

**Ay 20 - Fall 2004 - Lecture 17**

**Stellar Luminosity and Mass  
Functions**

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**History and Formation of  
Our Galaxy**

# Stellar Luminosity and Mass Functions

- Basic statistical descriptors of stellar populations: probability distribution for stellar luminosities (a function of the bandpass) and masses
- Most important: stellar Initial Mass Function (IMF) = mass function at the formation time
- Key in understanding and modeling star formation and galaxy evolution
- Needed in order to estimate stellar (baryonic?) mass content of galaxies (and other stellar systems) from the observed luminosities
- Very, very hard to do - depends on having lots of very reliable, well-understood data and calibrations

# Determining the IMF is a tricky business...

- Observed star counts
  - Understand your selection effects, completeness
  - Get the distances
  - Estimate the extinction
  - Correct for unresolved binaries
- Get the Present-Day Luminosity Function (PDLF)
  - Assume the appropriate mass-luminosity relation
  - It is a function of metallicity, bandpass, ...
  - Theoretical models tested by observations
- Get the Present-Day Mass Function (PDMF)
  - Assume some evolutionary tracks, correct for the evolved stars (also a function of metallicity, ...)
  - Assume some star formation history
- Get the Initial Mass Function (IMF)!

## Luminosity functions

Suppose we measure the distance and apparent magnitude  $m$  of all stars within some limiting distance  $d_{\text{max}}$  (a 'volume limited sample' - easier in theory than in practice!).

Convert from apparent magnitude to absolute magnitude  $M$  using the known distance  $d$  to each star and the definition:

$$M = m - 5 \log_{10} \left( \frac{d}{10 \text{ pc}} \right)$$

↑  
absolute magnitude in some  
waveband, e.g. in the visual  $M_V$

Finally count the number of stars with  $M$  between  $(M-0.5)$  and  $(M+0.5)$ , and divide by the volume  $\frac{4}{3}\pi d_{\text{max}}^3$  surveyed. Gives the **luminosity function**.

*(From P. Armitage)*

More formally:

$$\Phi(M)\Delta M = \text{number density (stars per pc}^3\text{) of stars with absolute magnitude } M \text{ between } M \text{ and } M+\Delta M$$



luminosity function

Identical concept applies to galaxies (though typically measure numbers of galaxies per  $\text{Mpc}^3$  rather than per  $\text{pc}^3$ ).

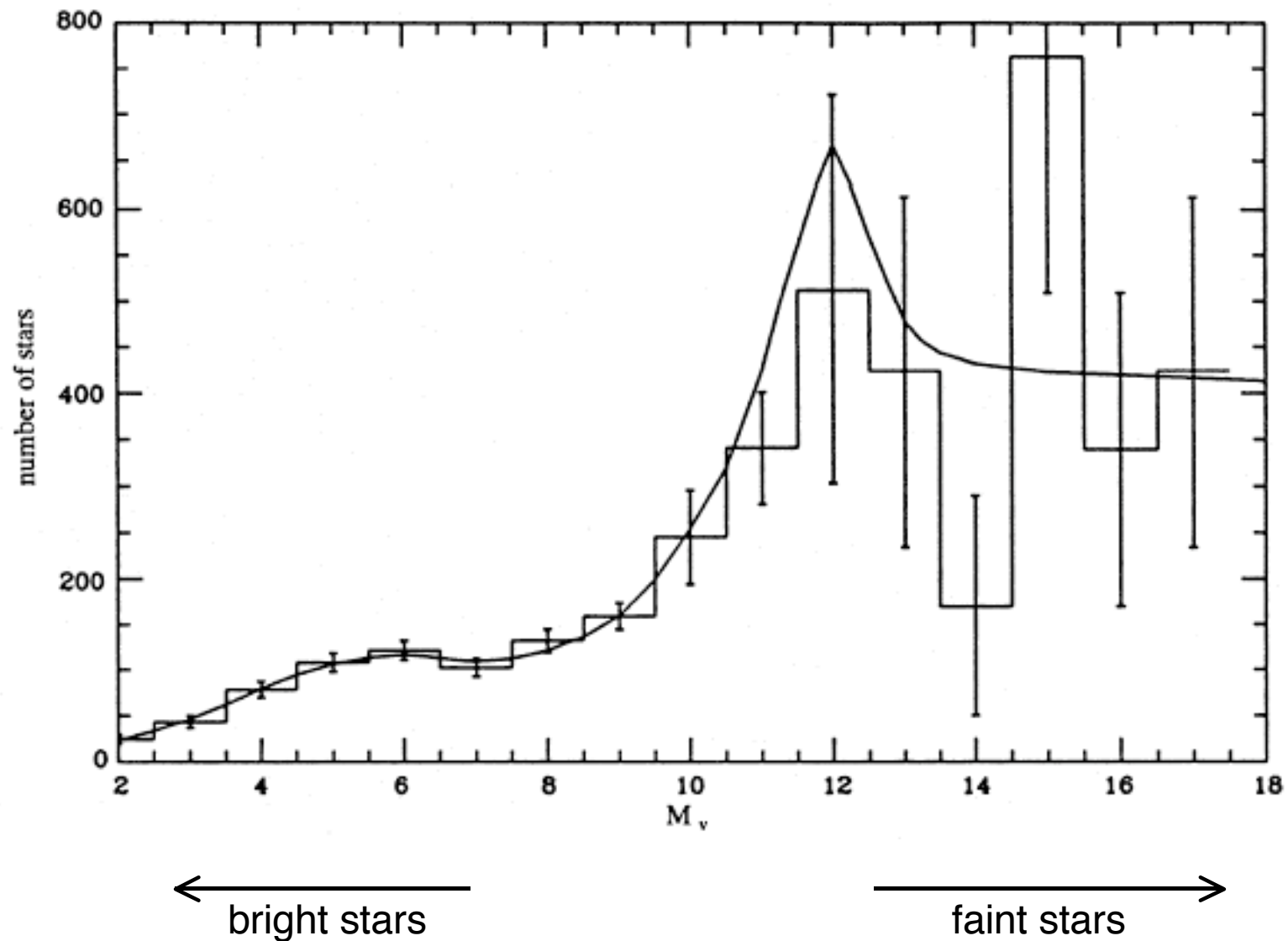
Can be hard to measure  $\Phi(M)$ :

- for very low mass stars ( $M$  large), which are dim unless very close to the Sun
- for massive stars ( $M$  small), which are rare

Luminosity function is the basic observable for studying a population of stars.

*(From P. Armitage)*

Local luminosity function (stars with  $d < 20$  pc) for the Milky Way measured by Kroupa, Tout & Gilmore (1993):



(From P. Armitage)

## Initial Mass Function (IMF)

Starting from the observed luminosity function, possible to derive an estimate for the Initial Mass Function (IMF). To define the IMF, imagine that we form a large number of stars. Then:

$$\xi(M)\Delta M = \text{the **number** of stars that have been born with initial masses between } M \text{ and } M+\Delta M \text{ (careful not to confuse mass and absolute magnitude here)}$$

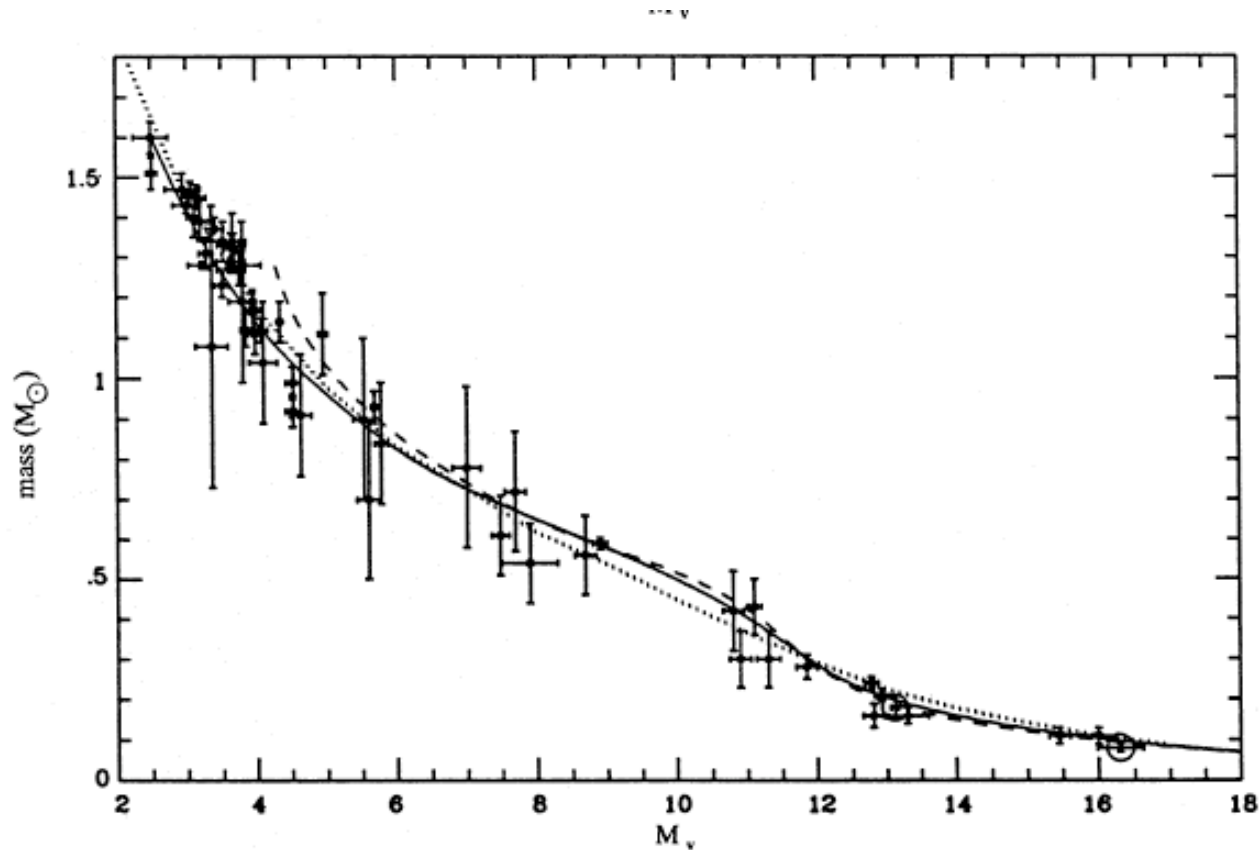
↑  
this is the Initial Mass Function or IMF

The IMF is a more fundamental theoretical quantity which is obviously related to the star formation process. Note that the IMF only gives the distribution of stellar masses immediately after stars have formed - it is **not** the mass distribution in, say, the Galactic disk today.

*(From P. Armitage)*

In practice: several obstacles to getting the IMF from the luminosity function:

Convert from absolute magnitude to mass



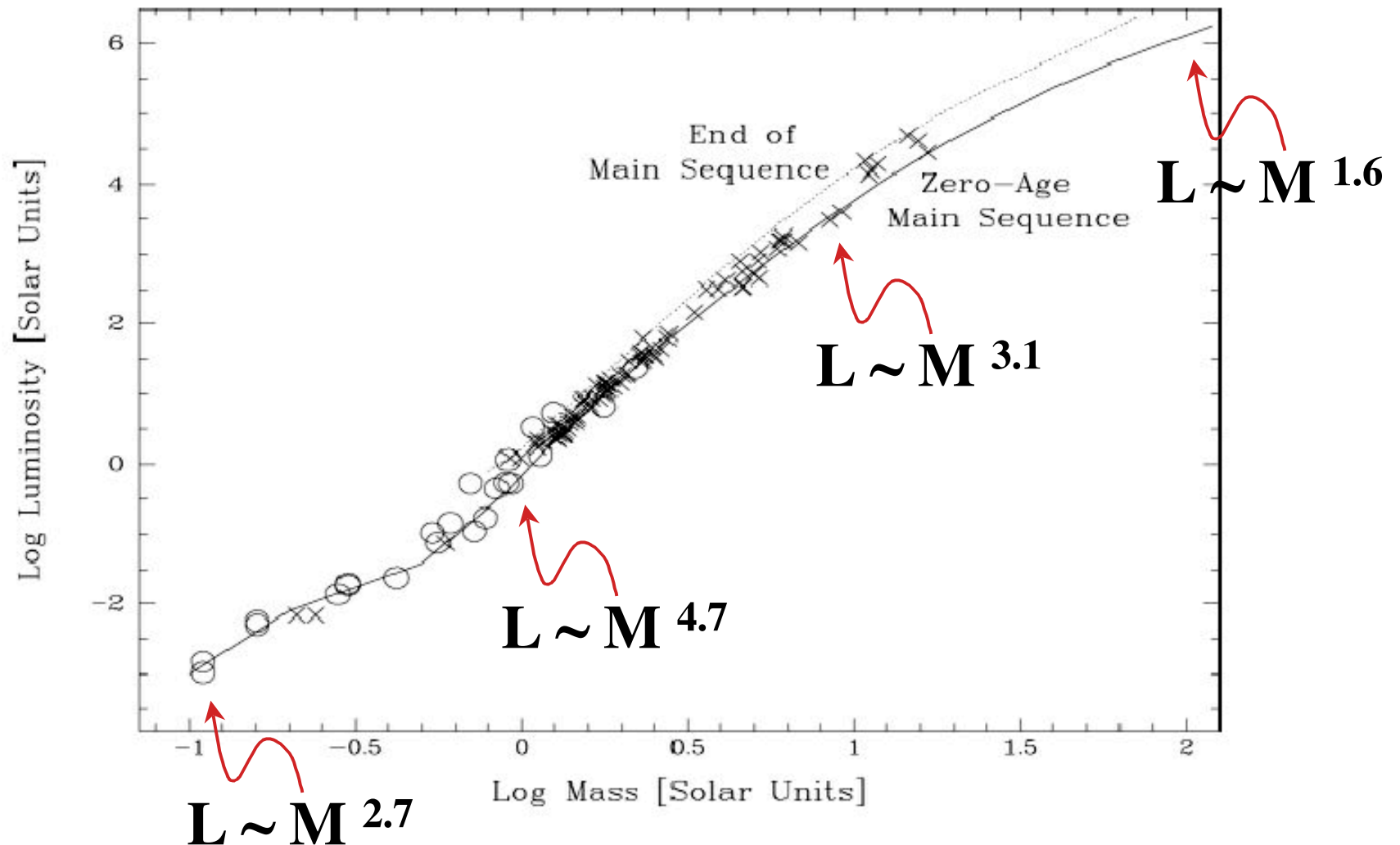
Need stellar structure theory, calibrated by observations of eclipsing binaries.

*(From P. Armitage)*



# The Mass-Luminosity Relation

Use it to convert stellar luminosities into masses



And, of course, it is a function of bandpass ...

## Massive stars have short lifetimes

Suppose we observe the luminosity function of an old cluster. There are **no** very luminous main sequence stars. But this does not mean that the IMF of the cluster had zero massive stars, only that such stars have ended their main sequence lifetimes.

More generally, we need to allow for the differing lifetimes of different stars in deriving the IMF. If we assume that the star formation rate in the disk has been constant with time, means we need to weight number of massive stars by  $1 / \tau_{\text{ms}}$ , where  $\tau_{\text{ms}}$  is the main sequence lifetime.

**Massive stars are doubly rare** - few are formed plus they don't live as long as low mass stars...

*(From P. Armitage)*

## Mass loss

For massive stars, mass loss in stellar winds means that the present mass is smaller than the initial mass.

These difficulties mean that although the local IMF is well determined for masses between  $\sim 0.5 M_{\text{sun}}$  and  $\sim 50 M_{\text{sun}}$ :

- not well determined at the very low mass end (mainly because the relation between luminosity and mass is not so well known)
- for very massive stars - simply too rare
- in other galaxies, especially in the distant Universe

*(From P. Armitage)*

## Salpeter Mass Function

The Initial Mass Function for stars in the Solar neighborhood was determined by Salpeter in 1955. He obtained:

$$\xi(M) = \xi_0 M^{-2.35} \quad \text{Salpeter IMF}$$

↑  
constant which sets the local stellar density

Using the definition of the IMF, the number of stars that form with masses between  $M$  and  $M + \Delta M$  is:  $\xi(M)\Delta M$

To determine the total number of stars formed with masses between  $M_1$  and  $M_2$ , integrate the IMF between these limits:

$$\begin{aligned} N &= \int_{M_1}^{M_2} \xi(M) dM = \xi_0 \int_{M_1}^{M_2} M^{-2.35} dM \\ &= \xi_0 \left[ \frac{M^{-1.35}}{-1.35} \right]_{M_1}^{M_2} = \frac{\xi_0}{1.35} \left[ M_1^{-1.35} - M_2^{-1.35} \right] \end{aligned}$$

*(From P. Armitage)*

Can similarly work out the total **mass** in stars born with mass  $M_1 < M < M_2$ :

$$M_* = \int_{M_1}^{M_2} M \xi(M) dM$$

Properties of the Salpeter IMF:

- most of the stars (by number) are low mass stars
- most of the **mass** in stars resides in low mass stars
- following a burst of star formation, most of the **luminosity** comes from high mass stars

Salpeter IMF must fail at low masses, since if we extrapolate to arbitrarily low masses the total mass in stars tends to infinity!

Observations suggest that the Salpeter form is valid for roughly  $M > 0.5 M_{\text{sun}}$ , and that the IMF 'flattens' at lower masses. The exact form of the low mass IMF remains uncertain.

*(From P. Armitage)*

## What is the origin of the IMF?

Most important unsolved problem in star formation. Many theories but no consensus.

Observationally, known that dense cores in molecular clouds have a power-law mass function rather similar to the IMF. So the IMF may be determined in part by how such cores form from turbulent molecular gas.

## Is the IMF `universal'?

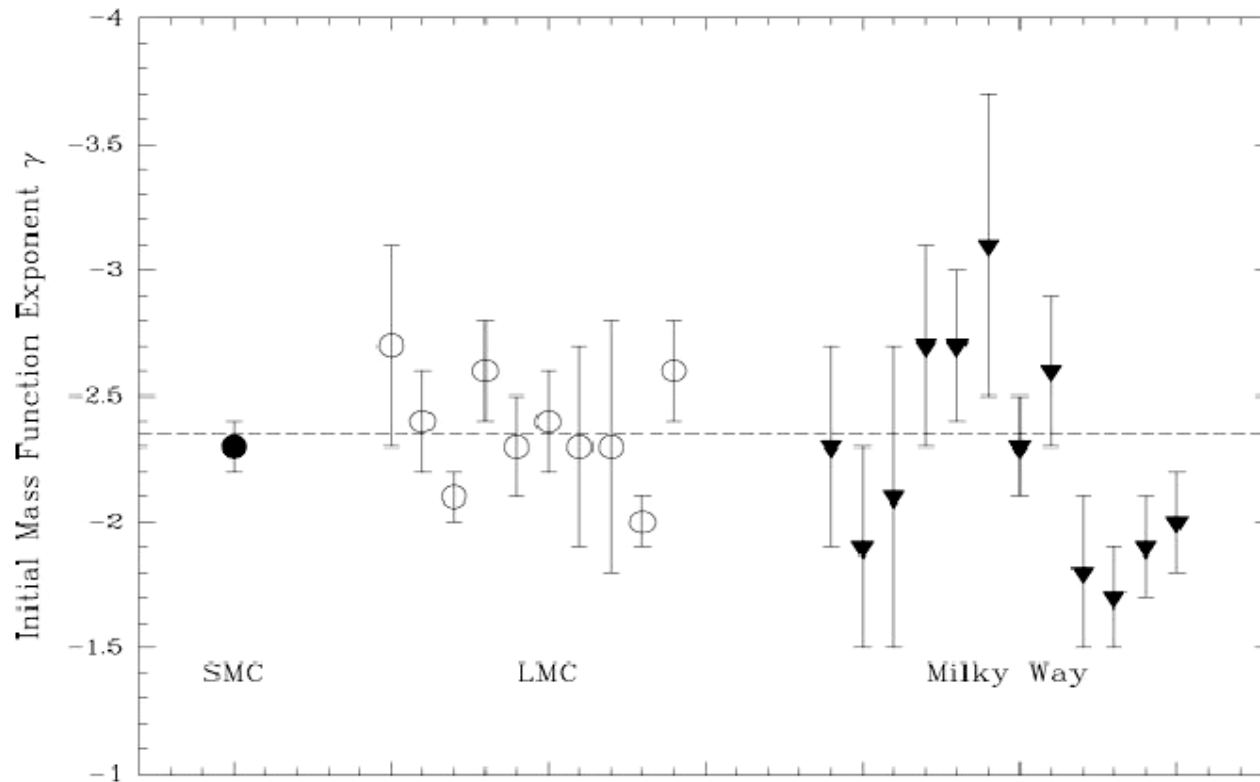
i.e. is  $\xi(M)$  the **same function** everywhere?

Most theorists say no. Predict that fragmentation is easier if the gas can cool, so primordial gas without any metals should form more massive stars.

Observationally, little or no evidence for variations in the IMF in our galaxy or nearby galaxies.

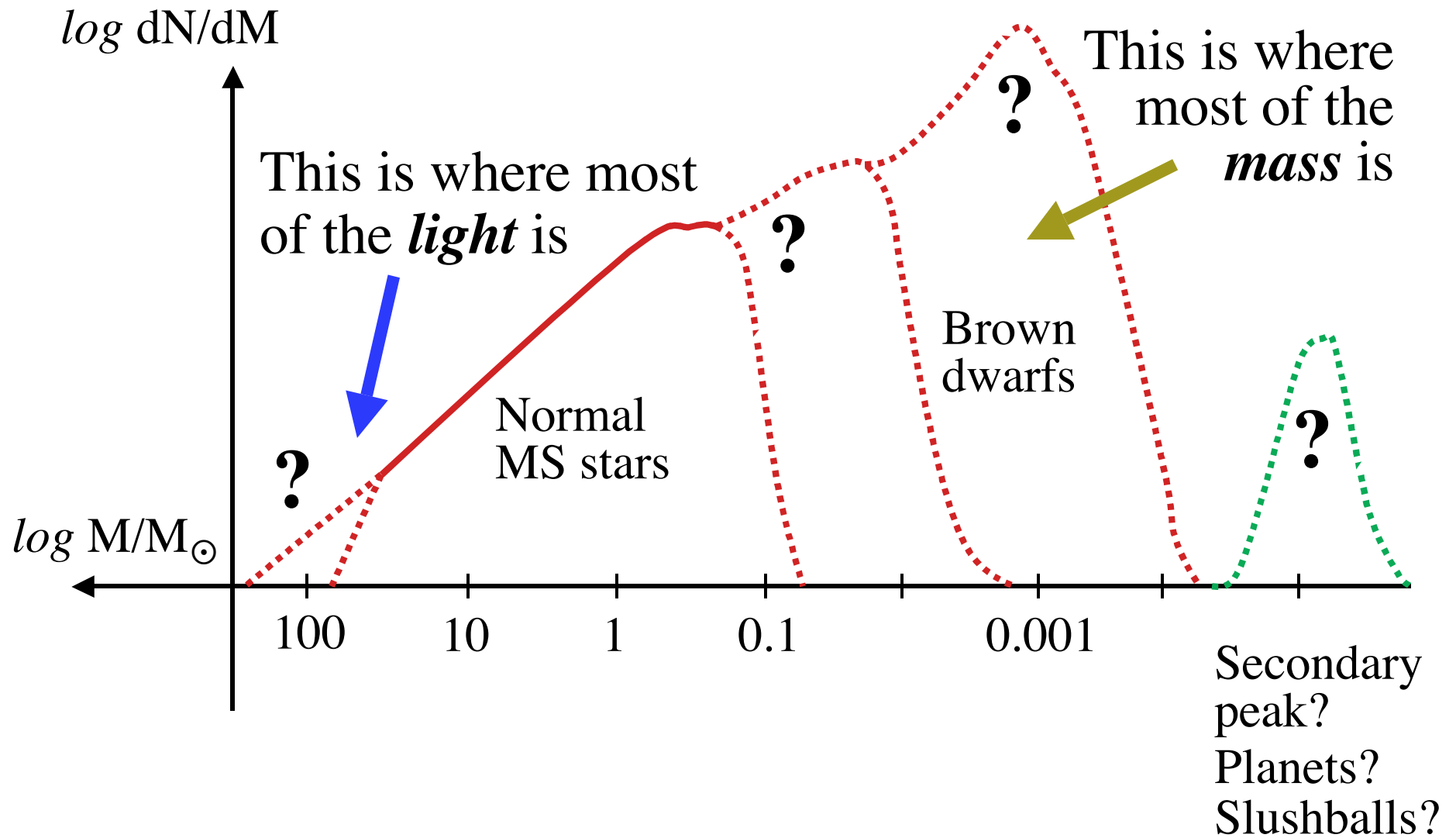
*(From P. Armitage)*

# The Universality of the IMF?



Looks fairly universal in the normal, star-forming galaxies nearby. However, it might be different (top-heavy?) in the ultraluminous starbursts (if so, the inferred star formation rates would be wrong). It might be also a function of metallicity: top-heavy for the metal-poor systems (including the primordial star formation).

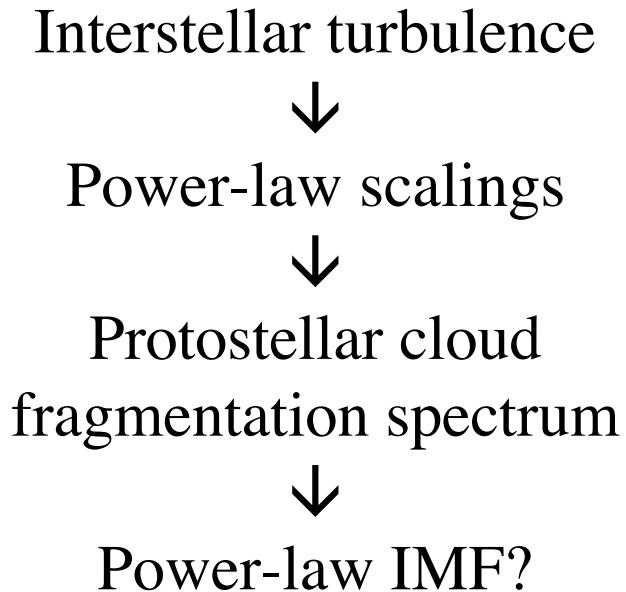
# Where are the Baryons?





# The Physical Origin of the IMF

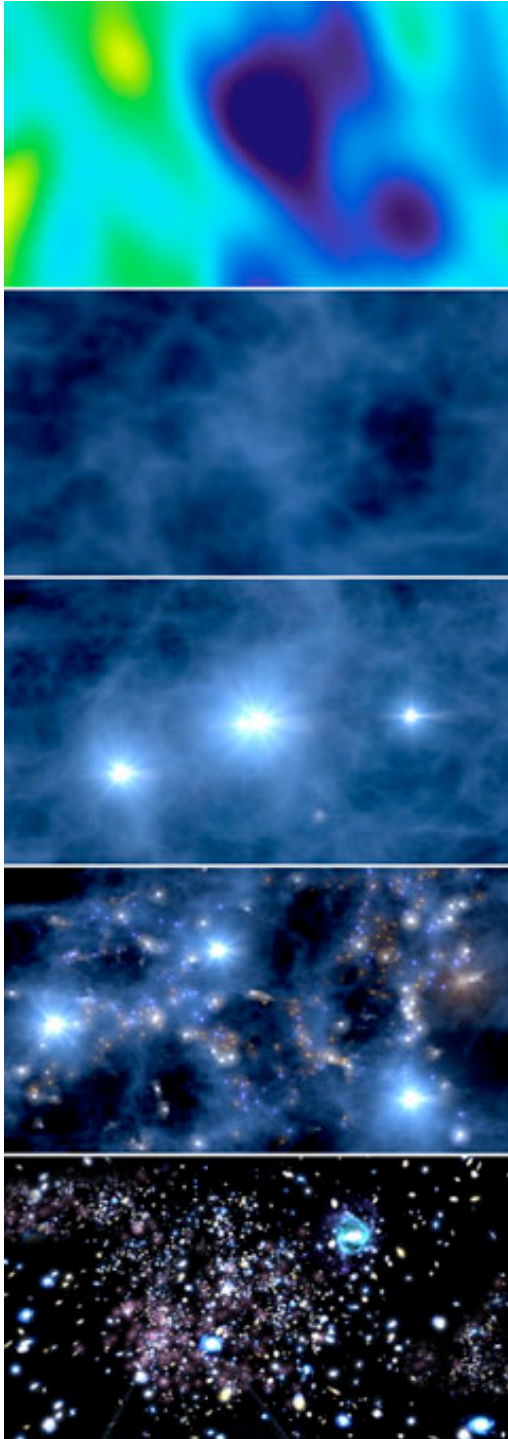
Not yet well  
understood ...



*Numerical simulation  
of ISM turbulence →*

# The Basic Paradigm of Structure Formation

- Structures (galaxies, clusters...) form from positive density fluctuations originating from the early universe, and grow through gravitational collapse, accretion, and merging
- Small scale fluctuations are erased before they can grow; the efficiency of this process depends on the nature of the dark matter (DM)
- Larger structures are assembled through hierarchical merging, which continues today
- DM dominates the mass assembly, and is dissipationless. Baryons follow, with dissipative energy release (star formation, AGN...)



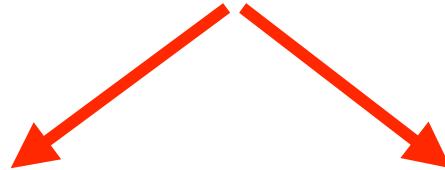
## The Early Cosmic History:

- Age 380,000 yrs: Recombination.  
The cosmic Dark Ages begin
- Dark matter condenses: seeds of the first stars and galaxies
- Age  $\sim$  500 million years: the first stars light up the universe
- Age  $\sim$  1 billion years: many galaxies, quasars, etc.  
Reionization complete
- From then on: galaxies and large-scale structures evolve

**Structure Formation:** Newtonian gravity, growth of density fluctuations, hierarchical merging, DM dominated, easy to simulate:

But what is *observed* is **Star Formation and AGN:** messy, poorly understood, difficult to simulate.

# The Two Fundamental Aspects of Galaxy Formation



## Assembly of the Mass

Collapse, infall, merging

Time scales  $\sim 10^8 - 10^9$  yr

Binding energy release

$$\frac{1}{2} |E_{\text{pot}}| \sim 10^{59} \text{ erg}$$

## Star Formation

Bursts and “steady”

Time scales  $\sim 10^7 - 10^{10}$  yr

Nuclear burning in stars

$$\epsilon \Delta X m c^2 \sim 10^{61} \text{ erg}$$

# Dissipative vs. Dissipationless Collapse

Density fluctuations initially expand with the expanding universe. Gravitationally bound density fluctuations (with  $E_{\text{kin}} < |E_{\text{pot}}|$ ) eventually turn around and collapse, on a *free-fall time scale*,  $t_{\text{ff}}$ .

Initially (at the turnaround)  $E_{\text{kin}} = |E_{\text{pot}}|$  :  $Mv^2/2 = GM^2/R_{\text{init}}$

After the virialization  $E_{\text{kin}} = 1/2 |E_{\text{pot}}|$  :  $Mv^2/2 = 1/2 GM^2/R_{\text{fin}}$

Thus,  $R_{\text{fin}} = 1/2 R_{\text{init}}$ , and  $\langle \rho \rangle_{\text{fin}} = 8 \langle \rho \rangle_{\text{init}}$

To achieve higher densities, one must *dissipate energy*, e.g., via radiative cooling. This has a characteristic time scale,  $t_{\text{cool}} = f(\text{density, temperature})$

If  $t_{\text{cool}} < t_{\text{ff}}$ , the collapse is *dissipative*, leading to high densities.

# Time Scales for Structure Formation

The free-fall time is:  $t_{\text{ff}} = (\pi^2/8G)^{1/2} R_{\text{init}}^{3/2} M^{-1/2}$

$$\approx 5 \times 10^8 \text{ yr } (R_{\text{init}} / 100 \text{ kpc})^{3/2} (M / 10^{12} M_{\odot})^{-1/2}$$

$$\approx 1.6 \times 10^9 \text{ yr } (R_{\text{init}} / 10 \text{ Mpc})^{3/2} (M / 10^{15} M_{\odot})^{-1/2}$$

For galaxies and protogalactic fragments,  $t_{\text{ff}} \sim 10^7 - 10^8$  yr (coincidentally  $\sim$  starburst time scale), but note also that for these systems  $t_{\text{cool}} < t_{\text{ff}}$

**→ Galaxies form quickly, at high redshifts** (but evolve on a slower time scales of mergers and stellar evolution,  $\sim 10^9$  yr)

For clusters,  $t_{\text{ff}} \sim 10^9 - 10^{10}$  yr

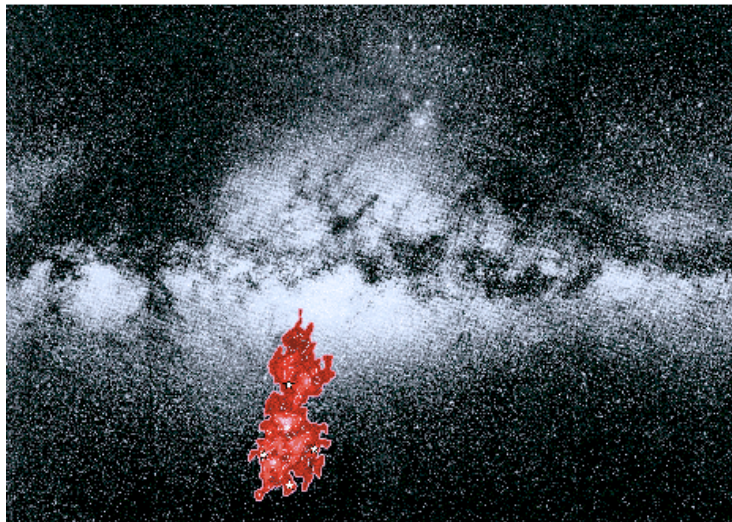
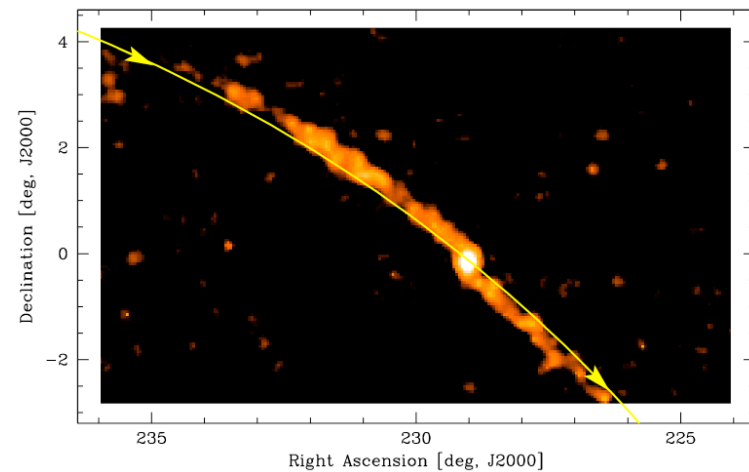
**→ Clusters are still forming today**

# Formation of the Milky Way Galaxy: The Early Models

- **The Monolithic Collapse** (Eggen, Lynden-Bell, & Sandage)
  - Galaxy bulge and halo form within a single free-fall time
  - Stars on more eccentric ( $\sim$  radial) orbits most metal-poor
  - Disk forms later
  - No longer believed, but some aspects may be right
- **Hierarchical Merging** (Searle & Zinn)
  - Old stars form in small systems ( $\sim$  dwarf galaxies), and get merged into the Galactic halo
  - Disk forms later
  - Now believed to be basically correct, but does not treat formation of the metal-rich bulge very well



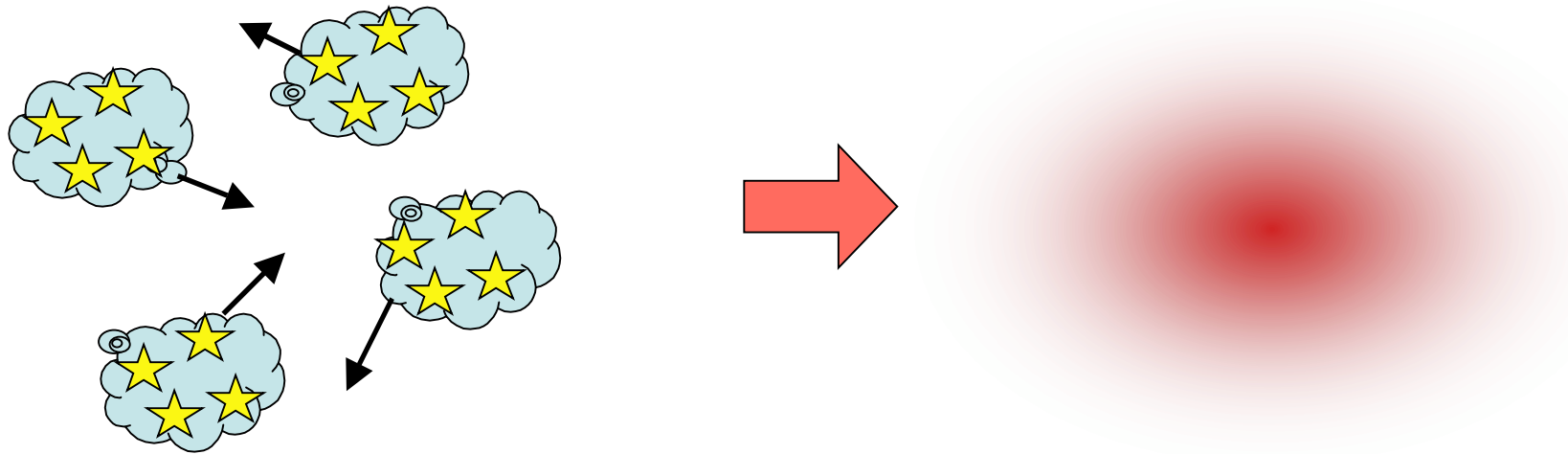
# Hierarchical Formation of the Galaxy's Halo Continues Today



↑ Palomar 5 globular  
cluster tidal tail

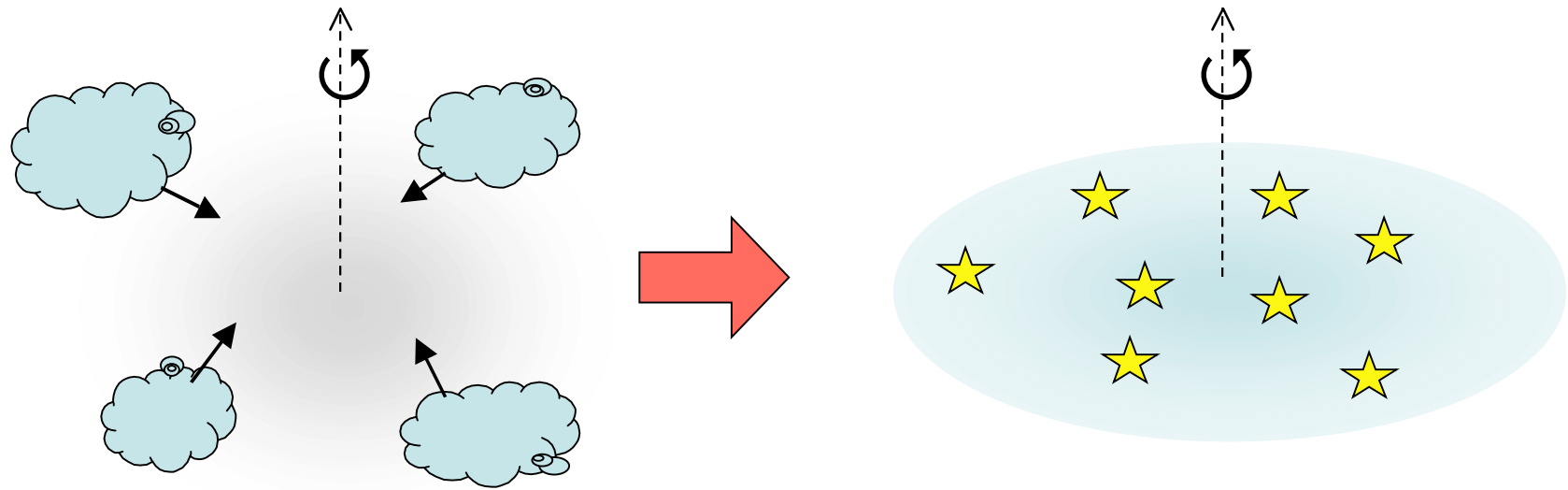
↖ Sagittarius dwarf  
galaxy

# Formation of Galaxy Spheroids and Dynamics of Stellar Populations



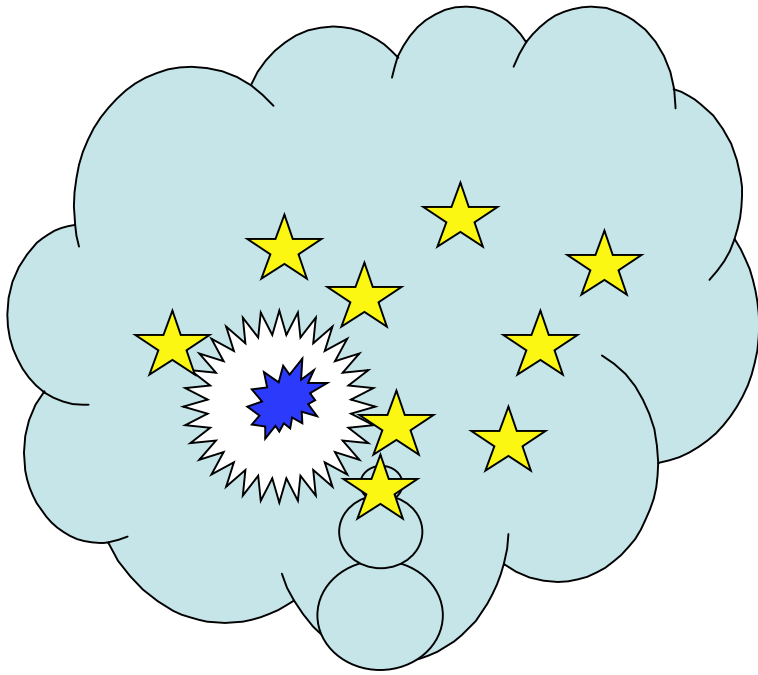
Stars “remember” the dynamics of their orbits at the time of formation, since dynamics of stellar systems is dissipationless. If stars form in dwarf protogalactic fragments which then merge, this will result in a pressure-supported system, *i.e.*, a spheroid (bulge or halo, or an elliptical galaxy). Their metallicities will reflect the abundances in their parent systems.

# Formation of Galaxy Disks and Dynamics of Stellar Populations

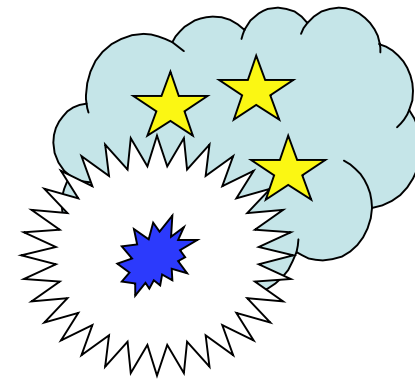


If protogalactic clouds merge dissipatively in a potential well of a dark halo, they will settle in a thin, rotating disk = the minimum energy configuration for a given angular momentum. If gas settles into a (dynamically cold) disk before stars form, then stars formed in that disk will inherit the motions of the gas (mainly an ordered rotation).

# Chemical Self-Enrichment in Young Stellar Systems



In a massive system, supernova ejecta are retained, and reused for subsequent generations of stars, which achieve ever higher metallicities.



In a low-mass system, supernova shocks and star winds from massive young stars expell the enriched gas and may supress any subsequent star formation. The system retains its initial (low) metallicity.

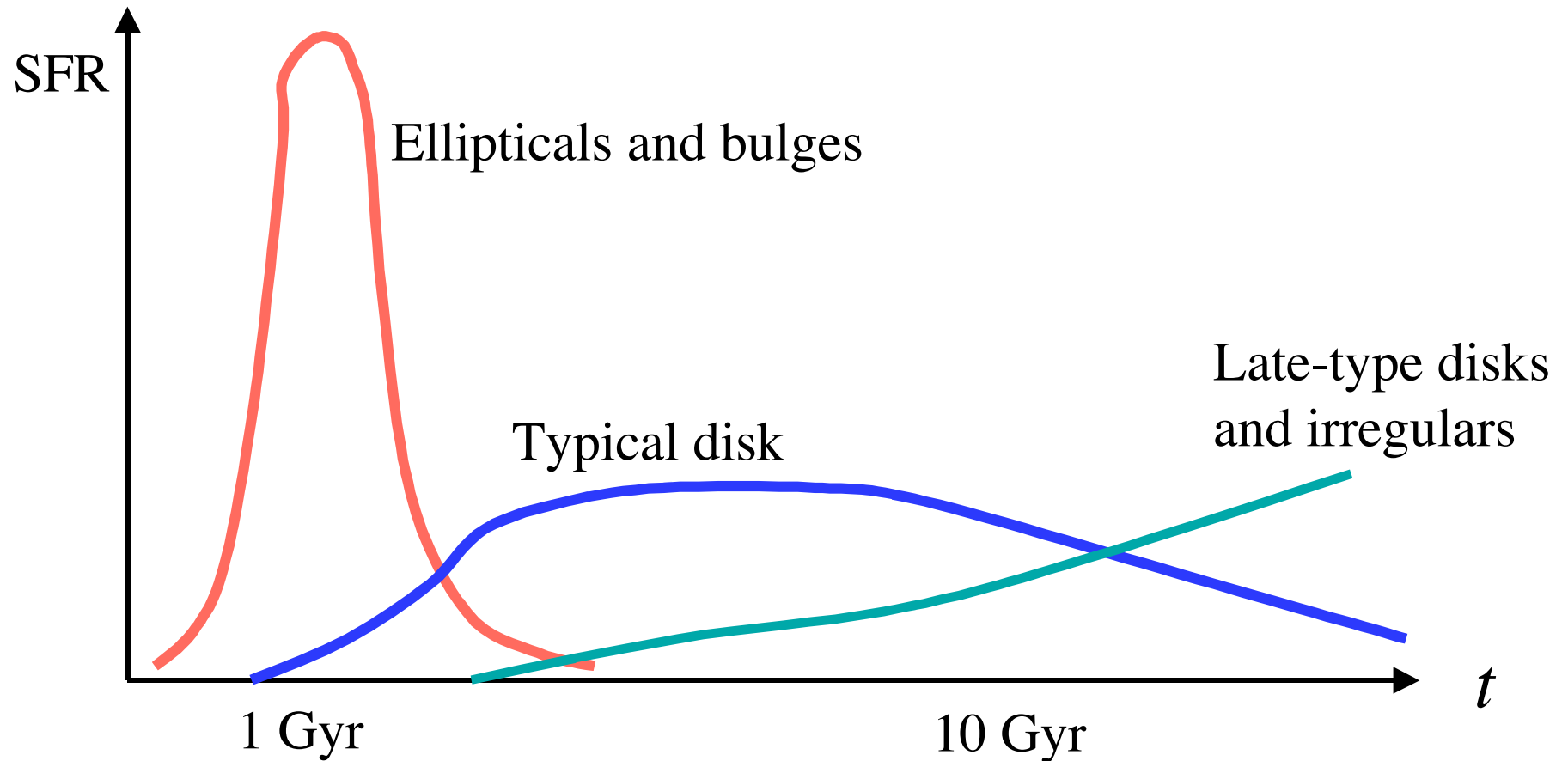
# Formation of Our Galaxy: A Summary

- Dark halo assembly by infall and merging of DM density fluctuations, with one dominant potential well
- Star formation ignites in densest fragments  $\sim 13$  Gyr ago
- In the proto-Bulge, SN ejecta are recycled, leading to an old, metal-rich stellar population
- In smaller fragments, most of which get tidally disrupted to make the stellar halo and globular clusters, and only the low, primordial abundances are retained
- These first star forming systems produce the pressure-supported stellar components of the Galaxy (bulge, halo)

# Formation of Our Galaxy: A Summary

- Subsequent dissipative accretion of gas forms the proto-disk of the Milky Way
- Star formation in the disk starts from the densest regions near the center and expands outward ( $\sim 9$  Gyr ago in the Solar neighborhood?)
- Enriched gas is recycled within the disk, leading to ever higher abundances
- No major mergers happened - otherwise the cold, thin disk would not have survived
- The origins of the thick disk are murky: an intermediate system between the bulge and the thin disk

# Star Formation History in Galaxies



The net star formation rate in the universe, after the initial rapid rise, has remained nearly constant for the first  $\sim 6$  to 8 Gyr, and has declined steeply (by an order of magnitude) since then