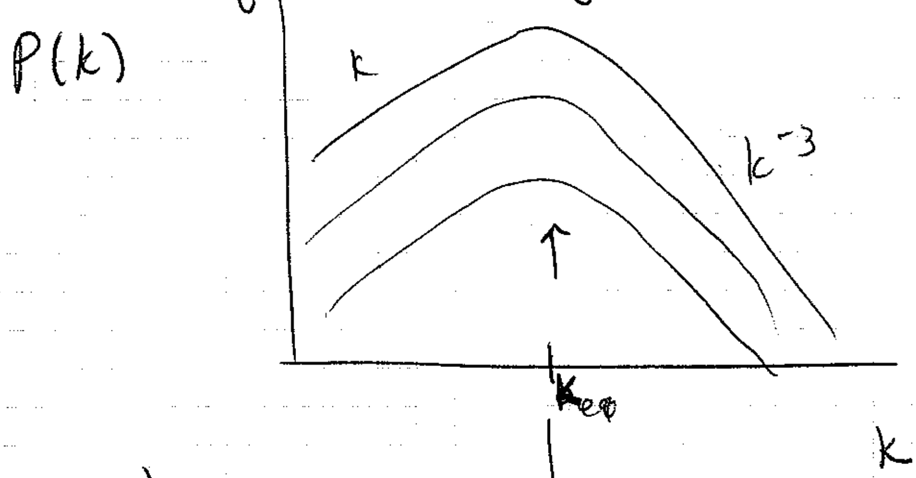
① let's define "large"



we said

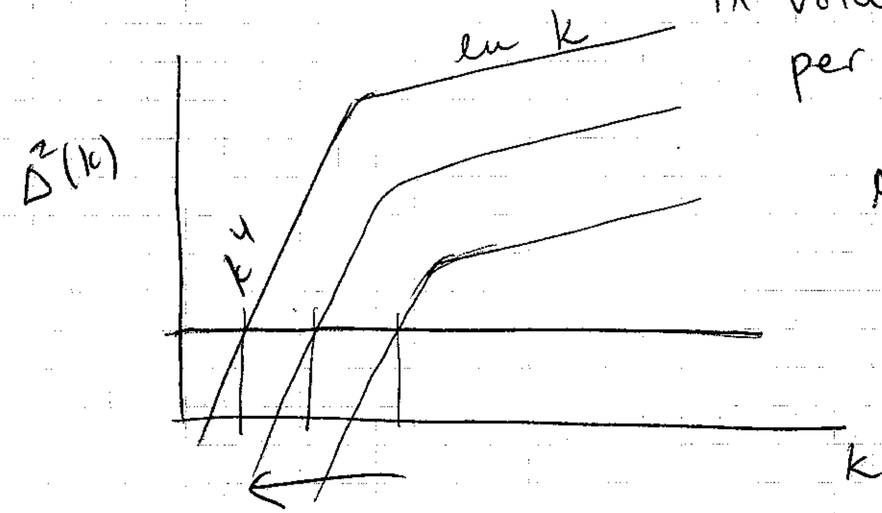
$$\delta(k,a) = \delta_0(k)\,T(k)\,G(a)$$

$$P(k,a) = |\delta(k,a)|^2$$

$$= |\delta_0(k)|^2 \underbrace{T^2(k)}_{\substack{\text{const}\\ \text{after}\\ a_{eq}}} G^2(a)$$

$\Delta^2(k)\, d\ln k \propto k^3\, P(k)\, d\ln k$

\# of waves that fit in volume with wave # k per ln k



wavelength is $\frac{2\pi}{k}$

$L = V^{1/3}$

\# of waves is $\left[\frac{V^{1/3}}{2\pi/k}\right]^3 \sim k^3$

nonlinear collapse was when $\delta \sim \delta_c = 1.69$ (last time)

thus, when $\Delta^2(k) \sim \delta_c^2$ we get collapse.

this intersection moves to <u>lower</u> K $\rightarrow$ <u>larger</u> R as universe evolves.

Today it's about $k \sim 0.02h/\mathrm{Mpc}$ or about $8\ \mathrm{Mpc}/h$
(~~set~~ norm by σ_8)

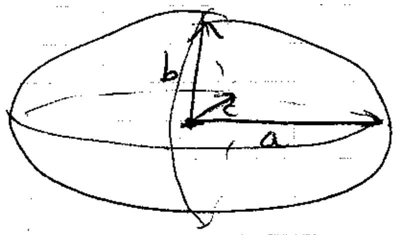
LSS $\equiv$ scales larger than $8\ \mathrm{Mpc}/h$.

$\rightarrow$ free-fall time of galaxy cluster? $\sim 10\ \mathrm{Mpc}$.

(2) What does univ. look like @ $\lambda > 10$ Mpc?

— SLIDES —

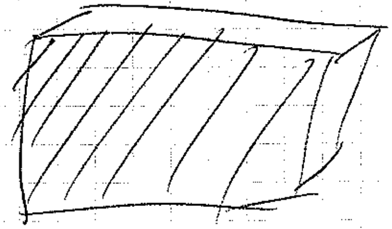
topology ~~elements~~ of structure $\leftrightarrow$ ellipsoidal collapse



$a > b > c$

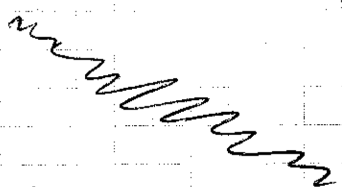
since $t_{ff} \sim r^{3/2}$,
difference is important @ large scales!

if only (c) collapses $\rightarrow$



wall (2D)

if b, c $\longrightarrow$



filament (1D)

if a, b, c $\longrightarrow$

cluster (0D)

~~(3) Features of LSS nearby~~

~~— slides —~~

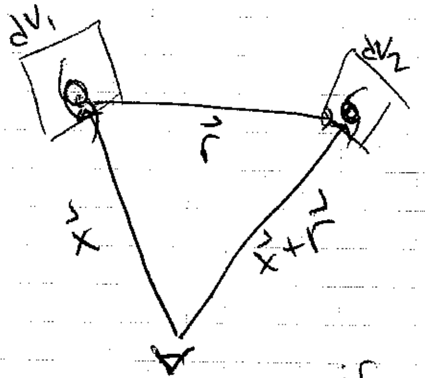
(4) From obsv → measurement of $\delta(k,a)$

Main workhorse: statistics of pairs of galaxies

Motivations – A) easy!

B) gaussian density field

↳ σ^2 describes sufficiently,



presumes that galaxies trace structure perfectly (not true – later)

if density is ρ_o everywhere,

then $P(\text{pair}|V_1,V_2) = P(\text{galaxy}|V_1)\,P(\text{galaxy}|V_2)$

$= \rho_o^2\, dV_1\, dV_2$

However, we have $\rho(\vec{x}) = \rho_o[1+\Delta(\vec{x})]$ to 1st order

so $P(\text{pair}|V_1,V_2) = \rho_o^2[1+\Delta(\vec{x})][1+\Delta(\vec{x}+\vec{r})]\,dV_1\,dV_2$

$$dN_{pair} = \left[\rho_o^2 + \rho_o^2\,\Delta(\vec{x}) + \rho_o^2\,\Delta(\vec{x}+\vec{r}) + \rho_o^2\,\Delta(\vec{x})\Delta(\vec{x}+\vec{r})\right] dV_1 dV_2$$

$$\langle N_{pair}\rangle_{Vol} = \rho_o^2\Big[V^2 + V\cancelto{0}{\int\Delta(\vec{x})\,dV} + V\cancelto{0}{\int\Delta(\vec{x}+\vec{r})\,dV}$$

$$+ V\int\Delta(\vec{x})\,\Delta(\vec{x}+\vec{r})\,d^3\vec{x}\Big]$$

$$= N^2 + N\rho_o\underbrace{\langle\Delta(\vec{x})\Delta(\vec{x}+\vec{r})\rangle_{Vol}}_{\xi(\vec{r})}$$

2-point CF $\xi(\vec{r})$ reflects underlying density perts Δ^2

4 con't From ξ to $P(k)$ presume $\Delta = \delta$

$$\xi(\vec{r}) = \int d^3\vec{x}\ \delta(\vec{x})\,\delta(\vec{x}+\vec{r})$$ convolution of δ

recall $\delta(\vec{x}) = \frac{V}{(2\pi)^3}\int \delta(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}}\, d^3\vec{k}$

$$\xi(\vec{r}) \propto \int d^3\vec{x} \left(\int \delta(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}}\, d^3\vec{k}\right)\left(\int \delta(\vec{k}')\, e^{-i\vec{k}'\cdot(\vec{x}+\vec{r})}\, d^3\vec{k}'\right)^*$$

$$\propto \int d^3\vec{k}\, d^3\vec{k}'\, \underbrace{d^3\vec{x}\ \delta(\vec{k})\,\delta^*(\vec{k}')\, e^{i\vec{k}'\cdot\vec{r}}\, e^{i(\vec{k}'-\vec{k})\cdot\vec{x}}}_{\delta^3_{dir}(\vec{k}'-\vec{k})}$$

$$\propto \int d^3\vec{k}\, \underbrace{\left|\delta(\vec{k})\right|^2}_{=P(k)} e^{i\vec{k}\cdot\vec{r}}$$

so $\xi(\vec{r})$, $P(\vec{k})$ are F.T. pair.

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(5) Parameters reminder:

norm:	σ_8	collapsing now
shape:	k_{eq}	collapsed at a_{eq}
tilt:	n	slope at inflation

- ~~Constraints from Surveys~~
  ~~(Slides)~~

- $\boxed{\text{Non-gaussianity} - \langle \xi\xi\xi \rangle \text{ etc.}}$

⑥ Problems with redshift $\leftrightarrow$ distance

- remember, $\vec{V}_{physical} = \vec{V}_{pec} + aH\vec{x}_{comoving}$

$$= \vec{V}_{pec} + H\vec{x}_{phys}$$

$$V_{phys,r} = V_{pec,r} + Hd$$

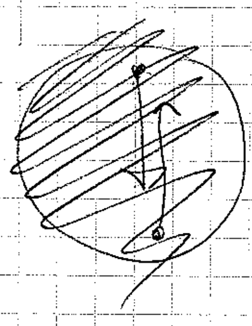
$$\Rightarrow d = \frac{V_{phys} - V_{pec}}{H}$$ ← presume this is zero.

what if $V_{pec} \neq 0$?

A) clusters have $\sigma^2 \sim (300\ km/s)^2$

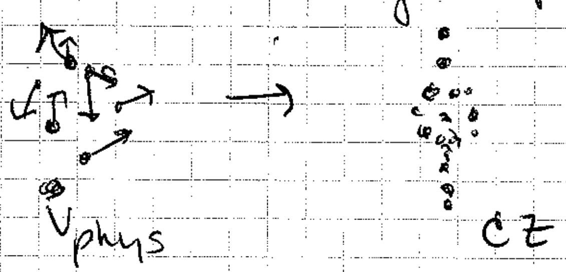
which is an effective redshift of

$$z \sim \frac{300\ km/s}{3\times10^5\ km/s} \sim 0.001$$

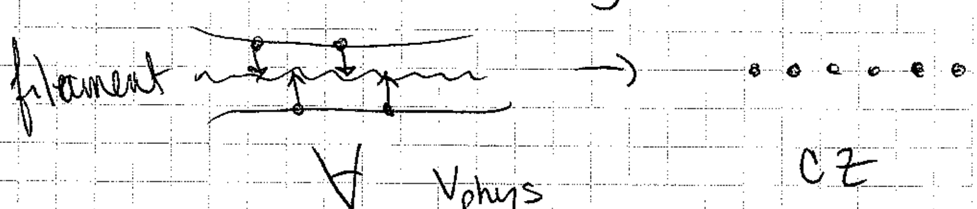


↳ cluster will be "smeared out" along l.o.s with this size

⇒ "fingers of god" effect



B) still collapsing structures have bulk flows



"great wall" effect

Redshift - space distortion

s: redshift space

$$n_s(\vec{x}_s)\, d^3\vec{x}_s = n(\vec{x})\, d^3\vec{x}$$

define $n_s(\vec{x}_s) = n(\vec{x})\, J$, $\quad J \equiv \left| \dfrac{d^3\vec{x}}{d^3\vec{x}_s} \right|$

using $z = H_0 x + \vec{V}_{pec} \cdot \hat{x}$

$$\hookrightarrow \quad x_s = x + \frac{\vec{V}_{pec} \cdot \hat{x}}{H_0} \quad \cdots \rightarrow$$

$$\frac{d^3\vec{x}}{d^3\vec{x}_s} = \frac{x^2\, dx\, \cancel{d\theta \sin\theta\, d\phi}}{x_s^2\, dx_s\, \cancel{d\theta \sin\theta\, d\phi}}$$

assume no distortion to angular patch

$$J = \left(\frac{x}{x_s}\right)^2 \frac{dx}{ds}$$

$$J = \underbrace{\left(1 + \frac{\vec{V}_{pec} \cdot \hat{x}}{H_0 x}\right)^{-2}}_{\mathcal{O}\left(\frac{V}{H_0 x}\right)} \underbrace{\left(1 + \frac{\partial}{\partial x}\left[\frac{\vec{V}_{pec} \cdot \hat{x}}{H_0}\right]\right)^{-1}}_{\mathcal{O}\left(\frac{kV}{H_0}\right)}$$

→ ratio is $\sim kx$

what are k and x?

x: size of survey

k: modes to be measured

$\left(\frac{2\pi}{k} \ll x, \text{ so } \underline{kx \gg 1}\right)$

so, derivative term is most important

$$J \simeq 1 - \frac{\partial}{\partial x}\left(\frac{\vec{V}_{pec} \cdot \hat{x}}{H_0}\right)$$

if $n = \bar{n}(1+\delta)$, $n_s = \bar{n}(1+\delta_s)$

$$1 + \delta_s \simeq (1+\delta)\left[1 - \frac{\partial}{\partial x}\left(\frac{\vec{V}_{pec} \cdot \hat{x}}{H_0}\right)\right] \rightarrow \delta_s \sim \delta - \frac{\partial}{\partial x}\left[\frac{\vec{V}(\vec{x}) \cdot \hat{x}}{H_0}\right]$$

<u>RSD</u> $\quad \delta_s \sim \delta - \frac{\partial}{\partial x}\left[\frac{\vec{v}_{pec}\cdot\hat{x}}{H_0}\right]$

assuming $\vec{v}\cdot\hat{x} \simeq \vec{v}\cdot\hat{z}$ for all galaxies
(distant obsv)

we can do FT to find — from $\vec{v}_{pec}$ definition

$$\delta_s(\vec{k}) \mathrel{\hat{=}} \delta(\vec{k})\left[1 + f(a)(\hat{z}\cdot\vec{k})^2\right] \quad \text{at low } z$$

$f(a)$: growth rate! $\sim \Omega_m^{0.6}$

$(\hat{z}\cdot\vec{k})$: component of mode along los

1) $f(a)(\hat{z}\cdot\vec{k})^2 > 0 \Rightarrow$ overdensities <u>larger</u> than real spc along los dir only → "great wall"

2) Power spectrum is also biased

$$P_s(k) \sim |\delta_s(k)|^2 = P(k)\left[1 + \beta(\hat{z}\cdot\vec{k})^2\right]^2$$

β: bias

$\beta = \frac{f}{b}$ where $b \equiv \frac{\delta_g}{\delta}$

note velocities sample mass, galaxies sample <u>biased</u> distrib^n of mass.

Slide showing RSD from 6dFGS

(5) Constraints from surveys (summarize 8 params)

1) Clustering analysis now uses anisotropic $\vec{k} = k_{\parallel} \hat{e}_{\parallel} + k_{\perp} \hat{e}_{\perp}$ (or $\vec{s}$)

→ incorporates RSD

→ constrains $f\sigma_8$ product

2) Use linear theory + ~~[illegible]~~ CMB power → BAO

→ r_d : distance to "CMB"

→ $H(z)$ and angular diam dist $D_M \equiv (1+z) D_A(z)$

via standard ruler (peak k is known)

Combination $D_V = \left(\dfrac{cz D_M^2(z)}{H(z)} \right)^{1/3}$

constrained by angle avg

A-P $F_{AP} = \frac{1}{c} D_M(z) H(z)$ clustering should match in angular + radial dir

Velocity correlations $\vec{v}_{phys} = \vec{v}_{pec} + aH\vec{x}_{com}$

$$\vec{\nabla} \cdot \vec{v}_{pec} = -aH\delta f(a) \qquad f(a) \equiv \frac{d\ln G}{d\ln a}$$

in Fourier space $\vec{k} \cdot \vec{v}_{pec}(\vec{k}) = -aH\,\delta(\vec{k}) f(a)$

so $v(k,a) \sim \dfrac{\delta(k,a)}{k} \Rightarrow$ larger scale modes have higher weight

Also, velocities are not biased (grav)

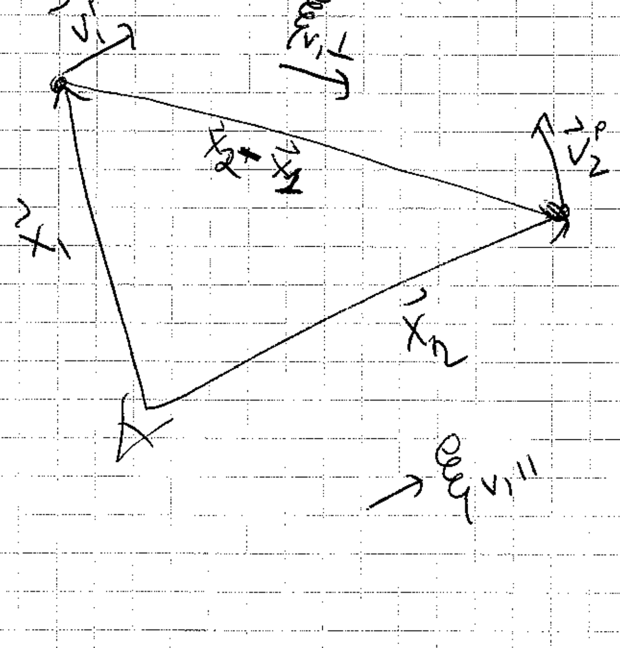
Velocity c.f.

$$\xi_v(\vec{x}_1, \vec{x}_2) = \langle \underbrace{\vec{v}(\vec{x}_1) \cdot \hat{x}_1}_{\text{along los}} \quad \underbrace{\vec{v}(\vec{x}_2) \cdot \hat{x}_2}_{\text{along los}} \rangle$$

SDSS uses this in P.S. form to break bias degeneracies

[+ cross - power spectrum $\xi^*_{gv}(\vec{x}_1, \vec{x}_2)$]

→ in practice, break up everything (spatial + velocity)



How to measure peculiar velocities

$$\vec{v}_{phys} = \vec{v}_{pec} + \underbrace{a H \vec{x}_{com}}$$

what you get from spectra

measure distance independently to get $\vec{x}$, then you know a, z, $H(z)$

$$\vec{v}_{pec} = \vec{v}_{phys} - cz(\vec{x})$$

Distance measures:

Fundamental Plane $\quad R_e \propto \sigma_0^a \langle I_e \rangle^b$

radius, velocity disp, mean SB

~~compare to~~:

$$\langle I_e \rangle R^2 = L$$

Tully - Fisher reln

$$L \propto \langle v \rangle^\alpha$$

compare to observed brightness to get distance

(see plots)

Constraining modified gravity with velocities

In ΛCDM, Ω_Λ is scale free so $f \sim \Omega_m^\gamma$

if $\exists$ scale-dependent modifications to gravity, $f(k,a)$ not just $f(a)$

one common param. is in terms of Φ, Ψ

$$ds^2 = -(1+2\Psi)\,dt^2 + a^2(1+2\Phi)(dx^2+dy^2+dz^2)$$

Newtonian potential

spatial curvature

$\boxed{\text{in Newtonian limit, } \Psi = \Phi}$

Ψ, Φ obey poisson eqns

$$\nabla^2\Psi = 4\pi G a^2 \bar{\rho}_m \delta_m$$

[From GR]

$$\nabla^2(\Phi+\Psi) = 8\pi G a^2 \bar{\rho}_m \delta_m$$

Gravity could modify these:

$$\nabla^2\Psi = 4\pi G a^2 \bar{\rho}_m \delta_m \times G_{matter}$$ ← Poisson

$$\nabla^2(\Psi+\Phi) = 8\pi G a^2 \bar{\rho}_m \delta_m \times G_{light}$$ ← light bending

see slides