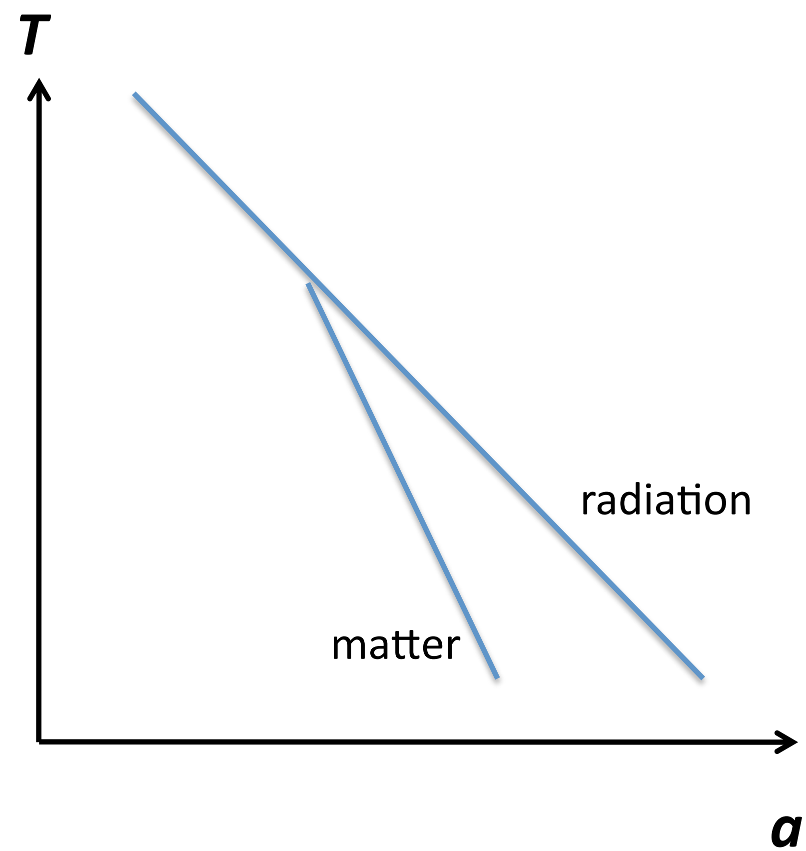
- Recall the timescale to equilibrate the baryon temperature with the CMB radiation:

$$t_b \sim \frac{m_e c^2 \left\langle E_\gamma \right\rangle^{-1}}{n_\gamma \sigma_T c x_e} = \frac{m_e c}{\sigma_T u_\gamma x_e} = 2 \times 10^{13} T_3^{-4} \text{ sec}$$

  - Now with $x_e$ factor since only free electrons contribute to Compton equilibrium.
- Compare to age of Universe which scales as $T^{-3/2}$ in matter era.
- There comes a time when the baryonic gas is no longer coupled to the CMB temperature. z=126, T=346 K (Scott & Moss 2009).

# Thermal Decoupling, Part 2

- Gas expands adiabatically. $T \sim \rho^{2/3}$ for monatomic gas, so the matter temperature $T_m \sim a^{-2}$.
- The matter cools below the CMB temperature, e.g. reaches 10 K at z=20 if there is no heat injection.



# Molecules?

- At z<400, formation of $H_2$ is thermodynamically favored.
- Reactions are slow however:

$$\mathrm{H} + \mathrm{H} \not\rightarrow \mathrm{H}_2 + \gamma$$

$$\mathrm{H} + e^- \leftrightarrow \mathrm{H}^- + \gamma$$

$$\mathrm{H}^- + \mathrm{H} \rightarrow \mathrm{H}_2 + e^-$$

- Pregalactic gas $H_2$:H ~ $6\times10^{-7}$ (Hirata & Padmanabhan 2006).

# Speed of Sound

- General: $c_s^2 = \left.\frac{\partial p}{\partial \rho}\right|_S$
- Before recombination:
  - Photons & baryons are tightly coupled. If compressed by δ ln V, we get:

$$\delta p = \delta p_r = \tfrac{1}{3}\delta u_r = -\tfrac{4}{9} u_r \delta \ln V$$

$$\delta\rho = \frac{\delta u_r}{c^2} + \delta\rho_b = -\left(\frac{4u_r}{3c^2} + \rho_b\right)\delta \ln V$$

$$c_s^2 = \frac{\tfrac{4}{9}u_r}{\frac{4u_r}{3c^2} + \rho_b} = \frac{c^2}{3\left(1 + \frac{3\rho_b c^2}{4u_r}\right)} \approx \frac{c^2}{3}$$

# Speed of Sound

- After recombination: monatomic gas, $p \sim \rho^{5/3}$:

$$c_s = \sqrt{\frac{5kT}{3m_H}} = 3.7 \times 10^5 T_3^{1/2} \text{ cm/s}$$

- Break in sound speed at z=126, cs=2×10$^5$ cm/s due to thermal evolution.
- Sound speed in DM is always zero.